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The Ultimate Cheat Sheet for Astrophysics Students

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Chapter 1

Physics

1.1 Motion

1.1.1 Velocity

- $$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} = \dot{\vec{x}}$$

1.1.2 Acceleration

- $$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} = \ddot{\vec{x}}$$

1.1.3 Newton's Laws

Newton's First Law

- When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a net force.

Newton's Second Law

- $$\vec{F}_{net} = m\vec{a} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

Newton's Third Law

- $$\vec{F}_A = -\vec{F}_B$$
- When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

1.1.4 Momentum

- $$\vec{p} = \gamma m\vec{v} \approx m\vec{v}$$
- $$\Delta\vec{p} = \vec{F}\Delta t$$
- $$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

1.1.5 Centripetal Force

- $$F_c = \frac{mv^2}{r}$$

1.1.6 Kinetic Energy

- $$K = \tfrac{1}{2}mv^2$$

1.1.7 Projectile Motion

- $v_y^2 = u_y^2 + 2a_y\Delta y$
- $x = u_x t$
- $\Delta y = u_y\Delta t + \frac{1}{2}a_y\Delta t^2 = u_y t + \frac{1}{2}\frac{F_y}{m}\Delta t^2$

1.1.8 Rotation

Angular Velocity

- $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \dot{\theta}$
- $\omega = \frac{v}{r}$
- $\vec{v} = \vec{r} \times \vec{\omega}$

Angular Acceleration

- $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \dot{\omega} = \ddot{\theta}$

Moment of Inertia

Point Mass

- $I = mr^2$

Several Point Masses

- $I = \sum mr^2$

Continuous mass

- $I = \int r^2 \ \mathrm{d}m$

Parallel axis theorem

- $I = I_{com} + md^2$

Thin disc rotating about centre

- $I = \frac{MR^2}{2}$

Thin hoop rotating about centre

- $I = MR^2$

Thin rod rotating about centre

- $I = \frac{ML^2}{12}$

Thin rod rotating about end

- $I = \frac{ML^2}{3}$

Rotational Kinetic Energy

- $K_{rot} = \frac{1}{2}I\omega^2$

Total Kinetic Energy

- $K_{tot} = K_{trans} + K_{rot} = \frac{1}{2}(mr_{com}^2 + I_{com})\omega^2$

Angular Momentum

- $$\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$$

Torque

- $\vec{\tau} = I\vec{\alpha} = \dfrac{\mathrm{d}L}{\mathrm{d}t} = \vec{r} \times \vec{F}$

1.1.9 Euler-Lagrange and the Hamiltonian

Lagrangian

- $$\ell = T - V = \sum_{lm} a(q)\dot{q}_l\dot{q}_m$$
- $$= K(\dot{q}_l) - U(q_l)$$

Generalised coordinates & momenta

- $$p_k \equiv \frac{\partial L}{\partial \dot{q}_k}$$

Euler-Lagrange Equation

- $$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \ell}{\partial \dot{x}} - \frac{\partial \ell}{\partial x} = 0$$

Action

- $$S[x(t)] = \int_{t_A}^{t_B} \ell(\dot{x}(t), x(t))\ \mathrm{d}t$$

Hamiltonian

- $$\mathcal{H} = \sum_l p_l\dot{q} - L$$
- $$\dot{P} = -\frac{\partial H}{\partial Q}$$
- $$\dot{Q} = \frac{\partial H}{\partial P}$$
- $$\dot{P} = -\omega^2 Q$$
- $$\dot{Q} = P$$

1.2 Oscillations

1.2.1 Springs

Force of a Spring

- $$\vec{F} = -k_s\vec{x}$$

Potential Energy of a Spring

- $$U_s = \frac{1}{2}k_s x^2$$

Angular Frequency of a Spring

- $$\omega = \sqrt{\frac{k_s}{m}}$$

1.3 Materials

1.3.1 Density

- $$\rho = \frac{m}{V} = \frac{\mathrm{d}m}{\mathrm{d}V}$$

1.4 Energy

1.4.1 Work

- $$W = \int_a^b \vec{F} \cdot \, \mathrm{d}\vec{l} \approx \vec{F} \cdot \vec{s}$$

1.5 Forces

1.5.1 Buoyancy (Archimedes' Principle)

- $F_{buoy} = m_{displaced} g = \rho_d V_d g$

1.5.2 Friction

- $F_K \approx \mu_K F_\perp$
- $F_S \leq \mu_S F_\perp$

1.6 Waves

- $a \sin(\omega t - kx + \phi)$
- $k = \frac{2\pi}{\lambda}$

1.6.1 Wavelength

- $v = f\lambda$

1.6.2 Angular Frequency

- $$\omega = \frac{2\pi}{T} = 2\pi f$$

1.7 Newtonian Gravity

1.7.1 Force of Gravity

- $$\vec{F}_G = \frac{GmM}{r^2}\hat{r} = -m\vec{\nabla}\Phi(\vec{r}) \approx -mg\hat{y} = m\vec{g}$$

1.7.2 Gravitational Potential (potential energy per unit mass)

- $$\Phi(\vec{r}) = -\sum_i \frac{GM(\vec{r}_i)}{|\vec{r} - \vec{r}_i|} = -\int \frac{G\mu(\vec{r'})}{|\vec{r} - \vec{r'}|} \, \mathrm{d}^3\vec{r'}$$

1.7.3 Gravitational field

- $$\vec{g}(\vec{r}) = \frac{GM}{r^2} = -\nabla\Phi(\vec{r})$$

1.7.4 Gravitational Potential Energy

- $$U_G = -\frac{GmM}{r} \approx mgh$$

1.7.5 Kepler's Third Law

- $$\frac{T^2}{r^3} = \frac{4\pi^2}{G(m+M)} = constant$$

1.8 Electromagnetism

1.8.1 Notation

- $$\vec{\mathscr{r}} = \vec{r} - \vec{r'}$$

1.8.2 Maxwell's Equations

	Integral form	**Differential form**
Gauss's Law	$\oiint_S \vec{E}\cdot \mathrm{d}\vec{a} = \frac{1}{\varepsilon_0}\iiint_V \rho \,\mathrm{d}V$ $= \frac{\sum Q_{enc}}{\varepsilon_0}$	$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$
Gauss's Law for Magnetism	$\oiint_S \vec{B}\cdot \mathrm{d}\vec{a} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
Maxwell-Faraday equation	$\oint_b \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B}\cdot \mathrm{d}\vec{a}$	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
Ampére's circuital law	$\oint_b \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0\iint_S \vec{J}\cdot \mathrm{d}\vec{a} + \mu_0\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{E}\cdot d\vec{a}$ $= \mu_0(I_{enc} + \varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\int_S \vec{E}\cdot \mathrm{d}\vec{a})$	$\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t})$

1.8.3 Lorentz Force

On a point charge

- $$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

On a current

- $$\mathrm{d}\vec{F} = I\int \mathrm{d}\vec{l}\times\vec{B}$$
- $$\vec{F} = \vec{I}L\times\vec{B}$$

1.8.4 Electric Field

- $$\vec{E} = \int_V \frac{\rho(\vec{r'})}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}\tau$$

From a single point charge

- $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

From a dipole

- $$|\vec{E}_a xis| \approx \frac{2p}{4\pi\varepsilon_0 r^3}$$
- $$|\vec{E}_\perp| \approx \frac{p}{4\pi\varepsilon_0 r^3}$$

1.8.5 Dipole moment

- $$\vec{p} = q\vec{d}$$

1.8.6 Electric potential

- $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\not{r}}$$
- $$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a single-point charge field

- $$\Delta(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

1.8.7 Electric potential difference

- $$\Delta(\vec{r}) = -\int_{\vec{b}}^{\vec{a}} \vec{E} \cdot \mathrm{d}\vec{l}$$

In a single-point charge field

- $$\Delta(\vec{r}) = \frac{1}{4\pi\varepsilon_0} Q(\frac{1}{b} - \frac{1}{a})$$

1.8.8 Electric potential energy

- $$U_E = q\Delta V = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\not{r}}$$

Energy stored in an electrostatic field distribution

- $U_E = \frac{1}{2} = \epsilon_0 E^2 \times volume$

1.8.9 Charge densities

Surface

- $$\sigma = \frac{\mathrm{d}q}{\mathrm{d}a} = \frac{Q}{A}$$

Line

- $$\lambda = \frac{\mathrm{d}q}{\mathrm{d}l} = \frac{Q}{L}$$

1.8.10 Current densities

Volume

- $$\vec{J} = \frac{\mathrm{d}\vec{I}}{\mathrm{d}\vec{a}_\perp} = \frac{I}{A_\perp} = \sigma(\vec{E} + \vec{v} \times B) = |q|nu(\vec{E} + \vec{v} \times B)$$
- $$\vec{\nabla} \cdot \vec{J} = 0$$

Surface

- $$\vec{K} = \frac{\mathrm{d}\vec{I}}{\mathrm{d}\vec{l}_\perp} = \frac{I}{l} = \sigma\vec{v}$$

1.8.11 Circuits

Electron drift velocity

- $\bar{v} = u\vec{E}_{net}$

Current per unit charge

- $i = nA_{cs}\bar{v} = nA_{cs}uE_{net}$

Current

- $$I = ei = enA_{cs}uE_{net} = \frac{\mathrm{d}q}{\mathrm{d}t}$$

Electrical Power

- $P = IV = I^2R$

Voltage (Electric potential difference)

- $V = \Delta V = IR = -\varepsilon$

Electromotive Force (EMF) from a Non-Coulomb force

- $$\epsilon = \frac{F_{NC}d}{e}$$

Resistance

- $$R = \frac{L\rho}{A} = \frac{L}{\sigma A}$$
- $R_{series} = R_1 + R_2 + ... + R_n$
- $$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$

1.8.12 Capacitors

Capacitance

- $$C = \frac{Q}{V} = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

Energy stored in a capacitor

- $$W = \frac{CV^2}{2}$$

Electric field in a capacitor

- $$E = \frac{Q}{\varepsilon_0 A}$$

Potential difference across a capacitor

- $$\Delta V = -\frac{dQ}{A\varepsilon_0}$$

1.8.13 Magnetic fields

- $\vec{B}(\vec{\mathscr{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, \mathrm{d}l$
- $\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0}{4\pi} \frac{I \, \mathrm{d}\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

Magnetic field due to a wire

- $\vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{r} \hat{\phi}$

Magnetic vector potential

- $\vec{A}(\vec{\mathscr{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r'})}{\mathscr{r}} \, \mathrm{d}\tau$
- $\vec{\nabla} \times \vec{A} = \vec{B}$
- $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\mu_0 \vec{J}$
- $\vec{\nabla} \cdot \vec{A} = 0$

1.8.14 Inductors

- $\varepsilon = -LI$

Energy stored in an inductor

- $W = \frac{LI^2}{2}$

1.8.15 Materials

Macroscopic Maxwell's Equations (Materials)

	Integral form	**Differential form**
Gauss's Laws	$\oiint_S \vec{P} \cdot \, \mathrm{d}\vec{a} = -\sum Q_B$ $\oiint_S \vec{D} \cdot \, \mathrm{d}\vec{a} = \sum Q_f$	$\vec{\nabla} \cdot \vec{P} = -\rho_B$ $\vec{\nabla} \cdot \vec{D} = \rho_f$
Gauss's Law for Magnetism	$\oiint_S \vec{B} \cdot \, \mathrm{d}\vec{a} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
Maxwell-Faraday equation	$\oint_b \vec{E} \cdot \, \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S \vec{B} \cdot \, \mathrm{d}\vec{a}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampére's circuital law	$\oint_b \vec{H} \cdot \, \mathrm{d}\vec{l} = I_{f,enc} + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot \, \mathrm{d}\vec{a}$	$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Dielectric constant

- $k = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$
- $\varepsilon = k\varepsilon_0 = \varepsilon_r \varepsilon$

Susceptibility

- $\chi_e = 1 - \varepsilon_r$

Polarisability

- $\vec{P} = \varepsilon_0 \chi_e \vec{E} = n\vec{p}$

Bound Charge

Surface

- $\sigma_B = \vec{p} \cdot \hat{n}$

Volume

- $\rho_B = -\vec{\nabla} \cdot \vec{P}$

Total

- $Q_B = \sigma_B + \rho_B = \vec{p} \cdot \hat{n} - \vec{\nabla} \cdot \vec{P}$

Electric displacement

- $\vec{D} = \varepsilon \vec{E} = k\varepsilon_0 \vec{E} = \varepsilon_0 \vec{E} + \vec{P}$

Magnetic field

- $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

Magnetic dipole

- $\vec{m} = I\vec{a}$

Bound current

- $\vec{J}_B = \vec{\nabla} \times \vec{M}$
- $\vec{K}_B = \vec{M} \times \hat{n}$

1.9 Special Relativity

1.9.1 Interval

- $\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$
- $\mathrm{d}s^2 = -c^2 \,\mathrm{d}t^2 + \,\mathrm{d}x^2 + \,\mathrm{d}y^2 + \,\mathrm{d}z^2$
- $\Delta s^2 < 0$ is a timelike interval. Events separated by this interval can be causally related.
- $\Delta s^2 = 0$ is a lightlike interval. Events separated by this interval can be causally related, but only by a lightspeed signal.
- $\Delta s^2 > 0$ is a spacelike interval. Events separated by this interval CANNOT be causally related.

Gamma Factor

- $\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}$
- $\gamma = \frac{dt}{d\tau}$

Mass-energy

- $E_{rest} = mc^2$
- $E = \gamma mc^2 = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} mc^2$

Relativistic kinetic energy

- $K = \gamma mc^2 - mc^2$

Length contraction

- $l_v = \frac{l_0}{\gamma} = l_0 \sqrt{1 - (\frac{v}{c})^2}$

Time dilation

- $t_v = \gamma t_0 = \frac{t_0}{\sqrt{1 - (\frac{v}{c})^2}}$

Mass dilation

- $m_v = \gamma m_0 = \frac{m_0}{\sqrt{1 - (\frac{v}{c})^2}}$

Relative Velocity

- $u'_x = \frac{\Delta x'}{\Delta t} = \frac{u_x - v_x}{1 - \frac{v_x u_x}{c^2}}$

Relativistic Momentum

- $\vec{p} = \gamma \vec{v} = \frac{m\vec{v}}{\sqrt{1 - (v/c)^2}}$

1.9.2 Four-vectors

Four-space

- $\mathbf{s} = \mathbf{x} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$

Four-velocity

- $\mathbf{u} = \frac{d\mathbf{s}}{d\tau} = \gamma \begin{bmatrix} c \\ v_x \\ v_y \\ v_z \end{bmatrix}$

Four-momentum

- $\mathbf{p} = \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix} = \gamma m \begin{bmatrix} c \\ v_x \\ v_y \\ v_z \end{bmatrix} = m\mathbf{u}$

1.9.3 Frames of Reference

Condition for an inertial frame

- $$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0$$

Galilean Transformations

- $x' = x + vt$
- $y' = y$
- $z' = z$
- All assuming x is along the axis of motion and $x = x$' when $t = 0$.

Lorentz Boosts

- $t' = \gamma(t - \frac{vx}{c^2})$
- $x' = \gamma(x - vt)$
- $y' = y$
- $z' = z$
- (x is along the axis of motion)
- $$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

General Lorentz transformation

- $$\begin{bmatrix} b'^0 \\ b'^1 \\ b'^2 \\ b'^3 \end{bmatrix} = \begin{bmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b^0 \\ b^1 \\ b^2 \\ b^3 \end{bmatrix}$$
- Motion along the x-axis.

Proper Time

- $$\tau = \int_{t_A}^{t_B} \frac{1}{\gamma}\, \mathrm{d}t = \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2(t)}{c^2}}\, \mathrm{d}t$$

1.9.4 Proper Velocity

- $$\mathbf{u} = \frac{\mathrm{ds}}{\mathrm{d}\tau}$$

1.10 General Relativity

1.10.1 Metrics

Minkowski

- $$\eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
- $\mathrm{d}s^2 = -c^2\ \mathrm{d}t^2 +\ \mathrm{d}x^2 +\ \mathrm{d}y^2 +\ \mathrm{d}z^2$

Schwarzschild

- $$g = \begin{bmatrix} -(1-\frac{2GM}{c^2r}) & 0 & 0 & 0 \\ 0 & (1-\frac{2GM}{c^2r})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}$$

- $$\mathrm{d}s^2 = -(1-\frac{2GM}{c^2r})c^2\ \mathrm{d}t^2 + (1-\frac{2GM}{c^2r})^{-1}\ \mathrm{d}r^2 + r^2\ \mathrm{d}\theta^2 + r^2\sin^2\theta\ \mathrm{d}\phi^2)$$

1.10.2 Rindler coordinates

Line element

- $$\mathrm{d}s^2 = -(1+\frac{gx'}{c^2})^2(c\ \mathrm{d}t')^2 + \ \mathrm{d}x'$$

1.10.3 Einstein notation

- Contravariant: e^α
- Covariant: e_α
- $$t_{\alpha\beta} = g_{\beta\gamma}t_\alpha{}^\gamma$$
- $$t_\alpha{}^\beta = g^{\beta\gamma}t_{\alpha\gamma}$$
- $$t'^\alpha{}_\beta = \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial x^\delta}{\partial x'^\beta}t^\gamma{}_\delta$$
- $$t'_\alpha{}^\beta = \frac{\partial x^\gamma}{\partial x'^\alpha}\frac{\partial x'^\beta}{\partial x^\delta}t_\gamma{}^\delta$$

Metrics

- $$\mathrm{d}s^2 = g_{\alpha\beta}\ \mathrm{d}x^\alpha\ \mathrm{d}x^\beta$$
- $$g^{\alpha\beta} = \frac{1}{g_{\alpha\beta}}$$
- $$\delta^\alpha_\beta = \begin{cases} 1 & \alpha = \beta \\ 1 & \alpha \neq \beta \end{cases}$$
- $$\delta^\alpha_\gamma a^\gamma = a^\alpha$$
- $$g^{\alpha\gamma}g_{\gamma\beta} = \delta^\alpha_\beta$$

Four-vector product

- $$\mathbf{a}\cdot\mathbf{b} = g_{\alpha\beta}a^\alpha b^\beta = a_\beta b^\alpha$$

1.10.4 Christoffel symbols

- $$\Gamma^\alpha{}_{\beta\gamma} = \tfrac{1}{2}g^{\alpha\delta}(\frac{\partial g_{\delta\beta}}{\partial x^\gamma} + \frac{\partial g_{\delta\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\delta})$$
- $$\Gamma_{\alpha\beta\gamma} = \tfrac{1}{2}(\frac{\partial g_{\delta\beta}}{\partial x^\gamma} + \frac{\partial g_{\delta\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\delta})$$
- $$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau} + \Gamma^\mu{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0$$

1.10.5 Covariant derivatives

- $\nabla_\gamma t^\alpha{}_\beta = \dfrac{\partial t^\alpha{}_\beta}{\partial x^\gamma} + \Gamma^\alpha{}_{\gamma\delta} t^\delta{}_\beta - \Gamma^\delta{}_{\gamma\beta} t^\alpha{}_\delta$
- $\nabla_\gamma t^{\alpha\beta} = \dfrac{\partial t^{\alpha\beta}}{\partial x^\gamma} + \Gamma^\alpha{}_{\gamma\delta} t^{\delta\beta} + \Gamma^\beta{}_{\gamma\delta} t^{\alpha\delta}$
- $\nabla_\gamma t_{\alpha\beta} = \dfrac{\partial t_{\alpha\beta}}{\partial x^\gamma} - \Gamma^\delta{}_{\gamma\alpha} t_{\delta\beta} - \Gamma^\delta{}_{\gamma\beta} t^{\alpha\delta}$
- $\nabla_\gamma t_\alpha{}^\beta = \dfrac{\partial t_\alpha{}^\beta}{\partial x^\gamma} - \Gamma^\delta{}_{\gamma\alpha} t_\delta{}^\beta + \Gamma^\beta{}_{\gamma\delta} t_\alpha{}^\delta$

1.10.6 Riemann curvature tensor

- $R^\alpha{}_{\beta\gamma\delta} = \dfrac{\partial \Gamma^\alpha{}_{\beta\delta}}{\partial x^\gamma} - \dfrac{\partial \Gamma^\alpha{}_{\beta\gamma}}{\partial x^\delta} + \Gamma^\alpha{}_{\gamma\epsilon}\Gamma^\epsilon{}_{\beta\delta} - \Gamma^\alpha{}_{\delta\epsilon}\Gamma^\epsilon{}_{\beta\gamma}$
- $R_{\alpha\beta\gamma\delta} = \frac{1}{2}(\dfrac{\partial^2 g_{\alpha\delta}}{\partial x^\beta \partial x^\gamma} - \dfrac{\partial^2 g_{\alpha\gamma}}{\partial x^\beta \partial x^\delta} - \dfrac{\partial^2 g_{\beta\delta}}{\partial x^\alpha \partial x^\gamma}) + \dfrac{\partial^2 g_{\beta\gamma}}{\partial x^\alpha \partial x^\delta}$
- $R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\gamma\delta}$
- $R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\delta\gamma}$
- $R_{\alpha\beta\gamma\delta} = R_{\delta\gamma\alpha\beta}$
- $R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta=0}$

1.10.7 Ricci curvature tensor

- $R_{\alpha\beta} = R^\gamma{}_{\alpha\gamma\beta}$
- $R = R^\alpha{}_\alpha$

1.10.8 Einstein's equations

- $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \dfrac{8\pi G}{c^4} T_{\mu\nu}$

1.11 Thermodynamics

1.11.1 Ideal Gases

Ideal Gas Law

- $pV = Nk_BT$

Heat / Thermal Energy

- $Q = mc\Delta T$

Heat Capacity

- $C = \dfrac{\mathrm{d}Q}{\mathrm{d}T}$

Specific Heat Capacity

- $c = \dfrac{C}{m}$

1.11.2 Microstates

- $\Omega = \dfrac{(q+N-1)}{q!(N-1)}$

1.11.3 Entropy

- $S = k_B \ln \Omega$

1.11.4 Black bodies

Energy of a photon

- $E = hf$

Wien's Displacement Law

- $\lambda_{max} = \frac{b}{T} = (2.8977729 \times 10^{-3})\frac{1}{T}$

Stefan-Boltzmann Law

- $I = \sigma T^4$

Spectrum

- $B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(\frac{hc}{\lambda k_B T} - 1)}$

- $B_\nu(T) = \frac{2h\nu}{c^2}\frac{1}{\exp(\frac{h\nu}{k_B T}) - 1}$

1.12 Quantum Mechanics

1.12.1 The Uncertainty Principle

- $\Delta x \Delta p \geq \frac{\hbar}{2}$

- $\Delta E \Delta t \geq \frac{\hbar}{2}$

1.12.2 Bras and Kets

- $|\psi\rangle = \langle\psi|^\dagger$

1.12.3 Rules for an Inner Product

- $\langle\psi|\phi\rangle \equiv (|\psi\rangle, |\phi\rangle)$
- Symmetric:
  $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$
- Linear in second component
- Anti-linear in first component

1.12.4 The Born Rule

- $P = |\langle\psi|\psi\rangle|^2$

1.12.5 Expectation

- $\langle A\rangle = \int A|\Psi(x,t)|^2 \, \mathrm{d}x$
- $\langle A\rangle = \langle\psi|A|\psi\rangle$

1.12.6 Variance

- $\text{var}(A) = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2$

1.12.7 Standard Deviation

- $\delta A = \sqrt{\text{var}(A)} = \sqrt{\langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2}$

1.12.8 Trace

- $\text{Tr}(A) = \sum_j \langle x_j|A|x_j\rangle$

1.12.9 Partial Trace

- $Tr_B(|a\rangle\langle a| \otimes |b\rangle\langle b|) \equiv |a\rangle\langle a|\text{Tr}(|b\rangle\langle b|)$
- $\text{Tr}(k_{AB}) = Tr_A(Tr_B(k_{AB})) = Tr_B(Tr_A(k_{AB}))$
- $\rho_B = Tr_A(\rho_{AB})$
- The partial trace is linear

1.12.10 The Schrödinger Equation

- $i\hbar\frac{\partial}{\partial t}\Psi(r,t) = \hat{H}\Psi(r,t)$
- $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$
- $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x,t) = E\psi(x)$
- $\hat{H}|\Psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle$

1.12.11 Heisenberg equation of motion

- $\frac{d}{dt}\hat{A}(t) = \frac{i}{\hbar}[\hat{H}, \hat{A}(t)]$

1.12.12 Operators

- $a_{jk} = \langle j|A|k\rangle$

Diagonalizable Operator

- $A = \sum_j \lambda_j|\lambda_j\rangle\langle\lambda_j|$

Normal Operator

- $A = \sum_j |\lambda_j\rangle\langle\lambda_j|$

Eigenstate Operators

- $(|\lambda_k\rangle\langle\lambda_k|)^n = |\lambda_k\rangle\langle\lambda_k|$

Identity

- $I = \sum_j |x_j\rangle\langle x_j|$

Projector

- $P = |\psi\rangle\langle\psi|$

Density operator

- $\rho \equiv \sum_j P_j |\psi_j\rangle\langle\psi_j|$
- Hermitian: $\rho^\dagger = \rho$
- Normalised: $\mathrm{Tr}(\rho) = 1$
- Positive Semi-Definite: $\langle\psi|\rho|\psi\rangle \geq 0, \ \forall \ |\psi\rangle \in \mathbf{H}$
- $purity = \mathrm{Tr}(\rho^2)$
  - $\frac{1}{d} \leq \mathrm{Tr}(\rho^2) \leq 1$
  - Pure: $\mathrm{Tr}(\rho^2) = 1$
  - Maximally mixed: $\mathrm{Tr}(\rho^2) = \frac{1}{d}$
- $\rho_A = Tr_B(\rho_{AB})$
- $\langle A\rangle = \mathrm{Tr}(\rho A)$

Pauli Operators

- $\sigma_x = X = |0\rangle\langle 1| + |1\rangle\langle 0| \doteq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\sigma_y = Y = i|1\rangle\langle 0| - i|0\rangle\langle 1| \doteq \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
- $\sigma_z = Z = |0\rangle\langle 0| - |1\rangle\langle 1| \doteq \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- $I = |0\rangle\langle 0| + |1\rangle\langle 1| \doteq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (Sometimes included)
- $Trx = \mathrm{Tr}(Y) = \mathrm{Tr}(Z) = 0$
- With respect to Hilbert-Schmidt Inner Product:
  $||X|| = ||Y|| = ||Z|| = ||I|| = \sqrt{2}$

Properties

- Unitary
- Hermitian
- $\lambda = \pm 1$

Photon Annihilation and Creation Operators

- $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
- $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$
- $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$
- $\langle\alpha|\hat{a}^\dagger = \alpha^*\langle\alpha|$

Atomic Energy Level Operators (for a two-level approximation)

- $\hat{\sigma}_+ = |e\rangle\langle g|$
- $\hat{\sigma}_- = |g\rangle\langle e|$
- $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$
- $\hat{\sigma}_+|g\rangle = |e\rangle$
- $\hat{\sigma}_-|e\rangle = |g\rangle$
- $\hat{\sigma}_+|e\rangle = 0$
- $\hat{\sigma}_-|g\rangle = 0$

Chapter 2

Astrophysics & Astronomy

2.1 Astrometry

2.1.1 Redshift

- $$\frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \approx \frac{v}{c}$$
- $$1 + z = \frac{\lambda_{obs}}{\lambda_{emit}}$$

2.1.2 Apparent magnitude

- $$m - m_0 = -2.5 \log_{10}(\frac{F}{F_0})$$

2.1.3 Absolute magnitude

- $$M = m - 5 \log_{10}(\frac{d}{10})$$

2.1.4 Relative magnitudes

- $$\frac{I_a}{I_b} = 100^{\frac{(m_b - m_a)}{5}}$$

2.1.5 Flux-magnitude relationship

- $$F = F_0 \times 10^{-0.4m}$$

2.1.6 Color

- $$-2.5 \log(\frac{F_{f1}}{F_{f2}})$$

2.1.7 Metallicity

- $$Z = \log_{10}(\frac{(Fe/H)_*}{(Fe/H)_\odot}) = \log_{10}(Fe/H)_* - log_{10}(Fe/H)_\odot$$

2.2 Stars

2.2.1 Stellar Structure Equations

Hydrostatic Equilibrium

- $$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{-GM_r\rho(r)}{r^2}$$

Mass Conservation

- $$\frac{M_r}{r} = 4\pi r^2 \rho$$

Energy Equation

- $$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2 \rho \varepsilon$$

Radiative Transport

- $$\frac{\mathrm{d}T}{\mathrm{d}r}|_{rad} = \frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

2.2.2 Timescales

Thermal / Kelvin-Helmholtz Timescale

- $$\tau_{KH} = \frac{|U_*|}{L_*} = \frac{GM_*^2}{R_* L_*}$$
- $$\tau_{KH\odot} \approx 50 millionyears$$

Nuclear Timescale / Main Sequence Lifespan

- $$\tau_N \approx \tau_\odot M^{-3} \approx 10^9 (\frac{M}{M_\odot})^{-3}$$

2.2.3 Gravitational potential energy

- $$U_* \approx \frac{-GM^2}{R}$$

2.2.4 Eddington Limit (hydrostatic equilibrium)

- $$L_{edd} = \frac{4\pi G M m_p c}{\sigma T} \approx 3.2 \times 10^4 (\frac{M}{M_\odot})\ [L_\odot]$$
- $$M_{edd} = 3.1 * 10^{-5} (\frac{L}{L_\odot})\ [M_\odot]$$

Eddington Rate (mass loss)

- $$\dot{M}_{edd} = \frac{L_{edd}}{\eta c^2} \approx 2.4 \times 10^{-8} (\frac{M}{M_\odot})\ [M_\odot / yr]$$

2.2.5 Mass-Luminosity Relationship

- $$\frac{L}{L_\odot} \approx b(\frac{M}{M_\odot})^a \ ; \qquad a, b = \begin{cases} 2.3,\ 0.23 & M < 0.43 M_\odot \\ 4,\ 1 & 0.43 M_\odot < M < 2 M_\odot \\ 3.5,\ 1.5 & 2 M_\odot < M < 20 M_\odot \\ 1,\ 32000 & M > 55 M_\odot \end{cases}$$

2.3 Galaxies

2.3.1 Hubble Eliptical Galaxy Classification

- $$10 \times (\frac{a-b}{a})$$

2.3.2 Sérsic Profile

- $$I(R) = I_0 \exp\{-b[(\frac{R}{R_e})^{\frac{1}{n}} - 1]\}$$

2.3.3 Density of stars in the Milky Way Galaxy

- $\rho(R, z) = \rho_0 e^{-z/z_0} e^{-R/h}$

2.4 Black Holes

2.4.1 Schwarszchild Radius

- $$r_S = \frac{2GM}{c^2}$$

2.5 Instrumentation

2.5.1 Lensmaker's equation

- $$\begin{aligned} \frac{1}{f} &= (n-1)[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}] \\ &\approx (n-1)[\frac{1}{R_1} - \frac{1}{R_2}] \qquad \text{(Thin lens approximation)} \end{aligned}$$

2.5.2 Focal ratio / Focal number

- $$N = \frac{f}{D}$$

2.5.3 Field of view

- $$FOV = \frac{w_D}{D_T N} = \frac{w_D}{f_{sys}}$$

2.5.4 Resolution Limits

Diffraction limit (Rayleigh criterion)

- $$\varepsilon_d = 1.22\frac{\lambda}{D}$$

Seeing limit (Rayleigh criterion)

- $$\varepsilon_s = 0.98\frac{\lambda}{r_0}$$

Total Resolution limit

- $$\varepsilon\sqrt{\varepsilon_d^2 + \varepsilon_s^2}$$

2.5.5 Nyquist sampling

- $$\frac{2p}{f_{sys}} = \frac{\lambda}{D_T} \qquad \text{(When diffraction limited)}$$
- $$N = \frac{2p}{\lambda}$$

2.5.6 Plate scale

- $$\frac{1}{f}[rad/m] = \frac{206265}{f}[arcsec/m]$$

2.5.7 Fitting error

- $$\sigma_{fit}^2 = a_f(\frac{d_{sub}}{r_0})^{\frac{5}{3}} = 0.26(\frac{d_{sub}}{r_0})^{\frac{5}{3}}$$

2.5.8 Adaptive optics error

- $\sigma_t otal^2 = 0.3(\frac{d_{sub}}{r_0})^{\frac{5}{3}} + (\frac{\theta}{\theta_0})^{\frac{5}{3}} + 28.4(\frac{\tau}{\tau_0})^{\frac{5}{3}} + C_{WFS}(\frac{\lambda}{F\tau d_{sub}})^2$

2.5.9 Signal-to-noise ratio

- $SNR = \frac{Ft}{\sqrt{Ft + (B_s n_p t) + (D n_p t) + (R^2 n_p)}}$

2.5.10 Atmospheric Extinction

- $m_\lambda = m_{\lambda,z} - a_\lambda(\sec z)$

2.5.11 Rocket science

Tsiolkovsky rockey equation

- $\Delta v = v_e \ln(\frac{m_0}{m_f})$

Chapter 3

Mathematics

3.1 Notation

- $[f(x)]_b^a = f(a) - f(b)$

3.2 Algebra

3.2.1 Factorisation

- $(a + b)^2 = a^2 + b^2 + 2ab^2$
- $(a - b)^2 = a^2 + b^2 - 2ab^2$
- $a^2 - b^2 = (a + b)(a - b)$
- $(a + b)(a + c) = a^2 + (b + c)a + bc$
- $(a + b)^3 = a^3 + 3ab^2 + 3a^2b + b^3$
- $(a - b)^3 = a^3 + 3ab^2 - 3a^2b - b^3$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $a^{2n} - b^{2n} = (a^n - b^n)(a^n + b^n)$

3.2.2 Absolute Value

- $|ab| = |a||b|$
- $|\frac{a}{b}| = \frac{|a|}{|b|}$
- $|a + b| \leq |a| + |b|$

3.2.3 Quadratics

Quadratic Formula

For $ax^2 + bx + c = 0$, $a \neq 0$:

- $\mathrm{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $b^2 - 4ac > 0$ - two real unequal solutions.
- $b^2 - 4ac = 0$ - repeated real solution.
- $b^2 - 4ac < 0$ - two complex solutions.

3.2.4 Logarithms

- $y = \log_b(x);\ x = b^y$
- $\log_b(xy) = \log_b x + \log_b(y)$
- $\log_b(\frac{x}{y}) = \log_b x - \log_b(y)$
- $\log_b(x^p) = p \log_b x$
- $\log_b(b^x) = x$
- $\log_b(a) = \frac{\log_d(a)}{\log_d(b)}$
- $\log_b(\sqrt[p]{x}) = \log$
- $p \log_b x + q \log_b(y) = \log(x^p y^q)$
- $b^{\log_b x} = x$
- $\log_b(b) = 1$
- $\log_b(1) = 0$

3.2.5 Vectors

Dot Product

- $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \ldots + a_n b_n = ab \cos\theta$

Cross Product

- $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\hat{i}\ + (a_3 b_1 - a_1 b_3)\hat{j} + (a_1 b_2 - a_2 b_1)\hat{k} = ab \sin\theta \hat{n}$
- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $\vec{a} \times \vec{b} = 0$

3.2.6 Factorials

- $n! = n(n-1)(n-2)...(2)(1)$
- $(n+1)! = (n+1)n!$

Stirling's approximation

- $n! \approx n \ln(n) - n + O(\ln(n))$

The Factorial Integral

- $\int\limits_0^\infty x^n e^{-x}\ \mathrm{d}x$

3.2.7 Inner product definition

1. Linear in first variable:

   $$(\alpha a + \beta b, c) = \alpha(a, c) + \beta(b, c)$$

2. Positive-definite:

   $$(a, a) \geq 0, (a, a) = 0 \iff a = 0$$

3. Conjugate symmetrical:

   $$(a, b) = (b, a)^*$$

   $$(a, b) = (b, a),\ b, a \in \mathbf{R}$$

3.2.8 Complex Numbers

- $$z = a + ib = \Re(z) + \Im(z)i$$

Euler's Formula

- $$e^{i\theta} = \cos\theta + i\sin\theta$$
- $$re^{i\theta} = |z|e^{i\theta} = r(\cos\theta + i\sin\theta)$$

De Moivre's Formula

- $$(\cos x + i\sin x)^n = \cos(nx) + i\sin(nx)$$

Complex Modulus

- $$r = |z| = |a + ib| = \sqrt{a^2 + b^2} = \sqrt{\Re^2(z) + \Im^2(z)}$$

Complex Conjugate

- $$\bar{z} = (a \bar{+} ib) = a - ib$$
- $$(a \bar{+} ib)(a + ib) = |a + ib|^2$$

Complex Argument

- $$\theta = \arg(z) = \arctan(\frac{a}{b}) = \arctan(\frac{\Im(z)}{\Re(z)})$$

3.2.9 Power Series

- $$f(x) = \sum_{n=0}^{\infty} f_n x^n$$

Notable Series

- $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$
- $$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ...$$
- $$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ...$$

3.2.10 Matrix Operations

Determinant

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Transpose

- $a^T_{jk} = a_{kj}$
- $(AB)^T = B^T A^T$
- Linear: $(A + B)^T = A^T + B^T$
- $(rA)^T = rA^T$

Hermitian Adjoint

- $A^\dagger \equiv (A^*)^T = (A^T)^*$
- $(AB)^\dagger = B^\dagger A^\dagger$
- $(A + B)^\dagger = A^\dagger + B^\dagger$

Trace

- $\text{Tr}(A) = \sum\limits_j^n a_{jj}$
- Cyclic: $\text{Tr}(AB) = \text{Tr}(AB)$
- Linear: $\text{Tr}(A + B) = \text{Tr}(A) + \text{Tr}(B)$
  $\text{Tr}(aB) = a\text{Tr}(B), a \in \mathbf{C}$
- $\text{Tr}(SAS^{-1}) = \text{Tr}(A)$

Hilbert-Schmidt Inner Product

- $(A, B) \equiv \text{Tr}(A^\dagger B)$

Rank-Nullity Theorem

- $\text{rk}(A) = \dim(\ker(A)) + \dim(\text{im}(A))$

3.2.11 Matrix Types

Real Matrix

- $a_{jk} \in \mathbf{R}$

Square Matrix

- $m = n$

Symmetric Matrix

- $A = A^T$
- $a_{jk} = a_{kj}$
- Square

Normal Matrix

- $A^\dagger A = AA^\dagger$
- Square
- Diagonalisable

Diagonal Matrix

- $a_{jk} = 0, j \neq k$
- $\lambda_j = a_{jj}$
- Square
- $e^D = \begin{bmatrix} e^{d_{11}} & 0 & \cdots & 0 \\ 0 & e^{d_{11}} & \cdots & 0 \end{bmatrix}$

Diagonalisable Matrix

- $A = PDP^{-1}$
- Square

Identity Matrix

- $IA = A$
- $i_{jj} = 1$
- $i_{jk} = 0, j \neq k$
- Real
- Square
- Diagonal
- Symmetric
- Hermitian

Hermitian Matrix

- $H = H^\dagger$
- $h_{jk} = h^*_{kj}$
- $h_{jj} \in \mathbf{R}$
- $\lambda \in \mathbf{R}$
- Square
- Normal
- All real, square matrices are Hermitian

Anti-Hermitian Matrix

- $H = -H^\dagger$
- $H_{jk} = -H^*_{kj}$
- Square

Orthogonal Matrix

- $A^T = A^{-1}$
- $AA^T = I$
- $(AA^T)_{jk} = \delta_{jk}$

Positive Semidefinite

- $A \geq 0$
- $\hat{A}^\dagger = \hat{A}, \hat{A} \geq 0$
- $B = \hat{A}^\dagger \hat{A}$ is positive semidefinite for any linear operator $\hat{A}$
- Positive semidefinite matrices are Hermitian

Projector

- $\hat{P}^2 = \hat{P}$
- $\lambda = 1 \; or \; 0$
- $P_1 P_2 \mapsto \mathbf{H}_1 \cap \mathbf{H}_2$
- Projectors are Hermitian

Real Matrix

- $A = A^*$
- $A_{jk} = A^*_{jk}$

Imaginary Matrix

- $A = -A^*$
- $A_{jk} = -A^*_{jk}$

Symmetric Matrix

- $A = A^T$
- $A_{jk} = A_{kj}$
- Square

Antisymmetric Matrix

- $A = -A^T$
- $a_{jk} = a_{kj}$
- $a_{jj} = 0$
- Square

Unitary Matrix

- $U^\dagger U = UU^\dagger = I$
- $U^\dagger = U^{-1}$
- $(U^\dagger U)_{jk} = \delta_{jk}$
- Square
- Normal
- Hermitian

3.2.12 Change of Basis Unitary

- Basis a is (?)
- $(V)_b = [U^\dagger]_a (V)_a$
- $[U]_a = [(b_0)_a (b_1)_a ... (b_n)_a]$

3.2.13 Commutator

- $[A, B] = AB - BA$
- $[A, A] = 0$
- $[A + B, C] = [A, C] + [B, C]$
- $[A, BC] = [A, B]C + B[A, C]$

3.2.14 Anticommutator

- $\{A, B\} = AB + BA$

3.2.15 Cauchy-Schwarz Inequality

- $|\langle \vec{u}, \vec{v} \rangle|^2 \leq \langle \vec{u}, \vec{u} \rangle \cdot \langle \vec{v}, \vec{v} \rangle$

3.3 Geometry

3.3.1 Pythagorean theorem

- $a^2 + b^2 = c^2$
- $a = \sqrt{b^2 + c^2}$

Higher dimensions

- $r = \sqrt{x^2 + y^2 + z^2}$
- $r = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} = \sqrt{\vec{r} \cdot \vec{r}}$

Distance between two points

In two dimensions

- $d_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

In higher dimensions

- $d_{ab} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + \cdots + (b_n - a_n)^2}$

3.3.2 Properties of shapes

	Area	**Circumference**
Circle	πR^2	$2\pi R$
Square	L^2	$4L$

3.3.3 Properties of solids

	Surface Area	**Volume**
Sphere	$4\pi R^2$	$\frac{4}{3}\pi R^3$

3.3.4 Circular formulae

Arc length

- $l = R\theta$

Area of a sector

- $A = \frac{R^2\theta}{2}$

Area of a segment

- $A = \frac{R^2}{2}(\theta - \sin\theta)$

3.3.5 Useful Functions

Parabola

- $f(x) = a(x-h)^2 + k$
- Vertex at (h, k)
- Up-concave if $a > 0$; down-concave if $a < 0$

- $f(x) = ax^2 + bx + c$
- Vertex at $(-\frac{b}{2a}, f(-\frac{b}{2a}))$
- Up-concave if $a > 0$; down-concave if $a < 0$

Hyperbola

- $(\frac{x-h}{a})^2 - (\frac{y-k}{b})^2 = 1$
- Centre at (h, k)
- Asymptotes through centre, slope $\pm\frac{b}{a}$

Circle

- $(x-h)^2 + (y-k)^2 = R^2$
- Centre at (h, k)

Ellipse

- $1 = (\frac{x-h}{a})^2 + (\frac{y-k}{b})^2$
- Centre at (h, k)
- Vertices a units right/left from the centre and vertices b units up/down from the center.

Sphere

- $R^2 = (x-h)^2 + (y-k)^2 + (z-l)^2$
- Centre at (h, k, l):

Ball

- $R^2 < (x-h)^2 + (y-k)^2 + (z-l)^2$
- Centre at (h, k, l):

3.3.6 Coordinates

Transformations to Cartesian coordinates

Cartesian	$x = x$	$y = y$	$z = z$	$dV = dx\ dy\ dz$
Polar (2D)	$x = r\cos\phi$	$y = r\sin\phi$	N/A	$dA = r\ drd\phi$
Cylindrical	$x = r\cos\phi$	$y = r\sin\phi$	$z = z$	$dV = r\ dr\ d\theta\ dz$
Spherical	$x = r\sin\theta\cos\phi$	$y = r\sin\theta\sin\phi$	$z = r\cos\theta$	$dV = r^2\sin\theta\ dr\ d\theta\ d\phi$

Transformations from Cartesian coordinates

Cartesian	$x = x$	$y = y$	$z = z$
Polar (2D)	$r = \sqrt{x^2+y^2}$	$\phi' = \arctan\lvert\frac{y}{x}\rvert$ (ϕ depends on quadrant)	N/A
Cylindrical	$r = \sqrt{x^2+y^2}$	$\phi' = \arctan\lvert\frac{y}{x}\rvert$ (ϕ depends on quadrant)	$z = z$
Spherical	$r = \sqrt{x^2+y^2+z^2}$	$\phi' = \arctan\lvert\frac{y}{x}\rvert$ (ϕ depends on quadrant)	$\theta = \arccos(\frac{z}{\sqrt{x^2+y^2+z^2}})$

3.3.7 Hyperbolic Functions

Hyberbolic Sine

- $\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x}-1}{2e^x} = \frac{1-e^{-2x}}{2e^{-x}}$
- $\sinh x = -i\sin(ix)$

Hyperbolic Cosine

- $\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x}+1}{2e^x} = \frac{1+e^{-2x}}{2e^{-x}}$
- $\cosh x = \cos(ix)$

Hyperbolic Tangent

- $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1} = \frac{1-e^{-2x}}{1+e^{-2x}}$
- $\tanh x = -i\tan(ix)$

Hyperbolic Cotangent

- $\coth x = \frac{1}{\tanh x} = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x}+1}{e^{2x}-1} = \frac{1+e^{-2x}}{1-e^{-2x}}$
- $\coth x = i\cot(ix)$

Hyperbolic Secant

- $\text{sech}x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}} = \dfrac{2e^x}{e^{2x} + 1} = \dfrac{2e^{-x}}{1 + e^{-2x}}$
- $\text{sech}x = \sec(ix)$

Hyperbolic Cosecant

- $\text{csch}x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}} = \dfrac{2e^x}{e^{2x} - 1} = \dfrac{2e^{-x}}{1 - e^{-2x}}$
- $\text{csch}x = i\csc(ix)$

Identities

- $\cosh^2 x - \sinh^2 x = 1$
- $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

3.4 Trigonometry

Definitions

- SOH CAH TOA
- $\sin\theta = \dfrac{opposite}{hypotenuse}$
- $\cos\theta = \dfrac{adjacent}{hypotenuse}$
- $\tan\theta = \dfrac{opposite}{adjacent} = \dfrac{\sin\theta}{\cos\theta}$
- $\cot\theta = \dfrac{adjacent}{opposite} = \dfrac{\cos\theta}{\sin\theta}$
- $\sec\theta = \dfrac{hypotenuse}{adjacent}$
- $\csc\theta = \dfrac{hypotenuse}{opposite}$

3.4.1 Identities

Pythagorean Identities

- $\cos^2\theta + \sin^2\theta = 1$
- $\tan^2\theta + 1\sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Reciprocals

- $\sin\theta = \dfrac{1}{\csc\theta}$
- $\cos\theta = \dfrac{1}{\sec\theta}$
- $\tan\theta = \dfrac{1}{\cot\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta}$

- $$\sec\theta = \frac{1}{\cos\theta}$$
- $$\csc\theta = \frac{1}{\sin\theta}$$

As complex exponentials

- $$\sin\theta = \frac{e^{ix} - e^{-ix}}{2i}$$
- $$\cos\theta = \frac{e^{ix} + e^{-ix}}{2}$$
- $$\tan\theta = \frac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$$
- $$\cot\theta = \frac{i(e^{ix} + e^{-ix})}{e^{ix} - e^{-ix}}$$
- $$\sec\theta = \frac{2}{e^{ix} + e^{-ix}}$$
- $$\csc\theta = \frac{2i}{e^{ix} - e^{-ix}}$$

Symmetries

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\csc(-\theta) = -\csc\theta$
- $\sec(-\theta) = \sec\theta$
- $\cot(-\theta) = -\cot\theta$

- $\sin(\frac{\pi}{2} - \theta) = \cos\theta$
- $\cos(\frac{\pi}{2} - \theta) = \sin\theta$
- $\tan(\frac{\pi}{2} - \theta) = \cot\theta$
- $\csc(\frac{\pi}{2} - \theta) = \sec\theta$
- $\sec(\frac{\pi}{2} - \theta) = \csc\theta$
- $\cot(\frac{\pi}{2} - \theta) = \tan\theta$

- $\sin(\pi - \theta) = \sin\theta$
- $\cos(\pi - \theta) = -\cos\theta$
- $\tan(\pi - \theta) = -\tan\theta$
- $\csc(\pi - \theta) = \csc\theta$
- $\sec(\pi - \theta) = -\sec\theta$
- $\cot(\pi - \theta) = -\cot\theta$

Angle sum and difference formulae

- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
- $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Half-angle formulae

- $\sin^2(\dfrac{\theta}{2}) = \dfrac{1-\cos\theta}{2}$
- $\cos^2(\dfrac{\theta}{2}) = \dfrac{1+\cos\theta}{2}$
- $\tan^2(\dfrac{\theta}{2}) = \dfrac{1-\cos\theta}{1+\cos\theta}$
- $\tan(\dfrac{\theta}{2}) = \dfrac{\tan\theta}{1+\sec\theta} = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

Double-angle formulae

- $\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Sum to Product

- $\sin\alpha + \sin\beta = 2\sin(\dfrac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$
- $\sin\alpha - \sin\beta = 2\cos(\dfrac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$
- $\cos\alpha + \cos\beta = 2\cos(\dfrac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$
- $\cos\alpha - \cos\beta = -2\sin(\dfrac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$

Product to Sum

- $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
- $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
- $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \cos(\alpha-\beta)]$
- $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Law of Sines

- $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$

Law of Cosines

- $a^2 = b^2 + c^2 - 2bc\cos\alpha$
- $b^2 = a^2 + c^2 - 2ac\cos\beta$
- $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Law of Tangents

- $$\frac{a-b}{a+b} = \frac{\tan(\frac{1}{2}[\alpha-\beta])}{\tan(\frac{1}{2}[\alpha+\beta])}$$
- $$\frac{b-c}{b+c} = \frac{\tan(\frac{1}{2}[\beta-\gamma])}{\tan(\frac{1}{2}[\beta+\gamma])}$$
- $$\frac{a-c}{a+c} = \frac{\tan(\frac{1}{2}[\alpha-\gamma])}{\tan(\frac{1}{2}[\alpha+\gamma])}$$

Mollweide's Formula

- $$\frac{a+b}{c} = \frac{\cos(\frac{1}{2}[\alpha-\beta])}{\sin(\frac{1}{2}\gamma)}$$

Small-angle approximations

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \frac{\theta^2}{2}$
- $\tan\theta \approx \theta$

Other identities

- $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$
- $\cos^2\theta = \frac{1}{2}(\cos(2\theta)+1)$

Averages

- $\overline{\sin}\, x = \overline{\cos}\, x = 0$
- $\overline{\sin^2}\, x = \overline{\cos^2}\, x = \frac{1}{2}$

Table of Identities

In terms of...	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\cot\theta$	$\csc\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\csc}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\pm\frac{\sqrt{\csc^2 -1}}{\csc\theta}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2 -1}}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\pm\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$	$\pm\sqrt{\csc^2\theta-1}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$	$\csc$

3.5 Calculus

3.5.1 Limits

3.5.2 Properties

- $$\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$$
- $$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$
- $$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$
- $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \qquad \lim_{x \to a} g(x) \neq 0$$
- $$\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$$

Useful Limits

- $$\lim_{x \to \infty} e^x = \infty$$
- $$\lim_{x \to -\infty} e^x = 0$$
- $$\lim_{x \to \infty} \ln(x) = \infty$$
- $$\lim_{x \to 0^-} \ln(x) = -\infty$$
- $$\lim_{x \to 0} x \log x = 0$$

L'Hôpital's rule

- $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Squeeze principle

- For $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$:
  $$\lim_{x \to a} f(x) = L$$

3.5.3 Differentiation

First Principles

- $$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Nature of derivatives

Derivative	**Function**
$f'x > 0$	Increasing
$f'x = 0$	Stationary
$f'x < 0$	Decreasing

Second Derivative	**Function**	**Stationary Points** $[f'x = 0]$
$f''x > 0$	Concave up	Local Minimum
$f'x = 0$	No information	Inflection Point
$f''x < 0$	Concave down	Local Maximum

Product Rule

- $$(uv)' = uv' + vu'$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} f(x)gx = f(x)g'(x) + f'(x)g(x)$$

Quotient Rule

- $$(\frac{u}{v})' = \frac{vu' - uv'}{v^2}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Chain Rule

- $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \frac{\mathrm{d}u}{\mathrm{d}x}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = f'(g(x))g'(x)$$

Useful Derivatives

- $$\frac{\mathrm{d}}{\mathrm{d}x} x^n = nx^{n-1}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} a^x = a^x \ln(a)$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{x},\ x > 0$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \ln |x| = \frac{1}{x},\ x \neq 0$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \ln(f(x)) = \frac{f'x}{f(x)}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \log_b x = \frac{1}{x \ln(b)},\ x > 0$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \cos x = -\sin x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \tan x = \sec^2 x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \sec x = \sec x \tan x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \csc x = -\csc x \cot x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \cot x = -\csc^2 x$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \cos^{-1} x = \frac{1}{-\sqrt{1 - x^2}}$$
- $$\frac{\mathrm{d}}{\mathrm{d}x} \tan^{-1} x = \frac{1}{-\sqrt{1 + x^2}}$$

3.5.4 Partial Differentiation

First Principles

- $$\frac{\partial}{\partial x} f(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

Jacobian Matrix

- $$D\vec{f}(\vec{a}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\vec{a}) & \frac{\partial f_1}{\partial x_2}(\vec{a}) & \cdots & \frac{\partial f_1}{\partial x_n}(\vec{a}) \\ \frac{\partial f_2}{\partial x_1}(\vec{a}) & \frac{\partial f_2}{\partial x_2}(\vec{a}) & \cdots & \frac{\partial f_2}{\partial x_n}(\vec{a}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1}(\vec{a}) & \frac{\partial f_m}{\partial x_2}(\vec{a}) & \cdots & \frac{\partial f_m}{\partial x_n}(\vec{a}) \end{bmatrix}$$

Definition of differentiability of a multivariable function

- $$\lim_{\vec{x}\to\vec{a}} = \frac{||f(\vec{x}) - f(\vec{a}) - Df(\vec{a})\cdot(\vec{x}-\vec{a})||}{||\vec{x}-\vec{a}||} = 0$$

3.5.5 The Differential

- $$\mathrm{d}F = \frac{\partial F}{\partial x}\ \mathrm{d}x + \frac{\partial F}{\partial y}\ \mathrm{d}y + \frac{\partial F}{\partial z}\ \mathrm{d}z$$

3.5.6 Line Element

- $\mathrm{d}S^2 = \ \mathrm{d}x^2 + \ \mathrm{d}y^2 + \ \mathrm{d}z^2$

3.5.7 Integration

Properties

- $\int f(x) \pm g(x)\ \mathrm{d}x = (\int f(x)\ \mathrm{d}x \int \pm g(x)\ \mathrm{d}x$
- $\int_a^a f(x)\ \mathrm{d}x = 0$
- $\int_a^b f(x)\ \mathrm{d}x = -\int_b^a f(x)\ \mathrm{d}x$
- $|\int_a^b f(x)\ \mathrm{d}x| \leq \int_a^b |f(x)|\ \mathrm{d}x$
- If $f(x) \geq g(x)$ over $[a,b]$, $\int_a^b f(x)\ \mathrm{d}x \geq \int_b^a g(x)\ \mathrm{d}x$
- If $f(x) \geq 0$ over $[a,b]$, $\int_a^b f(x)\ \mathrm{d}x > 0$
- If $m \leq f(x) \leq M$ over $[a,b]$, $m(b-a) \leq \int_a^b f(x)\ \mathrm{d}x \leq M(b-a)$

Integration by Parts

- $\int u'v = uv - \int uv'$
- $\int f'(x)g(x)\ \mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\ \mathrm{d}x$
- $\int_a^b f'(x)g(x)\ \mathrm{d}x = [f(x)g(x)]_a^b - \int_a^b f(x)g'(x)\ \mathrm{d}x$

Integration by Substitution

- $u = g(x);\ \ \mathrm{d}x = g'(x)\ \mathrm{d}x;\ \int_a^b f(g(x))g'(x)\ \mathrm{d}x = \int_{g(a)}^{g(b)} f(x)\ \mathrm{d}x$

Approximations

Trapezoid rule

- $\int_a^b f(x)\ \mathrm{d}x \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$

Simpson's rule

- $\int_a^b f(x)\ \mathrm{d}x \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

Useful Indefinite Integrals

- $\int k\ \mathrm{d}x = kx + C$
- $\int \log_b x\ \mathrm{d}x = x(\log_b x - \log_b(e)) + C = x(\log_b x - \frac{1}{\ln b}) + C$
- $\int \ln x\ \mathrm{d}x = x(\ln x - 1) + C$
- $\int e^x\ \mathrm{d}x = e^x + C$
- $\int x^n\ \mathrm{d}x = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
- $\int \frac{1}{x}\ \mathrm{d}x = \ln|x| + C$
- $\int \frac{1}{ax+b}\ \mathrm{d}x = \frac{1}{a}\ln|ax+b| + C$
- $\int \sin x\ \mathrm{d}x = -\cos x + C$
- $\int \cos x\ \mathrm{d}x = \sin x + C$
- $\int \tan x\ \mathrm{d}x = \ln|\sec x| + C$
- $\int \sec x\ \mathrm{d}x = \ln|\sec x + \tan x| + C$
- $\int \sec^2 x\ \mathrm{d}x = \tan x + C$
- $\int \csc^2 x\ \mathrm{d}x = -\cot x + C$
- $\int \sec x \tan x\ \mathrm{d}x = \sec x + C$
- $\int \csc x \cot x\ \mathrm{d}x = -\csc x + C$
- $\int \frac{1}{a^2+x^2}\ \mathrm{d}x = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$
- $\int \frac{1}{\sqrt{a^2+x^2}}\ \mathrm{d}x = \sin^{-1}(\frac{x}{a}) + C$

Useful Definite Integrals

- $$\int_0^\infty \frac{x^3}{e^x - 1}\ \mathrm{d}x = \frac{\pi^4}{15}$$
- $$\int_0^\infty x^n e^{-ax}\ \mathrm{d}x = \frac{n!}{a^{n+1}}$$

3.5.8 Vector Calculus

Vector derivative

- $\mathrm{grad}(f) = \vec{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$
- $$\int_a^b (\vec{\nabla} f) \cdot\ \mathrm{d}\vec{l} = f(a) - f(b)$$

The Laplacian

- $\delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Divergence

In Cartesian Coordinates

- $$\mathrm{div}(f) = \vec{\nabla} \cdot \vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

In Cylindrical Coordinates

- $$\vec{\nabla} \cdot \vec{f} = \frac{1}{r}\frac{\partial}{\partial r}(r f_r) + \frac{1}{r}\frac{\partial f_\phi}{\partial \phi} + \frac{\partial f_z}{\partial z}$$

In Spherical Coordinates

- $$\vec{\nabla} \cdot \vec{f} = \frac{1}{r}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(f_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial f_\phi}{\partial \phi}$$

Curl

In Cartesian Coordinates

- $$\mathrm{curl}(f) = \vec{\nabla} \times \vec{f} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} = (\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z})\hat{x} + (\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial z})\hat{y} + (\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y})\hat{z}$$

In Cylindrical Coordinates

- $$\vec{\nabla} \times \vec{f} = (\frac{1}{r}\frac{\partial f_z}{\partial \phi} - \frac{\partial f_\phi}{\partial z})\hat{r} + (\frac{\partial f_r}{\partial z} - \frac{\partial f_z}{\partial r})\hat{\phi} + \frac{1}{r}(\frac{\partial}{\partial r}(r f_\phi) - \frac{\partial f_r}{\partial \phi})\hat{z}$$

In Spherical Coordinates

- $$\vec{\nabla} \times \vec{f} = \frac{1}{r\sin\theta}(\frac{\partial}{\partial \theta}(f_\phi \sin\theta) - \frac{\partial f_\theta}{\partial \phi})\hat{r} + \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial f_r}{\partial \phi} - \frac{\partial}{\partial r}(r f_\phi)) + \frac{1}{r}(\frac{\partial}{\partial r}(r f_\theta) - \frac{\partial f_r}{\partial \theta})$$

Vector Second Derivatives

- $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0$
- $\vec{\nabla}(\vec{\nabla}(\vec{\nabla} \cdot \vec{f}))$

Vector Laplacian

- $\vec{\nabla}^2 \vec{f} = \vec{\nabla}(\vec{\nabla} \cdot \vec{f}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{f})$

Stokes' Theorem

- $\iint_S (\vec{\nabla} \times \vec{f}) \cdot d\vec{a} = \oint_B \vec{v} \cdot \, \mathrm{d}\vec{l}$

Divergence Theorem

- $\iiint_V (\vec{\nabla} \cdot \vec{f}) dV = \oint_S \vec{v} \cdot \, \mathrm{d}\vec{a}$

3.5.9 Dirac Delta Function

- $\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$
- $\delta(x-a) = \begin{cases} 0 & x \neq a \\ \infty & x = a \end{cases}$
- $\int\limits_{-\infty}^{\infty} \delta(x) \, \mathrm{d}x = 1$
- $f(x)\delta(x) = f(0)\delta(x)$

3.5.10 Approximations

- $f(x + \Delta x) \approx f(x) + \Delta x f'(x)$

Chapter 4

Statistics

4.1 Variance

- $$var(x) = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

4.2 Standard Deviation

- $$\sigma x = \sqrt{\text{var(x)}} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Appendix A

Values

A.1 Physics

A.1.1 Physical Constants

c: Speed of light

- $= \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad = 299\ 792\ 458\ m \cdot s^{-1} \quad \approx 3 \times 10^8\ m \cdot s^{-1}$

G: Universal gravitational constant

- $= 6.67408(31) \times 10^{-11}\ m^3 \cdot kg^{-1} \cdot s^{-2} \quad \approx 6.67 \times 10^{-11}\ m^3 \cdot kg^{-1} \cdot s^{-2}$

g: Average acceleration due to gravity at sea level on Earth

- $= 9.80665\ m \cdot s^{-2} \quad \approx 9.8\ m \cdot s^{-2}$

h: Planck constant

- $= 6.626\ 070\ 040 \times 10^{-34} J \cdot s \quad \approx 6.626 \times 10^{-34} J \cdot s$

$\hbar$: Reduced Planck constant

- $= \frac{h}{2\pi} = 1.054\ 571\ 726 \times 10^{-34} J \cdot s \quad \approx 1.055 \times 10^{-34} J \cdot s$

k_B: Boltzmann constant

- $= 1.3\ 806\ 488 \times 10^{-23}\ J \cdot K^{-1} \quad \approx 1.38 \times 10^{-23}\ J \cdot K^{-1}$

k_e: Coulomb's constant

- $= \frac{1}{4\pi\varepsilon_0} \quad = 8.987\ 551\ 787 \times 10^9\ N \cdot m \cdot C^{-2} \quad \approx 9 \times 10^9\ N \cdot m \cdot C^{-2}$

N_A: Avogadro constant

- $= 6.022\ 140\ 857(74) \times 10^{23}\ mol^{-1} \quad \approx 6.022 \times 10^{23}\ mol^{-1}$

ε_0: Vacuum permittivity

- $= \frac{1}{\mu_0 c^2} \quad = 8.854\ 187\ 817 \times 10^{-12}\ F \cdot m^{-1} \quad \approx 8.85 \times 10^{-6}\ F \cdot m^{-1}$

μ_0: Vacuum permeability

- $= 4\pi \times 10^{-7}\ N \cdot A^{-2} \quad = \frac{1}{\varepsilon_0 c^2} \quad \approx 1.257 \times 10^{-6}\ N \cdot A^{-2}$

A.1.2 Useful Quantities

Density of air (ρ_A): $1.2922\ kg \cdot m^{-3}$

Density of water (ρ_w): $10^3\ kg \cdot m^{-3}$

Mass of an electron (m_e): $9.10\ 938\ 291 \times 10^{-31}\ kg \quad \approx 9 \times 10^{-31}\ kg$

Mass of a neutron (m_n): $1.674\ 927\ 351 \times 10^{-27}\ kg \quad \approx 1.675 \times 10^{-27}\ kg$

Mass of a proton (m_p): $1.672\ 621\ 777 \times 10^{-27}\ kg \quad \approx 1.672 \times 10^{-27}\ kg$

Speed of sound in air: $343.2\ m \cdot s^{-1}$

Specific heat capacity of water: $4.186\ \times 10^3\ J \cdot kg^{-1} \cdot K^{-1}$

A.2 Astronomy

A.2.1 Useful Quantities

Surface Temperature of the Sun: ($T_\odot$): $= 5778\ K \quad = 5505\ °C$

Planetary Properties

Body	Mass	Average Radius	Semi-major axis	Eccentricity	Orbital period
Mercury ☿	$3.3011 \times 10^{23}\ kg$ $0.055\ M_\oplus$ $1.66 \times 10^{-7}\ M_\odot$ $1.74 \times 10^{-4}\ M_♃$	$2.4397 \times 10^6\ m$ $0.3829\ R_\oplus$	$5.790905 \times 10^{10}\ m$ $0.387098\ AU$	0.205630	0.240856 yr
Venus ♀	$4.8675 \times 10^{24}\ kg$ $0.815\ M_\oplus$ $2.447 \times 10^{-6}\ M_\odot$ $2.56 \times 10^{-3}\ M_♃$	$6.0518 \times 10^6\ m$ $0.9499\ R_\oplus$	$1.08208000 \times 10^{11}\ m$ $0.723332\ AU$	0.006772	0.615198 yr
Earth ⊕	$5.97237 \times 10^{24}\ kg$ $1\ M_\oplus$ $3.003 \times 10^{-6}\ M_\odot$ $2.67 \times 10^{-3}\ M_♃$	$6.3710 \times 10^6\ m$ $1\ R_\oplus$	$1.49598023 \times 10^{11}\ m$ $1.000001\ AU$	0.0167086	1.000017 yr
Mars ♂	$6.4171 \times 10^{23}\ kg$ $0.107\ M_\oplus$ $3.226 \times 10^{-7}\ M_\odot$ $3.38 \times 10^{-4}\ M_♃$	$3.3895 \times 10^6\ m$ $0.53\ R_\oplus$	$2.27939200 \times 10^{11}\ m$ $1.523679\ AU$	0.0934	1.88082 yr
Jupiter ♃	$1.8982 \times 10^{27}\ kg$ $317.8\ M_\oplus$ $9.55 \times 10^{-4}\ M_\odot$ $1\ M_♃$	$6.9911 \times 10^7\ m$ $10.97\ R_\oplus$	7.4052×10^{11} m $5.2044\ AU$	0.0489	11.862 yr
Saturn ♄	$5.6834 \times 10^{26}\ kg$ $95.159\ M_\oplus$ $2.86 \times 10^{-4}\ M_\odot$ $0.299\ M_♃$	$5.8232 \times 10^7\ m$ $9.14\ R_\oplus$	$1.43353 \times 10^{12}\ m$ $9.5826\ AU$	0.0565	29.4571 yr
Uranus ⛢	$8.68 \times 10^{25}\ kg$ $14.536\ M_\oplus$ $4.36 \times 10^{-5}\ M_\odot$ $0.046\ M_♃$	$2.5362 \times 10^7\ m$ $3.98\ R_\oplus$	$2.87504 \times 10^{12}\ m$ $19.2184\ AU$	0.046381	84.0205 yr
Neptune ♆	$1.0243 \times 10^{26}\ kg$ $17.147\ M_\oplus$ $5.15 \times 10^{-5}\ M_\odot$ $0.054\ M_♃$	$2.4622 \times 10^7\ m$ $3.86\ R_\oplus$	$4.5 \times 10^{12}\ m$ $30.11\ AU$	0.009456	164.8 yr

A.3 Mathematics

Euler's number (e): $\sum_{n=0}^{n} \frac{1}{n!} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots$

$= 2.71828182845904523536028747135266249775724709369995\ldots$

$\approx 2.7182 \qquad \approx 2.718$

Pi (π): $\frac{C}{d} = \frac{C}{2r}$

$= 3.14159265358979323846264338327950288419716939937510\ldots$

$\approx 3.14159 \qquad \approx 3.142$

Appendix B

Units of Measurement

B.1 Natural Units

Handy when you're dealing with small things.

Charge: elementary charge (e)

- The electric charge of a proton.
- $= 1.602\ 176\ 565 \times 10^{-19}\ C \qquad \approx 1.6 \times 10^{-19}\ C$

Energy: electron volt (eV)

- The work done to move an electron across one volt of potential.
- $= e \cdot V = 1.602\ 176\ 565 \times 10^{-19}\ J \qquad \approx 1.6 \times 10^{-19}\ J$

B.2 SI System

Universally acknowledged as the best system of units.

B.2.1 Base Units

Amount of Substance: mole (mol)

- The amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12.
- $= 6.022\ 140\ 857 \times 10^{23}$

Electric Current: ampere (A)

- The constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $210^{-7} N \cdot m^{-1}$ of length.
- $= C \cdot s^{-1}$

Force: newton (N)

- $= 0.224809\ lbf$

Length: metre (m)

- The distance traveled by light in vacuum in $\dfrac{1}{299\ 792\ 458}\ s$
- $= 3.2808\ ft$

Luminous intensity: candela (cd)

- The luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 5.4×10^{14} Hz and that has a radiant intensity in that direction of $\dfrac{1}{683}$ watt per steradian.

Mass: kilogram (kg)

- $= 2.205 lb$

Thermodynamic Temperature: kelvin (K)

- $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
- $= T[°C] + 273.15$

Time: second (s)

- The duration of 9 192 631 770 periods of rotation corresponding to the two hyperfine levels of the ground-state of the caesium-133 atom.

B.2.2 Derived Units

Angle: radian (rad)

- A full circle divided by 2π.
- $= m \cdot m^{-1} \quad = \dfrac{180}{\pi} \quad \approx 57.3° \quad = 206265\ arcsecs$

Electric Charge: coulomb (C)

- $= A \cdot s \quad = 6.242 \times 10^{18}\ e$

Electrical capacitance: farad (F)

- $= m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$

Electrical conductance: siemens (S)

- $= A \cdot V^{-1} \quad = kg^{-1} \cdot m^{-2} \cdot s^3 \cdot A^2$

Electrical inductance: henry (H)

- $= Wb \cdot A^{-1} \quad = kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$

Electrical potential difference / Voltage: volt (V)

- $= W \cdot A{-}1 \quad = kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$

Electrical resistance: ohm (Ω)

- $V \cdot A^{-1} \quad = kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$

Energy: joule (J)

- $= N \cdot m \quad = kg \cdot m^2 \cdot s^{-2}$

Force: newton (N)

- $= kg \cdot m \cdot s^{-2}$

Frequency: hertz (Hz)

- $= s^{-1}$

Illuminance: lux (lx)

- $= lm \cdot m^2 \quad = m^{-2} \cdot cd$

Luminous flux: lumen (lm)

- $= cd \cdot sr \qquad = cd$

Magnetic flux: weber (Wb)

- $= V \cdot s \qquad = kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$

Magnetic flux density: tesla (T)

- $= kg \cdot s^{-2} \cdot A^{-1}$

Power: watt (W)

- $= J \cdot s \qquad = kg \cdot m^2 \cdot s^{-3}$

Pressure: pascal (Pa)

- $= N \cdot m^{-2} \qquad = kg \cdot m^{-1} \cdot s^{-2}$

Radioactivity: becquerel (Ω)

- Decays per second
- $= s^{-1}$

Solid angle: steradian (sr)

- $= m^2 \cdot m^{-2}$

Temperature: degree Celcius ($^\circ C$)

- $T[C] = T[K] - 273.15$

B.3 CGS (centimetres-grams-seconds)

Commonly used in astronomy, to everyone's disappointment.

Acceleration: gal (Gal)

- $= cm \cdot s^{-2} \qquad = 10^-2 \ m \cdot s^{-2}$

Energy: erg (erg)

- $= g \cdot cm^2 \cdot s^{-2} \qquad = 10^{-7} \ J$

Force: dyne (dyn)

- $= g \cdot cm \cdot s^{-2} \qquad = 10^{-5} \ N$

Length: centimetre (cm)

- $= 0.01 \ m$

Mass: gram (g)

- $= 10^{-3} \ kg$

Power: erg per second (erg/s)

- $= g \cdot cm^2 \cdot s^{-2} \qquad = 10^{-7} \ W$

Pressure: barye (Ba)

- $= g \cdot cm^{-1} \cdot s^{-2} \qquad = 10^{-1} \ Pa$

Time: second (s)

Velocity: centimetre per second (cm/s)

- $= 10^{-2} \ m \cdot s^{-1}$

Viscosity (dynamic): poise (P)

- $= g \cdot cm^{-1}\ s^{-1} \qquad = 10^{-1}\ Pa \cdot s$

Viscosity (kinematic): stokes (St)

- $= g \cdot cm^{2}\ s^{-1} \qquad = 10^{-4}\ m^{2} \cdot s^{-1}$

Wavenumber: kayser (K)

- $= cm^{-1} \qquad = 100\ m^{-1}$

B.4 Astronomy units

B.4.1 Astronomical system

Distance: astronomical unit (AU)

- Roughly the distance from the Earth to the Sun.
- $= 1.4960 \times 10^{11}\ m \qquad = 4.8481 \times 10^{-6}\ pc \qquad = 1.5813 \times 10^{-5}\ ly$

Mass: solar mass ($M_\odot$)

- $= 1.98855 \times 10^{30}\ kg \approx 2 \times 10^{30}\ kg = 1048\ M_{♃} = 332\ 950\ M_\odot$

Time: Day

- $= 86\ 400 s$

Complimentary units

Distance: Solar radius ($R_\odot$)

- $= 6.957 \times 10^{8}\ m \qquad = 695\ 700\ km \qquad \approx 7 \times 10^{8}\ m$

Distance: parsec (pc)

- The distance at which the parallax of an object over the course of the Earth's orbit is one arcsec.
- $= 3.0857 \times 10^{16}\ m \qquad = 2.0626 \times 10^{5}\ AU \qquad = 3.26156\ ly$

Distance: light year (ly)

- The distance travelled by light in a vacuum in a year.
- $= 9.4607 \times 10^{15}\ m \qquad = 6.3241 \times 10^{4}\ AU \qquad = 0.3066\ pc$

Mass: Earth mass ($M_\odot$)

- $= 5.9722 \times 10^{24} kg \qquad \approx 6 \times 10^{27} kg$

Mass: Jupiter mass ($M_{♃}$)

- $= 1.898 \times 10^{27} kg \qquad \approx 1.9 \times 10^{27} kg$

Specific Flux: Jansky (Jy)

- $= 10^{-26}\ W \cdot m^{-2} \cdot Hz^{-1}$

B.4.2 Equatorial Coordinate System

Right Ascension (α)

Hour (^{h}): $\frac{1}{24}\ circle \quad = 15°$

Minute (^{m}): $\frac{1}{60}^{h} \quad = \frac{1}{1440}\ circle \quad = 15'$

Second (^{s}): $\frac{1}{60}^{m} \quad = \frac{1}{3600}^{h} \quad = \frac{1}{86400}\ circle \quad = 15"$

Declination (δ)

Declination is measured using normal degrees (see *Degrees of Angle*) from the equator.

B.5 United States customary units (aka Imperial Units)

B.5.1 Length

Point (p): $= \frac{127}{360}\ mm$

Pica ($P/$): $= 12\ p \quad = \frac{127}{30}\ mm$

Inch (in **or** "): $= 6\ P/ \quad = 25.4\ mm$

Foot (ft **or** $'$): $= 12\ in \quad = 0.3048\ m$

Yard (yd): $= 3\ ft \quad = 0.9144\ m$

Mile (Mi): $= 5280\ ft \quad = 1760\ yd \quad = 1.609344\ km$

B.6 Degrees of Angle

Degree ($°$): $\frac{1}{360}\ circle \quad = \frac{\pi}{180}\ rad \quad \approx 0.0174532925199433\ rad$

Minute of arc ($arcmin$ **or** $'$): $\frac{1}{60}^{\circ} \quad = \frac{1}{21600}\ circle \quad = \frac{\pi}{10800}\ rad$

Second of arc($arcsec$ **or** "): $\frac{1}{60}\ arcmin \quad = \frac{1}{3600}^{\circ} \quad = \frac{1}{206265}\ circle \quad = \frac{\pi}{648000}\ rad$

B.7 Miscellaneous Units

B.7.1 Pressure

Bar (bar): $= 10^5\ Pa \quad \approx 0.9869\ atm$

Atmosphere (atm): $= 101325\ Pa \quad = 1.01325\ bar$

Torr ($torr$): $= \frac{1}{760}\ atm \quad = \frac{101325}{760}\ Pa \quad \approx 133.3224\ Pa$

B.8 Prefixes

atto (a) = $\times 10^{-18}$

femto (f) = $\times 10^{-15}$

pico (p) = $\times 10^{-12}$

nano (n) = $\times 10^{-9}$

micro (μ) = $\times 10^{-6}$

milli (m) = $\times 10^{-3}$

centi (c) = $\times 10^{-2}$

deca (da) = $\times 10^{1}$

hecto (h) = $\times 10^{2}$

kilo (k) = $\times 10^{3}$

mega (M) = $\times 10^{6}$

giga (G) = $\times 10^{9}$

tera (T) = $\times 10^{12}$

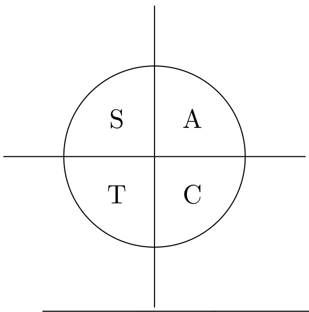
peta (P) = $\times 10^{15}$

exa (E) = $\times 10^{18}$

Appendix C

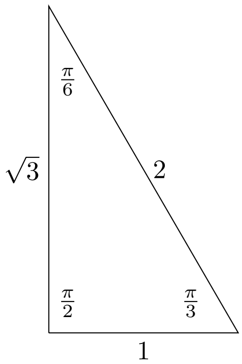
Mathematical Stuff

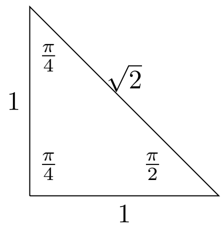
C.1 Trigonometric Values



rad	°	sin	cos	tan
0	0	0	1	0
$\pi/6$	30	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	45	$1/\sqrt{2}$	$1/\sqrt{2}$	1
$\pi/3$	60	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	90	1	0	$\pm\infty$
$2\pi/3$	120	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
$3\pi/4$	135	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1
$5\pi/6$	150	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$
π	180	0	-1	0
$7\pi/6$	210	$-1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$5\pi/4$	225	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1
$4\pi/3$	240	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$
$3\pi/2$	270	-1	0	$\pm\infty$
$5\pi/3$	300	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$
$7\pi/4$	315	$-1/\sqrt{2}$	$1/\sqrt{2}$	-1
$11\pi/6$	330	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$
2π	360	0	1	0

C.1.1 Pythagorean Triples





Appendix D

Boring stuff

D.1 Version History

v 0.1 2016: This project is begun in a trio of physical exercise books as *The Little Book of Physics Formulae*, *The Little Book of Mathematics Formulae*, and *The Little Book of Astronomy Formulae*

v 0.6 2016: The process of transferring the formulae from paper to Latex is initiated, but abandoned (or drifted away from).

v 0.7 2018-03-20: The project is resurrected (probably because the author started MRes), uploaded to Overleaf, and cleaned up.

v 0.8 2018-07-12: All remaining formulae imported from the original books.

v 0.9 2018-07-26: Formulae imported from undergrad formula sheets.

v 1.0 2018-07-27: First public release, with some additions from 0.9.

D.2 Licensing

D.3 Contact

Visit www.webofworlds.net for science fiction, science fact, geeky opinions, and maybe some Python code.
Suggestions or corrections are welcome at webofworlds@gmail.com

D.4 Credits

Unit Circle: By Jim.belk [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0) or GFDL (`http://www.gnu.org/copyleft/fdl.html`)], from Wikimedia Commons (`https://commons.wikimedia.org/wiki/File:Unit_circle_angles_color.svg`)

Periodic Table: By Dmarcus100 [CC BY-SA 4.0 (`https://creativecommons.org/licenses/by-sa/4.0`)], from Wikimedia Commons (`https://commons.wikimedia.org/wiki/File:Periodic_Table_Of_Elements_Atomic_Mass_Black_And_White.jpg`)

1 H Hydrogen 1.008																		2 He Helium 4.003
3 Li Lithium 6.94	4 Be Beryllium 9.012												5 B Boron 10.81	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
11 Na Sodium 22.990	12 Mg Magnesium 24.305												13 Al Aluminum 26.982	14 Si Silicon 28.085	15 P Phosphorus 30.974	16 S Sulfur 32.06	17 Cl Chlorine 35.45	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078		21 Sc Scandium 44.956	22 Ti Titanium 47.867	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.630	33 As Arsenic 74.922	34 Se Selenium 78.97	35 Br Bromine 79.904	36 Kr Krypton 83.798
37 Rb Rubidium 85.468	38 Sr Strontium 87.62		39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium [97]	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.760	53 Te Tellurium 127.60	53 I Iodine 126.904	54 Xe Xenon 131.293
55 Cs Cesium 132.905	56 Ba Barium 137.327	* 57 - 70	71 Lu Lutetium 174.967	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	78 Ir Iridium 192.217	79 Pt Platinum 195.084	80 Au Gold 196.997	81 Hg Mercury 200.592	81 Tl Thallium 204.38	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [209]	85 At Astatine [210]	86 Rn Radon [222]
87 Fr Francium [223]	88 Ra Radium [226]	** 89 - 102	103 Lr Lawrencium [262]	104 Rf Rutherfordium [267]	105 Db Dubnium [270]	106 Sg Seaborgium [269]	107 Bh Bohrium [270]	108 Hs Hassium [270]	109 Mt Meitnerium [278]	110 Ds Darmstadtium [281]	111 Rg Roentgenium [281]	112 Cn Copernicium [285]	113 Nh Nihonium [286]	114 Fl Flerovium [289]	115 Mc Moscovium [289]	116 Lv Livermorium [293]	117 Ts Tennessine [293]	118 Og Oganesson [294]

*Lanthanide series	57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.242	61 Pm Promethium [145]	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.500	67 Ho Holmium 164.930	68 Er Erbium 167.259	69 Tm Thulium 168.934	70 Yb Ytterbium 173.045
**Actinide series	89 Ac Actinium [227]	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium [237]	94 Pu Plutonium [244]	95 Am Americium [243]	96 Cm Curium [247]	97 Bk Berkelium [247]	98 Cf Californium [251]	99 Es Einsteinium [252]	100 Fm Fermium [257]	101 Md Mendelevium [258]	102 No Nobelium [259]