Chapter 5

Applying Newton's Laws

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Newton's Laws

1st Law: An object at rest or traveling in uniform motion will remain at rest or traveling in uniform motion unless and until an external force is applied

2nd Law:

$$\vec{F}_{net} = m\vec{a}$$

$$F_{net,x} = ma_x, \quad F_{net,y} = ma_y, \quad F_{net,z} = ma_z$$

3rd Law: For every Action, there is an equal but opposite Reaction

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Part 1

Some particular forces

Some particular forces

In Chapter 5 we deal with following forces

- ✓ Gravitational force F_g
- ✓ Normal force N
- ✓ Tension T
- ✓ Frictional force f
- ✓ Centripetal force F_c

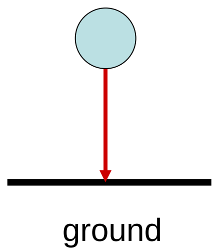
new concepts

The Gravitational Force

A gravitational force on a body is a pull that is directed toward a second body.

In chapter 5 we do not discuss the nature of this force, and we usually consider that the second body is Earth

We mean that the gravitational force pulls directly toward the center of Earth – that is, directly down toward the ground



Magnitude: $F_g = mg$

Direction: toward the ground

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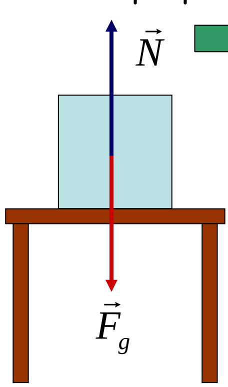
new concepts

The Normal Force

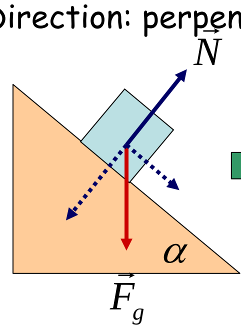
When a body presses against a surface, the surface deforms and pushes on the body with a normal force that is perpendicular to the surface



$N = mg$

Magnitude: comp. of mg $\perp$ to the surface

Direction: perpendicular to the surface



$N = mg\cos(\alpha)$

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new concepts

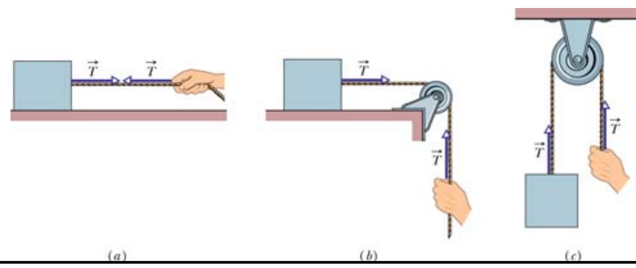
Tension force

When a cord (a rope, cable, …) is attached to a body a pulled taut, the cord pulls on the body with force T.

Normally: the cord is considered massless (comparing to the body's mass) and unstretchable (it si only a connection between bodies)

Magnitude: T

Direction: away from the body and along the cord



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Part 1b

Frictional forces

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new concepts

Frictional Forces

Just one example

The bad:

About 20% of the gasoline used in an automobile is needed to counteract friction in the engine and in the drive train.

Then … air resistance

The good

Without frictional forces we could not get an automobile to go anywhere

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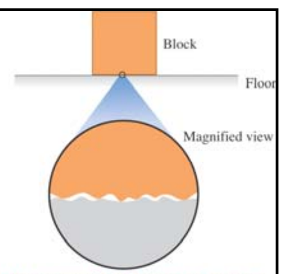
Frictional Forces (cont)



On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.

Microscopically, friction is a very complicated phenomenon
(spikes, cold welding, …)

Macroscopically, at large-scale level, it is relatively simple

Phenomenologically (empirically), force of friction acting on a body is directly proportional in magnitude to normal force of the surface on the body:

$$\left|\vec{f}\right| \propto \left|\vec{N}\right|$$

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Properties of Friction

Property 1: If the body does not move, then the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface balance each other.

Property 2: The magnitude of $\vec{f}_s$ has a maximum value $f_{s,max}$ that is given by

$$f_{s,\max} = \mu_s N$$

where μ_s is a **coefficient of static friction**, and N is the magnitude of the normal force on the body from the surface

Force of static friction: one must overcome (exceed) it in order to initiate motion of the body along the surface

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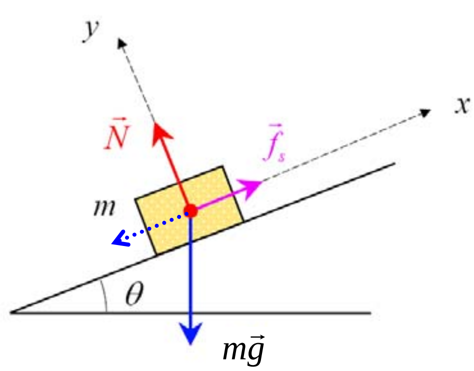
Formal derivation

When block is not moving, friction force compensates x-component of gravitational force: $f_s = mg*\sin(\theta)$.
However by definition $f_s=\mu_s N = \mu_s mg*\cos(\theta)$.
Then $mg*\sin(\theta) = \mu_s mg*\cos(\theta)$, and $\mu = \tan(\theta)$



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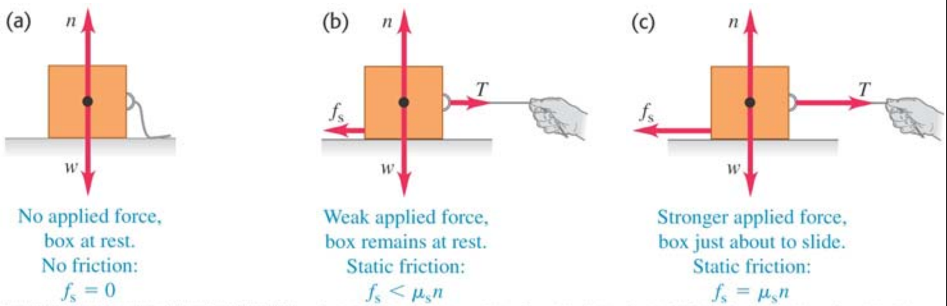
$$f_{s,\max} = \mu_s N$$

Static Friction



Properties of Friction (more)

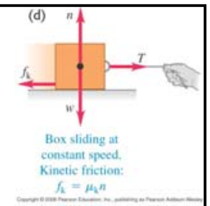
Property 3: If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

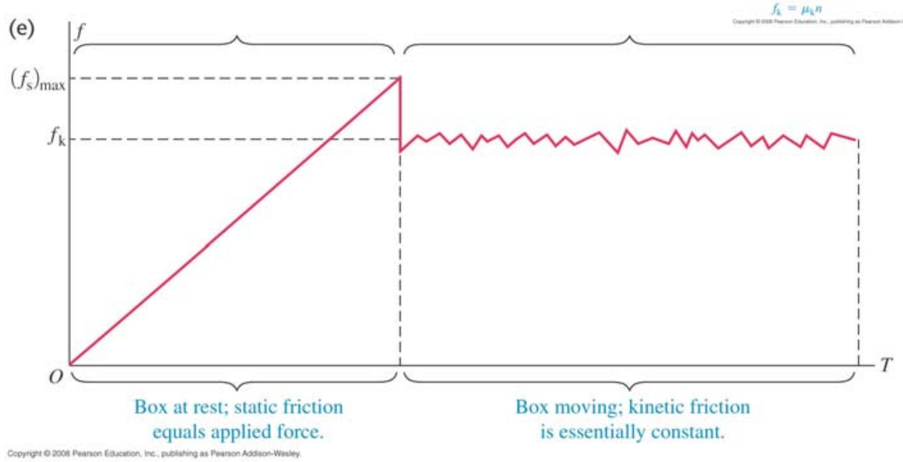
$$f_k = \mu_k N$$

where μ_k is **the coefficient of kinetic friction**

Force of kinetic friction: acts while the body moves along the surface

Static and Kinetic Friction





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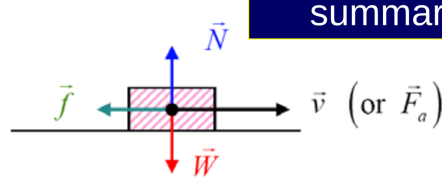
summary

Frictional Forces



Static Friction

Direction: parallel to the surface, and is directed opposite the component of an external force.

Magnitude $f_s = \mu_s N$

Kinetic Friction

Direction: always opposite to direction of velocity

Magnitude $f_k = \mu_k N$

Frictional force is independent of the area of contact between the body and the surface.

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How about motion in fluids

A fluid is anything that can flow – either a gas or liquid

When a body moves through the fluid (or the fluid moves past the body), the body experience a **drag** force that opposes the relative motion.

At low speed: $f = k_1 v$

At high speed $f = k_2 v^2 = D = 0.5 C\rho A v^2$

Terminal speed: when a body falls at constant speed, i.e.

$$0.5 C\rho A v^2 = mg$$

$$v = \sqrt{\frac{2mg}{C\rho A}}$$

C – the drag coefficient (0.2 – 1.0)
A – effective cross section
ρ - the air density

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Terminal speed

$$v = \sqrt{\frac{2mg}{C\rho A}}$$

object	speed (m/s)	speed (mph)	distance (m) 95%
shot	145	316	2500
sky diver	60	130	430
baseball	42	92	210
basketball	20	44	47
raindrop	7	15	6
parachutist	5	11	3

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Part 2

Free-Body Diagrams

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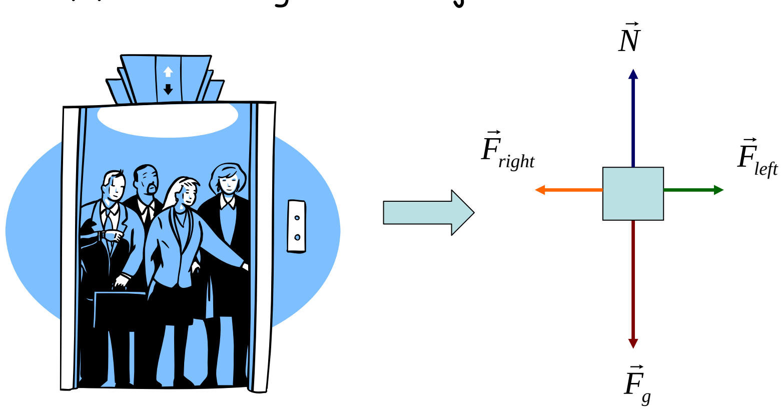
new concepts

Free-body diagram as a powerful tool

IDEA: replace an actual environment of an object as a set of forces acting on that object



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Free-body diagram as a powerful tool

Free-body diagrams show all forces acting on a body

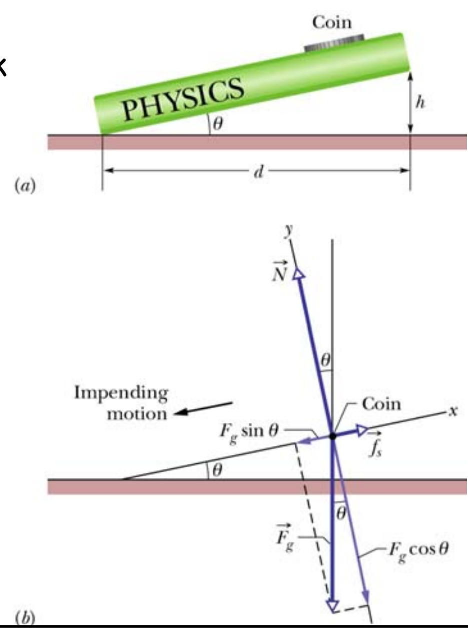
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots \vec{F}_n = \sum_i^n \vec{F}_i$$

Key ideas for drawing a free-body diagram:

1. Include: ALL forces acting on the body matter.
2. When a problem includes more than one body – draw a separate free-body diagram for each body.
3. Not to include: any forces that the body exerts on any other body.
4. Not to include: non-existing forces (no object – no force).

example

a coin on a book



Part 3

Problems with gravitational, normal and tension forces

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Objects in equilibrium

According to Newton's first law for an object in equilibrium (at rest)

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots \vec{F}_n = \sum_i^n \vec{F}_i = 0$$

$$F_x = F_{x1} + F_{x2} + \ldots F_{xn} = \sum_i^n F_{xi} = 0$$

$$F_y = F_{y1} + F_{y2} + \ldots F_{yn} = \sum_i^n F_{yi} = 0$$

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Objects in motion

According to Newton's second law

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots \vec{F}_n = \sum_i^n \vec{F}_i = m\vec{a}$$

$$F_x = F_{x1} + F_{x2} + \dots F_{xn} = \sum_i^n F_{xi} = ma_x$$

$$F_y = F_{y1} + F_{y2} + \dots F_{yn} = \sum_i^n F_{yi} = ma_y$$

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example

example 1: one object in 1D

an object on a string (equilibrium)

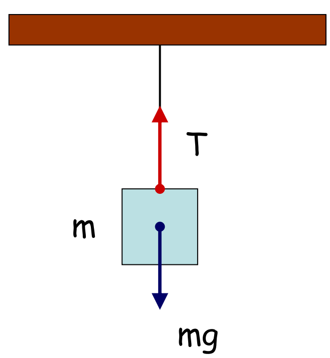
Given: m

Unknown: T

$$\vec{F}_{net} = \vec{T} + m\vec{g} = 0$$

$$\begin{cases} ma_x = & 0 \\ ma_y = & T - mg = 0 \end{cases}$$

$$T = mg$$



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example

example 2: two objects in 1D

two objects on a string (equilibrium)

Given: m_1, m_2

Unknown: T_1, T_2

$$\vec{F}_{net,1} = \vec{T}_1 + \vec{T}_2 + m_1\vec{g} = 0$$

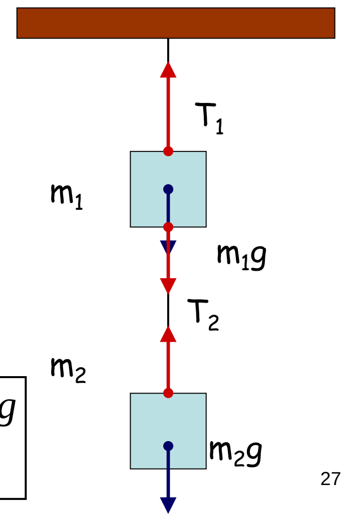
$$\vec{F}_{net,2} = \vec{T}_2 + m_2\vec{g} = 0$$

$$T_1 - T_2 - m_1 g = 0$$

$$T_2 - m_2 g = 0$$

$$T_1 = (m_1 + m_2)g$$

$$T_2 = m_2 g$$



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example

example 3a: 2D case

one object in equilibrium

Given: m, angle

Unknown: T_1, T_2, T_3

Objects: the engine & the ring

$$\vec{F}_{net,1} = \vec{T}_1 + m\vec{g} = 0$$

$$\vec{F}_{net,2} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = 0$$

$$y_1: \quad T_1 - mg = 0$$

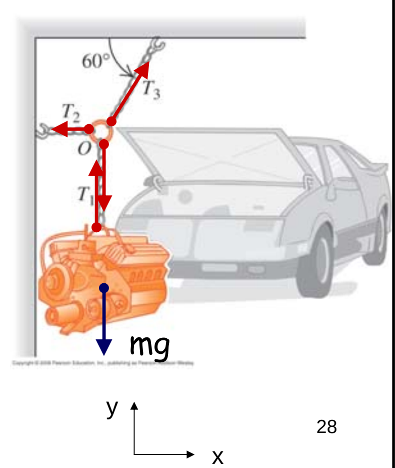
$$x_2: \quad T_3\cos(\alpha) - T_2 = 0$$

$$y_2: \quad T_3\sin(\alpha) - T_1 = 0$$

$$T_1 = mg$$

$$T_3 = mg/\sin(\alpha)$$

$$T_2 = mg\cos(\alpha)/\sin(\alpha)$$



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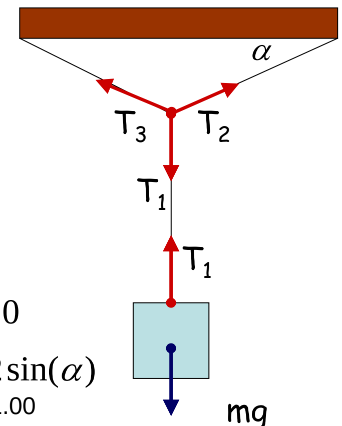
example

example 3b: 2D case

Given: m, angles

Unknown: T, T_2

$$\vec{F}_{net,1} = \vec{T}_1 + m\vec{g} = 0$$
$$\vec{F}_{net,2} = \vec{T} + \vec{T}_2 + \vec{T}_3 = 0$$



$y_1: \quad T_1 - mg = 0$

$x_2: \quad T_2\cos(\alpha) - T_3\cos(\alpha) = 0$

$y_2: \quad T_2\sin(\alpha) + T_3\sin(\alpha) - T_1 = 0$

$$T_1 = mg$$

$$T_2 = T_3 = \frac{mg}{2\sin(\alpha)}$$

α	$1/2\sin(\alpha)$
30^0	1.00
10^0	2.88
5^0	5.74
1^0	28.6

practical applications?

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example

example 4a: one object in 2D

object in equilibrium

Given: m, angle

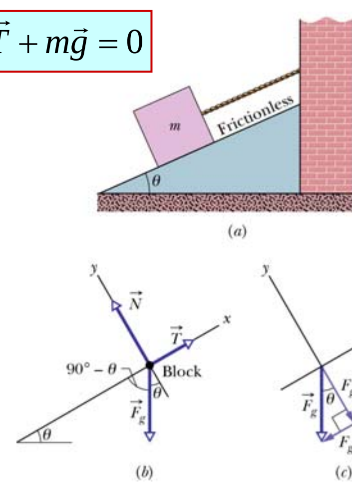
Unknown: T, N

$$\vec{F}_{net} = \vec{N} + \vec{T} + m\vec{g} = 0$$

$x: \quad T - mg\sin(\theta) = 0$

$y: \quad N - mg\cos(\theta) = 0$

$$T = mg\sin(\theta)$$
$$N = mg\cos(\theta)$$



example

example 4b: one object in 2D

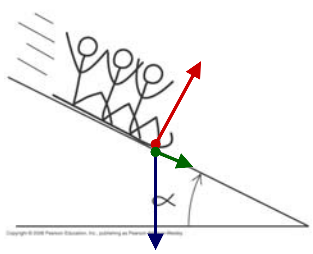
Moving object (no friction)

Given: m, angle

Unknown: a

$$\vec{F}_{net} = \vec{N} + m\vec{g} = m\vec{a}$$

(a) The situation



$$x: \quad mg\sin(\theta) = ma_x$$

$$y: \quad N - mg\cos(\theta) = ma_y = 0$$

$$a_x = g\sin(\theta)$$

$$N = mg\cos(\theta)$$

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example

example 5: two objects in 2D

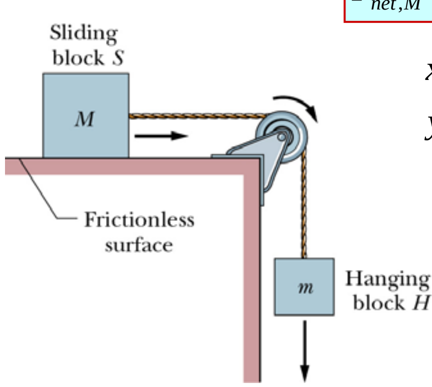
Moving objects (no friction)

Given: M, m

Unknown: T, a

$$\vec{F}_{net,m} = \vec{T} + m\vec{g} = m\vec{a}$$

$$\vec{F}_{net,M} = \vec{T} = M\vec{a}$$



$$x: \quad T = Ma$$

$$y: \quad T - mg = -ma$$

$$a = \frac{m}{m+M}g$$

$$T = \frac{mM}{m+M}g$$

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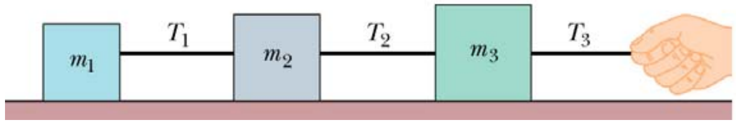
example

example 6: three objects in 1D

Moving objects (no friction)

Given: T_3, m_1, m_2, m_3

Unknown: a, T_1, T_2



Attention: all blocks have the same acceleration

$$T_1 = m_1 a$$
$$T_2 - T_1 = m_2 a$$
$$T_3 - T_2 = m_3 a$$

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Part 4

Problems with gravitational, normal, tension forces and frictional forces

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example

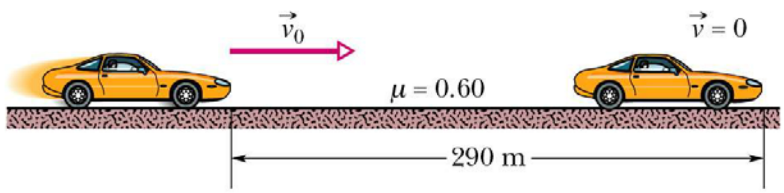
example: one object in 1D

The record for the longest skid marks on a public road was recorded in 1960 by a Jaguar on the M1 highway in England – the marks were 290 meters long.

Given: L = 290 m, kinetic friction 0.6

Unknown: v_0.

Assume: the car's deceleration was constant during the breaking



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example

example: one object in 1D

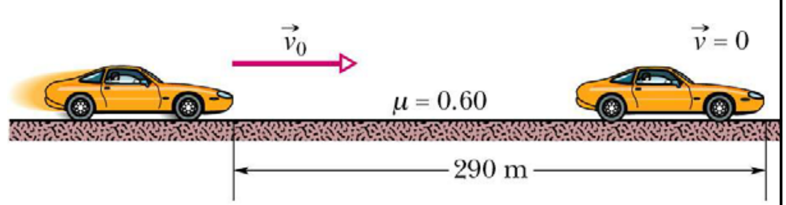
Given: L = 290 m, kinetic friction 0.6

Unknown: v_0.

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$-ma = f_k = \mu_k mg$$
$$a = -\mu_k g$$
$$v_0 = \sqrt{2\mu_k g(x - x_0)}$$

v_0 = 58 m/s = 130 mph

The result does NOT depend on the car's mass!!!



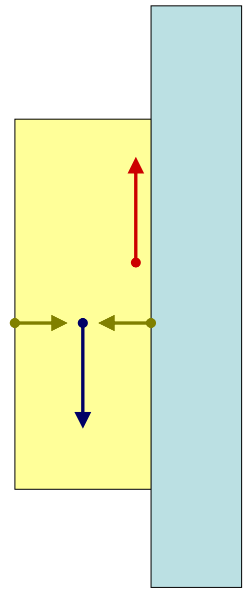
example

example: a book against a wall

Given: book 2 kg, static friction 0.4

Unknown: force to keep the book

$$mg = f_s$$
$$f_s = \mu N = \mu F$$
$$F = mg / \mu_s$$



Part 5

Dynamics of Uniform Circular Motion

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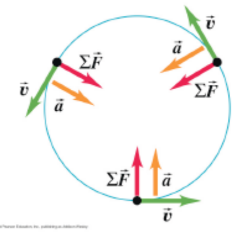
Dynamics of Uniform Circular Motion



From kinematics of uniform circular motion

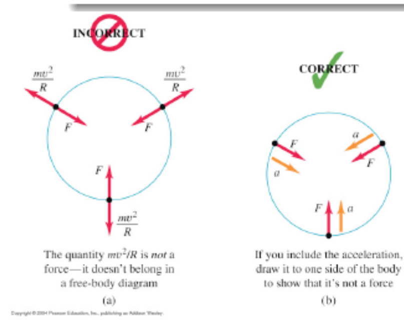
$\vec{a} = -r\omega^2\,\hat{\imath}_r = (-v^2/r)\,\hat{\imath}_r$

According to 2nd Law

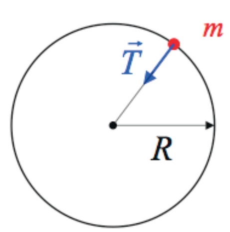
$\vec{F} = m\vec{a} = (-mv^2/r)\,\hat{\imath}_r$

Circular motion is possible only if there is an applied force

in radial direction producing required centripetal acceleration



Mass on a string in uniform motion



From 2nd Law

$T\hat{\imath}_r = -mr\omega^2\hat{\imath}_r$

String tension

$T = -mr\omega^2 = -m\,\frac{v_\theta^2}{R}$

Tension causes particle to move in circular path with constant speed

Important:

Force pulls particle toward center, rather than prevents it from moving radially outward

If string breaks, particle would move along tangent to circular path

example

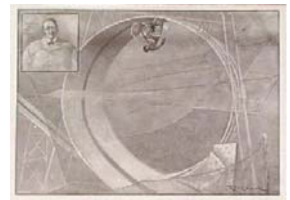
example: "Dare Devil 1901"

In 1901 circus performance

Given: R, M

Unknown: v

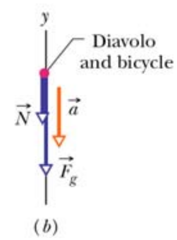
$$\vec{N} + m\vec{g} = m\vec{a}$$



$$-N - mg = -m\frac{v^2}{R}$$

$$N = m\frac{v^2}{R} - mg$$

$$N = 0: \quad v = \sqrt{gR}$$



for R = 2.7 m   v = 5.1 m/s

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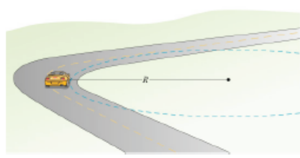
Car on circular path with flat roadbed



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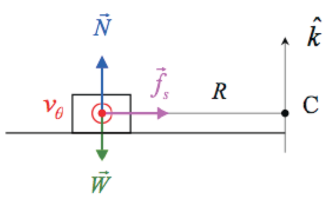
Entire centripetal force is provided by static friction



Maximum possible speed is reached when friction is maximal: $f_s = \mu_s N$

2nd Law requires that

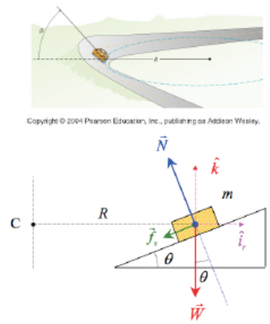
$\mu_s N = -m\,\frac{v_\theta^2}{R}$
and $N = mg$

Combining equations

$v_\theta = \sqrt{\mu_s g R}$
or $v_\theta = \sqrt{gR\tan\theta_{\max}}$
RECALL: $\mu_s = \tan\theta_{\max}$

NOTE: circular motion of car is impossible without friction

Car on circular path with banked roadbed



Centripetal force is provided
by static friction $\vec{f}_s$ and
normal force $\vec{N}$

Apply 2$^{\rm nd}$ Law:

$$-m\,\frac{v_\theta^2}{R}\,\hat{\imath}_r = \vec{N} + \vec{f}_s + \vec{W}$$

In terms of unit vectors:

$$-m\,\frac{v_\theta^2}{R}\,\hat{\imath}_r = \hat{k}\,N\,\cos\theta - \hat{\imath}_r\,N\,\sin\theta$$
$$-\hat{\imath}_r\,f_s\,\cos\theta - \hat{k}\,f_s\,\sin\theta$$
$$-\hat{k}\,mg$$

In component form:

$$m\,\frac{v_\theta^2}{R} = N\,\sin\theta + f_s\,\cos\theta$$
$$0 = N\,\cos\theta - f_s\,\sin\theta - mg$$

NOTE: circular motion of car is possible without friction

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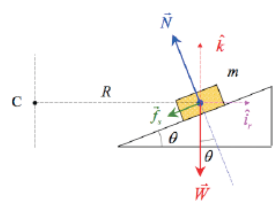
Car on frictionless ramp



When $\vec{f}_s = 0$, centripetal force
is provided by normal force $\vec{N}$ ony

Set of equations:

$$N\,\sin\theta = m\,\frac{v_\theta^2}{R}$$
$$N\,\cos\theta = mg$$

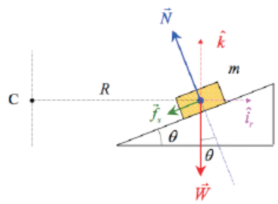
Combining equations:

$\tan\theta = v_\theta^2/gR$
or
$v_\theta = \sqrt{gR\tan\theta}$

For given radius R and ramp angle θ,
there is a speed $v_\theta = \sqrt{gR\tan\theta}$ at which car can hold frictionless ramp

For given speed v and ramp angle θ,
there is a radius $R = v^2\cot\theta/g$ at which car can hold frictionless ramp

Car on ramp with friction



Returning to general case:

$$m\,\frac{v_\theta^2}{R} = N\,\sin\theta + f_s\,\cos\theta$$
$$N\,\cos\theta = f_s\,\sin\theta + mg$$

Adding static friction increases centripetal force

$\Rightarrow$ car can travel ramp at higher speed

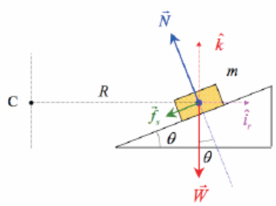
Maximum speed is reached when friction is raised to its maximum value

$$f_s = \mu_s N$$

Set of equations:

$$m\,\frac{(v_\theta)^2_{\text{MAX}}}{R} = N\,\sin\theta + \mu_s\,N\,\cos\theta$$
$$0 = N\,\cos\theta - \mu_s\,N\,\sin\theta - mg$$

Solution



Set of equations:

$$m\,\frac{(v_\theta)^2_{\text{MAX}}}{R} = N\,\sin\theta + \mu_s\,N\,\cos\theta$$
$$mg = N\,\cos\theta - \mu_s\,N\,\sin\theta$$

Eliminating N:

$$\frac{(v_\theta)^2_{\text{MAX}}}{gR} = \frac{\sin\theta + \mu_s\,\cos\theta}{\cos\theta - \mu_s\,\sin\theta}$$

Recall that

$$\mu_s = \tan\theta_{\text{max}} = \frac{\sin\theta_{\text{max}}}{\cos\theta_{\text{max}}}$$

Trig algebra exercise:

$$\frac{(v_\theta)^2_{\text{MAX}}}{gR} = \frac{\sin\theta\cos\theta_{\text{max}} + \sin\theta_{\text{max}}\cos\theta}{\cos\theta\cos\theta_{\text{max}} - \sin\theta_{\text{max}}\sin\theta}$$

$$= \frac{\sin(\theta+\theta_{\text{max}})}{\cos(\theta+\theta_{\text{max}})} = \tan(\theta + \theta_{\text{max}})$$

Maximal speed:

$$(v_\theta)_{\text{MAX}} = [gR\,\tan(\theta + \theta_{\text{max}})]^{1/2}$$

Summary of ramp experience

No speed:

$\theta \leq \theta_{\max}$

$\tan\theta_{\max} = \mu_s$

If car has zero velocity, $v_\theta = 0$,
it would not slide down the ramp
when angle θ of the ramp is smaller
than the critical angle $\theta_{\max}$

No friction:

$v_\theta = [gR\,\tan\theta]^{1/2}$

If car moves along a frictionless ramp
then its velocity v_θ should be
strongly correlated with the ramp
radius R and angle θ

Maximal speed:

$(v_\theta)_{\mathrm{MAX}} =$
$[gR\,\tan(\theta + \theta_{\max})]^{1/2}$

If car moves along ramp with friction
coefficient $\mu_s = \tan\theta_{\max}$, then
maximal velocity $(v_\theta)_{\mathrm{MAX}}$
is determined by ramp
radius R, its angle θ and $\theta_{\max}$