

# **Theoretical Nuclear Physics**

**John M. Blatt    Victor F. Weisskopf**

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## PREFACE

The last twenty years have witnessed an enormous development of nuclear physics. A large number of data have accumulated and many experimental facts are known. As the experimental techniques have achieved greater and greater perfection, the theoretical analysis and interpretation of these data have become correspondingly more accurate and detailed. The development of nuclear physics has depended on the development of physics as a whole. While there were interesting speculations about nuclear constitution as early as 1922, it was impossible to make any quantitative theory of even the simplest nucleus until the discovery of quantum mechanics on the one hand, and the development of experimental methods sufficiently sensitive to detect the presence of a neutral particle (the neutron) on the other hand. The further development of our understanding of the nucleus has depended, and still depends, on the development of ever more powerful experimental techniques for measuring nuclear properties and more powerful theoretical techniques for correlating these properties. Practically every "simple," "reasonable," and "plausible" assumption made in theoretical nuclear physics has turned out to be in need of refinement; and the numerous attempts to derive nuclear forces and the properties of nuclei from a more "fundamental" approach than the analysis of the data have proved unsuccessful so far. Nuclear physics is by no means a finished edifice. It is very much to be hoped that simple fundamental laws can be discovered which will account for all the known properties of nuclei, and will allow us to predict new, unknown properties successfully. At present we must restrict ourselves to the investigation and correlation of all known nuclear properties on a semi-empirical basis.

This book is devoted in its entirety to this task. Its subject matter is *theoretical nuclear physics*, by which we mean the theoretical concepts, methods, and considerations which have been devised in order to interpret the experimental material and to advance our ability to predict and control nuclear phenomena.

Obviously, this book does not pretend to cover all aspects of theoretical nuclear physics. We are forced to omit many details and special developments. The omissions are due partly to the lack of space and partly to the authors' lack of special knowledge. We hope

that the study of this book will make it somewhat easier for the reader to understand the original literature containing the material which is not covered in this book.

We have restricted ourselves to phenomena involving energies below about 50 Mev, a region which is sometimes called classical nuclear physics. Thus we exclude the nuclear phenomena in cosmic rays as well as the phenomena associated with the production and absorption of mesons. The only exception to this rule is Chapter IV, which deals with nucleon-nucleon scattering experiments at energies up to 350 Mev and their interpretation in terms of nuclear forces.

In general, we have omitted theoretical considerations which are not concerned directly with the properties of the nucleus itself. Thus we exclude, for example, the theory of the stopping of charged particles in matter, the theory of the diffraction and slowing down of neutrons, the theory underlying molecular beam and magnetic resonance experiments, and the theory of atomic hyperfine structure. Although all these subjects are important from an experimental point of view, their inclusion would have lengthened the book too much. We have also excluded subjects generally referred to as nuclear engineering, such as the theory of nuclear reactors. In so far as the relevant material has been declassified, adequate textbooks are already available. Unfortunately, the theory of nuclear fission (which properly belongs in this book) could be treated only in a very cursory manner, since too many relevant facts are still unavailable.

We have completely omitted the discussion of the theories of nuclear forces based on the various meson field theories. The numerous attempts to predict nuclear forces on the basis of meson fields have led to brilliant insights and predictions regarding mesons, but they have failed so far to reproduce quantitatively the observed forces between nuclear particles. This subject seemed to us not yet sufficiently developed to warrant a systematic treatment in this book.

It was our constant aim throughout the book to keep it on a level which is understandable to the experimental physicist who works in the field of nuclear physics or to a graduate student who knows the essential concepts and problems of nuclear physics. A one-term course in quantum mechanics, based on a book such as Schiff (49) should suffice as a prerequisite. Some parts of the book may be hard reading for students unaccustomed to theoretical work. Sections which are difficult and can be omitted without loss of understanding of subsequent material are indicated by the symbol ►.

We have concentrated our efforts on a better understanding and a critical analysis of the different subjects. This has led in some cases

to new developments which are not yet published elsewhere. Since we are dealing with a growing and changing part of science, we must expect that many ideas which today are considered valid will turn out to be incorrect before long. Hence, many assumptions and statements found in this book should be regarded as preliminary. It contains a great deal of information of which we are far from sure, and which is included only because nothing better is available at present. A characteristic example is the information about nuclear level densities given in Chapter VIII. In some instances recent developments have changed the emphasis from one way of description to another more successful one. For example, nuclear spectroscopy today is being based to an increasing extent upon the shell theory of nuclear structure, as indicated in Chapter XIV. At the time this book was conceived the concepts of Wigner's supermultiplet theory were the main tools for the understanding of nuclear spectra; they are the basis of the discussions in Chapter VI.

We make no pretense of having a complete list of references. We have tried to include the basic theoretical papers in each field, such other theoretical papers as we happened to come across, and experimental papers only in so far as they illustrate some points made in the discussion or substantiate values of nuclear constants used in the text. We have not made a systematic search of the whole literature. This applies especially to papers which appeared in journals other than *The Physical Review*. If a relevant reference is missing, it is very probable that we did not know of its existence. The manuscript was revised for the last time in the spring of 1951; it contains only occasional references to later work.

We are quite aware of the possibility that this book contains errors, not all of which may be trivial or typographical in nature. We have tried to keep a reasonable balance between the effort to eliminate errors and the effort to understand the subject matter and to clarify its presentation.

At the end of every chapter is a list of symbols with a short explanation of the meaning of each symbol, and the number of the equation in which the symbol is introduced and defined. If the symbol in question is defined in the body of the text, the section number is given in the symbol list. We have made an effort to have these lists of symbols comprehensive, but some minor symbols, which occur only a few times in the chapter, have been omitted.

It would be impossible to acknowledge in detail the enormous amount of generous help and constructive criticism which we have received from friends and colleagues. We would like, however, to

express our special gratitude to H. Bethe, M. Deutsch, H. Feshbach, and E. Wigner for their extensive advice. We are greatly indebted to Miss Inge Reethof for her constant help and patience in the typing and retyping of the manuscript.

J. M. BLATT  
V. F. WEISSKOPF

*June, 1952*

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