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The Ultimate Load on Pile Foundations

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Paper No. 1175

THE ULTIMATE LOAD ON PILE FOUNDATIONS:
A STATIC THEORY.

BY JOHN H. GRIFFITH, ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. LUTHER WAGONER, AND
JOHN H. GRIFFITH.

Introduction.—In one of his discussions as to the ultimate bearing power of pile foundations, the late E. Sherman Gould, M. Am. Soc. C. E., stated that the theories of Goodrich had mathematically exhausted the subject, referring, of course, to a dynamic analysis. It is interesting, therefore, to note an entire departure from the usual procedure in a treatment proposed by Desmond* in which he studies a concrete pile purely by static methods.

A perfected static analysis would appear to have certain advantages over the older methods in that it will either eliminate altogether, or relegate to a sphere of minor importance, a number of elements the real significance of which, even in a most precise dynamic theory, is destined to be rather vague and indeterminate. One might cite, for example, the case where the pile bounds back or slowly rises after driving, owing possibly to a resiliency or sponginess of the soil, or perhaps to a buoyant effect of the latter on the pile. Such a phenomenon as brooming of the head might likewise be cited. When the engineer analyzes such perplexing problems as compression of the hammer or the pile, questions of impact, friction of the guides, measurements of velocity, and the like, the real import of any one of which will require involved

* *Transactions*, Am. Soc. C. E., Vol. LXV, 1909. Discussion on paper, "Concrete Piles" p. 498, by Mr. Thomas C. Desmond.

analyses by the accomplished physicist, he may often be constrained to take the viewpoint of such eminently practical engineers as Haswell and Gould as to some of these matters. In fact, with any final working formula, to measure such an uncertain element as the penetration and neglect altogether the earth factors (as is tacitly done in any of the representative Sanders' expressions) would seem to seek a sort of negative magnification of the effect, reading, as it were, through the wrong end of the telescope, or taking observations at the short arm of the lever. Goodrich remarks* that:

“The liability to error is so enormous with small penetrations that no penetration should be trusted much less than 1 in., and no formula can be guaranteed within a reasonable percentage of error for less penetrations.”

He shows that: “With a total penetration as large as 4 ins. (which is seldom observed), a variation of $\frac{1}{8}$ in. would make this penetration liable to 3% error.”

Such a static theory will further endeavor to eliminate what Maxwell has called the historical element. The analysis of Desmond, for example, is not concerned with the load status a minute after driving, nor a year after, but rather in that indefinite period of time when the condition of the earth may be said to correspond with that minimum of stored energy which exists or tends to exist in Nature for stable equilibrium; or, if this element is to enter the analysis explicitly, it can only serve to render the problem more determinate. The dynamic analysis at best can only cover the situation in the period immediately after driving.

Then there are such formidable questions as the number of blows to refusal, the effect of the earth clinging to the pile, and many items of like moment.

In a larger sense, however, the static treatment should be viewed as complementary to the older method. A perfected theory of the pile will neither be confined exclusively to a study of the left-hand member of the equation of work, nor, in the other case, to the $\int P ds$ of the right-hand member, but, taking a unitary conception of the problem, will seek to include all variables and a determination of their effect on the status of ultimate load.

* *Transactions*, Am. Soc. C. E., Vol. XLVIII, p. 205.

It is to be hoped that Desmond's discussion may be the nucleus for a literature considering this larger view; further, that it may stimulate engineers to extend their experiments on earth pressures, hitherto confined to retaining walls, to include examinations of pile phenomena as well, the pile being in many respects a sort of inverted retaining wall in its analytical features.

The able engineers who have followed exclusively in the paths pioneered by Rankine and Moseley seem finally to have reached the proverbial blind alley in their attempts to solve the pile problem purely as a dynamic proposition; but Rankine* himself, it should be considered, at least implicitly suggested the static method in his attempt to figure the drawing power of screw-piles and the pressure on foundations. Any advance, however, in this field, seems to have been restricted, at least in America, by a too close adherence to his ellipse of stress principle, a rather subsidiary relation in the paper mentioned, which, while it may serve its purpose in elementary problems of the retaining wall, is not an efficient tool for a general investigation in the theory of earth pressure.

The writer will offer herein a few criticisms on the static method as it has been presented to date, and will outline some views as to its development along rational and empirical lines. In doing this, the paper will necessarily be confined to little more than an examination of the premises of the older authorities and an attempted statement of the problem. Owing to the scarcity of experimental data directly bearing on this subject, and an inadequate literature, such an investigation must be largely *a priori* in its nature, paving the way for a more rigorous analysis and suitable experimentation by others.

Existing Methods.—In the first and later editions of his “Civil Engineering” (1895), Patton gives the following equations for the “total bearing power of the pile”:

$$P = Awx \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 + \frac{Sfwx}{2} \left(\frac{1 - \sin. \phi}{1 + \sin. \phi} \right) \text{ minimum,}$$

$$P' = Awx \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 + \frac{Sfwx}{2} \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right) \text{ maximum,}$$

* *Philosophical Transactions*, Royal Society, 1857.

where w = the weight of a cubic foot of the material,

A = cross-section of the pile at the bottom,

x = depth of the pile in the soil,

S = area of exterior surface of the pile,

f = coefficient of friction of earth on the pile surface.

The expression, $wx \left(\frac{1 \mp \sin. \phi}{1 \pm \sin. \phi} \right)$, is the intensity of lateral normal pressure, minimum and maximum, on the surface of the pile. When multiplied by the proper coefficient of friction of wood on earth, this resulting tangential stress, when summed over the whole peripheral surface of the pile, gives, according to the Patton theory, the frictional resistance of the soil. The first terms in the right-hand members of each equation give the pressure on the base. Patton remarks:

“If proper values of ϕ , S , and f in equations above are determined by experiment, it would seem that these formulæ would produce better and more reliable results than the more common formulæ would.”

The solution given is the earliest direct attempt to solve the problem (other than that given by Rankine, before mentioned) that has come to the writer’s attention.

Very recently, Professor Vierendeel (University of Louvain, Flanders) has treated the subject in more detail, together with the dynamic method, in a comprehensive work* in which he gives the formula:

$$R = \pi D f w \frac{1 + \sin. \alpha}{1 - \sin. \alpha} \frac{L^2}{2} = 1.5 D f w L^2 \frac{1 + \sin. \alpha}{1 - \sin. \alpha},$$

which he deduces by the principle of work, where R = the ultimate load, D = the mean diameter of the pile, L = the depth of penetration, w is the unit weight, and α is the natural talus.

It will be seen by a little study that the foregoing methods are practically in agreement with the aforementioned treatment by Desmond, in that each makes use of the ordinary Rankine relation, multiplies by a friction factor, and integrates the stress in one form or another over the entire surface in contact with the soil.

* “Cours de Stabilité des Constructions” (Tome VI, 1907).

Viewed as an empirical expedient, such equations should commend themselves to engineers for practical use in fixing load limits. In this capacity, they will doubtless excel the ordinary Sanders' energy formulas, if constants are properly evaluated from test loadings, as suggested by Patton.

A true empirical basis for the study of the pile problem may be established by actual laboratory tests more easily than in the case of the retaining wall; for if loads at incipient motion are measured on a model pile which passes entirely through a reservoir of sand, having a hole in the base for egress of the pile, actual values of the total peripheral friction may be obtained and studied with respect to its variation for a variety of perimeters. Combined effects of basal and lateral stress could be obtained, of course, by independent experiments. It is important, however, that the base and lateral effects should be differentiated if they are to be studied and analyzed.

If, however, the methods given by these authors are to be construed as rational propositions, then, in their present form, they appear to be open to serious criticism, because, in making use of Rankine's expression for the intensity of stress, they violate his principle of conjugate stresses, which in this particular case makes the expression of the form, $wx \left(\frac{1 \mp \sin \phi}{1 \pm \sin \phi} \right)$, a principal stress, that is, one purely normal to the surface of the pile and having its maximum value. Consequently, the notion advanced by these writers of multiplying this principal stress by a friction factor is incompatible with the well-known principles of mechanics of stress.

Empirically, however, there is as much justification for the use of such types of formulas as there is for any of the present-day column formulas or some of the beam applications. The forms of the expressions are correct enough, as far as Rankine's intensity of lateral pressure is concerned, but, of course, the angle, ϕ , must be considered as an arbitrary parameter to be determined for certain soils, and not as the angle of repose or internal friction. Just what the deviation of this parameter from the angle of internal friction will be must be determined by such experiments as have been suggested or by actual tests in the field for ultimate loading.

A general criticism, of course, is that the problem in its final analysis will not lend itself to any such elementary forms as a Rankine solution may be expected to give. Any theory must experience that evolution

characteristic not alone of the dynamic analysis of the pile and the retaining wall, but of all the classical problems in engineering. In such an evolution the Rankine theory rightfully assumes its place as a primitive, true enough under its own premises, but of which the premises are not general enough to include the whole range of phenomena either of the pile or of the retaining wall.

The Rankine Theory.—In view of the fact that the Rankine theory has already taken its place as the basis for a static analysis of the pile, it is important that it be rigorously stated. The following is conceived to be an exact solution, with no assumptions except those contained in Rankine's premises.

Consider a pulley-shaped foundation, with data as indicated in Fig. 1, which, as in the case treated by Desmond, may be a concrete or timber pile jetted or driven to place. Any form of cross-section might be taken, but, for simplicity, it is assumed as circular.

The dotted lines may be considered to represent displacement filaments passing out from the horizontal rims to the free surface around the head of the pile. The position of these lines can only be inferred from the treatises, say Ketchum's or Vierendeel's, as few if any precise investigations have been made along this line.

At incipient motion of the pile, it being assumed that it is at its final depth, any increment of the load will cause an actual displacement of the particles, and this will manifest itself as an increment or surface displacement to the upheaval surface which has

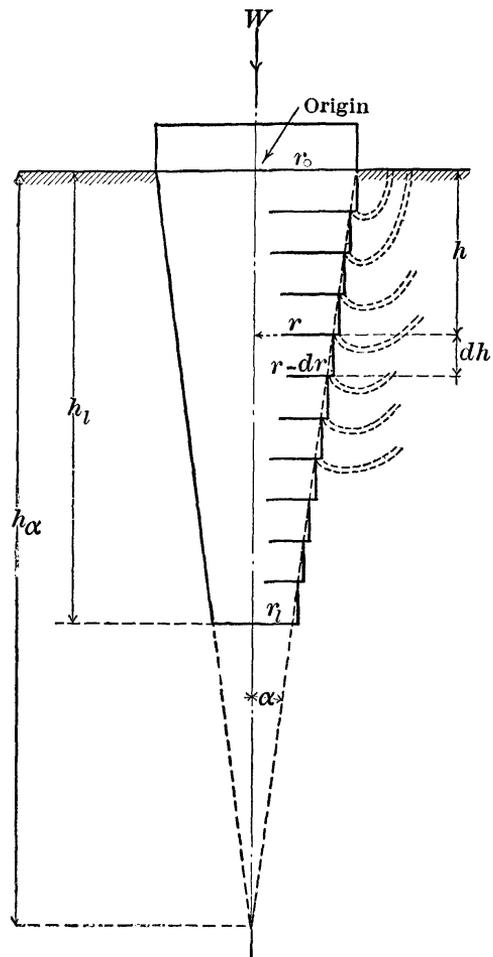


FIG. 1.

formed around the head of the pile in driving. This assumption is necessary under the Rankine hypothesis of incompressible particles, although it has been the writer's experience that the phenomenon is often difficult to observe at such a stage. The load at this time is considered to be the ultimate carrying capacity, by the Rankine law.

The area of a small rim of variable radius, r , and width, $dr = 2\pi r dr$.

Let p = the intensity of pressure on this rim element.

Then $p = wh \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2$ for a maximum,

where w = the weight of a cubic unit of earth,

and ϕ = the angle of internal friction, assumed as constant.

The total pressure on the element = $2\pi r dr \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 wh$.

Now substitute $r = (h_\alpha - h) \tan. \alpha$,

and $dr = -\tan. \alpha dh$,

where h_α represents the distance from the surface to the vertex of the cone formed by the surface of the pile, h_l = the actual length in the earth, and α = the angle of slope of the conical surface. The total pressure on the rim element becomes

$$2\pi w \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 \tan.^2 \alpha \int_{h_l}^0 -(h_\alpha - h)h dh.$$

In order to take account of a principle of continuity, which in this case will manifest itself in the law of pressure varying as a function of the depth, one may conceive that, as the elementary rim pressure exceeds the amount above given, the pile will tend to subside under this, so that each rim will take its proportionate quota of stress in turn. The total buoyant effect is at the limit when the pulley-shaped foundation becomes a conical-shaped pile. The value of the integral becomes:

$$\int_{h_l}^0 -(h_\alpha - h)h dh = \left[-\left(h_\alpha \frac{h^2}{2} - \frac{h^3}{3} \right) \right]_{h_l}^0 = h_\alpha \frac{h_l^2}{2} - \frac{h_l^3}{3},$$

and, substituting this in the previous expression,

$$P_{(lat.)} = 2\pi w \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 \tan.^2 \alpha \left[h_\alpha \frac{h_l^2}{2} - \frac{h_l^3}{3} \right],$$

where the expression, $P_{(lat.)}$ represents the entire upward pressure on the lateral surface of the pile. To this must be added the basal pressure, giving, for the total load, P , which the pile can sustain according to Rankine's theory:

$$P = 2\pi w \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2 \tan.^2 \alpha \left[h_\alpha \frac{h_l^2}{2} - \frac{h_l^3}{3} \right] + \pi r_l^2 w h_l \left(\frac{1 + \sin. \phi}{1 - \sin. \phi} \right)^2.$$

In the case of the "butt end down," the weight of the variable column of earth may be similarly summed and added to the load on the pile, and this equated to the bearing power of the base.

Such an analysis assumes, of course, that the earth conditions, absence of cohesion, etc., will warrant a treatment by the Rankine method. It is believed to give all that can consistently be demanded of the hypothesis.

Limitations of the Theory.—It will be seen that the above application is quite limited in its efficiency as a working method. Specifically, it neglects the friction on the vertical projections of the face. Indeed, the Rankine premises do not take cognizance of any foreign body, such as the pile, but confine the problem to an indefinite extent of the material.

While it assumes the existence of displacement tubes, it makes no analytical provision as to their zone of action, unless one may take any series of vertical and horizontal lines as defining the field.

The usual applications of this theory assume a constant coefficient of friction, which, in the light of experiment, is only approximately tenable; but, confining the problem to its own more particular domain, the chief limitation is the necessity of the assumption of Moseley's law of least resistance as Rankine referred to it, at once either the element of weakness or of strength in his method, as one may prefer to call it.

Consider an ordinary wedge element of the material, Fig. 2, with vertical and horizontal faces and an inclined face the normal of which, n , is inclined at an angle, θ , with the horizontal. The area of this θ -face may be conveniently taken as unity.

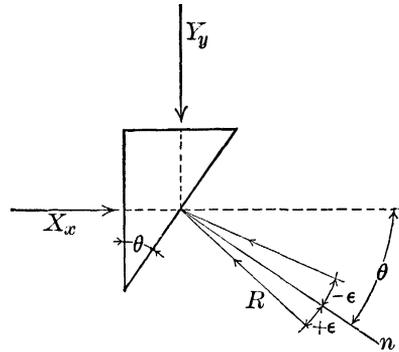


FIG. 2.

Let the intensity of the vertical stress be considered in this particular case as due to a column of earth of length y feet below the surface of the ground, the value of which is Y_y . The corresponding intensities upon the x - and θ -planes, respectively, are X_x and R . The stress, R , has an obliquity of $\pm\varepsilon$ from the normal. By compounding stresses, by any of the elementary methods, there results the general expression:

$$X_x = \frac{\tan. \theta}{\tan.(\theta \pm \varepsilon)} Y_y.$$

To evaluate X_x , another condition is required. Rankine sought to supply this condition through the Moseley assumption, taking the obliquity, $\pm\varepsilon$, as having its maximum value, ϕ , at impending motion of the particles. By seeking the maximum and minimum values of $\frac{\tan. \theta}{\tan.(\theta \pm \varepsilon)}$ on this basis, there results then, for the particular values of θ where Rankine's value of X_x may be assumed to hold:

$$\begin{aligned} \theta &= \text{multiples of } \frac{\pi}{4} - \frac{\phi}{2}, \text{ for } X_x \text{ a maximum,} \\ \theta &= \text{multiples of } \frac{3\pi}{4} - \frac{\phi}{2}, \text{ for } X_x \text{ a minimum,} \end{aligned}$$

for positive values of ϕ , and in a similar manner when ϕ is negative. For example, taking a common value of $\phi = 30^\circ$, one receives $X_x = \frac{1}{3}Y_y$ and $3Y_y$, as in the ordinary case. For the above given values of θ , Rankine's solution may be considered to hold, but for all other values the problem is absolutely indeterminate. The common practice of engineers, in applying this method as a general solution to problems of earthwork, is quite in keeping with that practice which seeks the deportment of a column within the elastic limit from tests to destruction.

Neither will the common defense, of the law being on the safe side, hold in all cases. For instance, it has already been pointed out by Boussinesq* that, in the case of a retaining wall when it is in its ordinary position of equilibrium, otherwise than at the time of incipient motion, as predicated by Rankine, although the particles are less forcibly retained, they nevertheless exert upon the structure a greater thrust than that given by Rankine.

* "Essai théorique sur l'équilibre des massifs pulvérulents, comparé à celui de massifs solides, et sur la poussée des terres sans cohésion," (1876), p. 5.

A number of practical phases of this indeterminateness might be cited, showing the shortcomings of the method as a theoretical device. This is made apparent in the packing of balls. For example, a rather low angle of repose may be expected for fine shot if it is dropped from a short height, but had one the patience to arrange the shot particle by particle in a pyramidal array, according to the geometry of packing spheres, a much higher angle might be obtained for the slope of the pyramid, and this would be entirely independent of the condition of the balls, that is, whether rough or frictionless. Further, taking the old problem of the thousand 1-in. balls* packed in cubical array in a 10-in. cubical box, it is quite possible to conceive of an angle of repose of 90° if the sides of the box could be gently removed, although, of course, in such a case, the equilibrium would be very unstable. In the latter cases, the Moseley assumption would be quite justifiable. However, taking another extreme, say, the thrust of barrels on the walls of a warehouse, only the exigency of an occasional earthquake could render the application of the method theoretically permissible. The law is inoperative.

It is such limitations as have been cited that render the Rankine method of rather doubtful utility for any general rational treatment, either of the pile or the retaining wall. European and other than American authorities have ceased treating the Rankine formula as a general solution for all problems involving the lateral pressure of earth, and prefer to give it its more proper position as defining one particular kind of equilibrium. Even in its own special field, a solution approaching nearer the facts may doubtless be secured in many cases by the more determinate method of Greenhill,[†] as in the instance of barrel thrust.

Theoretical Position of the Method.—In order, then, to give to the Rankine theory applied to the pile that definiteness of position which attaches, say, to that of Euler's formula in the column theory, it may be defined as the theory of an infinitely smooth shaft afloat on a medium deporting in several respects as a sort of generalized fluid, where the particles are subject to negative normal stresses or pressures and to tangential or friction stresses, but where no permanent shearing resistance exists. In such a theory the vertical pressures may be assumed to follow the hydrostatic law. The horizontal pressures will also follow

* Quoted by Greenhill from "Cosmos," September, 1887, "Hydrostatics," p. 52.

† Greenhill's "Hydrostatics," pp. 45 *et seq.*

this law, but, owing to friction, the effect is such as would occur with a reduced specific weight, $w\left(\frac{1 \pm \sin. \phi}{1 \mp \sin. \phi}\right)$, where ϕ is the angle of internal friction, or, as Rankine referred to it usually, the angle of repose. The (\pm) signs are to be used in the above for the case of maximum loadings, in which case the pressure exerted by the pile is a so-called "active force," as the term is used by Rankine. The (\mp) signs are for minimum loads on the pile, namely, if the "buoyancy" of the surrounding earth (viewing this now as an active force) is greater than the load on the pile, as prescribed by this theory, the pile will tend to rise, and may actually do so, especially if the medium contains more or less water.

Accordingly, it will be seen that the laws of pulverulent masses will agree well with the theory originally advanced by Boussinesq,* and given later by Flamant,† Greenhill,‡ and others, in that they are intermediate in their properties between fluids and solids. Fresh cement, in its ordinary condition, will follow closely the hydrostatic law, but, under pressure, will take on the properties of elastic bodies. Even the Rankine equations, if consistently interpreted, find analogies in the theory of stress and strain in solids on the one hand (Tresca), and agree with the hydrostatic law for $\phi = 0$, on the other.

A dynamics of pulverulence is quite possible to formulate under such a notion, and would probably find practical applications in designing orifices for the discharge of grain, etc. Under this caption such phenomena as have been described by Vierendeel§ as occurring at the circumference of disk piles, and by Le Conte|| and Goodrich,¶ as "cones" and the like, forming under the bases of models, would probably find interpretation as suppressed vortex or eddy effects.

Practical Utility of the Rankine Formula.—While the Rankine theory is little more than an abstraction, and if consistently and rationally applied to a single pile can only be expected to give a fraction of the real carrying power, its utility to the practicing engineer may still exist in the fact that a multiple-pile system may be tested by Rankine's

* "History of the Elasticity and Strength of Materials," Vol. II, Pt. II, Article on Boussinesq, by Karl Pearson.

† "Stabilité des Constructions," p. 111.

‡ "Hydrostatics," pp. 45 *et seq.*

§ "Cours de Stabilité des Constructions," Vol. VI, p. 246.

|| *Transactions*, Am. Soc. C. E., Vol. XLII, p. 284.

¶ *Transactions*, Am. Soc. C. E., Vol. XLVIII, p. 181.

equations about as logically as they may be applied to any ordinary foundation. In such a multiple pile the integrity of the structure is usually preserved by suitable framing; but, if this were not so, the material in the cusp-like interstices between the piles can be expected to be much more compressed, and consequently to have a considerably higher friction factor, than the less restrained material at the periphery of the composite structure, thus tending to maintain this unity of action.

In a multiple pile great reliance is placed on the increased density of the soil, due to the driving, with the corresponding increase in the friction coefficient. As the condensation under the Rankine premises is purely inelastic, an approximate idea of the increase in density may be found by an equation between the displacement of the pile and the upheaval mass around the head.

Nearly all writers, with the exception of Vierendeel, in discussing the bearing power of foundations, follow Rankine in ignoring the stresses on the side walls, and confine their analysis solely to the base. Accordingly, on the common theory, a designer of a multiple pile would neglect the peripheral friction on the composite structure in comparison with the presumably larger pressure on the base. In this case such a procedure can be viewed as giving only crudely approximate results. It is believed that the phenomenon of dilatancy of media composed of rigid particles, as studied by Professor Osborne Reynolds,* may even warrant the belief that this lateral friction is larger than supposed, especially in water-bearing strata. The writer will revert to this point later.

The Elastic Theory.—Nearly all the structural problems of engineering find their ultimate analysis in the elastic hypothesis. This is true of the arch, and in a large measure of the retaining wall. Just as the beam, on account of the labors of de St. Venant and his contemporaries, owes its truly rational position to such elastic studies, quite independent of the empiricists, the column theory, with of course a few possible exceptions, may be said to have made no consistent advances since the days of Euler by departing therefrom.

While to place such an apparently crude and sordid problem as the pile in this field will undoubtedly seem inopportune, it is believed

* *Philosophical Magazine* (London, E., and D.), Vol. XX, 1885, p. 469. "On the Dilatancy of Media Composed of Rigid Particles in Contact;" also Reynolds' Works, Vol. III.

that, in the end, such a step will avoid a great deal of useless effort and incorrect thinking. It is thought important to bring out a few arguments *pro* and *con* as to the advisability of such procedure.

In the first place, to make the problem of the lateral pressure of earth truly determinate, the idea of strain is involved. Its introduction into the analysis is due to Boussinesq. Take the case previously cited, of the problem of the balls packed in the 10-in. box in cubical array. The problem of their lateral pressure against the walls owing to their own weight becomes at least theoretically determinate, provided all the stress and strain constituents of the material are known, and the elastic deformation of the walls is understood. Although such elastic solutions are in many cases extremely difficult to obtain, on the other hand, they have the advantage of a high degree of certainty of result, and will tend to obviate that endless modification so common, say, in column and pile formulas.

As contributing data toward such a final and correct analysis, ideal problems, approximating in part toward the actual conditions, may be solved. For example, it may be shown that:

“If a vertical cylindrical hole of circular section is cut in a rigid body, and an elastic cylinder of density ρ , which, if freed from the action of gravity, would exactly fit the hole, is placed in it and stands upon the bottom, * * * the sides of the hole suffer the same hydrostatic pressure as if it were filled with a liquid of density $\rho(m - n)(m + n)$.” (Ibbetson.)

Slichter,* in commenting along this line, remarks:

“It is important * * * that we should have before us the solution of as many problems as possible, since the most likely method by which we shall be able to solve a new problem is by reducing it to one of the cases in which a similar problem has been constructed by the inverse process. Indeed, one must often be content to secure an approximate solution in a given case by searching among problems already solved for one whose equipotential lines or surfaces have a form somewhat resembling the given boundary, and then so to modify the problem by tentative methods as to produce conditions more nearly corresponding to those of the given problem. For this reason it is desirable to solve all possible kinds of problems * * * whether they seem to be ‘practical’ or not.”

* “Theoretical Investigation of the Motion of Ground-Waters,” by C. S. Slichter (Government Printing Office, Washington, D.C., 1899), p. 333.

Accordingly, Coulomb, Rankine, Weyrauch, Levy, Boussinesq, Kötter, and others, have contributed much in their study of various kinds of equilibrium. The work of Boussinesq, while furnishing valuable researches in the whole field, seems to be carefully ignored by the practicing profession.

Such an elastic hypothesis, it has been urged, is less applicable to the case of earth pressure than in the case of any other medium, it being difficult to predicate continuity laws of the medium and the existence of derivatives, as is done in hydrodynamic and elastic theories. As sufficiently typical of such criticism, the remarks of Darwin* are closely to the point.

“It has always been assumed by previous writers that the tangential action across an ideal interface in a mass of loose earth is of the same nature as the statical friction between solids, and that when the tangential stress has attained in magnitude a certain fraction of the normal stress, the equilibrium is on the point of breaking down. * * * A little consideration will show that the hypothesis cannot be exact, even with an ideal sand with incompressible grains, and absolutely devoid of coherence. For imagine a mass of sand thrown loosely together; then if the grains are of irregular shape a certain portion of them will be resting on points and angles, thus occupying more space than they might do.

“If the sand be now compressed, many of the grains will slip and rotate, and fall into interstices; in fact a considerable amount of re-arrangement will take place, and the density of the mass will rise considerably—by quite 10 per cent. if the re-arrangement be thorough, as found experimentally.

“Even if all the grains were spherical a considerable amount of change would take place, and when they are angular of course much more. * * *

“Hence it is clear that the coefficient of internal friction of sand is a function of the pressure, and not merely of the pressure then existing, but also of the pressure and shaking to which at some previous period that portion of the mass of sand has been subjected. * * *

“It is quite impossible to say how much these causes will vitiate any mathematical theory of the equilibrium of sand, but experience seems to show that the vitiation is extensive.”

* *Minutes of Proceedings*, Inst. C. E., Vol. LXXI, 1883, “On the Horizontal Thrust of a Mass of Sand,” pp. 374 *et seq.*

On the other hand, in the elastic theory, the researches of Boussinesq* show that pulverulent material when under pressure—such as may occur in this particular case of the pile owing to impacted soil through driving, even more than in the retaining wall—resists a change of form with a force which is proportional to the mean of the three principal stresses acting on the particle. He takes the coefficient of rigidity, μ , as varying with this mean pressure. As the weight on the particle increases, either owing to its own “head,” or, in this case, to the compressed soil in driving, the surrounding medium approaches an elastic body in its properties. Under great pressure, of course, it becomes perfectly so, thus justifying geologists or physicists in calculating earth stresses, delta pressures, faults, etc., by known elastic methods.

Now, the writer believes that there is a tacit notion, prevalent among representative engineers, which is quite conformable to such an hypothesis, and in support of this belief would quote the remarks of Goodrich:†

“When a pile is supported entirely by the frictional resistance, the actual region supporting the load is some deep ground level at which the frictional resistance holding the pile has been transferred through the earth in the shape of a conoid of pressure, the base of which gives a total bearing value equal to the load and a unit bearing value which the earth at that lower level will support. Each kind and degree of compactness of earth will give a different angle for the slope of the conoidal surface.”

Again, he says:

“When supported by frictional resistance, they [the piles] must be driven so far apart, or to such a depth, that the increased area of bearing developed by the conoid of pressure having the required altitude of frictional resistance meets a level which will afford the required support before intersecting the conoid of a neighboring pile.”

Such a description would seem to show analogies with the “fan” distribution of Stokes and Carus Wilson,‡ with the local perturbations of Boussinesq,‡ or some of the equipollent effects of de St. Venant.

* “Essai théorique * * *,” p. 6.

† *Transactions*, Am. Soc. C. E., Vol. XLVIII, “Supporting Power of Piles,” pp. 182 *et seq.*

‡ *Proceedings*, Physical Society of London, Vol. XI, 1891, p. 194, “The Influence of Surface-Loading on the Flexure of Beams.”

It is natural to ask, however, how the inelastic distortions of Darwin can be made to harmonize with the other views. The answer would be by postulating or defining the medium. Slichter,* in a somewhat related problem involving a study of the flow of ground-waters through a soil, has attacked his problem very successfully by the assumption of a mean soil. The size of the grains in a soil having the same transmission power as the more complex soil he calls the "effective" size. He says:

"There probably exists a tendency in every such soil toward a certain average size and mean arrangement of grains which the theory of probabilities would justify us in setting up as an ideal soil to replace a given soil in the investigation."

The same remarks may be applied to the analysis of the pile and related phenomena. It is this idealization of the problem which is tacitly done in all the problems of engineering, perhaps, however, with less justification at this time in the theory of earth pressure, on account of the lack of physical investigation.

On the whole, the opinion of elasticians, Darwin and de St. Venant included, would seem to be favorable to an elastic analysis of the problem of the lateral pressure of earth and pulverulent material. Pearson,† in his critique of the elastic analysis of Boussinesq, has said:

"They appear to contain the most complete scientific theory yet given of the stability of such a mass * * * indeed, they are perhaps the limit to what elastic theory can provide in these directions."

In view of the dearth of knowledge of strain and friction factors, little progress can be made. It is believed, however, that as engineers direct their attention to the static outlook and conduct experiments along this line, a great many features now rather obscure will clear up. Such a study also affords another angle of vision upon the pile viewed dynamically and the retaining wall.

General Notions.—For purposes of discussion, consider a pile driven or jettied to place and carrying, say, its maximum load. It is desired to investigate the mechanical state of the soil as it reacts upon the pile and prevents its further subsidence under the load. The principles of

* "Theoretical Investigation of the Motion of Ground-Waters," p. 305.

† "History of Elasticity and Strength of Materials," Vol. II, Pt. II, pp. 313 and 357.

mechanics* furnish the following well-known equations for the static equilibrium of a volume element of the material surrounding the pile, say, a small parallelepiped the co-ordinates of which, as shown in Fig. 3, are x , y , and z .

$$\begin{aligned} \frac{\delta X_x}{\delta x} + \frac{\delta X_y}{\delta y} + \frac{\delta X_z}{\delta z} + \rho X &= 0 \\ \frac{\delta Y_x}{\delta x} + \frac{\delta Y_y}{\delta y} + \frac{\delta Y_z}{\delta z} + \rho Y &= 0 \\ \frac{\delta Z_x}{\delta x} + \frac{\delta Z_y}{\delta y} + \frac{\delta Z_z}{\delta z} + \rho Z &= 0 \end{aligned}$$

Using the Kirchoff notation, as preferable to that of Lamé, the expression, X_x , represents the intensity of normal stress on the elementary area, $dy dz$, of the parallelepiped, that is, the stress acting in the direction of the x -axis upon the plane element, $dy dz$, perpendicular to this plane. Briefly, X_x represents the x -stress upon the x -plane. In a similar manner, X_y is the x -stress on the y -plane, a shearing or tangential stress; Z_z is a normal stress upon the z -plane, and so on. Since the shears at right angles are always equal, then $Y_x = X_y$, $Z_x = X_z$, $Z_y = Y_z$; but, for convenience of mental retention, the symmetrical notation commends itself. The equivalence can be asserted as desired in calculations of any particular problem.

The expressions, ρX , ρY , and ρZ , where ρ is the density, represent "body forces," such as gravity or "centrifugal" force. In this case, these volumetric forces may be the components of gravity in the direction of the co-ordinate axes, or, as these are taken in Fig. 3, $\rho X = \rho Z = 0$, and $\rho Y =$ the weight of the earth per cubic foot in the engineer's notation. For sand charged with water, this would be, say, 110 lb., or as the case might be.

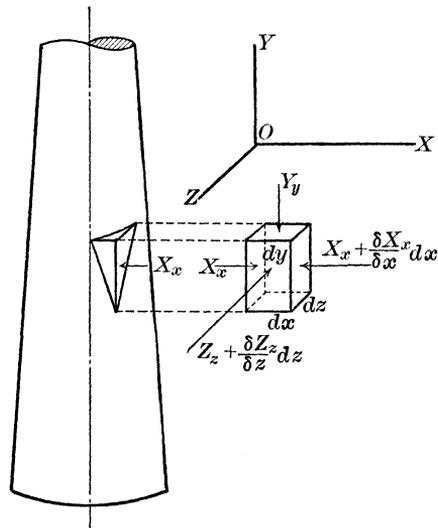


FIG. 3.

* "Theory of Elasticity," by Love, Chapter V, pp. 122 *et seq.*

The foregoing equations, as used by Rankine* in his original paper, are rigid body equations. Boussinesq,† by introducing a comprehensive theory of strain, formulates an independent system for the theory of earth pressure. There are, of course, relations of compatibility in the general problem which will show analytically, as already shown for the Patton equations, that the engineer cannot choose his values at random.

Thus far, as the writer has discovered, few practical data in the matter of strain are accessible for different earths. Reference to this must be brief. The theory, then, will be founded on stress relations, as in the ordinary beam formula, for practical purposes.

In the case of conical or cylindrical piles, the equations for static equilibrium in a final analysis will be best expressed in the well-known cylindrical co-ordinates, the notation being similar to that used before, namely:

$$\begin{aligned} \frac{\delta Y_y}{\delta y} + \frac{\delta Y_r}{\delta r} + \frac{\delta Y_\phi}{r d\phi} + \frac{Y_r}{r} + \rho Y &= 0 \\ \frac{\delta R_y}{\delta y} + \frac{\delta R_r}{\delta r} + \frac{\delta R_\phi}{r d\phi} + \frac{R_r - \phi_\phi}{r} + \rho R &= 0 \\ \frac{\delta \phi_y}{\delta y} + \frac{\delta \phi_r}{\delta r} + \frac{\delta \phi_\phi}{r d\phi} + \frac{\phi_r + R_\phi}{r} + \rho \phi &= 0 \end{aligned}$$

In these equations the capital letters give the direction of action of the stress and the subscripts refer to the planes on which they act. For example, ϕ_y represents the intensity in the direction of the normal to the plane, $dr dy$, on the y -plane, that is, the plane, $dr r d\phi$, etc. The shears in rectangular directions, as in the previous case, are equal.

In this more complicated case, however, owing to the symmetry about the y -axis, or axis of the pile, the "hoop compression" becomes constant around any particular ring element of the radius, r . The shears

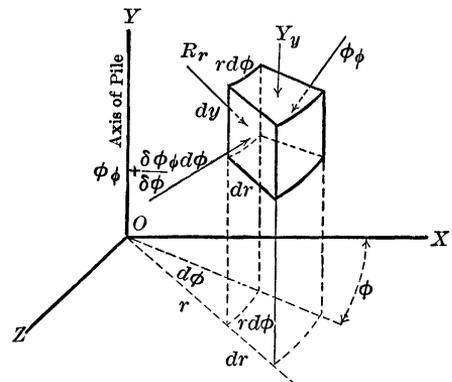


FIG. 4.

* *Philosophical Transactions*, Royal Society, 1857.

† "Essai théorique sur l'équilibre d'élasticité des massifs pulvérulents * * *," p. 24, etc.

also vanish on the ϕ -planes, that is, any of the faces, dy , dr . This distribution of stresses, when a solution of the various particular intensities is obtained, may ultimately be used for obtaining the tubes of stress, their intensities and slopes at any point, in the conoid described by Goodrich.

Now, to make any particular problem a determinate one, such types of equilibrium equations as have been given are to be satisfied for all values of the variables and for certain boundary conditions, namely, at the upheaval surface and at the entire periphery of the pile. In a continuous beam the analogy exists in the state at the supports. Similarly, a correct column formula must not only satisfy such equilibrium equations along the axis, but also hold for very long and very short columns. The problem under discussion is relatively more determinate than in the column problem, as the best that may ultimately be expected in the latter case is a least-square solution.

At the surface of the pile the following type of equations must be satisfied, as well as for the upper surface.*

$$\begin{aligned} X_n &= X_x \cos.(xn) + X_y \cos.(yn) + X_z \cos.(zn) \\ Y_n &= Y_x \cos.(xn) + Y_y \cos.(yn) + Y_z \cos.(zn) \\ Z_n &= Z_x \cos.(xn) + Z_y \cos.(yn) + Z_z \cos.(zn) \end{aligned}$$

To make these expressions clear, it may be remarked that the surface of the pile, being in the general case the surface of a cone, will transform the volume element of earth, $dx dy dz$ (Fig. 3), into a tetrahedral element. And these equations assert the equilibrium of all stresses on the tetrahedron in the directions, x , y , and z , respectively.

Call the surface element of the pile, that is, the inclined face of the tetrahedron element, the n -face, because its normal is, say, n . Let its area be unity, for convenience of discussion. Then the other faces, namely, the x , y and z -faces, respectively, are $\cos.(xn)$, $\cos.(yn)$, and $\cos.(zn)$, where (xn) , (yn) , and (zn) are the angles between the x , y , and z -directions and the normal of the n -face or n . Accordingly, X_n is the resultant stress component in the x -direction on the surface element of the pile. A similar set holds for the ground surface, but becomes very much simpler owing to vanishing of terms when the upheaval surface is assumed as horizontal.

* "Theory of Elasticity," by Love, Chapter V, pp. 122 *et seq.*

Now, in a precise and finished analysis involving the strain relations, both the boundary equations just given and the equilibrium equations are usually expressed in terms of these strains. Just as they are neglected in the derivation of the beam formula, they will be neglected here. The two sets of equations will be used solely as stress relations, as given, to keep the problem within working bounds.

A two-dimensional solution only can be attempted at this time, on account of the analytical difficulties involved in the more general treatment. It is believed, however, that a general solution exists in the case where the "immersed" length of pile is zero in the Boussinesq* problem of the distribution of stress and strain due to a rigid cylinder resting upon an infinite elastic solid, combined, of course, with suitable superpositions to provide for the weight of the soil. Moreover, since the strain in the earth at some distance from the body is quite independent of the manner of distribution of the peripheral stresses, but will depend rather on the resultant statically equivalent to them, it is thought that this solution for immersion of length zero may actually be taken for finite lengths of the pile. It would seem to the writer that the existence of the "cone" under the base will approximately justify this.

All authors, from Barlow and Rankine to the present time, have pleaded a lack of experimental data with which to correlate their mathematical investigations. The writer has felt this constraint in his attempts to get any trustworthy results from the case given, after analyzing the problem from different points of view; but, while these efforts have been largely fruitless, they have afforded certain lines of approach in analyzing the "conoid."

One of these is that, in the case of experiment, instead of restricting the investigation solely to the special case of granular or pulverulent media, as all engineers have heretofore done, the problem should be generalized to include media which have elastic properties within limits, say clay, hardpan, spongy soils, and very probably sand in its most compact position, especially when it is charged with water. It is believed that, eventually, when more experiments have been made, these premises will be easier to work to than in the case of granular media.

* "Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques," 1885.

"History of Theory of Elasticity," Todhunter-Pearson, Vol. II, Pt. II, p. 237.
Love's "Theory of Elasticity," Chapter VIII.

In some preliminary experiments along this line, made for the purpose of throwing light upon more precise efforts to be undertaken, C. J. Green, *Jun. Am. Soc. C. E.*, and the writer used rather fine and compacted saw-dust, in a duplication of the Goodrich* experiment made with sand. Such a saw-dust medium will permit a considerable magnification of the strain that may be expected in an actual case, when a small vertical motion of the model pile is made in the medium, keeping the "pile" close to the glass wall of the box. Leygue,† in his experiments on retaining walls, used a series of strata of a different colored medium to bring out the faults in the sand and confirm his notion of a curved surface for the interior face of the Coulomb wedge. In like manner, this notion has been tried by "sprinkling" a series of co-ordinate lines of any convenient medium on the face of the glass wall when laid flat with the "pile" in place, and laying over this the saw-dust, with a view of showing the strained lines when a small vertical displacement of the "pile" occurs. The original positions of the co-ordinates are marked on the glass with a wax pencil.

Two limiting aspects are to be studied: First, the strained condition for a very smooth or polished prism with a flat base, and then that for one with serrated or notched faces next to the saw-dust. The first case simulates that where the pressure on the sides is normal. The second case approximates the actual status of a pile in a cohesive soil where the full friction exists. While little of this has been carried out, it is believed that qualitative data of value will be obtained by using, not only straight prisms, but also wedges of rectangular cross-section with the faces next the material inclined to and from the vertical. It is hoped in the first case to obtain the deportment of the material under the pile. Preliminary experiments seem to confirm, partially at least, such a flow of stress as has been already derived both experimentally and analytically by Hertz‡ in the well-known problem of the pressure between two elastic bodies in contact. It is thought, by carrying out the Goodrich experiment as thus described, not only for sand, but also for other "more springy" media, that a great deal of light may be afforded, not only on the basal action of the pile, but also on the related problems

* *Transactions*, Am. Soc. C. E., Vol. XLVIII, p. 181.

† *Annales des Ponts et Chaussées*, 1885.

‡ Hertz, "Miscellaneous Papers," Translation by Jones and Schott.

Hertz, *J. F. Math. (Crelle)*, Bd. 92 (1881).

Love's "Theory of Elasticity," p. 195.

of surface loading, as in beams, etc. Here, analysis is already far ahead of experiment, at least for elastic bodies.

In the second case, it is desired to discover the zone of action in regard to the lateral friction in a cohesive and elastic soil. In the subsequent analysis this can only be assumed for the case of pulverulent material.

Two-Dimensional Stress Relations.—With the Rankine premises the uniplanar or two-dimensional case is easily extended to three dimensions by the assumption of a vertical axis of symmetry, namely, his ellipse of stress relation becomes an ellipsoid of stress; but, when the influence of a body such as the pile is concerned, the problem becomes greatly complicated, involving a solution in the case of stress alone of the equations of equilibrium in cylindrical co-ordinates subject to proper boundaries, as has been shown. The writer has been unable to obtain general solutions for these, as has been already remarked.

It is proposed, in accordance with the suggestion of Slichter, to attempt an approximate solution as the best available at this time. Such a solution, accordingly, may be considered to be a second approximation to that already given by Patton and by Desmond, but it will avoid largely the Rankine inconsistencies. This may then be used in studying the experimental data at hand with a view to discovering the general law, if such law does not already exist, at least for short piles in the Boussinesq problem of the rigid cylinder.*

In a two-dimensional case, either of the sets of equilibrium equations may be applied as it were to a pile of very large radius, or, taking as equivalent, a stretch of sheet-piling. Accordingly, the piling partakes more or less of the nature of the retaining wall.

Two-dimensional treatments of the equilibrium equations have already been given, in the case of

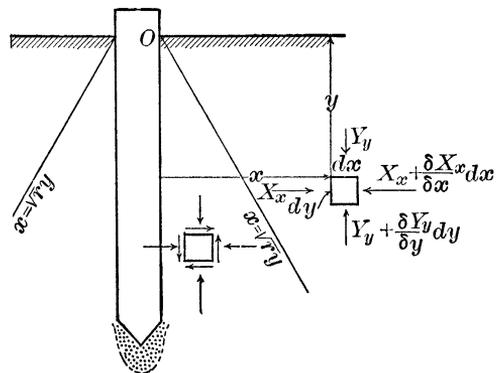


FIG. 5.

* "Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques," 1885 (Gauthier-Villars, Paris).

the retaining wall, by Kötter* and Boussinesq. As the latter has discussed local effects particularly, it is believed his results may be applied to the pile.†

The equilibrium equations, being independent of z , or the direction in the length of the wall or piling, reduce to

$$\begin{aligned} \frac{\delta X_x}{\delta x} + \frac{\delta X_y}{\delta y} &= 0 \text{ for the } x\text{-direction,} \\ \frac{\delta Y_x}{\delta x} + \frac{\delta Y_y}{\delta y} + (\rho y = w) &= 0 \text{ for the } y\text{-direction.} \end{aligned}$$

The region of perturbation is supposed to extend to the line the equation of which is

$$x = \sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}} y = \sqrt{r} y,$$

where the coefficient \sqrt{r} , is the square root of the Rankine ratio. This must here be tentatively assumed. (See Fig. 5.)

The general Rankine relation is assumed to hold outside of this region at a distance from the pile, namely,

$$\sin^2 \phi = \frac{(X_x - Y_y)^2 + 4X_y}{(X_x + Y_y)^2},$$

which states the expression for the “stability of a mass of earth in terms of the pressure at a point referred to any pair of rectangular axes, OX' and OY' , in the plane of greatest and least pressure.”‡ Taking $4X_y = 0$, the common expression is easily derived.

As a justification of the use of the above, it is assumed here that the plunger and cylinder experiments of Goodrich give a fair confirmation. (Whether $\frac{1}{5000}$ in. \pm movement of the “plug” will permit the inference that the pressure on the plug is of the same intensity as that on the walls of the cylinder has always raised a query in the writer’s mind.§)

* “Erddrucken auf Stützmauern,” Müller-Breslau (Stuttgart, 1906), pp. 107 *et seq.*

† *Annales des Ponts et Chaussées*, T. III, pp. 625–643.

See also “Theory of Elasticity,” Todhunter-Pearson, Vol. II, Pt. II, p. 347; and *Minutes of Proceedings*, Inst. C. E., Vol. LXV, p. 212.

‡ *Philosophical Transactions*, Royal Society, Vol. 147, p. 18.

§ *Transactions*, Am. Soc. C. E., Vol. LIII, p. 283.

Since the weight at the surface, assumed flat, is zero, the boundary relations become, by the vanishing of terms in the equations for the tetrahedron:

$$Y = 0, \quad Y_x = X_y = 0.$$

At the sheet-piling (the retaining wall in the problem of Boussinesq), there results $X_y = \tan. \phi_1 X_x$, where $\tan. \phi_1$ is the tangent of the angle of obliquity of the resultant pressure on the vertical face at the pile.

Boussinesq assumes "that in practice sustaining walls are generally sufficiently rough to render a thin stratum of the pulverulent mass stationary upon them. Hence the angle of friction between wall and mass really reduces to the angle of friction of the pulverulent mass upon itself."* This certainly is the maximum value. Kötter differentiates, however, the obliquity upon the x -face from that on the θ -face. In the case of sand, on account of dilatancy, the writer will follow the Boussinesq assumption, but for soils not granular or pulverulent will obtain such values for the later numerical computations as may be had from actual tests. Cain, Darwin, and others follow Boussinesq in retaining-wall design in this respect.

The following solution is given for the equations of equilibrium:

$$\begin{aligned} X_x &= -\frac{1 - \sin. \phi}{1 + \sin. \phi} wy, \\ X_y &= 0, \\ Y_y &= -wy, \end{aligned}$$

to apply without the region limited by $x - \sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} y$, and these are the ordinary Rankine relations. Within this region, or in the zone of perturbation of the pile, the following equations hold:

$$\begin{aligned} X_x &= -\frac{1 - \sin. \phi}{1 + \sin. \phi} \frac{(y + x \tan. \phi)w}{1 + \sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} \tan. \phi} \\ Y_y &= -\frac{(y + x \tan. \phi)w}{1 + \sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} \tan. \phi} \end{aligned}$$

* "History of Elasticity," Todhunter-Pearson, Vol. II, Pt. II, p. 336.

$$Y_x = X_y = \frac{\tan. \phi_1 \sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} \left(\sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} y - x \right) w}{1 + \sqrt{\frac{1 - \sin. \phi}{1 + \sin. \phi}} \tan. \phi}.$$

In the above set of constituents, the stresses, X_x and $Y_x = X_y$, are induced stresses, that is, they are called into play on a hypothetical infinitesimal motion outward of a retaining wall by the pressure head, Y_y . In the Rankine language, they stand to each other in the relation of “cause to effect.” The pressure head of earth is “active,” and the induced lateral stress is “passive.”

In the case of piling, however, Y_x is the “active” stress. Accordingly, one would assume, very consistently, that the resultant stress on the x -face of a small element at the piling is active. To provide for this case, one might proceed in the ordinary manner of Rankine, namely, take $X \left(\frac{1 - \sin. \phi}{1 + \sin. \phi} \right)^2 \leq Y_y$. This would appear to introduce ambiguities into the problem. The writer will proceed as follows:

Call the passive or smaller ratio of Rankine r_p and the active or larger ratio r_a . If Y_x and X_x are active, it seems reasonable to assume that the zone of perturbation due to pile action is larger. The wedge defining this region, the slant height of which is $x = \sqrt{r}y$, must intersect the head of the pile at the ground, because, whether “active” or “passive,” the shears vanish at the pile for $y = 0$. Let $x = \frac{1}{\sqrt{r_p}} y = \sqrt{r_a}y$. The following is still true: At any point without the region the general Rankine relations hold. The constituents hold in general for all values of the variable within the region; the intensities become zero at the surface; while, at the pile, for $\phi = 0$, the ordinary Rankine relations still hold, the more general relations hold for ϕ_1 .

To obtain a direct application of this, it is necessary to integrate the intensity, Y_x , over the surface of the pile at $x = 0$. First call

$$Y_{x0} = \tan. \phi_1 \frac{1 + \sin. \phi}{1 - \sin. \phi} \frac{wy}{1 + \sqrt{\frac{1 + \sin. \phi}{1 - \sin. \phi}} \tan. \phi_1} = \frac{fr_a}{1 + f\sqrt{r_a}} wy$$

for simplicity of expression, where f = the coefficient of friction at the pile, w = the specific weight of the earth, and y = the variable

depth. To apply this intensity in practice, where cylindrical and slightly tapering piles are used, the assumption of Vierendeel and the others is made, that the tangential intensity is independent of the shape of the perimeter of the pile, a common enough assumption in other branches of engineering.

By integration there results for a working formula comparable with the Sanders' type in simplicity, but based on static considerations:

$$W = \frac{fr_a}{1 + f\sqrt{r_a}} w\pi D \int_0^L y dy,$$

$$\text{or } W = \frac{fr_a}{1 + f\sqrt{r_a}} w\pi D \frac{L^2}{2},$$

where πD is the mean circumference, L is the length of pile, w is the weight of a cubic unit of earth, f is the coefficient of friction, and r_a is the larger Rankine ratio, namely, $\frac{1 + \sin. \phi}{1 - \sin. \phi}$, ϕ being the angle of internal friction, or so-called angle of repose.

Now $w\pi D \frac{L^2}{2}$ is the normal hydrostatic pressure on a cylinder for $w =$ specific weight. Accordingly, $\frac{fr_a}{1 + f\sqrt{r_a}}$ is a more or less rational friction factor for the same. While the formula is quite as simple as Vierendeel's, it would seem to possess a more rational derivation.

Effect of the Base.—In the above working formula, upward pressure on the base and sides, other than that due to tangential stresses, has been disregarded as relatively negligible. This will need to be discussed.

First, in the case of stiff earths possessing some elastic properties, where a more or less well-defined "conoid of pressure" may be assumed to exist, the pressure over the base of this conoid is naturally assumed to be continuous. The principle of equipollent loads (de St. Venant) shows that it is only in the region of the point that the real distribution of stress has any effect. In the case of a peg driven into a wooden beam and carrying a load on its head acting longitudinally to the axis of the peg, the local effect of the stress would be much the same whether the point of the peg entered a small knot-hole or butted against sound wood. The assumption, then, will be that the pressure under the pile is practically that which exists a foot or two horizontally away. In the horizontal projection of the lateral surface, the Y_y is assumed to be that for $x = 0$, $y = y_1$.

In the case of the Goodrich experiment, with the box and glass walls, when the model pile is pushed down in the sand close to the glass face the inverted paraboloid forming under the squared end of the pile is only two or three end diameters of the pile in height. The “eddy” action is largely confined to this small region. The Rankine hypothesis, of necessity, assumes that the action is felt at the surface, by reason of incompressible molecules arranged in most compact space. It is believed, however, as has been shown by Bauschinger, Darwin, and others, that considerable interstitial free space exists in any pulverulent soil; accordingly, when the pressure occurs it simply compacts the soil in the immediate region concerned. The assumption, then, for semi-liquid materials, it would seem to be reasonable, may be similar to that of the previous paragraph, namely, that the pressure at the point and sides suffers no sudden breaks or discontinuities from that a short distance away.

Accordingly, it is thought that the base and lateral buoyancy, when the point is down, may be amply provided for by taking L a few diameters longer, say to the point of the inverted paraboloid, instead of to the point of the pile, and using this length with the mean diameter of the pile. Such data, of course, would need to be determined experimentally; or, perhaps it might be better to consider the friction factor, $\frac{fr_a}{1 + f\sqrt{r_a}}$, simply as an empirical parameter to be determined for various cases.

Some Data.—In lieu of any precise coefficients of friction and angles of friction, no great precision can be expected in fitting the formula to actual cases. In the following the formula has been applied to the Annapolis* tests, J. P. Carlin, Assoc. M. Am. Soc. C. E., Engineer in Charge, also to the well-known Louisiana† pile (Proctorville, La., 1856–57).

Case 1 is worked out below in full, to show the effect of vertical pressure on the side and base. The developed surface of contact

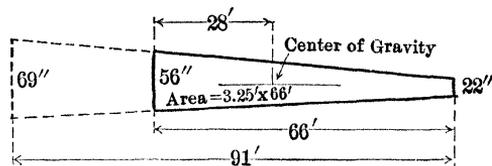


FIG. 6.

* *The Engineering Record*, May 11th, 1901; also *Transactions*, Am. Soc. C. E., Vol. XLVIII, pp. 215 and 218.

† Baker's "Masonry," 8th ed., p. 247.

TABLE 1.—ANNAPOLIS TESTS.

Number.	Length.	Point.	Butt.	Hammer.	Fall.	Actual test load.	Formula.	Goodrich $\frac{10WH}{Sp}$	Remarks.
1	91	7	22	2300	22	75 000	105 200	96 500	$L = \left\{ \begin{array}{l} 60 \text{ ft. mud} \\ 6 \text{ ft. sand} \end{array} \right\} = 66 \text{ ft.}$
2	91	7	22	"	22	85 090	133 610	112 000	$L = \left\{ \begin{array}{l} 60 \text{ ft. mud} \\ 12 \text{ ft. sand} \end{array} \right\} = 72 \text{ ft.}$
3	73	9	18	"	$33\frac{1}{2}$	34 000	95 450	67 000	$L = 61 \text{ ft. of mud}$
4	30	12	8	"	22	38 000	54 400	84 500	Sand.
5	32	13	9	"	22	110 000	66 500	168 700	Sand.

For Cases 1, 2, and 3, $f = 0.1$ and $\phi = 15^\circ$ was used, for Cases 4 and 5, $f = 0.268 = \tan. \phi$, and $\phi = 15^\circ$.

(See Patton's "Civil Engineering," 1st ed., p. 487, for actual test for f in liquid mud.) w is taken at 110 lb. per cu. ft.

is a trapezoid. The projected area on a horizontal plane is 1.45 sq. ft. The area of the base is 0.267 sq. ft.

$$\frac{1 + \sin. 15^\circ}{1 - \sin. 15^\circ} = \frac{1.259}{0.741} = 1.70$$

$$\sqrt{1.70} \times 0.1 = 0.131$$

$$\frac{0.1 \times 1.70}{1.131} \times 110 \times 3.25 \times 66 \times 28 = 99\,500, \text{ friction on side,}$$

$$\frac{66}{1.131} \times 110 \times 0.267 = 1\,700, \text{ pressure on base,}$$

$$\frac{28}{1.131} \times 110 \times 1.46 = 4\,000, \text{ pressure on projected face,}$$

$$W = 105\,200, \text{ total calculated load.}$$

The common hydrostatic methods of area multiplied by mean head is used instead of the integration. The Rankine pressure on the base and projected side is 5 600 and 13 000 lb., respectively, for $\phi = 15$ degrees.*

For the Louisiana case, Baker's "Masonry" gives the following data: Pile was 12 in. square throughout, driven 29.5 ft., and bore 29.9 tons

* Note that 100 lb. instead of 110 lb. per cu. ft. will give about 10 000 lb. smaller.

without settlement. It settled slowly under 31.2 tons. The same values of f and ϕ are used as in Cases 4 and 5 of the Annapolis test, namely, $f = 0.268$ and $\phi = 15^\circ$, or

$$\frac{0.268 \times 1.70}{1 \times \sqrt{1.70} \times 0.268} \times 110 \times 4 \times \frac{29.5^2}{2} = 64\,800, \text{ friction on sides,}$$

$$\frac{29.5}{1 \times \sqrt{1.70} \times 0.268} \times 110 \times 1.0^2 = 2\,320, \text{ pressure on base,}$$

$$W = 67\,120, \text{ total calculated load.}$$

The static treatment presented gives an average deviation from fact about commensurate with that of the most rational dynamic formula. It is thought, however, that by obtaining actual experimental factors, based on the physical qualities of the pile, a much closer agreement would be possible. Most of the recorded data, being made solely with reference to their availability for comparison and study of dynamic formulas, omit such information.

The formula presented, being of the form of that given by Vierendeel, who neglects the basal action, it should be easy, by drawing tests, to ascertain the friction, expressed as a function of length and mean diameter, for different soils.

Dilatancy of Granular Media.—In his interesting discussion of the Goodrich paper, the late Mr. Gould remarked:*

“Another element which makes for safety, but which baffles calculation, is the clinging action of the material through which the pile is driven, and which action is set up immediately after it has been allowed to come to rest. It is often impossible to draw a defective pile even a very short time after it has been driven, unless a few blows be given by the hammer to start it, when it may come up very easily.”

It is believed that the theory of the dilatancy of media composed of rigid particles in contact, as proposed by Professor Osborne Reynolds,[†] will account for this phenomenon noticed and recorded by many engineers. While the theory was formulated to account for the sub-mechanics of the universe, not the least of its claims is that it will place the theory of earth pressures on a true foundation. He says:

* *Transactions*, Am. Soc. C. E., Vol. XLVIII, p. 214.

† *Philosophical Magazine* (London, E., & D.), Vol. XX, 1885, pp. 469 *et seq.*; also Reynolds' Works, Vol. III.

“I will point out the existence of a singular fundamental property of such granular media which is not possessed by known fluids or solids. * * * I have called this unique property of granular masses ‘dilatancy,’ because the property consists in a definite change of bulk, consequent on a definite change of shape or distortional strain, any disturbance whatever causing a change of volume and generally dilation.

“In the case of fluids, volume and shape are perfectly independent; and although in practice it is often difficult to alter the shape of any elastic body without altering its volume, yet the properties of dilation and distortion are essentially distinct, and are so considered in the theory of elasticity. In fact there are very few solid bodies which are to any extent dilatible at all.

“With granular media, the grains being sensibly hard, the case is, according to the results I have obtained, entirely different. So long as the grains are held in mutual equilibrium by stresses transmitted through the mass, every change of relative position of the grains is attended by a consequent change of volume; and if in any way the volume be fixed, then all change of shape is prevented.”

The mathematics of this is long and difficult, in general. The essential features, as it is desired to apply them in reference to Mr. Gould’s remarks, may be illustrated by the following experiment:

“If we have in a canvas bag any hard grains or balls, so long as the bag is not nearly full it will change its shape as it is moved about; but when the sack is approximately full a small change of shape causes it to become perfectly hard. There is perhaps nothing surprising in this, even apart from familiarity; because an inextensible sack has a rigid shape when extended to the full, any deformation diminishing its capacity, so that contents which did not fill the sack at its greatest extension fill it when deformed. On careful consideration, however, many curious questions present themselves.

“If, instead of a canvas bag, we have an extremely flexible bag of india-rubber, this envelope, when filled with heavy spheres (No. 6 shot), imposes no sensible restraint on their distortion; standing on the table it takes nearly the form of a heap of shot. This is apparently accounted for by the fact that the capacity of the bag does not diminish as it is deformed. In this condition it really shows us less of the qualities of its granular contents than the canvas bag. But as it is impervious to fluid, it will enable me to measure exactly the volume of its contents.

“Filling up the interstices between the shot with water so that the bag is quite full of water and shot, no bubble of air in it, and carefully closing the

mouth, I now find that the bag has become absolutely rigid in whatever form it happened to be when closed.

“It is clear that the envelope now imposes no distortional constraint on the shot within it, nor does the water. What then, converts the heap of loose shot into an absolutely rigid body? Clearly, the limit which is imposed on the volume by the pressure of the atmosphere.

“So long as the arrangement of the shot is such that there is enough water to fill the interstices the shot are free, but any arrangement which requires more room is absolutely prevented by the pressure of the atmosphere * * *.

“The very finest quartz sand, or glass balls $\frac{3}{4}$ in. in diameter, all give the same results.”

It would seem that such a state of affairs would tend to exist after the driving, in the final rearrangement of the particles in granular soils, and that the phenomenon may throw light on the case, as cited by Mr. Gould. It would further seem to favor an elastic theory, especially as one, to use another of Professor Reynolds' illustrations, may note the firmness of a sandy beach after the recession of a wave, in contradistinction to the quite fluid effect of the dry sand. The phenomenon deserves to be studied in its relation to the pile.

The writer was led to appreciate the importance of the static point of view, in the theory of the pile, through the suggestions of G. S. Williams, M. Am. Soc. C. E. He was introduced to the Boussinesq and Kötter theories* by Professor Alexander Ziwet. In making acknowledgment to these authorities and to Professor A. B. Pierce for discussion and criticism of these theories, the writer does not wish to be construed as committing them to these views.

* A number of solutions involving different phases of this problem may be easily found. Among these the writer would call attention to a treatment of the glacier by Hopkins, Cambridge Phil. Soc. *Transactions*, Vol. 8, 1849, in which he treats the glacier as an elastic body forcing its way between the walls of the valley, exerting lateral forces and friction on the sides of the stream quite comparable to the action of a pile in the earth. The Boussinesq literature, however, affords the most suggestion in addition to that of Kötter.

DISCUSSION.

Mr.
Wagoner.

LUTHER WAGONER, M. AM. SOC. C. E. (by letter).—The writer believes this to be a timely paper, and that, with sufficient data concerning the physical properties of the resisting media, it will be possible to construct a satisfactory formula for computing the safe bearing load of a pile by purely statical methods. The assumption of uniform values of the angle, ϕ , is probably incorrect, and in what follows the writer will consider a soil composed of mud to an indefinite depth. The top layer of such a mud offers but little resistance to penetration, and the resistance appears to increase more rapidly than that of the static pressure due to depth. In tests made by the writer in Islais Creek, which is an arm of San Francisco Bay, the top mud is seen to be full of blow-holes, and relatively there is much water to each unit of solid. A sample of the material taken at a depth of 10 ft., or 15 ft. below mean tide, weighed 105 lb. per cu. ft. When dried at steam heat it lost 34% of its weight, or 35.7 lb. of water per cu. ft., which left 0.428 cu. ft. of volume for 69.3 lb. of solids, or $\frac{69.3}{0.428} = 162$ lb. per cu. ft. for the weight of the solid material, which is about the weight of the rock from which the mud is derived.

From the foregoing this important deduction can be made: The mud is the débris of rocks, in an exceedingly fine state of subdivision, mixed with some clay. Any unit of this material possesses an enormous surface as compared with its volume, and to this surface the water is held by capillary action. Both the water and the separate units of the mud are practically incompressible. The mixture is also incompressible, and as long as it is saturated it must continue to behave as an imperfect fluid. For example, a pile driven into such material does not compress the soil laterally to any extent, but elevates the soil near the pile. A blanket of rock or earth can be floated upon the mud, and there will be little or no subsidence if the lateral flow can be prevented by retaining walls, or, if a flat enough slope is given to the mud thus covered it will stand. A sharp distinction must be made for soils above and below the permanent ground-water level; in the first case, the soil has voids, once occupied by water, and is compressible by pile-drivers, and it is in such cases that a conical pile gives good results. In the second case, there can be no compression, but there is an upward displacement.

Mr.
Wagoner.

The mass of mud may be thought of as made up of small flattish particles of solids, arranged, as a rule, nearly level, and the interstitial space filled with water, or as a solid having an infinite number of capillary tubes of variable diameters. It is obvious that under sufficient pressure the water will move laterally through the tubes, and the solids will be brought closer together, at the same time, the diameters of the tubes will be reduced, and a greater head or pressure will be required to produce further flow; but, as compression ensues, and the diameters of the tubes are reduced, the rate of subsidence must become slower; and, if for total subsidence we take y , and for time x , it is clear that the resulting curve must be an asymptote to the x axis.

That the above is sound reasoning is borne out by various known settlements, during long periods of time, where structures have been founded upon clay or muds. In San Francisco a notable example of settlement has occurred: A roughly semicircular body of land, one mile long and half a mile wide, was reclaimed from the bay, and upon this the business section of the city is built. A sea-wall, made by dumping rock into a dredged trench, marks the outer boundary. Inside of the sea-wall the streets have sunk slowly below grade, while adjacent buildings upon long piles have remained intact; a total subsidence of several feet, perhaps from 5 to 8 ft., has taken place in the past thirty years. The subsidence of the sea-wall has been very small in amount, certainly not greater than one-tenth of the landward subsidence. This appears to the writer to be a case of steady loading causing the water to percolate slowly seawards, thus allowing settlement to occur.

S. W. Hoag, Jr., M. Am. Soc. C. E., in testing the soil for the Chelsea Docks, New York City, where the mud is 180 ft. deep, drove four groups of four piles each to a penetration of 50 ft., loaded each group with concrete blocks, and noted the rate of subsidence. The experiment ended at 51 days, but the curves of subsidence show a total movement of from $1\frac{1}{2}$ to 3 in. where the load was 18 tons on a plain pile and 34.6 tons on a lagged pile. The curves mentioned above are markedly asymptotic to the time axis. These experiments deserve careful study by any one desiring data regarding the behavior of piles in a mud soil.*

It should not be forgotten that a formula may give correctly the immediate or present ultimate bearing load of a pile, and yet serious damage may arise from slow and long-continued settlements; and it is

* Report to John A. Bensel, Engr. in Chief, N. Y. Docks, Nov. 24th, 1902.

Mr. Wagoner. the writer's belief that the attention of the investigator should be turned toward the physics of the soil; thus, probably in a rational formula, he will be able to forecast settlement as well as immediate loads.

Mr. Griffith. JOHN H. GRIFFITH, ASSOC. M. AM. SOC. C. E.—Since this paper was printed, the writer's attention has been called to the fact that, in the Annapolis tests, by Mr. Carlin, some of the piles had several feet of water above the top surface of the earth surrounding them. In a more general treatment, such a case should be considered, that is, the boundary relation at the surface should be satisfied for the head of water existing, instead of zero, as in the case worked out. If the different strata of soil are considered, this will complicate the problem still further.

At the time of this criticism, the writer made some approximate calculations modifying the result for Case 1 which would indicate that the water pressure would cause an increase in the value given for this pile of about 10 000 to 12 000 lb.

In a practical theory, it will be necessary to assume a homogeneous or isotropic soil medium, as has been done heretofore in engineering studies of earth pressure; also to adhere to the common assumption of an upper surface free from stress, as this will apply to the greater number of cases in experience. Otherwise, it will be necessary to increase the specific weight of the earth a proportionate amount, or actually modify the stress constituents. All the problems of engineering are really beyond the realms of analysis in the rigorous aspects, and it is only by defining the media specifically, or averaging the conditions, that practical solutions may be effected.

Another point has been raised, regarding the weight of the pile, or dead load, which did not appear in the calculations. Since it has been remarked to the writer that the specific weight of yellow pine or oak when soaked with water may even exceed that of water, so that such timbers will sometimes sink below the surface, the dead load will compensate the value given in the first criticism to the amount of several thousand pounds.

By far the most important factor in an elementary static analysis is the question of friction and internal angle of friction. The writer believes that in the main the experimental methods of approach heretofore in vogue are often more or less unsatisfactory because based on false premises of operation. Space will only suffice for one illustration in the

pile theory, and that is the common notion advanced by many engineers that the friction in drawing a pile is equal to that encountered in driving it. This conclusion, if not absolutely erroneous, is believed to be not true in general. The argument can only be imperfectly stated, as follows: When the pile is driven the tubes of stress will start out from the periphery of the pile and spread over a correspondingly large area, probably in a conoidal distribution such as has been well described by Goodrich. Assuming that a cohesive material under pressure will be subject to the laws of elastic analysis, such a notion would appear to be confirmed by the well-known tests of Professors Carus-Wilson* on the beam for surface loading, and of Marston† for the roller problem, if the principle of equipollence is strained a little from its usual applications. On the other hand, in withdrawing the pile, such is not the case. One might indeed conceive of the conoid as inverted, with its base at the free surface of earth, in which case, for the pulverulent material, the friction on the periphery would be equated to the weight of the material under discussion.

The treatment of Patton will illustrate this point in the fact that he uses equations for minimum and maximum loadings, respectively, taking $\frac{1 - \sin. \phi}{1 + \sin. \phi}$ for the minimum and $\frac{1 + \sin. \phi}{1 - \sin. \phi}$ for the maximum lateral factor, according to Rankine. Now, if the pile is being driven, the $\frac{1 + \sin. \phi}{1 - \sin. \phi}$ is operative while $\frac{1 - \sin. \phi}{1 + \sin. \phi}$, the reciprocal, holds if for any reason the pile tends to rise. Accordingly, assuming the coefficient of friction to be the same, as in the ordinary cases in other fields of engineering, the total friction on the pile in the case of withdrawal is less than that in driving. But, for a cohesive soil, it would naturally be expected that a closer agreement would hold between the two cases.

From the discussion by Mr. Gould on the paper by Mr. Goodrich, quoted previously, the writer would be led to infer that a considerable arching action of the ring elements around the pile, which are subject to hoop compression, exists; and, after the pile moves upward, it encounters little resistance, due to the earth friction, by reason of this; but, whether the friction at the start in withdrawal is equal to that

* *Proceedings*, Physical Society of London, Vol. XI, 1891, p. 194. Also Sir G. G. Stokes' works. A good discussion is in Bovey's "Mechanics on Surface Loading of Beams."

† *Transactions*, Am. Soc. C. E., Vol. XXXII, pp. 99 and 273.

Mr. Griffith. in driving, opens a path for discussion by the profession. It is hoped that engineers who have the opportunity to withdraw piles may compare their results with the loading the pile previously carried, or was supposed to carry.

The whole static problem would appear to be a more general statement of the problem usually ascribed to Cerruti and amplified in its various phases by Boussinesq, Hertz, and various mathematicians, viz.:

When an elastic cylinder or cone of revolution, initially free from stress, is inserted in an elastic medium of relatively large extent, the upper surface of which is flat or nearly so, and normal to the axis, what are the strains and stresses in both due to vertical loading of the cylinder?

For the practical purposes of the engineer, he will specialize this problem to the case of a rigid pile and a pulverulent medium, and ignore the discontinuity at the base for a first approximation. Professor Burr, in his "Mechanics of Materials," has given a derivation of the equations of equilibrium in cylindrical co-ordinates which will be more easily understood by the practical engineer than by attempting the transformation of co-ordinates usually given in the treatises. The tubes of stress may be considered to start from the surface of the pile with an angle of inclination to the axis equal to the angle of friction of the earth on the pile. The principal stress is co-axial with this tube. It will travel downward and radially until ultimately the integrity of the tube is destroyed and displacement of the particles takes place at some distance from the pile, in conformity with the Rankine analysis.

The Rankine equations will satisfy the equations at a distance from the pile, but cannot be assumed to hold locally on account of the tangential or friction stresses which must be distributed into the earth around the pile. Hence a correction or parameter must be inserted in the Rankine stress values, and the resulting values inserted in the equations of equilibrium. The values of these corrections must be those which are compatible with the conditions at the ground and at the pile. The resulting constituents must hold for all values within the region affected by the tangential stresses. What is this region? The writer's experiments thus far have shown rather unsatisfactory results. In the first approximation it will be natural to take the region as limited by a cone, but this does not seem to give close agreement with fact. The

matter deserves investigation by those interested in placing the pile on a rational basis, where it belongs. Mr. Griffith.

The writer has followed Mr. Wagoner's discussion with interest. His citation of experiments for getting the specific weight, etc., of a soil in its ocean bed is quite pertinent at this time.

With regard to the friction angle, the writer is prepared to agree that the assumption of "uniform values" of this is not rigorous, as has already been intimated by various investigators, say, Darwin and Wilson; but, for the purposes of the practical engineer, it must be assumed as constant for any particular medium, for reasons of expediency. Such expressions for the variation of the coefficient of friction, as have been given heretofore, are too complicated to introduce into the theory in the early stages of its evolution, but must ultimately find a place in a rational analysis of earth pressure. Most engineers will be content if they can approximate to the true status of loading, with a probable error of, say, 15 or 20% \pm , such as one might expect in good bridge design.

The discussion of time rate of strain variation, involving questions of subsidence, soil viscosity, etc., enters a comparatively virgin field of study and experiment. Such investigations, considered in connection with Slichter's studies, already cited, will undoubtedly receive an important place in a final analysis of the pile, from the static point of view.

SUMMARY.

The practical points which are brought out in this paper are as follows:

a.—Attempts to get at the loading on a pile by a dynamic theory are indirection of effort. The direct method is static, and should carry out the work inaugurated by Patton, Vierendeel, Desmond, and others. Both methods will co-operate in fixing load limits, and will serve as check operations.

b.—Their methods, while not rational of form, give an efficient means of first approximation. Their value may be augmented in efficiency by abandoning the Rankine ratio altogether and replacing by a constant, which constant is to be selected on the basis of actual tests from ultimate loads for similar soils, or from drawing tests when properly interpreted. Using this constant with the "hydrostatic pressure"

Mr. Griffith. due to earth, a simple integration gives the real load value empirically when multiplied by the coefficient of friction; or, more directly, owing to the uncertainty of this coefficient, it may be included in the determination of the constant.

c.—The notion of using the full Rankine pressure on the base is probably in error, except for short piles and shallow foundations. The Vierendeel form of neglecting this in comparison with that on the periphery would seem to be more favorable to fact in a continuous medium. This does not apply to a discontinuity at the base such as would be implied by a rock foundation under the pile.

d.—The pile is susceptible to an elastic treatment to a considerable extent. Engineers should determine the moduli of soils, pulverulent, plastic, or other, just as in steel or wooden structures. The fact of average values of moduli may apply as properly as it does in concrete or wood.

e.—The Rankine theory should ultimately be displaced in favor of an elastic treatment, because, by reason of strain, viscosity, and cohesion, it can never fit the facts.

f.—The question of dilatancy should be studied in its relation to the pile.

g.—Those who are interested in the development of a correct theory of the pile should preserve the physical data for a static analysis, such as that of the pile periphery, its slope, and diameters, also the soil data and stratification. It has been the writer's experience, in attempting to correlate figures with facts, that most of this has been rejected in the dynamic analysis.

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