Calculus I
Formula Sheet
Chapter 4

Section 4.1

1. Definition of the Extrema of a function:
   Let f be defined on interval I :
   - $f(c)$ is abs min when $f(c) \leq f(x)$ on I
   - $f(c)$ is abs max when $f(c) \geq f(x)$ on I
2. Exterme Value Theorem:
   If f is cts on $[a,b]$
   Then f has both max/min on $[a,b]$
3. Definition of Relative Extrema:
   - If $f(c)$ is max on (a,b) (open interval)
     Then $f(c)$ is rel max
   - If $f(c)$ is min on (a,b) (open interval)
     Then $f(c)$ is rel min
4. Definition of a Critical Number:
   Let f be defined at c
   Then c is a critical number if
     - $f'(c) = 0$ or
     - $f'(c)$ DNE
5. Relative extrema occur only at c.n.
6. Find extrema on $[a,b]$:
   - f cts on $[a,b]$
     - Find c.n. on (a,b)
     - Eval f at: a, all c.n., b
     - Smallest = abs max
     - Largest = abs min

Section 4.2

7. Rolle's Theorem
   - f cts on $[a,b]$
   - f diff on (a,b)
   - $f(a) = f(b)$
     $\Rightarrow$ there is at least one c in (a,b) such that $f'(c) = 0$
8. Mean Value Theorem
   - f cts on $[a,b]$
   - f diff on (a,b)
   $\Rightarrow$ there exists a c in (a,b) such that
   $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Section 4.3

9. Definition of Increasing and Decreasing
   - Increasing: $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
   - Decreasing: $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$
10. Test for Increasing and Decreasing
    - f cts on $[a,b]$
    - f diff on (a,b)
      - $f'(x) > 0$ on (a,b) $\Rightarrow$ increasing
      - $f'(x) < 0$ on (a,b) $\Rightarrow$ decreasing
      - $f'(x) = 0$ on (a,b) $\Rightarrow$ constant
11. Find interval of increasing and decreasing
    - f cts on (a,b)
    - Find c.n. on (a,b)
    - Create intervals
    - Find the sign of $f'(x)$ on each interval
      - $+ \Rightarrow$ increasing
      - $- \Rightarrow$ decreasing
12. The First Derivative Test
    - c is a c.n. in (a,b)
    - f cts on (a,b)
    - f diff on (a,b) except possibly at c
      - $f'(c)$ change – to +
        $\Rightarrow f(c)$ is rel min
      - $f'(c)$ change + to –
        $\Rightarrow f(c)$ is rel max
      - + to + or – to –
        $\Rightarrow$ neither max nor min

Section 4.4

13. Definition of Concavity
- f diff on (a,b)
  - $f'(x)$ increasing
    $\Rightarrow$ concave upward
  - $f'(x)$ decreasing
    $\Rightarrow$ concave downward

14. Test for Concavity
- Find Intervals using
  - $f''(x)=0$
  - f'' DNE
  - f undefined
- Write Intervals
- f'' exists on interval (a,b)
  - $f''(x)>0 \Rightarrow$ concave upward
  - $f''(x)<0 \Rightarrow$ concave downward

15. Definition of Point of Inflection
- f cts on (a,b)
- c in (a,b)
- Graph of f has tangent line at c
- Graph changes from:
  - Concave up to concave down
  - Concave down to concave up

  $\Rightarrow (c, f(c))$ is a point of inflection

16. Find possible points of inflection:
    If $(c, f(c))$ is a point of inflection
    Then either
    - $f''(c)=0$ or
    - $f''(c)$ DNE

17. Second Derivative Test
- $f'(c)=0$
- $f''(x)$ exists on (a,b)
  - $f''(c)>0 \Rightarrow$ rel min at $(c, f(c))$
  - $f''(c)<0 \Rightarrow$ rel max at $(c, f(c))$
  - $f''(c)=0 \Rightarrow$ test fails
    rel min, rel max, neither??

Section 4.6

18. Slant Asymptote
- Rational function $f(x)=\dfrac{poly}{poly}$
- Degree of numerator is exactly one more than degree of denominator
- Divide – throw away the remainder
- $y=$ what's left is the SA

19. See "Summary of Graphing" sheet under "Notes" on website

Section 4.7

20. Optimization
- Primary equation – the equation involving the variable to be maximized or minimized.
- Secondary equation – the equation used to substitute into the primary equation to make the primary equation a function of only one variable.

Section 4.8

21. Tangent line approximation at $(c, f(c))$
    $y-f(c)=f'(c)(x-c)$
    $\Rightarrow y=f(c)+f'(c)(x-c)$
22. Differential of x: $dx=$ any nonzero real number
23. Differential of y: $dy=f'(x)dx$
24. Measurement error: $\Delta x=dx$
25. Propagated error: $\Delta y=f(x+\Delta x)-f(x)$
26. Relative error (volume example): $\dfrac{dV}{V}$
27. Percent error: relative error as %