

# **Algebra III**

## **Lesson 33**

**Quadrilaterals – Properties of  
Parallelograms – Types of Parallelograms  
– Conditions for Parallelograms -  
Trapezoids**

# Quadrilaterals

What is a quadrilateral?

Quad means?      4

Lateral means?      side

A quadrilateral is a 4 sided polygon (figure).

Types of quadrilaterals

- 1) None of the sides are parallel.
- 2) One pair of opposite sides are parallel.
- 3) Both sets of opposite sides are parallel.

## Properties of Parallelograms

Parallelograms fall under case three.

Both pairs of opposite sides are parallel.

Current postulates about parallelograms.

- 1) Both pairs of opposite sides are parallel.
- 2) Both pairs of opposite angles are congruent.
- 3) The diagonals bisect each other.
- 4) Any pair of consecutive angles is supplementary.

These properties can be used in proofs as a reason for a step.

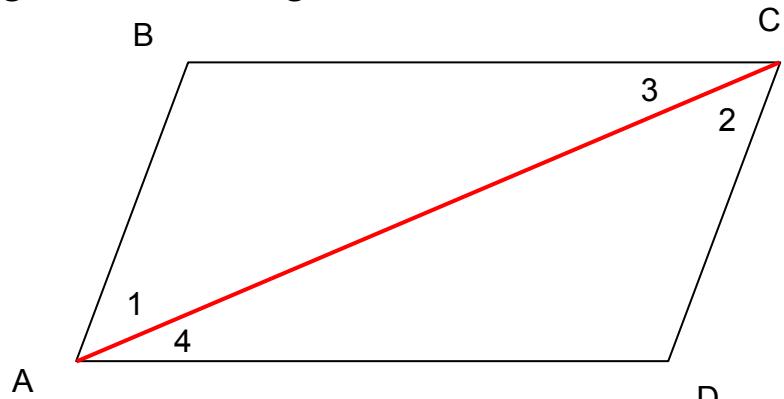
## Example 33.1

Prove that the opposite sides of a parallelogram are congruent.

Given: Parallelogram ABCD

Prove:  $\overline{BC} \cong \overline{AD}$

$\overline{AB} \cong \overline{DC}$

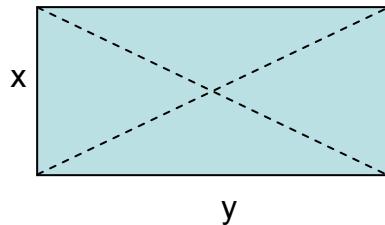


Statements	Reasons
1) Parallelogram ABCD	1) Given
2) Construct $\overline{AC}$	2) Def. of a line
3) $\overline{AC} \cong \overline{AC}$	3) Reflexive
4) $\overline{BC} \parallel \overline{AD}$ & $\overline{AB} \parallel \overline{DC}$	4) Def. of parallelogram
5) $\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$	5) Alt. int. angles of parallel lines
6) $\triangle BAC \cong \triangle DCA$	6) ASA
7) $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	7) CPCTC

## Types of Parallelograms

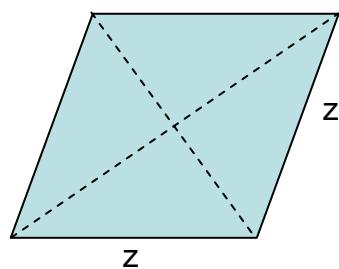
What else has both sides parallel? (3 names of special parallelograms)

Rectangle



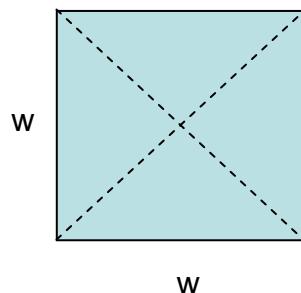
- 1) Parallelogram
- 2) Right angles
- 3) Diagonals are congruent

Rhombus  
(Diamond)



- 1) Parallelogram
- 2) All sides congruent
- 3) Diagonals are perpendicular
- 4) Diagonals bisect vertices (corners)
- 5) Diagonals divide rhombus into 4 congruent right triangles

Square

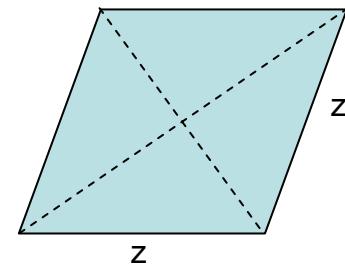


- 1) Rectangle
- 2) Rhombus
- 3) Diagonals form 4 congruent isosceles right triangles

## Example 33.2

The diagonals of a certain parallelogram are of unequal length. Which of the following could the parallelogram be?

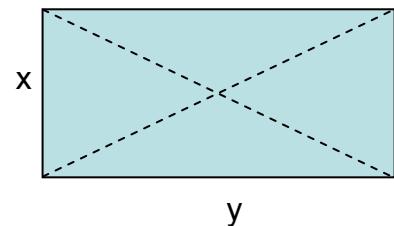
- A) Square
- B) Rectangle
- C) Rhombus



## Example 33.3

The diagonals of a certain parallelogram are not perpendicular. Which of the following could the parallelogram be?

- A) Rectangle
- B) Rhombus
- C) Square



## **Conditions for parallelograms**

What it takes to prove a quadrilateral is a parallelogram or a specific subtype of parallelogram.

### **Quadrilateral → Parallelogram** (Any of the following)

- A) Both pairs of opposite sides are parallel
- B) One pair of sides are both parallel and congruent
- C) The diagonals of the quadrilateral bisect each other
- D) Both pairs of opposite angles are congruent

### **Quadrilateral → Rectangle** (Either of the following)

- A) If a quadrilateral is a parallelogram and its diagonals are congruent, then the parallelogram is a rectangle.
- B) If all four angles of a quadrilateral are right angles, then the quadrilateral is a rectangle.

### **Quadrilateral → Rhombus** (Either of the following)

- A) If a quadrilateral is a parallelogram and either of its diagonals bisects the vertices of the angles it connects, then the parallelogram is a rhombus.
- B) If the diagonals of a quadrilateral are perpendicular, then the quadrilateral is a rhombus.

### **Example 33.4**

Given a parallelogram whose diagonals are congruent, which of the following must be true?

- A) All four sides have the same length.
- B) The diagonals are perpendicular bisectors of each other.
- C) All four angles of the parallelogram are right angles.

Congruent diagonals eliminates the rhombus.

This leaves the rectangle or the square.

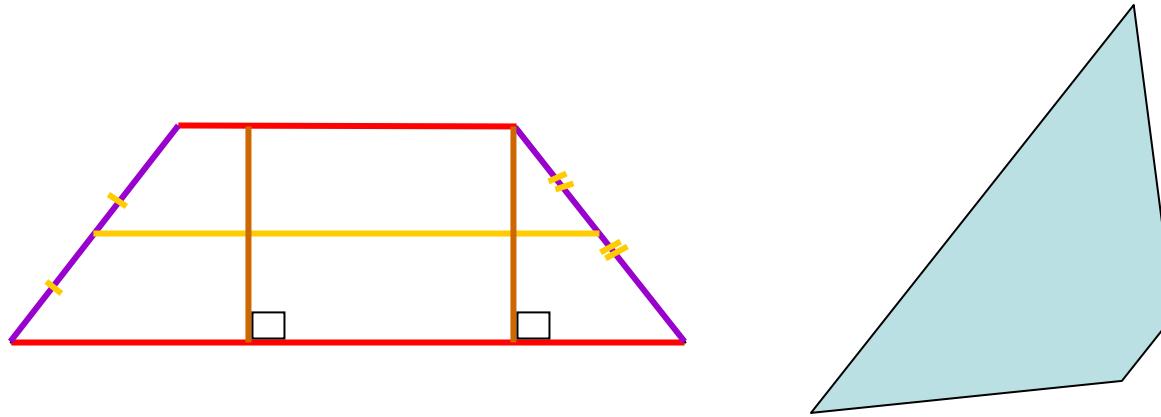
Since the square is a special rhombus, it is also eliminated.

Leaving the rectangle as the answer.

A & B refer to the rhombus specifically, C refers to the rectangle. So, C is the answer.

## Trapezoids

A quadrilateral with only one pair of parallel sides.



The parallel sides are called the **bases** of the trapezoid.

The non-parallel sides are called the **legs**.

The mid-points of the legs are connected by the **median**.

A line that connects the two bases and is perpendicular to one base is called an **altitude**.

## **Properties of a Trapezoid**

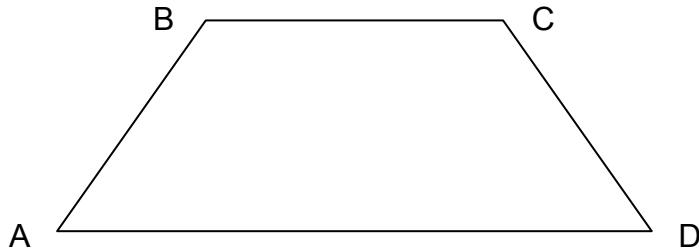
- 1) The area of a trapezoid equals one half the product of the length of the altitude (height) and the sum of the bases.

$$A = \frac{h(b_1 + b_2)}{2}$$

- 2) The median of a trapezoid is parallel to the bases, and its length is one half of the sum (average) of the lengths of the bases.

## Isosceles Trapezoids

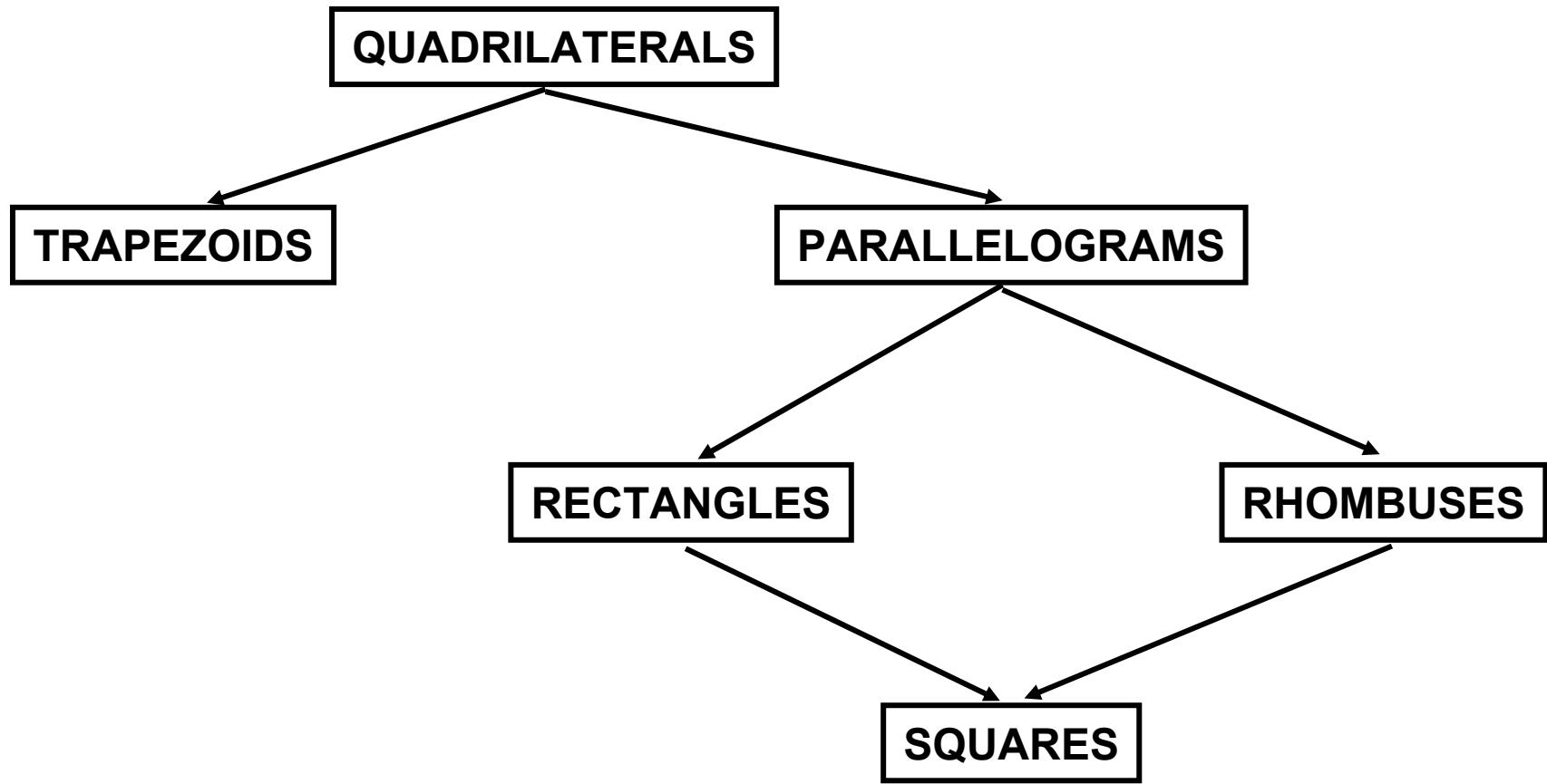
A trapezoid where the legs are of equal length is an isosceles trapezoid.



In this trapezoid, B & C are the upper base angles and A & D are the lower base angles.

### Properties of an isosceles trapezoid

- 1) The lower base angles are congruent.
- 2) The upper base angles are congruent.
- 3) The diagonals are congruent.
- 4) Any lower base angle is supplementary to any upper base angle.



## Example 33.5

Derive a general formula for the area of a trapezoid using the figure shown.

$$A = \frac{h(b_1 + b_2)}{2}$$

This is the formula,  
we need to prove  
it.

Since we're making the area formula for this shape, we'll have to use some other shape.

Triangles

Draw a diagonal to make two triangles.

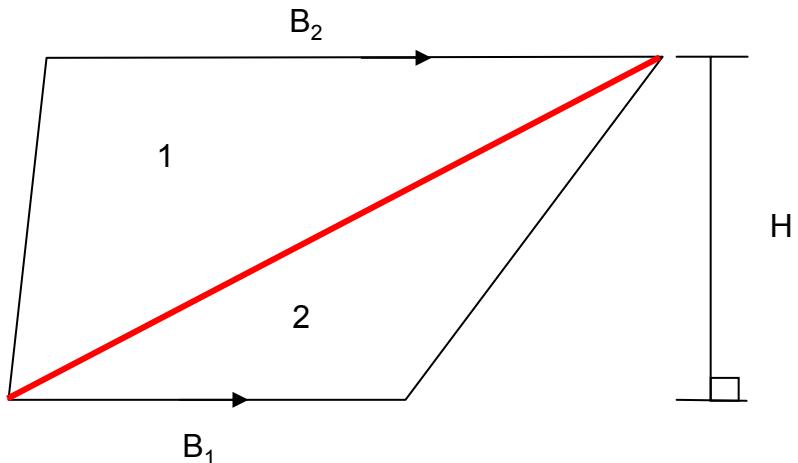
$$A_{\text{trap}} = A_{\Delta 1} + A_{\Delta 2}$$

$$A_{\Delta 1} = \frac{1}{2} bh$$

$$A_{\Delta 2} = \frac{1}{2} bh$$

$$A_{\Delta 1} = \frac{1}{2} B_2 H$$

$$A_{\Delta 2} = \frac{1}{2} B_1 H$$



$$\begin{aligned} A_{\text{Trap}} &= \frac{1}{2} B_2 H + \frac{1}{2} B_1 H \\ &= \frac{1}{2} H(B_2 + B_1) \end{aligned}$$

Done

### Example 33.6

The figure shown is a trapezoid whose area is  $175 \text{ cm}^2$ . Find  $x$ . Dimensions are in centimeters.

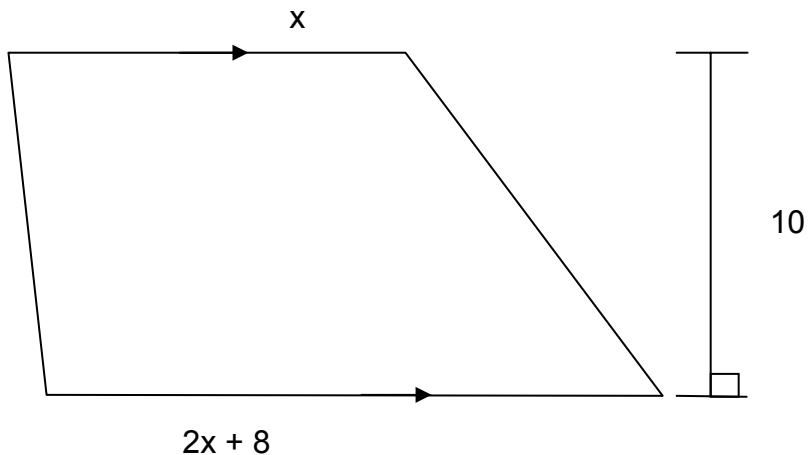
Use the area formula since we have the area.

$$A = \frac{h(b_1 + b_2)}{2}$$

$$175 = \frac{10(x + 2x + 8)}{2}$$

$$175 = 5(3x + 8)$$

$$35 = 3x + 8$$



$$27 = 3x$$

$$x = 9$$

### Example 33.7

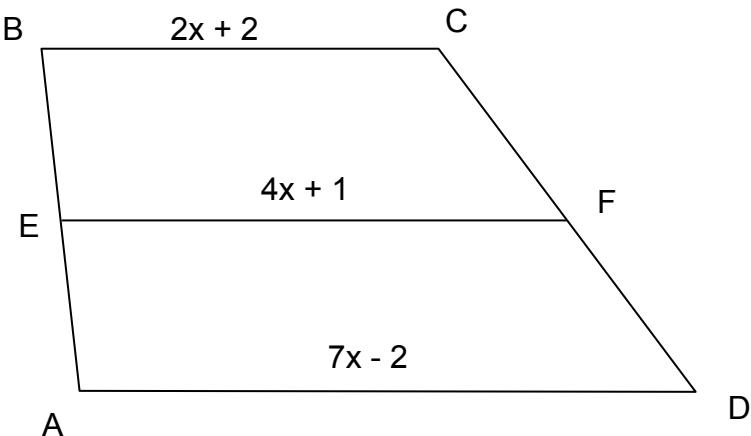
In the figure shown,  $\overline{EF}$  is the median of trapezoid ABCD. Find x.

By definition,  
median =  $\frac{1}{2} (B_1 + B_2)$

$$4x + 1 = \frac{1}{2} ([2x + 2] + [7x - 2])$$

$$8x + 2 = 9x$$

$$x = 2$$



## Practice

- a) All four sides of a certain quadrilateral have the same length. Which of the following must the quadrilateral be?
- 1) Rectangle      2) Square      3) Rhombus      4) Trapezoid

Four equal sides knocks out?      Rectangle & Trapezoid

Leaving Square & Rhombus.

Since a square is a rhombus then no matter what rhombus is at least one answer.

But to be a square we need information about the angles.

So, Rhombus is the only certain answer.

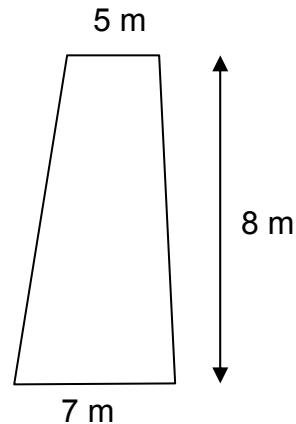
b) Find the area of this trapezoid.

$$A = \frac{h(b_1 + b_2)}{2}$$

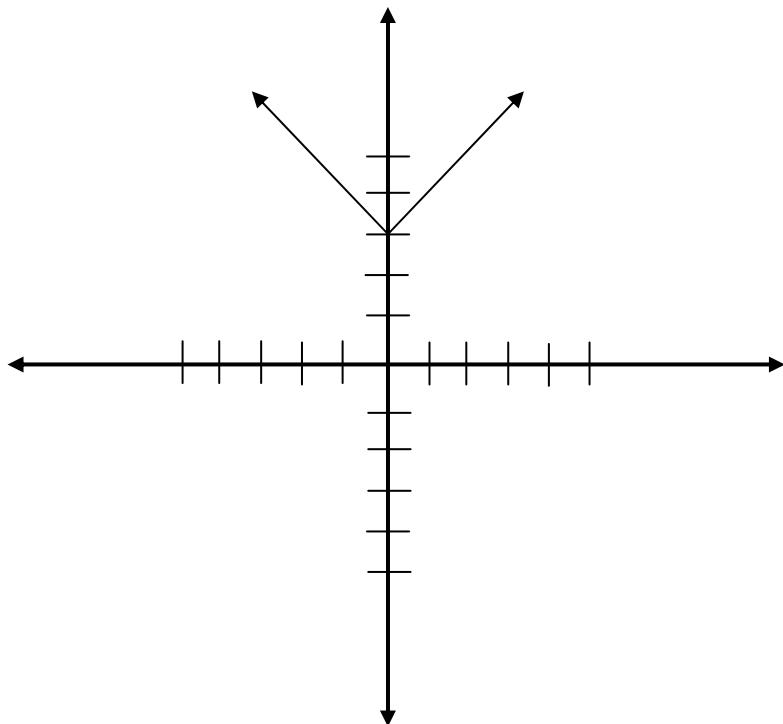
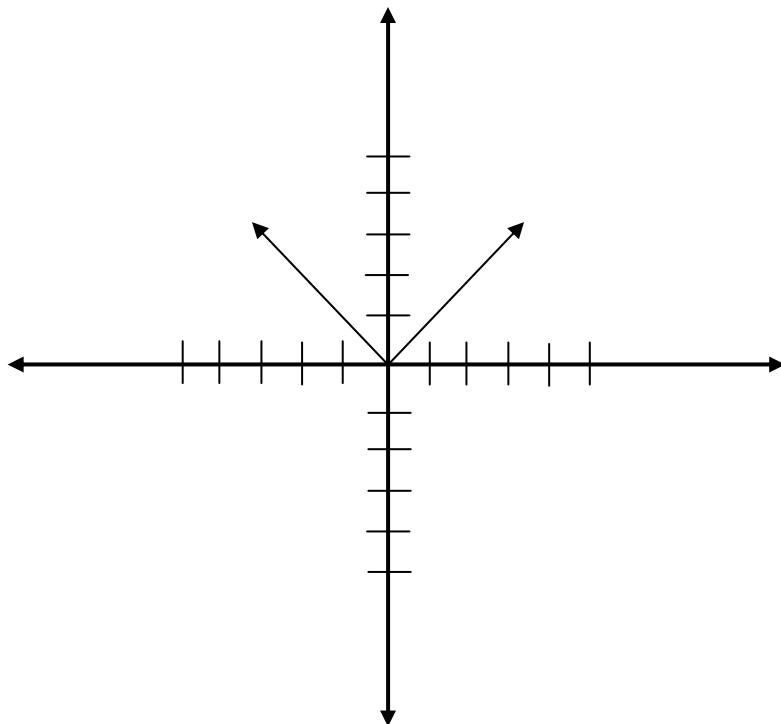
$$A = \frac{8(5 + 7)}{2}$$

$$A = 4(12)$$

$$A = 48 \text{ m}^2$$



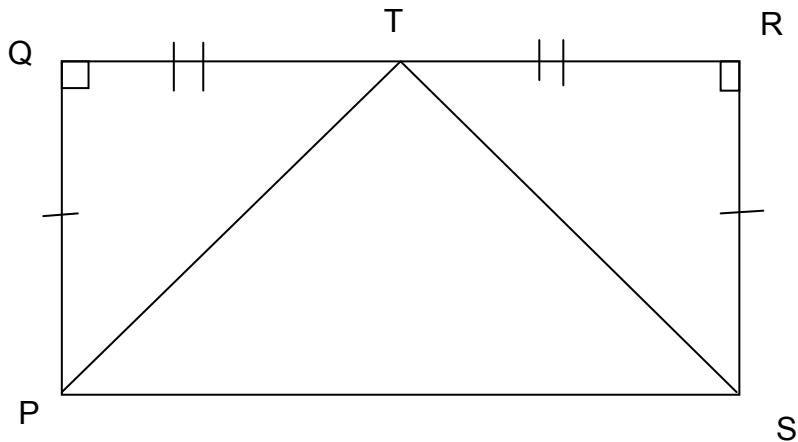
c) The graph of the function  $f(x) = |x|$  is shown on the left below. The graph on the right is the same graph translated three units up. Write the equation of the graph on the right.



$$f(x) = |x| + 3$$

d) Givens: PQRS is a rectangle.  
T is a midpoint of  $\overline{QR}$ .

Prove:  $\Delta PQT \cong \Delta SRT$



Statements	Reasons
1) PQRS is rectangle. T is a midpt of $\overline{QR}$	1) Given
2) $\overline{PQ} \cong \overline{SR}$	2) Def. of rectangle
3) $\overline{QT} \cong \overline{RT}$	3) Def. of midpt
4) $\angle Q$ & $\angle R$ are rt angles	4) Def of rectangle
5) $\angle Q \cong \angle R$	5) Def of rt angles
6) $\Delta PQT \cong \Delta SRT$	6) SAS