

ANSWER KEY

1. Algebraically determine the true point(s) of intersection for the following polar graphs where $\theta = [0, 2\pi]$. Then provide a POLAR sketch and label the true point(s) of intersection.

a. $r_1 = 3 - 2\cos\theta$ and $r_2 = 5\cos\theta$

$$\begin{aligned} 3 - 2\cos\theta &= 5\cos\theta \\ 3 &= 7\cos\theta \\ \frac{3}{7} &= \cos\theta \\ \pm 1.128 &= \theta \end{aligned}$$

$$r = 3 - 2\cos(1.128)$$

$$r = 2.143$$

true points of intersection
(2.143, 1.128)
(2.143, -1.128)

b. $r_1 = 3 - 3\cos\theta$ and $r_2 = 3\cos\theta$

$\text{II} + \text{III}$

$$3 - 3\cos\theta = 3\cos\theta$$

$$\begin{aligned} 3 &= 6\cos\theta \\ -\frac{3}{6} &= \cos\theta \end{aligned}$$

$$2\frac{\pi}{3}, \sqrt{3} = \theta$$

$$r = 3\cos\frac{2\pi}{3}$$

$$r = -\frac{3}{2}$$

true points of intersection
(-3/2, 2π/3) (-3/2, 4π/3)

c. $r_1 = 3 - 2\sin\theta$ and $r_2 = 5\cos 2\theta$

$$3 - 2\sin\theta = 5\cos 2\theta$$

$$3 - 2\sin\theta = 5(1 - 2\sin^2\theta)$$

$$3 - 2\sin\theta = 5 - 10\sin^2\theta$$

$$10\sin^2\theta - 2\sin\theta - 2 = 0$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

Quad Formula

$$\sin\theta = -\frac{1}{358}$$

$$\sin\theta = .558$$

$$\theta_1 = 366, 2.775$$

$$\theta_2 = 592, 2.550$$

$$r_1 = 2.284$$

$$r_2 = 3$$

d. $r_1 = 2\sin\theta$ and $r_2 = 5\cos\theta$

$\text{II} + \text{I}$

$$2\sin\theta = 5\cos\theta$$

$$\frac{2}{5} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{2}{5} = \cot\theta$$

$$1.190, 4.332 \quad (\cancel{2\pi} = \theta)$$

$$r = 2\sin 1.190$$

$$r = 1.857$$

$$(1.857, 1.190)$$

$$(4.332, 1.857)$$

e. $r_1 = 2\sin\theta$ and $r_2 = 5\cos\theta$

①c) (2.284, .364)
(2.284, 2.775)

(3, .592)
(3, 2.55)

2. Sketch each conic and then algebraically transform the polar equation into Cartesian form.

a. $r = \frac{12}{3 - \cos\theta}$

$$r(3 - \cos\theta) = 12$$

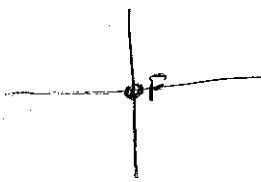
$$3r - r\cos\theta = 12$$

$$3\sqrt{x^2+y^2} - x = 12$$

$$(3\sqrt{x^2+y^2})^2 = (12+x)^2$$

$$9x^2+9y^2 = 144 + 24x + x^2$$

$$8x^2 - 24x + 9y^2 - 144 = 0$$



b. $r = \frac{6}{1 - 3\sin\theta}$

$$r(1 - 3\sin\theta) = 6$$

$$r - 3r\sin\theta = 6$$

$$\sqrt{x^2+y^2} - 3y = 6$$

$$(\sqrt{x^2+y^2})^2 = (6+3y)^2$$

$$x^2+y^2 = 36 + 12y + 9y^2$$

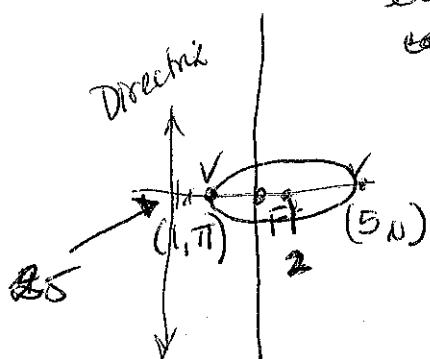
$$0 = -x^2 + 8y^2 + 36y + 36$$



3. Find the polar equation of the conic with its focus at the pole and,

- a. vertices at $(5, 0)$ and $(1, \pi)$,

ellipse



$$2a = 6$$

$$a = 3$$

$$c = 2$$

$$e = \frac{c}{a} = \frac{2}{3}$$

$$r = \frac{p(2/3)}{1 - 2/3 \cos\theta} = \frac{\frac{5}{2}(2/3)}{1 - 2/3 \cos\theta} \left(\frac{3}{3} \right)$$

$$= \frac{5}{3 - 2\cos\theta}$$

p = distance directrix is from the pole!

$$e = \frac{c}{a} = \frac{2}{3} \rightarrow \frac{2}{3} = \frac{3}{d} \rightarrow \frac{2d}{3} = 9 \rightarrow d = \frac{9}{2} \rightarrow p = 2.5$$

- b. vertices at $(2, 0)$ and $(8, 0)$

Hyperbola

$$2a = 6$$

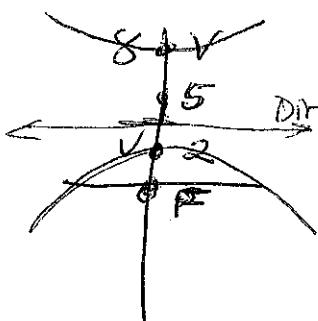
$$a = 3$$

$$c = 5$$

$$e = \frac{c}{a} = \frac{5}{3}$$

$$r = \frac{p(5/3)}{1 + 5/3 \sin\theta} = \frac{\left(\frac{16}{5}\right)\left(\frac{5}{3}\right)}{1 + 5/3 \sin\theta} \left(\frac{3}{3} \right)$$

$$= \frac{16}{3 + 5\sin\theta}$$



$$P = ?$$

$$e = \frac{c}{a} = \frac{5}{3} \rightarrow \frac{5}{3} = \frac{5}{d} \rightarrow \frac{5d}{3} = 5 \rightarrow d = 3$$

$$\frac{5d}{3} = 5$$

$$5 - 9/5 = P$$

$$\frac{16}{5} = P$$