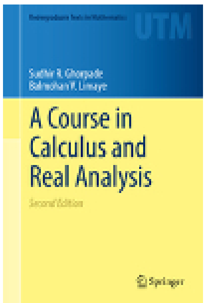
A Course in Calculus and Real Analysis, 2nd ed.



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[Reviewed by Jonathan Lewin, on 04/15/2019]

The style of presentation of this book by Ghorpade and Limaye is unusual. The book straddles the world of basic calculus and the world of real analysis and it includes a wide range of topics, each of which is presented clearly and rigorously and it also includes some interesting historical background on these topics. I was particularly interested to see that Chapter 10 includes the Arzela bounded convergence theorem for Riemann integration and some nice applications of this theorem. I think that this book would be a valuable asset to a university library and that many instructors would do well to have a copy of this book in their personal libraries. In addition, I believe that some students would benefit if they possessed a copy of this book to use for reference purposes.

Having said this, I must add that I am not convinced that the writing style of this book is optimized for use as a primary textbook for introductory courses in real analysis. Students who are encountering their first course in real analysis in a typical university curriculum often see their first course in real analysis as a difficult hurdle and I believe that this course needs to be presented gently and slowly with lots of illustrative examples and exercises that are presented on the spot to help the student through the study process. I do not think that the student should have to turn to the end of each chapter to find those exercises. The book by Ghorpade and Limaye covers many of its topics quite tersely and it frequently says too much at once. The following examples may illustrate what I mean.

1. Section 1.1 piles the idea of bounds, supremum, infimum and the completeness property into less than a page and then immediately gives a quick and unfriendly proof that every nonempty set that is bounded below must have an infimum. Right after that, it zips through the Archimedean property of the system of real numbers. I would think that, after giving the definition of an upper bound of a set, we need to give some practice exercises and examples to illustrate the definition. We could ask why, if $x < 1$, then x must fail to be an upper bound of the interval $[0, 1)$. We could ask why, if two sets A and B are bounded above, then their union $A \cup B$ must be bounded above. We need to teach them to say: Choose an upper bound u of A and an upper bound v of B. Define w to the larger of the two numbers u and v. Then they need to explain why this w must be an upper bound of $A \cup B$. We need to teach them to prove that, if a set E of real numbers is nonempty and bounded above, then E will have a largest member if and only if $\sup E$ belongs to E. Simple exercises such as these can provide a valuable lesson on mathematical writing.

2. The title of Section 2.2 is “Subsequences and Cauchy Sequences”. I find that title rather strange. Why are these two topics introduced in the same breath? Subsequences are introduced very quickly and then

the student is told that “it is easy to see” that a sequence $\{a_n\}$ has a limit a if and only if every one of its subsequences converges to a. After saying that the fact is “easy to see”, the book supplies a quick proof (too quick in my opinion.) I don’t think the proof should come after the appearance has been given that it will be left to the reader.

I think there is too much emphasis in this book on subsequences and I think it was unwise to base so much of the theory of limits of sequences on the fact that every sequence of real numbers must have a monotone subsequence. That property of sequences is not easy and, in fact, subsequences are not easy to study and teach. I think we could get along quite nicely without them.

The notion of “cluster point” is defined in Section 2.3 as a number that is a limit of a subsequence. Some people use the term “subsequential limit” for this purpose. The usual notion of a cluster point is equivalent to the idea of a subsequential limit in simple spaces but not in general topology and so the language being used is not forward compatible to more advanced courses.

3. The presentation of limits and continuity of functions in Chapter 3 also looks quite strange to me. I do not think that definitions based on limits of sequences are able to reveal the concepts efficiently and, again, we have a definition that is not forward compatible to more advanced courses.

Section 3.2, sneaks in the definition of a closed set while the reader is trying to study properties of continuous functions. In my opinion, concepts such as closed set, open set, interior point, neighborhood, closure, and limit point should be presented slowly and carefully in a chapter of its own before either sequences or functions are introduced. I find it odd that the standard properties of continuous functions, and even the concept of uniform continuity, are introduced before the idea of limit of a function has been presented.

When the book finally gets around to limits of functions, it suddenly sneaks in a definition of limit point of a set and I think that the definition given in terms of sequences is unwise.

4. Section 4.1 introduces the idea of a derivative and we are told that it is obvious that the limits

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

and

$$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

are the same. Is this so obvious? My students have to write a careful proof of this equivalence. I notice that, at the point at which the derivative is introduced, the book sneaks in a definition of an interior point of a set. I wonder, also, why the definition was restricted to interior points of the domain of the function.

I could give many more such illustrations but I’ll stop at this point. I’ll sum up by saying that everything works, but it is all fired from the hip. It comes too fast to be friendly enough for many students. My feeling is that none of the criticisms that I have made detract from the value of the book as a reference source but, as I have said, I would not want to use the book as a primary textbook in one of my courses.

Jonathan Lewin is currently Professor of Mathematics at Kennesaw State University where he has been for the past 36 years. He did his undergraduate study at the University of the Witwatersrand in South Africa and then obtained his PhD at the University of Wisconsin in 1970. He has held faculty positons in South Africa and Israel, and in the USA.

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