CHAPTER 2 PROBLEMS AND SOLUTIONS

Problems for Sections 2.2 and 2.3

2.1 A force vector forms a 30° angle with the x axis. It is often convenient to work in a coordinate system along a line of action, such as the $x' - y'$ coordinate system shown. Calculate the relationship between the two coordinate systems.

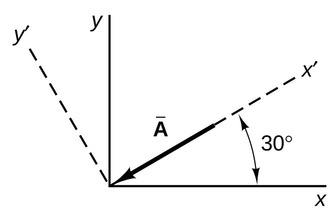

FIGURE P2.1

Solution: From the drawing and refering to mathmatics window 2.1

$$\underline{x = x' \cos 30^\circ}$$

or

$$\underline{x' = x / \cos 30^\circ = x \csc 30^\circ}.$$

Likewise:

$$\underline{y = y' \cos 30^\circ}$$

or

$$\underline{y' = y \csc 30^\circ}$$

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2.2 Suppose the vector in problem 2.1 corresponds to a 100-N force. Calculate the $x - y$ coordinates of this force. (That is, calculate the components of the force in the x and y directions). What are the components in the $x' - y'$ directions?

Solution: Following along the development for Fig. 2.15,

$$A = 100, \quad \theta_x = 30^\circ$$

so that the x component is:

$$A_x = A\cos 30^\circ = (-100)(\mathrm{N})(.8660) = -86.6\mathrm{N},$$

the y component is:

$$A_y = A\sin 30^\circ = (-100)(\mathrm{N})(.5) = -50\mathrm{N}.$$

Note that the minus sign arises because the 100 N vector points in the negative x, y direction. The x' component is simply -100N and the y' component is zero. ∎

2.11 A barge is pulled by two tug boats. To move the barge along in the water properly, the tug boats must exert a resultant force of 5000 lb along the direction of motion of the barge. (a) First determine, the tension in each rope if the position of tug B is such that $\beta = 45°$. Second, suppose that tug B can move anywhere such that

$$0 < \beta < 90°.$$

Determine the angle β where the tension in tug B's rope is (b) maximum and again where it has a (c) minimum value still maintaining the resultant of 5000 lb along the x direction on tug A.

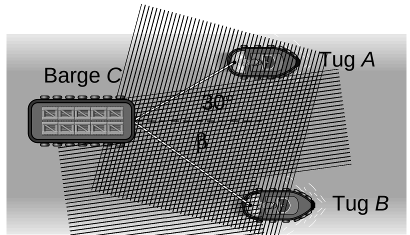

FIGURE P2.11

Solution: (a) for $\beta = 45°$ the geometry of the vector sum becomes as illustrated in the vector diagram.

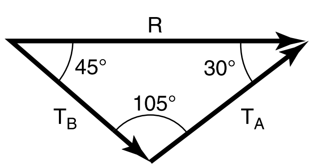

FIGURE S2.11

Since the angles must sum to 180° the angle across from R is 105°. Repeated use of the law of sines yield

$$\frac{T_B}{\sin 30°} = \frac{5000\text{lb}}{\sin 105°} \text{ or } T_B = \frac{50000\text{lbs}}{\sin 105°} \sin 30° = \underline{3660 \text{ lb}}$$

and likewise

$$T_A = \frac{5000 \text{ lb}}{\sin 105°} \sin 45° = \underline{2590 \text{ lb}}.$$

(b) With the angle β as a variable, the tension T_B can be written from the law of sines as

$$T_B = \frac{5000 \sin 30}{\sin(105 - \beta)}$$

as long as tug A does not change its orientation. This expresses T_B as a function of β. From calculus the maximum and minimum of $T_B(\beta)$ occurs when the first derivative of T_B with respect to β vanishes. Hence

$$\frac{dT_B}{d\beta} = 0 \text{ or } \frac{d}{d\beta}\left(\frac{5000\sin 30^\circ}{\sin(105-\beta)}\right) = \frac{\cos(105-\beta)(-1)}{\sin^2(105-\beta)} = 0$$

which is satisfied when $105-\beta = 90^\circ$ (i.e., when $\cos(105-\beta) = 0$ or $\beta = 15^\circ$ yields either a maximum value of T_B or a minimum value.

$$T_B = 5000\sin 30^\circ / \sin 105^\circ = 25882$$

at $\beta = 0$. Also

$$T_B = 5000\sin 30^\circ / \sin(105^\circ - 90^\circ) = 9,659.3$$

at $\beta = 90^\circ$. Next

$$T_B = 5000\sin 30^\circ / \sin 90^\circ = 2,500\text{lbs}$$

at $\beta = 15^\circ$, the value where $dT_B/d\beta = 0$. Thus the value of $\underline{\beta = 15^\circ}$ is a minimum (recall that a minimum is reached either at an end point or at the point where the first derivative vanishes).

c) Since $dT_B/d\beta = 90$ yields a minimum, the max value on the interval $0 \le \beta \le 90^\circ$ occurs at the end point which in the case is $\underline{\beta = 90^\circ}$ where

$$\underline{T_B = 9659.3 \text{ lbs}} > T_B(0^\circ) = 2588 \text{ lbs.}$$

■

2.15 A guy wire is used to stabilize a power line pole. Find the proper position (x) to place the wire (A) if the wire can provide a 600-N force and the force F due to the power line is 500 N, by examining the components of F along A and P. Also calculate the force along the pole P.

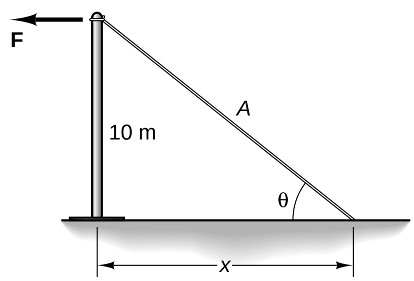

FIGURE P2.15

Solution: Again the geometry of the system yields the appropriate angles in the parallelogram sum of the vector:

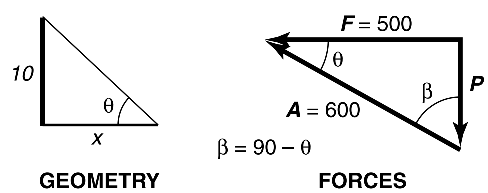

FIGURE S2.15

From the law of sines, or from the formulas for a right triangle:

$$\cos\theta = \frac{F}{A} = \frac{500}{600} = 0.833 \text{ or } \theta = 33.6^\circ.$$

From the definition of the tangent and the length of the pole:

$$\tan\theta = \frac{10}{x} \text{ or } x = \frac{10}{\tan 33.6^\circ} = \underline{15.05 \text{ m}}.$$

Thus the guy wire should be placed 15.05 meters out from the bottom of the pole to correspond to the required 33.6° angle. From the vector diagram the component of A along P is

$$P = \sin\theta = 600\sin 33.6^\circ, \text{ or } \underline{P = 332 \text{ N}}.$$

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2.21 Four concurrent forces act on the center of mass of a landing airplane. Calculate the resultant force and the angle it makes with the x axis.

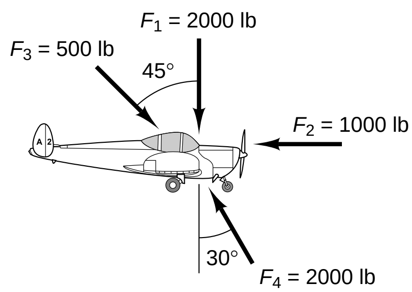

FIGURE P2.21

Solution: First find $R_1 = F_3 + F_1$

$$R_1 = \sqrt{500^2 + 2000^2 - (2)(500)(2000)\cos 135^\circ} = \underline{2380 \text{ lb}}$$

making an angle of

$$45^\circ + \sin\left(\frac{2000 \sin 135^\circ}{2380}\right) = \underline{81.5^\circ}$$

counterclockwise from x (the vertical). Next find $R_2 = R_1 + F_4$ which is drawn in the top figure. Applying the law of cosines to the figure yields

$$R_2 = \sqrt{(2000)^2 + (2380)^2 - 2(2380)(2000)\cos(21.5^\circ)} = \underline{898.2 \text{ lb}}$$

which makes an angle of

$$\theta = 180^\circ - 81.5^\circ - \sin^{-1}\left((2000)\frac{\sin 21.5^\circ}{898.2}\right) = \underline{43.8^\circ}$$

counterclockwise for x. Last, $R = R_2 + F_2$ which is drawn in the bottom figure.

Apply the law of cosines to get

$$R = \sqrt{(1000)^2 + (898.2)^2 - 2(1000)(898.2)\cos 136.2^\circ} = \underline{1761 \text{ lb}}$$

which makes an angle of

$$43.8^\circ - \alpha = 43.8^\circ - \sin^{-1}\left(\frac{1000 \sin 136.2^\circ}{1761 \text{ lb}}\right) = \underline{20.5^\circ}$$

counterclockwise from the x axis. ∎