MECHANICAL ENGINEERING/11

FLAT AND CORRUGATED DIAPHRAGM DESIGN HANDBOOK

MARIO DI GIOVANNI

FLAT AND CORRUGATED DIAPHRAGM DESIGN HANDBOOK

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FLAT AND CORRUGATED DIAPHRAGM DESIGN HANDBOOK

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Foreword

In writing the foreword to this book on diaphragm design, I am pleased to note that the author attempts to teach the subject more on a scientific basis than the empirical one normally used, especially in the design of corrugated diaphragms.

The theory of the performance of flat and corrugated plates is well known but rarely used because it is too theoretical and not readily available for diaphragm design and performance.

This book brings together for the first time, under one cover, a comprehensive, cohesive method of designing diaphragms quickly and accurately using equations which are common and intimately related to flat and corrugated diaphragms. Thus the performance equations for both flat and corrugated diaphragms, though similarly expressed, differ in the numerical values of their "stiffness coefficients" which may be easily obtained from the many graphs and tables included in the text.

The true measure of the scientific method is inductive logic, whereas empiricism leads the research ultimately into blind alleys. The author, keenly aware of this, presents his information inductively, starting with the flat diaphragm and proceeding step by step to diaphragms with rigid centers, to diaphragms with transverse loading, to diaphragms with a snap action, to bellows, and so forth. Corrugated diaphragms are similarly treated, making this book perhaps one of the best of its kind ever written.

R. L. Noland
President, Ametek, Inc.

Preface

This book has been written to provide instrument designers with simple, practical methods of calculating the performance characteristics of flat and corrugated diaphragms.

The theory of flat round diaphragms is generally available from many books on elasticity or strength of materials; that for corrugated diaphragms is dismally sparse and impractical for instrument designers. But theory is one thing and application of that theory is something else. This book, then, also provides some useful and practical information based on my many years of experience in the design and application of diaphragms.

The theoretical treatment of diaphragms, or "plates," as normally referred to in books on strength of material, involves calculations to determine the deflection and stress analysis usually of interest to the structural engineer. The instrument designer, however, is also interested in linearity, accuracy, hysteresis, stability, repeatability, and so on. In effect, the designer is interested in a diaphragm-type element which will respond to a physical stimulus in a predictable and accurate manner. It follows, then, that a choice of material becomes more important to an instrument designer than to a structural engineer who could not care less if the material exhibited hysteretic tendencies or whether the material's metallurgical constituent is martensitic or ferritic. Also, the structural engineer would have little concern in the manufacture and installation of "plates." Little would the structural engineer care about thermal, mechanical, or residual stresses generated upon installing the plates. For an instrument designer, lack of concern with these "minor problems" would spell trouble.

Part I of this book basically covers material selection as material affects diaphragm performance. It also covers basic definitions and discusses the effects of production processes on the stability of diaphragms.

Part II covers flat diaphragms of various shapes: round, rectangular, convex diaphragms for snap-action effects, and bellows.

Part III covers the performance characteristics of corrugated diaphragms, including stress analysis and frequency response. It also describes forming methods, overload protection, and capsular elements.

Of added interest is the final chapter on the computer solution of diaphragm performance characteristics. Ready answers are given on diaphragms of various sizes and profiles ranging from diameters of 1.00 to 2.5 in., material thickness of 0.002 to 0.005 in., and of various corrugation depths.

Considerable emphasis has been placed on the direct use of graphical data for quick solutions of diaphragm performance problems. In optimizing a design, the reader is encouraged to make a first approximation by using the graphical data. A final, more exact solution through the use of a mathematical analysis can then be made.

Many problems and solutions are provided to develop confidence and skill in performance analysis.

The main objective of this book is to present all available information currently available and to show all the factors which influence the design and hence the performance of diaphragms for instrument use.

In some instances, semiempirical approaches had to be used, as, for example, in estimating the linearity of rectangular diaphragms in Part III. A more rigorous and exact method involving Fourier series coefficients and partial differential equations could have been used, but higher mathematics has no place in a design handbook.

I am confident that this, the first book of its kind ever written entirely on the design of diaphragms, will be of great value not only to instrumentation engineers but to students and all multidisciplinary design engineers as well.

Mario Di Giovanni

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Contents

FLAT AND CORRUGATED DIAPHRAGM DESIGN HANDBOOK

Part I

Diaphragm Performance and Materials

Diaphragms are round flexible plates which undergo elastic deflection when subjected to pressure or axial loading. They are sometimes referred to as "pressure collectors" or "pressure summing devices." They may be flat or corrugated for larger axial displacement.

Diaphragms, whether used as pressure collectors or flexural suspensions, are important links in instruments. As pressure collectors they measure pressure and establish the range and accuracy of the measuring instrument. In such cases the accuracy, reliability, and safety of the instrument depend on the designer's ability to develop a quality diaphragm.

Users of instruments are demanding better performance and more reliability from instruments today because the related equipment in the overall measurement system is itself extremely accurate and depends on the instrument to pick up the signal from the measurand faithfully and accurately.

The design of diaphragms in the past has been performed to some degree by trial and error as the available plate theory, developed for structural engineers, dealt primarily with allowable stresses and deflections and not with accuracy and dynamic performance which is of primary interest to the instrument designer.

Historically, the theory of the symmetrical bending of circular plates was originally given by Lagrange in 1811 and a little later in 1829 by Poisson. Clebsch, Saint Venant, Michell, and Love proposed additional rigorous theories from which Timoshenko and other investigators have written some excellent textbooks. Unfortunately, the related design information is either too theoretical or too empirical for instrument design use.

A method of diaphragn design utilizing performance ratios and coefficients has been used with great success by the author for the past

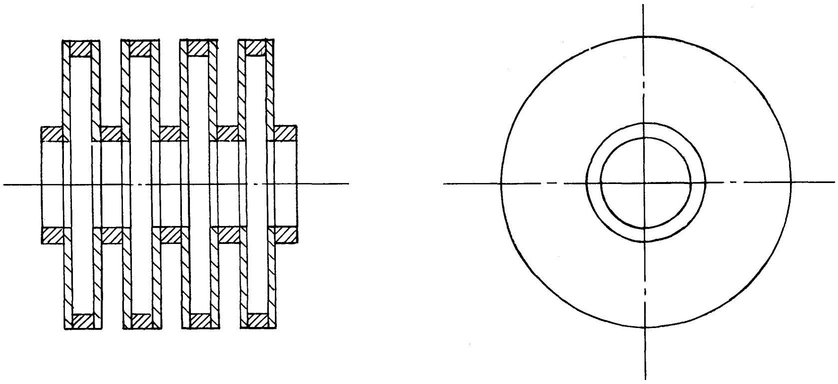
25 years in his practice and recently in his classes at Loyola Marymount University.

The method is simple, as all diaphragms, whether flat, corrugated, or with rigid centers, use the same basic equations. Only the coefficients change for the particular profile and configuration. Since the coefficients may be obtained from graphs, the performance characteristics of any "normal" diaphragm may be calculated easily and expeditiously.

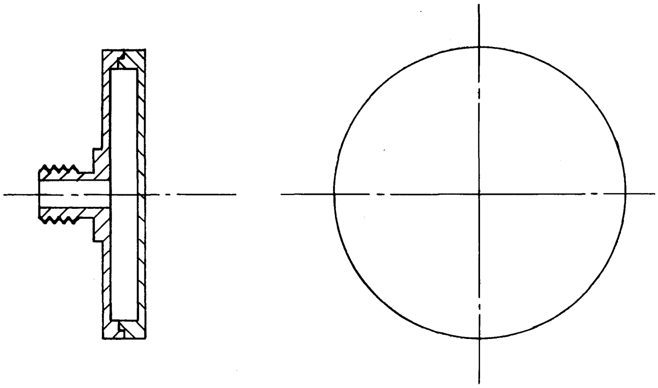
The fundamentals of the theory of flat diaphragms has been fully covered by many researchers, notably by Timoshenko and Woinowsky-Krieger (1959), Way (1934), Andreeva (1962), and many others and will not be repeated in this text.

The fundamentals of the theory of corrugated diaphragms from which most of the data for this text have been taken have been presented by L. A. Haringx (1950, 1956, 1957). L. A. Andreeva (1955a, 1955b, 1956, 1958), Y. T. Feodos'ev (1945, 1949), and others listed in the Bibliography at the back of this book.

Flat diagrams are widely used in pressure-measuring instruments because of their simplicity and high frequency response. With proper design they can react with great accuracy to pressures from thousands of psi to less than 1 psi. Flat diaphragms are linear only for very small deflections as for this condition the median plane of the diaphragm endures almost no elongation. However, as the diaphragm gets very thin, as is the case for low-range instruments, flexural rigidity diminishes and tensile stresses in the median plane become dominant. In this condition the diaphragm becomes nonlinear and unfit for most applications. The designer than must resort to corrugated diaphragms to produce a linear instrument.



Flat diaphragms mounted in series.



Flat diaphragm capsule.

However, flat diaphragms can be mounted in series to increase the linear range of the elastic element, the total axial displacement being the sum of the individual elements. Observe also that many flat diaphragms mounted in series constitute a bellows, as illustrated. A common elastic element consisting of two diaphragms is called a capsule, also illustrated.

1

The Performance of Diaphragms

THE DIAPHRAGM AS A PRIMARY SENSING ELEMENT

The most common method of measuring pressure is to balance it against an elastic force provided by an elastic member. The construction variations of these elastic members are many. Typically diaphragms, bellows, or Bourdon tubes are most commonly used; the choice depending primarily on the amount of elastic deflection required for the data presentation element. Thus, if the data presentation element is the conventional scale and pointer usually used in dial gage instruments where the pointer must rotate 270° or more, then the sensing element must have a high deflection range. The choice would then be bellows for low-pressure range or Bourdon tubes for high-pressure range. On the other hand, if the deflection is small, the diaphragm would be the best choice. The diaphragm is not only the simplest to construct of all the elastic members but is the best sensing element that can be used in a high-vibration environment. Moreover, its performance is predictable and it lends itself to many design variations, as will be shown in this text. Generally speaking, diaphragms are used where the deflection is less than 0.005 in. (depending on the diameter) where the dynamic response of the instrument must be high and where simple overload stops must be provided. Nearly all pressure transducers of recent design utilize the advantages of the diaphragm to improve their performance.

LINEARITY OF DIAPHRAGMS

In designing diaphragms the designer is interested not only in its deflection for a given presssure but also in a linear deflection-pressure relationship. It should be stressed that if the diaphragm's calibration curve for given inputs is not a straight line, the diaphragm may still be accurate and serviceable. However, a linear deflection-pressure relationship is highly desirable and easier to handle in calculations and data reduction. Therefore, performance conformity to straight-line behavior are highly desirable and universally sought by designers.

In instrument engineering language, "linearity" is defined as "the closeness to which a curve approximates a straight line." It is usually measured as nonlinearity and expressed as "linearity,"* e.g., a maximum deviation between the calibration curve and a straight line.

Linearity is usually expressed as independent linearity, terminal-based linearity or zero-based linearity. When expressed simply as linearity, it is assumed to be independent linearity. The Scientific Apparatus Makers Association (SAMA) in its SAMA Standard PMC20.1-1973, endorsed by the Instrument Society of America, specifies the following definitions:

Independent linearity—the maximum deviation of the actual characteristic (average of upscale and downscale readings) from a straight line so positioned as to minimize the maximum deviation (see Fig. 1.1a).

Terminal-based linearity—the maximum deviation of the actual characteristic (average upscale and downscale readings) from a straight line coinciding with the actual characteristic at upper and lower range values (see Fig. 1.1b).

Zero-based linearity—the maximum deviation of the actual characteristic (average of upscale and downscale readings) from a straight line so positioned as to coincide with the actual characteristic at the lower range value and to minimize the maximum deviation (see Fig. 1.1c).

HYSTERESIS

It is generally accepted that, at low stresses, most materials exhibit elastic behavior in accordance with Hooke's law and that, in this range, the strains are reversible with stress. This would suggest that, if we were to put pressure on a diaphragm and calibrate it from zero to full scale within the elastic range, the ascending and descending curves would be superimposed. This is not the case because of a phenomenon known as hysteresis. Recall that hysteresis effects also show in elec-

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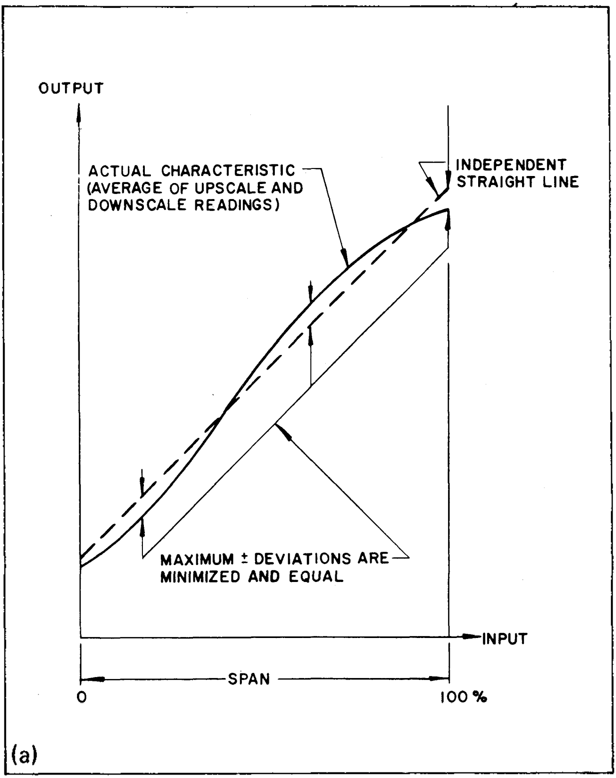
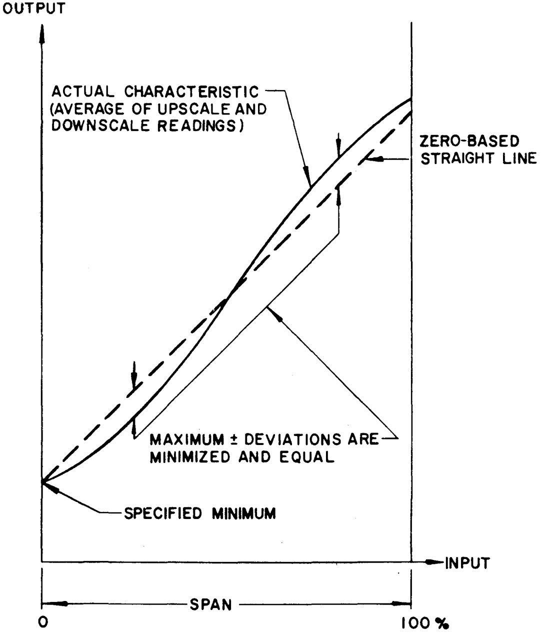
*Notwithstanding SAMA's admonition that "nonlinearity" be expressed as linearity, the term "nonlinearity" is still very much used in the instrument industry, especially by transducer manufacturers.


OUTPUT
ACTUAL CHARACTERISTIC (AVERAGE OF UPSCALE AND DOWNSCALE READINGS)
INDEPENDENT STRAIGHT LINE
MAXIMUM ± DEVIATIONS ARE MINIMIZED AND EQUAL
INPUT
SPAN
0
100 %
(a)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ACTUAL CHARACTERISTIC
(AVERAGE OF UPSCALE AND
DOWNSCALE READINGS)
ZERO-BASED
STRAIGHT LINE
MAXIMUM ± DEVIATIONS ARE
MINIMIZED AND EQUAL
SPECIFIED MINIMUM
INPUT
SPAN
0
100 %

(b)

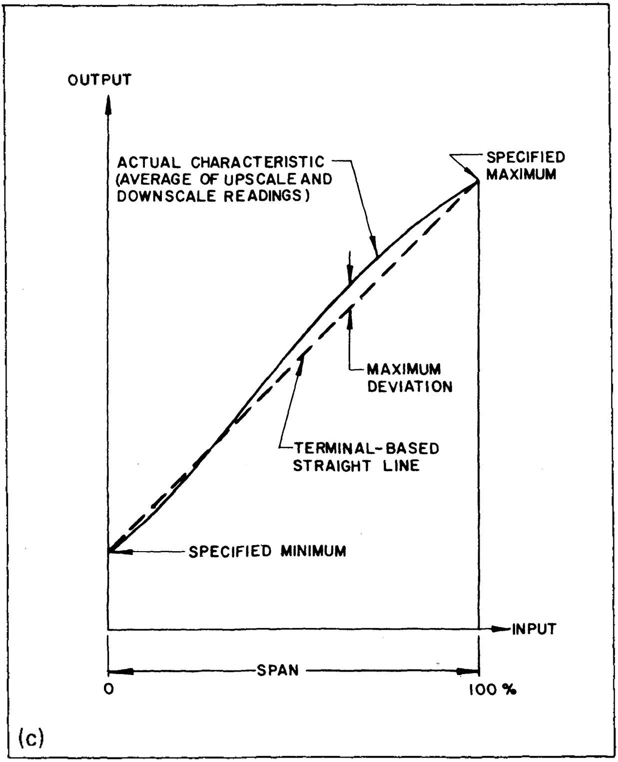

OUTPUT
ACTUAL CHARACTERISTIC (AVERAGE OF UPSCALE AND DOWNSCALE READINGS)
SPECIFIED MAXIMUM
MAXIMUM DEVIATION
TERMINAL-BASED STRAIGHT LINE
SPECIFIED MINIMUM
INPUT
SPAN
0
100 %
(c)

FIG. 1.1 Types of linearity: (a) independent; (b) zero-based; (c) terminal-based. [From SAMA Standard PMC20-1-1973. Reprinted by permission.]

trical phenomena as when a ferromagnetic material such as iron is magnetized. The relationship between the flux density and the magnetizing force in the complete magnetization cycle is represented by a hysteresis loop, and the property of a magnetic substance in which the flux density lags behind its previous value while the magnetizing force returns to its former value is called hysteresis. But the mechanism which induces "mechanical" hysteresis is due to energy absorption in the elastic member produced by molecular friction and appears as heat in dynamic cycling. Late evidence indicates that hysteresis is due to the homogeneous displacement of all the atoms in the crystal lattice from their equilibrium positions. Figure 1.2 shows a typical hysteresis curve. The numerical value of hysteresis can be specified by specifying the difference of the ascending and descending value, usually at midscale, as a percentage of full scale.

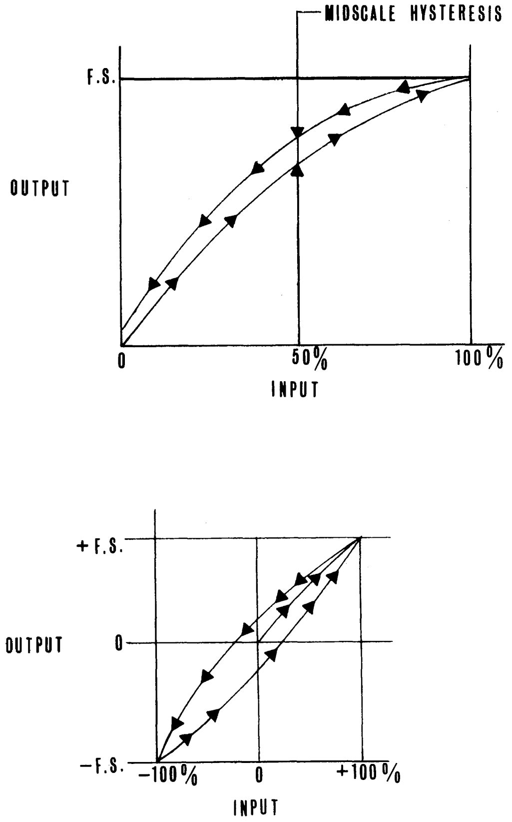


FIG. 1.2 Hysteresis effects.

Hysteresis depends on the magnitude of the applied stress. Since it decreases with a decrease in stress, the value of the maximum allowable stress in a diaphragm is determined by how much hysteresis error can be tolerated and not on structural requirements.

Hysteresis cannot be calculated, but can be minimized or nearly eliminated by proper choice of material and stress level. However, the designer may not have the choice of designing with low-hysteresis materials, as compatibility with process media may dictate materials poor in hysteresis characteristics. For example, corrosion resistance with pressure media may dictate materials such as tantalum or even platinum—materials which cannot be heat-treated or which are of unsuitable hardness level. Here, the recourse available is to keep the stress level as low as possible or to design a diaphragm which can be backed up with a heat-treated elastic member of a higher spring constant—thus complicating the problem.

Generally speaking, crystalline materials yield low hysteresis errors. In metals, materials heat-treated after fabrication to a martensitic microstructure are best. Diaphragms made of monocrystalline silicon chips by the author have shown indiscernible levels of hysteresis.

SPRING RATE AND SENSITIVITY

The relationship between applied load and deflection of a diaphragm may be represented in tabular or graphical forms. Since diaphragms are elastic links, the spring rate and its reciprocal, the sensitivity, are normally used in evaluating their performance characteristics.

The spring rate k is defined as the ratio of the applied load to the corresponding deflection y. If the load is a force F, then

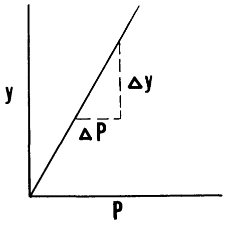
$$k_f = \frac{F}{y} \quad \text{usually in lb/in. or kg/mm} \qquad (1.1a)$$

If the load is a pressure,

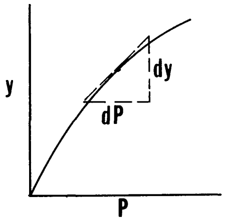
$$k_p = \frac{P}{y} \quad \text{psi/in. or kg/cm}^2\text{/mm} \qquad (1.1b)$$

The sensitivity s is defined as the ratio of the deflection to the load. That is, it gives the displacement y of the diaphragm produced by unit load F:

$$s_f = \frac{y}{F} \quad \text{usually in in./lb or mm/kg} \qquad (1.2a)$$



Sensitivity for Linear System $= \frac{\Delta y}{\Delta P}$



Sensitivity for Nonlinear System $= \frac{dy}{dP}$

FIG. 1.3 Definition of sensitivity: (top) linear system; (bottom) nonlinear system.

If the load is pressure,

$$s_p = \frac{y}{P} \quad \text{in./psi or mm/kg/cm}^2 \tag{1.2b}$$

It is apparent that, when an input-output calibration such as that in Fig. 1.3 is made, the sensitivity of the diaphragm can be defined as the slope of the calibration curve. If the curve is not a straight line, the sensitivity will vary with the input value and therefore is written in the differential form:

$$s = \frac{dy}{dP}$$

COMBINATION OF ELEMENTS

Diaphragms Mounted in Series

It is common, where high sensitivity is desired, to mount diaphragms in series as is the case with bellows and capsules (see illustrations in the Introduction). For this condition, the sensitivity of the system as a whole is equal to the sum of the sensitivities of the component elements

$$s = \sum_{i=1}^{n} s_i \qquad (1.3)$$

The formula for the spring rate for the same series system is

$$k = \frac{1}{\sum_{i=1}^{n} (1/k)} \qquad (1.3a)$$

Diaphragms Mounted in Parallel

In some rare applications, diaphragm elements may be mounted in parallel. This is usually the case, for example, when two or more diaphragms are used as shock-absorbing elements. For this condition, the spring rate will be the sum of the spring rates of the individual elements:

$$k = \sum_{i=1}^{n} k_i \qquad (1.4)$$

The sensitivity of the system will be the reciprocal of the spring rate, or

$$s = \frac{1}{\sum_{i=1}^{n} (1/s_i)} \qquad (1.4a)$$

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