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# MECHANICAL ENGINEERING THERMAL AND FLUID SYSTEMS STUDY PROBLEMS 

## MASS BALANCES, PSYCHROMETRICS \& HVAC

## How to use this book

The exam specifications in effect since April 2017 state that approximately 4 problems from the "Mass Balance Principles" topic will be in the morning "Principles" portion of your exam. As examples of mass balance principles, the official NCEES exam lists the HVAC processes of evaporation, dehumidification, and mixing. Additionally, psychrometrics is one of the four components of the Supportive Knowledge topic in the morning "Principles" portion of your exam. Reviewing all the problems in this book will prepare you for all these problems in the morning "Principles" portion.
The exam specifications also state that "Cooling/Heating" comprise 6 of the 24 problems ( $25 \%$ ) in the afternoon "Energy/Power System Applications" portion. Furthermore, cooling towers are listed as one of the relevant examples of Energy/Power Equipment in the afternoon "Energy/Power System Applications" portion. Reviewing all the problems in this book will prepare you for these problems in the "Energy/Power System Applications" portion of the exam.

## How it works

This study problems book works on what we call the "principle of progressive overload". With this technique you start with very easy problems and smoothly progress towards more complex problems. A good example of progressive overload is the story of the famous wrestler Milo of Croton in ancient Greece. This extraordinarily strong man was allegedly capable of carrying a fully grown bull on his shoulders. He was reported to have achieved this tremendous strength by walking around town with a new born calf on his shoulders every single day. As the calf grew, so did the man's strength.

We recommend you work the problems in this book in the order they are presented. Within each section of the book, the first problems will feel "light", like carrying that baby calf - you might even be tempted to skip them. We strongly urge you to resist this temptation. As you progress, the problems become harder, but the work you've been putting in with all the previous problems will bear fruit. You will be pleasantly surprised at how relatively easy those "hard" problems will seem. You will soon be carrying intellectual bulls on your shoulders! The problems that are considered "exam-level difficult" are denoted with an asterisk.

This book is comprised of the following sections:

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For the most part, these sections are not independent and build from the previous ones. We recommend you go through them in the order presented, and be sure to review them all. Each section begins with a brief discussion of the relevant concepts and equations. These discussions are laser-focused on the aspects that are relevant to the P.E. exam and do not go into derivations with academic rigor.

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## SECTION 01: Properties of Moist Air

Psychrometrics is the science involving thermodynamic properties of moist air and the effect of atmospheric moisture on materials and human comfort.
Air is a mixture of nitrogen, oxygen, and small amounts of some other gases. Air in the atmosphere normally contains some water vapor (or moisture) and is referred to as atmospheric air. Conversely, air that contains no water vapor is called dry air. At the relatively low atmospheric temperature we can treat dry air as an ideal gas with constant specific heat. The water vapor in atmospheric air is typically at very low pressures (i.e., the partial pressure of water in the mixture ${ }^{1}$ ) and thus can be treated as an ideal gas without loss of accuracy (even if it is saturated vapor).
Consider a container with 10 pounds of dry air and 0.1 pounds of water vapor, thoroughly mixed, at atmospheric pressure ( 14.7 psia ) and $75^{\circ} \mathrm{F}$. The humidity ratio $\omega$ (also known as specific humidity) is the ratio of the mass of water vapor to mass of dry air:

$$
\begin{equation*}
\omega=\frac{m_{v}}{m_{a}} \tag{1-1}
\end{equation*}
$$

where the subscript " $v$ " denotes water vapor, and " $a$ " denotes dry air. In our example, $\omega=0.1 / 10=0.01 \mathrm{lbm} \mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air . The mass of water vapor is sometimes expressed in grains, which is a unit of mass such that 7,000 grains $=1$ pound-mass. Therefore, in our example, $\omega=70$ grains $\mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air . The humidity ratio is not to be confused with the water vapor mass fraction: $m f_{v}=m_{v} /\left(m_{v}+m_{a}\right)=\omega /(1+\omega)$

In our example, the moles of dry air $N_{a}=m_{a} / M_{a}=10 \mathrm{lbm} / 28.97(\mathrm{lbm} / \mathrm{lbmol})=0.3452 \mathrm{lbmol}$ and the moles of water vapor $N_{v}=m_{v} / M_{a}=0.1 \mathrm{lbm} / 18(\mathrm{lbm} / \mathrm{lbmol})=0.0056 \mathrm{lbmol}$. Therefore, the mole fraction of water vapor $y_{v}=0.0056 /(0.0056+0.3452)=0.01584$. We can can thus obtain the partial pressure of the water vapor (or vapor pressure) from its definition:

$$
\begin{equation*}
p_{v}=y_{v} p \tag{1-2}
\end{equation*}
$$

thus $p_{v}=0.01584 \times 14.7=0.2328 \mathrm{psia}$ in our example. At this pressure, the saturation temperature is $57.3^{\circ} \mathrm{F}$, so the vapor in the mixture is actually superheated vapor with a superheat of $75-57.3=$ $17.7^{\circ} \mathrm{F}$.

[^0]We can repeat these calculations for a mixture of 10 pounds of dry air and this time with 0.15 pounds of water (i.e, we are increasing the amount of water in the mixture by $50 \%$, so now $\omega=0.015 \mathrm{lbm} \mathrm{H} \mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air ). In this case;

$$
\begin{gathered}
N_{a}=m_{a} / M_{a}=10 \mathrm{lbm} / 28.97(\mathrm{lbm} / \mathrm{lbmol})=0.3452 \mathrm{lbmol} \\
N_{v}=m_{v} / M_{a}=0.15 \mathrm{lbm} / 18(\mathrm{lbm} / \mathrm{lbmol})=0.00833 \mathrm{lbmol} \\
y_{v}=0.00833 /(0.00833+0.3452)=0.02357 \\
p_{v}=0.02357 \times 14.7=0.3465 \mathrm{psia}
\end{gathered}
$$

We see that for this mixture (which contains more water) the vapor pressure is higher, and the saturation temperature corresponding to it is lower. For $0.3465 \mathrm{psia}, T_{\text {sat }}=68.6^{\circ} \mathrm{F}$ and thus the superheat is reduced to $75-68.6=6.4^{\circ} \mathrm{F}$. If the mass of water is further increased again, this time to 0.1873 pounds, the vapor pressure will be 0.43017 psia - the saturation pressure at $75^{\circ} \mathrm{F}$ - thus the water vapor in the mixture will be saturated vapor.

This example shows that, as we increase the mass of vapor in the mixture the closer that vapor comes to the saturated vapor state. We denote with $m_{g}$ the mass of water in the mixture such that the water vapor is saturated (i.e., not superheated, that is, when the vapor pressure corresponds to the saturation pressure at the mixture temperature). In our example, $m_{g}=0.1873 \mathrm{lbm}$. Saturated air is a mixture of dry air and saturated water vapor. The amount of water vapor in moist air varies from zero in dry air to a maximum value $m_{g}$ (which depends on the pressure and temperature) in saturated air (when the vapor is saturated.) The relative humidity $\phi$ (sometimes also referred to as "rh") is defined as the ratio of the mass of water in the air to the mass of water in saturated air at the same temperature and pressure:

$$
\begin{equation*}
\phi=\frac{m_{v}}{m_{g}} \tag{1-3}
\end{equation*}
$$

In our example for the container with 10 pounds of dry air and 0.1 pounds of water, the relative humidity would be $\phi=0.1 / 0.1873=0.534=53.4 \%$. When the mass of water vapor is increased to 0.15 pounds, the relative humidity increases to $\phi=0.15 / 0.1873=0.8=80 \%$.
Note that, since $m_{g}$ depends on temperature, we can change the relative humidity of the mixture by changing the temperature, without adding or removing any water. Consider our example of 10 pounds of dry air and 0.1 pounds of water vapor $(\omega=0.01 \mathrm{lbm} / \mathrm{lbm})$ at 14.7 psia and $75^{\circ} \mathrm{F}$. We saw earlier
that $N_{a}=0.3452 \mathrm{lbmol} ; N_{v}=0.0056 \mathrm{lbmol}$ and $y_{v}=0.01584$. Consider what happens when we cool the mixture down to $65^{\circ} \mathrm{F}$, without adding (or removing) any water vapor: At $65^{\circ} \mathrm{F}$, the saturation pressure for water is 0.306 psia . We would like to determine $m_{g}$, the mass of water vapor that would be required to make the water in the mixture a saturated vapor. A vapor pressure of .306 psia means that the mole fraction of water is: $y_{v}=p_{v} / p=0.306 / 14.7=0.02081$. It can be shown that this mole fraction is obtained when the mass of water vapor is 0.13212 pounds. That is, for a mixture temperature of $65^{\circ} \mathrm{F}$, $m_{g}=0.13212 \mathrm{lbm}$ if $m_{a}=10 \mathrm{lbm}$. So, when the mixture temperature is decreased to $65^{\circ} \mathrm{F}$, $\phi=0.1 / 0.13212=0.7569=75.7 \%$.
So, humidity ratio provides a measure of the concentration of water vapor in atmospheric air and relative humidity provides a measure of how close the vapor in atmospheric air is to being saturated vapor. The relative humidity of air changes with temperature even when its humidity ratio remains constant. The amount of moisture in the air has a definite effect on how comfortable people feel in an environment. However, the comfort level depends more on how close the water vapor is to being saturated; that is, the comfort level is more related to the relative humidity.
We can re-cast equation (1-1) by inserting the ideal gas law:

$$
\omega=\frac{m_{v}}{m_{a}}=\frac{\frac{p_{v} V}{R_{v} T}}{\frac{p_{a} V}{R_{a} T}}=\frac{R_{a}}{R_{v}} \frac{p_{v}}{p_{a}}=0.622 \frac{p_{v}}{p_{a}}
$$

Now use Dalton's law of additive pressures: $p=p_{a}+p_{v}$,

$$
\begin{equation*}
\omega=0.622\left(\frac{p_{v}}{p-p_{v}}\right) \tag{1-4}
\end{equation*}
$$

which can be rearranged as:

$$
\begin{equation*}
p_{v}=\left(\frac{\omega}{\omega+0.622}\right) p \tag{1-5}
\end{equation*}
$$

Similarly, we can use the ideal gas law with equation (1-3):

$$
\begin{equation*}
\phi=\frac{m_{v}}{m_{g}}=\frac{\frac{p_{v} V}{R_{v} T}}{\frac{p_{g} V}{R_{v} T}}=\frac{p_{v}}{p_{g}} \tag{1-6}
\end{equation*}
$$

where $p_{g}=p_{\text {sat@ } T}$ is the saturation pressure at the mixture temperature. Combining (1-6) with (1-5) we obtain:

$$
\begin{equation*}
\phi=\left(\frac{\omega}{\omega+0.622}\right) \frac{p}{p_{g}} \tag{1-7}
\end{equation*}
$$

or alternatively:

$$
\begin{equation*}
\omega=\frac{0.622 \phi p_{g}}{p-\phi p_{g}} \tag{1-8}
\end{equation*}
$$

The humidity ratio at saturation $\omega_{s}-$ which is the amount of water vapor in saturated air at a specified temperature and pressure - can be determined from equation (1-4) by replacing $p_{v}$ by $p_{g}$, the saturation pressure of water at that temperature, or by setting $\phi=1$ in equation (1-8).
The degree of saturation $\mu$ is the ratio of the actual humidity ratio $\omega$ to the humidity ratio $\omega_{s}$ of saturated air at the same temperature and pressure:

$$
\begin{equation*}
\mu=\omega / \omega_{s} \tag{1-9}
\end{equation*}
$$

Relative humidity and specific humidity are frequently used in engineering and atmospheric sciences, and it is desirable to relate them to easily measurable quantities such as temperature and pressure. The term dry-bulb temperature $T$ (sometimes simply referred to as the temperature) is the temperature measured by an ordinary thermometer placed in atmospheric air. The wet-bulb temperature $T_{\mathrm{wb}}$, is read from an ordinary liquid-in-glass thermometer whose bulb is enclosed by a wick moistened with water. When unsaturated air passes over the wet wick, some of the water in the wick evaporates. As a result, the temperature of the water drops, creating a temperature difference (which is the driving force for heat transfer) between the air and the water. After a while, the heat loss from the water by evaporation equals the heat gain from the air, and the water temperature stabilizes. The thermometer reading at this point is the wet-bulb temperature. The wet-bulb temperature is an excellent approximation to another concept known as the adiabatic saturation temperature $T_{\text {as }}$. It can be shown that relative and absolute humidity can be obtained if $T_{\text {as }}, T$, and $p$ are all known.

Consider a mass of moist air at a known temperature $T$ and pressure $p$. Equation (1-7) indicates that these two variables are not enough to determine $\phi$ and $\omega$. The additional relationship involves $\omega$ and $T_{\mathrm{as}}$ (and therefore $T_{\mathrm{wb}}$ because $T_{\mathrm{as}} \approx T_{\mathrm{wb}}$ ). In customary US units, this relationship is:

$$
\begin{equation*}
\omega=\frac{\left(1,093-0.556 T_{\text {w.b. }}\right) \omega_{s}-0.24\left(T-T_{\text {w.b. }}\right)}{1,093+0.444 T-T_{\text {w.b. }}} \tag{1-10}
\end{equation*}
$$

where temperatures are in ${ }^{\circ} \mathrm{F}$, and $\omega_{s}^{*}$ is the humidity ratio at saturation for the wet bulb temperature. The condensation of part of the water vapor in atmospheric air when the temperature is reduced is an important aspect of the behavior of moist air. This is encountered in the condensation of vapor on window panes and on cold pipes, as well as condensation of water vapor from cool flue gases. The dew-point temperature $T_{\mathrm{dp}}$ is defined as the temperature at which condensation begins when moist air is cooled at constant pressure. In other words, $T_{\mathrm{dp}}$ is the saturation temperature of water corresponding to the vapor pressure, $T_{\mathrm{dp}}=T_{\text {sat }}\left(p_{v}\right)$.

In most practical applications, the amount of dry air in the air-water-vapor mixture remains constant, but the amount of water vapor changes. Therefore, the enthalpy of atmospheric air is expressed per unit mass of dry air instead of per unit mass of the air-water vapor mixture. The enthalpy of a mixture of perfect gases is equal to the sum of the individual partial enthalpies of the components. The enthalpy of moist air is then:

$$
h=h_{a}+\omega h_{v}
$$

where $h_{a}$ is the specific enthalpy for dry air and $h_{v}$ is the specific enthalpy for water vapor at the temperature of the mixture. For most HVAC applications, $h_{v} \approx h_{g}$ where $h_{g}$ is the enthalpy of saturated vapor at the mixture temperature, thus:

$$
\begin{equation*}
h=h_{a}+\omega h_{g} \tag{1-11}
\end{equation*}
$$

For most HVAC applications, the following correlation in the form of equation (1-11) works quite well:

$$
\begin{equation*}
h=0.24 T+\omega(1,061+0.444 T) \tag{1-12}
\end{equation*}
$$

which yields the enthalpy in $\mathrm{Btu} /\left(\mathrm{lbm}\right.$ of dry air) as long as $T$ is in ${ }^{\circ} \mathrm{F}$ and $\omega$ is in $\left(1 \mathrm{bm} \mathrm{H}_{2} \mathrm{O}\right) /(\mathrm{lbm}$ dry air). The specific volume $v$ of a moist air mixture is expressed in terms of a unit mass of dry air:

$$
\begin{aligned}
& p_{a} V=m_{a} R_{a} T \\
& \Rightarrow \frac{V}{m_{a}}=\frac{R_{a} T}{p_{a}} \\
& \Rightarrow v=\frac{R_{a} T}{p_{a}}
\end{aligned}
$$

and using Dalton's law, we get:

$$
\begin{aligned}
v & =\frac{R_{a} T}{p-p_{v}} \\
& =\frac{R_{a} T}{p\left(1-\frac{p_{v}}{p}\right)}
\end{aligned}
$$

which can be combined with equation (1-5):

$$
v=\frac{R_{a} T}{p\left(1-\left(\frac{\omega}{\omega+0.622}\right)\right)}
$$

thus yielding:

$$
\begin{equation*}
v=\frac{R_{a} T}{p}\left(1+\frac{\omega}{0.622}\right) \tag{1-13}
\end{equation*}
$$

where

$$
R_{a}=\left\{\begin{array}{l}
53.34 \mathrm{ft} \cdot \mathrm{lbf} / \mathrm{lbm}^{\circ} \cdot{ }^{\mathrm{R}} \\
0.3704 \mathrm{psia} \cdot \mathrm{ft}^{3} / \mathrm{lbm} \cdot{ }^{\circ} \mathrm{R} \\
640.08 \mathrm{psia} \cdot \mathrm{in}^{3} / \mathrm{lbm} \cdot \cdot \mathrm{R} \\
0.0686 \mathrm{Btu} / \mathrm{lbm}^{\circ} \cdot{ }^{\circ} \mathrm{R} \\
0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{array}\right\}
$$

is the gas constant for dry air.

## PROBLEMS

01-01. A sample of air at 14.7 psia and $75^{\circ} \mathrm{F}$ has a humidity ratio of $0.015 \mathrm{lbm} \mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air . Determine:
a) the partial pressure of water vapor, in psi.
b) the dew point temperature, in ${ }^{\circ} \mathrm{F}$.
c) the relative humidity.
d) the degree of saturation.
e) the enthalpy in Btu/lbm dry air
f) the specific volume in $\mathrm{ft}^{3} / \mathrm{lbm}$ dry air

01-02. A sample of air at 14.7 psia and $75^{\circ} \mathrm{F}$ has a dew point of $52^{\circ} \mathrm{F}$. Determine the humidity ratio and the relative humidity.

01-03. A sample of air at 14.7 psia and $75^{\circ} \mathrm{F}$ has a wet bulb temperature of $55^{\circ} \mathrm{F}$. Determine the humidity ratio and the relative humidity.

01-04. A sample of air is at $70^{\circ} \mathrm{F}, 50 \% \mathrm{rh}$, and 14.7 psia . What is the lowest temperature to which the air can be cooled at constant pressure if condensation is to be avoided?

01-05*. The conditions at the intake of an air conditioning system are $95^{\circ} \mathrm{F}, 30 \% \mathrm{rh}$, and 14.7 psia . The air flows at a rate of 1,000 cubic feet per minute ( cfm ). The rate $(\mathrm{lbm} / \mathrm{hour})$ at which water vapor enters the system is most nearly:
(A) 0.74
(B) 10.5
(C) 44.5
(D) 55.4

01-06*. The air conditions at the intake of an air compressor are $70^{\circ} \mathrm{F}, 50 \% \mathrm{rh}$, and 14.7 psia . The air is compressed to 50 psia , then sent to an intercooler. If condensation of water vapor from the air is to be prevented, the lowest temperature $\left({ }^{\circ} \mathrm{F}\right)$ to which the air can be cooled in the intercooler is most nearly:
(A) 31
(B) 46
(C) 51
(D) 86

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## SECTION 02: The Psychrometric Chart

Many of the properties of moist air discussed in the previous section can be estimated from a psychrometric chart, a plot of dry-bulb temperature (horizontal axis) and humidity ratio (vertical axis). In other words, the chart is a graphical representation of all the equations in the previous section. Psychrometric charts for sea level altitude are available for free download from the "Free Resources" section of www.SlayThePE.com. Lines of constant humidity ratio are horizontal on the graph. Lines of constant (dry-bulb) temperature exhibit a slight tilt to the left of vertical, with the degree of tilt increasing with lower temperature. The left top portion of the plot is terminated at the saturation line, which represents both the $100 \%$ relative humidity curve and the plot for dew-point temperature. Lines of constant enthalpy appear as straight lines that slope down as the temperature increases. Lines of constant wet-bulb temperature are nearly parallel to lines of constant enthalpy. Lines of constant specific volume also slope down as temperature increases but at a much greater angle compared to enthalpy and wet-bulb lines. Lines of constant relative humidity curve up as temperature increases. Processes performed with air can be plotted on the chart for quick visualization, as well as for determining changes in significant properties such as temperature, humidity ratio, and enthalpy for the process.
To demonstrate the use of the chart, obtain one for sea level altitude and locate the point corresponding to a dry-bulb temperature of $80^{\circ} \mathrm{F}$ and a wet-bulb temperature of $67^{\circ} \mathrm{F}$ :


Using this condition as a starting point on the chart, make sure you can verify that:

- Humidity ratio-Move right horizontally to the axis and read, $\omega \approx 0.0112 \mathrm{lbm} / \mathrm{lbm}$ ( 78 grains $/ \mathrm{lbm}$ )

- Relative humidity—Interpolate between the $\phi=50 \%$ and $\phi=60 \%$ lines, $\phi \approx 52 \%$.

- Enthalpy—Follow a constant enthalpy line and read from the enthalpy scale, $h \approx 31.5 \mathrm{Btu} / \mathrm{lbm}$

- Dew point-Move left horizontally to the saturation curve ( $\phi=100 \%$ ) and read $T_{\mathrm{dp}} \approx 60.5^{\circ} \mathrm{F}$

- Specific volume-Interpolate between the 13.5 and 14.0 lines, to get $v=13.85 \mathrm{ft}^{3} / \mathrm{lbm}$



## PROBLEMS

02-01. A sample of air at 14.7 psia and $75^{\circ} \mathrm{F}$ has a humidity ratio of $0.015 \mathrm{lbm} \mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air . Use a psychrometric chart to determine:
a) the dew point temperature, in ${ }^{\circ} \mathrm{F}$.
b) the relative humidity.
c) the degree of saturation.
d) the enthalpy in Btu/lbm dry air
e) the specific volume in $\mathrm{ft}^{3} / \mathrm{lbm}$ dry air

02-02. Use a sea-level psychrometric chart to complete the following table:

| Dry Bulb <br> ${ }^{\circ} \mathrm{F}$ | Wet Bulb <br> ${ }^{\circ} \mathrm{F}$ | Dew Point <br> ${ }^{\circ} \mathrm{F}$ | Humidity Ratio <br> $\mathrm{lbm} / \mathrm{lbm}$ | R.H. <br> $\%$ | Enthalpy <br> $\mathrm{Btu} / \mathrm{lbm}$ | Specific Volume <br> $\mathrm{ft} / \mathrm{l} / \mathrm{bm}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 55 |  |  |  |  |  |
| 100 |  | 70 |  |  |  |  |
|  |  |  |  | 40 | 40 |  |
|  |  |  | 0.01 |  |  | 13.8 |
| 40 | 60 | 40 |  |  |  |  |
|  |  |  |  | 50 |  |  |
| 85 |  |  |  |  | 30 |  |
| 80 | 80 |  | 0.012 |  |  |  |

02-03*. The conditions at the intake of an air conditioning system are $95^{\circ} \mathrm{F}, 30 \% \mathrm{rh}$, and 14.7 psia . The air flows at a rate of 1,000 cubic feet per minute (CFM). The rate ( $\mathrm{lbm} / \mathrm{hour}$ ) at which water vapor enters the system is most nearly:
(A) 0.74
(B) 10.5
(C) 44.5
(D) 55.4

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## SECTION 03: Simple Cooling and Heating

The air in heating systems is heated by circulating it through a duct that contains a heat exchanger (for the hot combustion gases) or electric resistance wires. The amount of moisture in the air remains constant during this process since no moisture is added to or removed from the air. That is, the specific humidity of the air $\omega$ remains constant during a heating (or cooling) process with no humidification or dehumidification. Such a heating process proceeds in the direction of increasing dry-bulb temperature following a line of constant specific humidity on the psychrometric chart, which appears as a horizontal line.

Notice that the relative humidity of air decreases during a heating process even though the specific humidity remains constant. Therefore, the relative humidity of heated air may be well below comfortable levels, causing dry skin, respiratory difficulties, and an increase in static electricity.
As air flowing at a rate $\dot{m}_{a}$ is heated from 1 to 2 , an amount of heat is added at a rate $\dot{Q}$. This could occur in the heating section of an HVAC device. This could also occur as cold air warms as it flows from
 the supply register(s) in a room towards the return register(s). An energy balance reveals that:

$$
\dot{Q}=\dot{m}_{a}\left(h_{2}-h_{1}\right)
$$

(heating)
where $h_{1}$ and $h_{2}$ are enthalpies per unit mass of dry air at the inlet and the exit of the heating section, respectively.
A cooling process at constant specific humidity is similar to the heating process discussed above, except the dry-bulb temperature decreases and the relative humidity increases during such a process. Cooling can be accomplished by passing the air over some coils through which a refrigerant or a chilled coolant flows. The heat transfer rate for a simple cooling process can be obtained from an energy balance on the cooling section as:

$$
\begin{equation*}
\dot{Q}=\dot{m}_{a}\left(h_{1}-h_{2}\right) \tag{cooling}
\end{equation*}
$$

Since the specific volume of air varies with temperature, all calculations should be made with the mass of air instead of the volume. Nevertheless, volume values are required when selecting coils, fans, ducts, and other components. One method of using volume while still including mass is to use volume values based on measurement at standard air conditions. ASHRAE defines one standard condition as dry air at $20^{\circ} \mathrm{C}$ and $101.325 \mathrm{kPa}\left(68^{\circ} \mathrm{F}\right.$ and 14.7 psia$)$. Under that condition the density of dry air is about 1.204 $\mathrm{kg} / \mathrm{m}^{3}\left(0.075 \mathrm{lbm} / \mathrm{ft}^{3}\right)$ and the specific volume is $0.83 \mathrm{~m}^{3} / \mathrm{kg}\left(13.3 \mathrm{ft}^{3} / \mathrm{lbm}\right)$. Saturated air at $15^{\circ} \mathrm{C}$ $\left(59.5^{\circ} \mathrm{F}\right)$ has about the same density or specific volume. Thus, in the range at which air usually passes through the coils, fans, ducts, and other equipment, its density is close to standard and is not likely to require correction.

The heat transfer to or from the air in simple heating and cooling processes is known as sensible heat which corresponds to the change in dry-bulb temperature for a given airflow (standard conditions). The sensible heat in Btu/h as a result of a difference in temperature $\Delta T \underline{\text { in }}{ }^{\circ} \mathrm{F}$ between the incoming air and leaving air flowing at ASHRAE standard conditions is:

$$
\dot{Q}_{\text {sens }}=\dot{m}_{a} \Delta h
$$

Which can be written in terms of volumetric flow, and combined with equation (1-12):

$$
\dot{Q}_{\mathrm{sens}}=\rho_{a} \dot{V}_{a}(0.24+0.444 \omega) \Delta T\left[{ }^{\circ} \mathrm{F}\right]
$$

Now, use $\rho_{a}=0.075 \mathrm{lbm} / \mathrm{ft}^{3}$, the density of dry air at standard conditions and define a new variable, CFM which is the volumetric flow rate in cubic feet per minute. Then the following equation yields the sensible heat in Btu/h:

$$
\dot{Q}_{\text {sens }}[\mathrm{Btu} / \mathrm{h}]=0.075 \frac{\mathrm{lbm}}{\mathrm{ft}^{3}} \times \mathrm{CFM} \times\left|\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right|(0.24+0.444 \omega) \Delta T\left[{ }^{\circ} \mathrm{F}\right]
$$

Since $\omega \approx 0.01$ in many air-conditioning problems, the sensible heat is typically approximated by:

$$
\begin{equation*}
\dot{Q}_{\text {sens }}[\mathrm{Btu} / \mathrm{h}]=1.1 \times \mathrm{CFM} \times \Delta T\left[{ }^{\circ} \mathrm{F}\right] \tag{3-1}
\end{equation*}
$$

If standard conditions are specified as $70^{\circ} \mathrm{F}$ and $50 \% \mathrm{RH}$, replace 1.1 with 1.08 in equation (3-1). For the purposes of the P.E. exam, equation (3-1) yields results that are accurate enough. However, you must exercise caution if the air experiences a very large temperature difference across the heating or cooling section (say, $30^{\circ} \mathrm{F}$ or greater) in which case it is advisable to use the mass flow based equations, $\dot{Q}=\dot{m}_{a}(\Delta h)$.

If the actual volumetric airflow is needed at any particular condition or point, the corresponding
specific volume is obtained from the psychrometric chart (or equation 1-13) and the volume at standard conditions is multiplied by the ratio of the actual specific volume to the standard value of 13.3. For example, assume the outdoor airflow rate at ASHRAE standard conditions is $1,000 \mathrm{cfm}$. The actual outdoor air condition is $95^{\circ} \mathrm{F}$ dry bulb, and $75^{\circ} \mathrm{F}$ wet bulb $\left[\mathrm{v}=14.3 \mathrm{ft}^{3} / \mathrm{lb}\right]$. The actual volume flow rate at this condition would be $1,000(14.3 / 13.3)=1,080 \mathrm{cfm}$.

## PROBLEMS

03-01. Air is heated to $80^{\circ} \mathrm{F}$ without adding water, from $60^{\circ} \mathrm{F}$ dry-bulb and $50^{\circ} \mathrm{F}$ wet-bulb temperature. Use a sea-level psychrometric chart to find:
(a) relative humidity of the original mixture, (b) original dew-point temperature, (c) original humidity ratio, (d) initial enthalpy, (e) final enthalpy, (f) the heat added, and (g) final relative humidity.

03-02. Air at 14.7 psia is cooled down to $55^{\circ} \mathrm{F}, 90 \%$ rh from $90^{\circ} \mathrm{F}$. Use a sea-level psychrometric chart to find the enthalpy change in Btu per pound of dry air.

03-03.* A gas furnace produces $60,000 \mathrm{Btu} / \mathrm{h}$ with an airflow of $2,800 \mathrm{cfm}$ heated air with an inlet condition of $65^{\circ} \mathrm{F}, 45 \% \mathrm{rh}$. The relative humidity of the outlet air is most nearly:
(A) 23
(B) 46
(C) 62
(D) 85

03-04.* A stream of 500 cfm of saturated air at $50^{\circ} \mathrm{F}$ and 14.7 psia is heated without adding or removing water until its relative humidity drops to $40 \%$. The heat input (Btu/h) required of the heater is most nearly:
(A) 245
(B) 9,200
(C) 14,540
(D) 25,500

03-05.* A heating section consists of a 15 -in.-diameter duct that houses an electric resistance heater. Air enters the heating section at $14.7 \mathrm{psia}, 50^{\circ} \mathrm{F}$, and $40 \% \mathrm{rh}$ at a velocity of $25 \mathrm{ft} / \mathrm{s}$ and is discharged with a relative humidity of 31 percent. The heater input $(\mathrm{kW})$ is most nearly:
(A) 4.3
(B) 6.2
(C) 80.3
(D) 14,000

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[^0]:    1 For a review of the concept of partial pressure and the modeling of mixtures of ideal gases consult our "Thermodynamics and Energy Balances" e-book

