Precalculus
Student Solution Manual

Chapter 1

Try It

1.1. a. yes; b. yes. (Note: If two players had been tied for, say, 4th place, then the name would not have been a function of rank.)

1.2.

$w = f(d)$

1.3. yes

1.4.

$g(5) = 1$

1.5.

$m = 8$

1.6.

$y = f(x) = \frac{\sqrt[3]{x}}{2}$

1.7.

$g(1) = 8$

1.8.

$x = 0$ or

$x = 2$

1.9. a. yes, because each bank account has a single balance at any given time; b. no, because several bank account numbers may have the same balance; c. no, because the same output may correspond to more than one input.

1.10.

a. Yes, letter grade is a function of percent grade;
b. No, it is not one-to-one. There are 100 different percent numbers we could get but only about five possible letter grades, so there cannot be only one percent number that corresponds to each letter grade.

1.11. yes

1.12. No, because it does not pass the horizontal line test.

1.13.

$\{-5,\ 0,\ 5,\ 10,\ 15\}$

1.14.

$(-\infty,\ \infty)$

1.15.

$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

1.16.

$\left[-\frac{5}{2}, \infty\right)$

1.17.

a. values that are less than or equal to –2, or values that are greater than or equal to –1 and less than 3;
b. $\{x | x \leq -2 \text{ or } -1 \leq x < 3\}$;
c. $(-\infty,\ -2] \cup [-1,\ 3)$

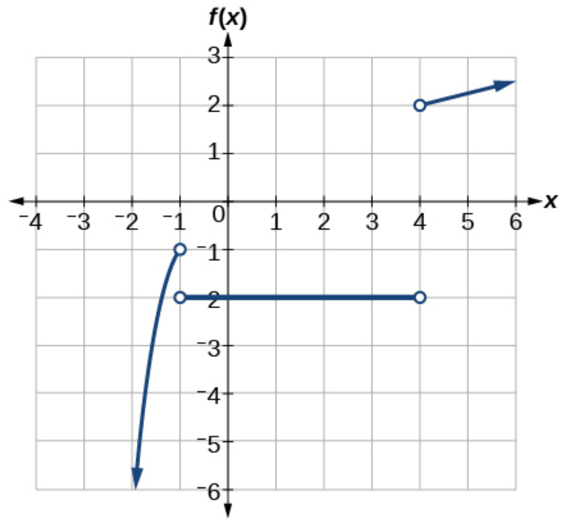
1.18. domain =[1950,2002] range = [47,000,000,89,000,000]

1.19. domain:

$(-\infty, 2];$ range:

$(-\infty, 0]$

1.20.

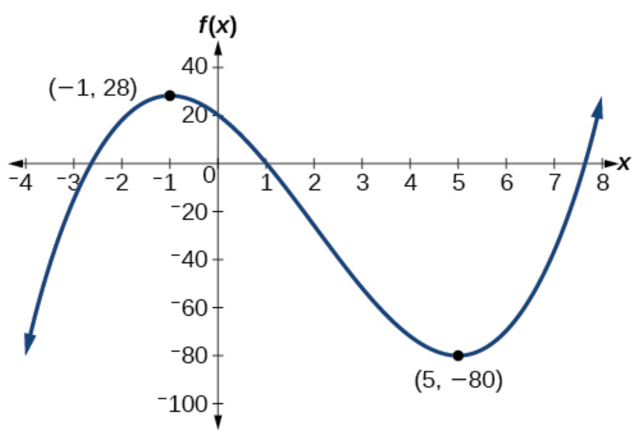


1.21.
$$\frac{\$2.84 - \$2.31}{5 \text{ years}} = \frac{\$0.53}{5 \text{ years}} = \$0.106$$ per year.

1.22.
$$\frac{1}{2}$$

1.23.
$$a + 7$$

1.24. The local maximum appears to occur at $(-1, 28),$ and the local minimum occurs at $(5, -80).$ The function is increasing on $(-\infty, -1) \cup (5, \infty)$ and decreasing on $(-1, 5).$



1.25.
$$(fg)(x) = f(x)g(x) = (x-1)\left(x^2-1\right) = x^3 - x^2 - x + 1$$
$$(f-g)(x) = f(x) - g(x) = (x-1) - \left(x^2-1\right) = x - x^2$$ No, the functions are not the same.

1.26. A gravitational force is still a force, so $a(G(r))$ makes sense as the acceleration of a planet at a distance *r* from the Sun (due to gravity), but $G(a(F))$ does not make sense.

1.27.
$f(g(1)) = f(3) = 3$ and
$g(f(4)) = g(1) = 3$

1.28.

$$g(f(2)) = g(5) = 3$$

1.29. a. 8; b. 20

1.30.

$$[-4,\ 0) \cup (0,\ \infty)$$

1.31. Possible answer:

$$g(x) = \sqrt{4 + x^2}$$

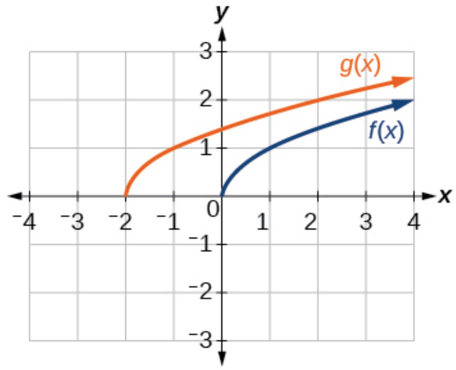
$$h(x) = \frac{4}{3 - x}$$

$$f = h \circ g$$

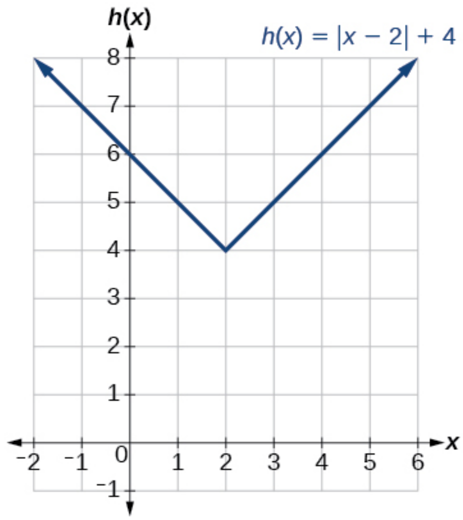
1.32. (1.76)

$$b(t) = h(t) + 10 = \ -4.9t^2 + 30t + 10$$

1.33. The graphs of $f(x)$ and $g(x)$ are shown below. The transformation is a horizontal shift. The function is shifted to the left by 2 units.



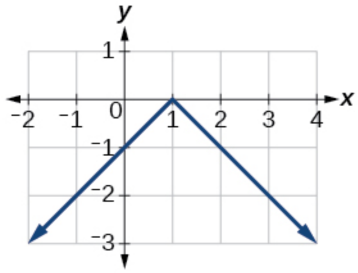
1.34.

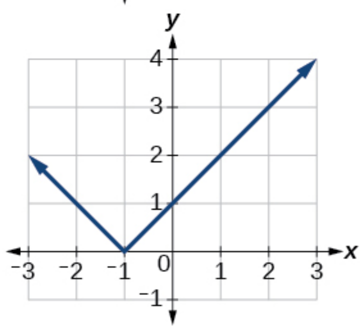


1.35.

$$g(x) = \frac{1}{x - 1} + 1$$

1.36.

a. 

b. 

1.37.

a.

$g(x) = -f(x)$

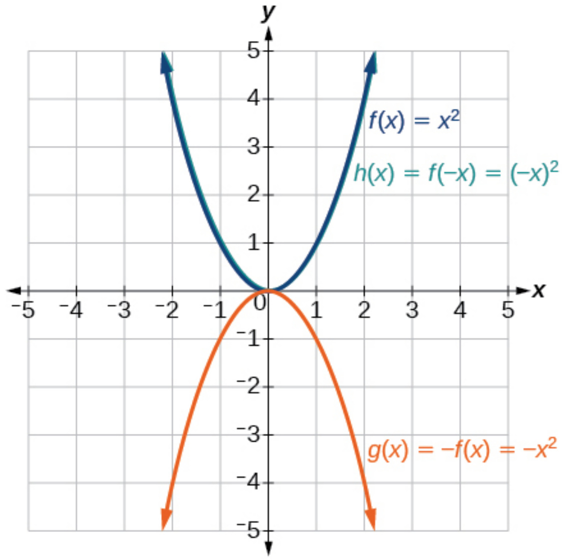
x	-2	0	2	4
$g(x)$	-5	-10	-15	-20

b.

$h(x) = f(-x)$

x	-2	0	2	4
$h(x)$	15	10	5	unknown

1.38.



Notice:
$g(x) = f(-x)$ looks the same as
$f(x)$.

1.39. even

1.40.

x	2	4	6	8
$g(x)$	9	12	15	0

1.41.
$g(x) = 3x - 2$

1.42.
$g(x) = f\left(\frac{1}{3}x\right)$ so using the square root function we get
$g(x) = \sqrt{\frac{1}{3}x}$

1.43.
$|x - 2| \leq 3$

1.44. using the variable
p for passing,
$|p - 80| \leq 20$

1.45.
$f(x) = -|x + 2| + 3$

1.46.
$x = -1$ or
$x = 2$

1.47.
$f(0) = 1,$ so the graph intersects the vertical axis at
$(0, 1).$
$f(x) = 0$ when
$x = -5$ and
$x = 1$ so the graph intersects the horizontal axis at

$(-5, 0)$ and
$(1, 0)$.

1.48.
$4 \leq x \leq 8$

1.49.
$k \leq 1$ or
$k \geq 7;$ in interval notation, this would be
$(-\infty, 1] \cup [7, \infty)$

1.50.
$h(2) = 6$

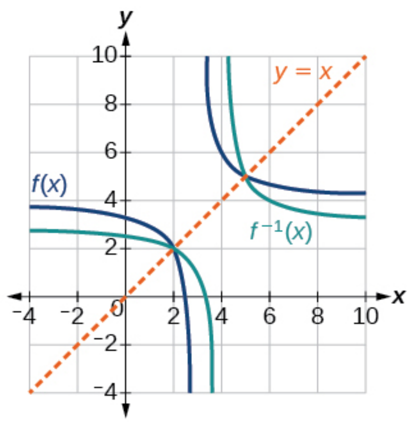
1.51. Yes
1.52. Yes
1.53. The domain of function
f^{-1} is
$(-\infty, -2)$ and the range of function
f^{-1} is
$(1, \infty)$.

1.54.
a.
$f(60) = 50.$ In 60 minutes, 50 miles are traveled.
b.
$f^{-1}(60) = 70.$ To travel 60 miles, it will take 70 minutes.

1.55. a. 3; b. 5.6
1.56.
$x = 3y + 5$

1.57.
$f^{-1}(x) = (2 - x)^2;$ domain of $f:$ $[0, \infty);$ domain of $f^{-1}:$ $(-\infty, 2]$

1.58.



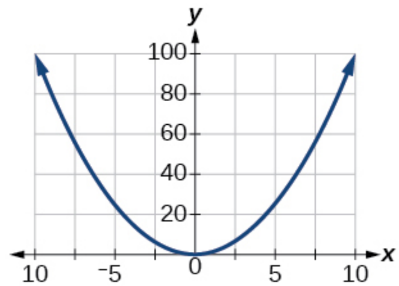
Section Exercises

1. A relation is a set of ordered pairs. A function is a special kind of relation in which no two ordered pairs have the same first coordinate.
3. When a vertical line intersects the graph of a relation more than once, that indicates that for that input there is more than one output. At any particular input value, there can be only one output if the relation is to be a function.
5. When a horizontal line intersects the graph of a function more than once, that indicates that for that output there is more than one input. A function is one-to-one if each output corresponds to only one input.
7. function
9. function
11. function
13. function
15. function

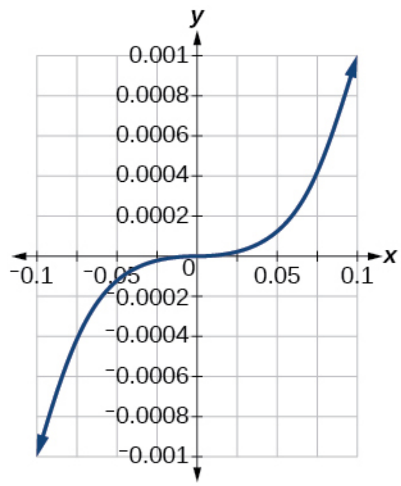
17. function
19. function
21. function
23. function
25. not a function
27.
$f(-3) = -11;\ f(2) = -1;\ f(-a) = -2a-5;\ -f(a) = -2a+5;\quad f(a+h) = 2a+2h-5$
29.
$f(-3) = \sqrt{5}+5;\ f(2) = 5;\ f(-a) = \sqrt{2+a}+5;\ -f(a) = -\sqrt{2-a}-5;\quad f(a+h) =$
$\sqrt{2-a-h}+5$
31.
$f(-3) = 2;\ f(2) = 1-3 = -2;\ f(-a) = |-a-1|-|-a+1|;\ -f(a) = -|a-1|+|a+1|;\quad f(a+h) = |a+h-1|-|a+h+1|$

33.
$\frac{g(x)-g(a)}{x-a} = x+a+2,\ x \neq a$
35. a.
$f(-2) = 14;$ b.
$x = 3$
37. a.
$f(5) = 10;$ b.
$x = -1$ or
$x = 4$
39. a.
$f(t) = 6-\frac{2}{3}t;$ b.
$f(-3) = 8;$ c.
$t = 6$
41. not a function
43. function
45. function
47. function
49. function
51. function
53. a.
$f(0) = 1;$ b.
$f(x) = -3,\ x = -2$ or
$x = 2$
55. not a function so it is also not a one-to-one function
57. one-to- one function
59. function, but not one-to-one
61. function
63. function
65. not a function
67.
$f(x) = 1,\ x = 2$
69.
$f(-2) = 14;\ f(-1) = 11;\ f(0) = 8;\ f(1) = 5;\ f(2) = 2$
71.
$f(-2) = 4;\qquad f(-1) = 4.414;\ f(0) = 4.732;\ f(1) = 4.5;\ f(2) = 5.236$
73.
$f(-2) = \frac{1}{9};\ f(-1) = \frac{1}{3};\ f(0) = 1;\ f(1) = 3;\ f(2) = 9$
75. 20

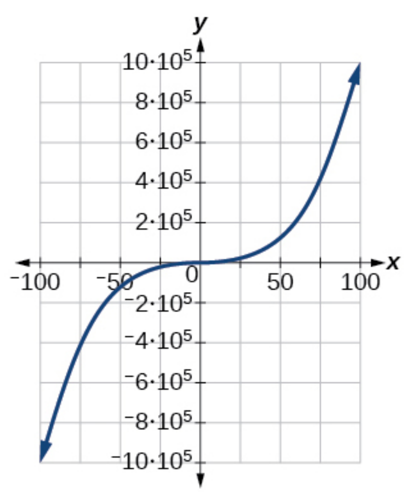
77.
$[0, \ 100]$



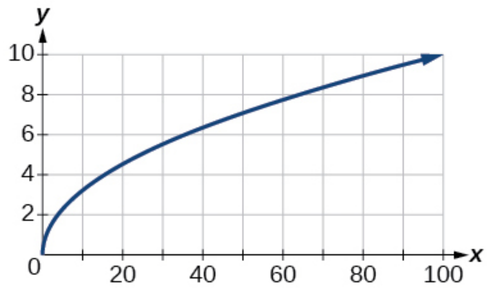
79.
$[-0.001, \ 0.001]$



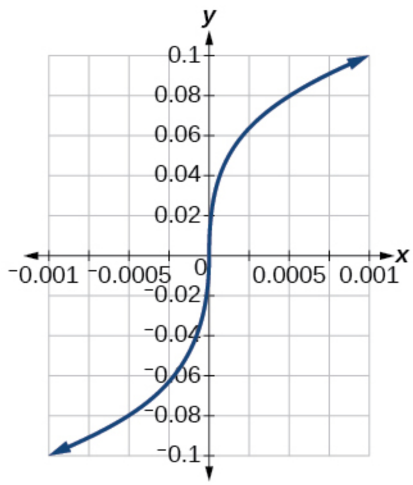
81.
$[-1, 000, 000, \ 1{,}000{,}000]$



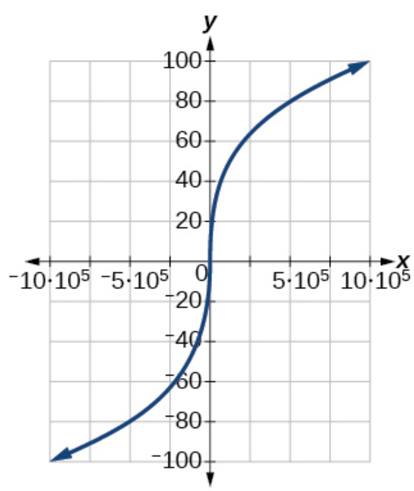
83.
$[0, \ 10]$



85.
$[-0.1,\ 0.1]$



87.
$[-100,\ \ 100]$



89. a.
$g(5000) = 50;$ b. The number of cubic yards of dirt required for a garden of 100 square feet is 1.
91. a. The height of a rocket above ground after 1 second is 200 ft. b. the height of a rocket above ground after 2 seconds is 350 ft.
93. The domain of a function depends upon what values of the independent variable make the function undefined or imaginary.
95. There is no restriction on x for $f(x) = \sqrt[3]{x}$ because you can take the cube root of any real number. So the domain is all real numbers, $(-\infty,\ \infty).$ When dealing with the set of real numbers, you cannot take the square root of negative numbers. So x -values are restricted for $f(x) = \sqrt{x}$ to nonnegative numbers and the domain is $[0,\ \infty).$

97. Graph each formula of the piecewise function over its corresponding domain. Use the same scale for the x -axis and y -axis for each graph. Indicate inclusive endpoints with a solid circle and exclusive endpoints with an open circle. Use an arrow to indicate $-\infty$ or $\infty.$ Combine the graphs to find the graph of the piecewise function.

99.
$(-\infty,\ \infty)$

101.
$(-\infty,\ 3]$

103.
$(-\infty,\ \infty)$

105.
$(-\infty,\ \infty)$

107.
$(-\infty,\ -\frac{1}{2}) \cup (-\frac{1}{2},\ \infty)$

109.
$(-\infty,\ -11) \cup (-11,\ 2) \cup (2,\ \infty)$

111.
$(-\infty,\ -3) \cup (-3,\ 5) \cup (5,\ \infty)$

113.
$(-\infty,\ 5)$

115.
$[6,\ \infty)$

117.
$(-\infty,\ -9) \cup (-9,\ 9) \cup (9,\ \infty)$

119. domain: $(2,\ 8],$ range $[6,\ 8)$

121. domain: $[-4,\ 4],$ range: $[0,\ 2]$

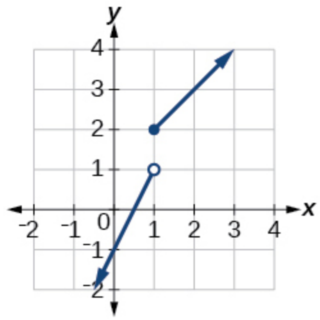
123. domain: $[-5,\ 3),$ range: $[0,\ 2]$

125. domain: $(-\infty,\ 1],$ range: $[0,\ \infty)$

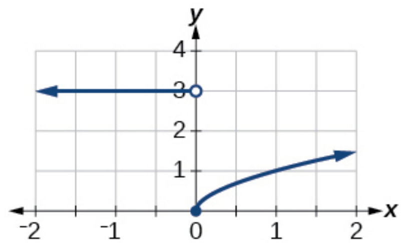
127. domain: $\left[-6,\ -\frac{1}{6}\right] \cup \left[\frac{1}{6},\ 6\right];$ range: $\left[-6,\ -\frac{1}{6}\right] \cup \left[\frac{1}{6},\ 6\right]$

129. domain: $[-3,\ \infty);$ range: $[0,\ \infty)$

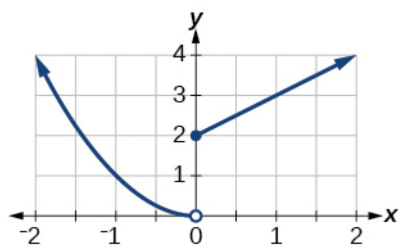
131. domain: $(-\infty,\ \infty)$



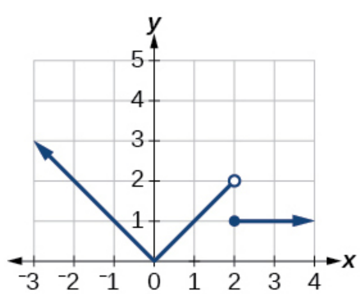
133. domain:
$(-\infty, \infty)$



135. domain:
$(-\infty, \infty)$



137. domain:
$(-\infty, \infty)$



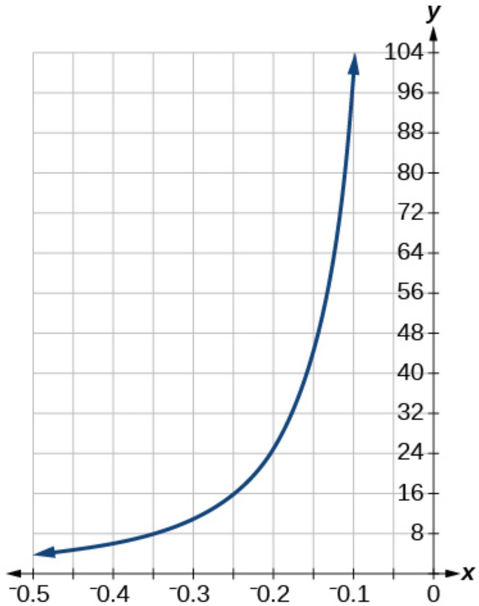
139.
$f(-3) = 1;\ f(-2) = 0;\ f(-1) = 0;\ f(0) = 0$

141.
$f(-1) = -4;\ f(0) = 6;\ f(2) = 20;\ f(4) = 34$

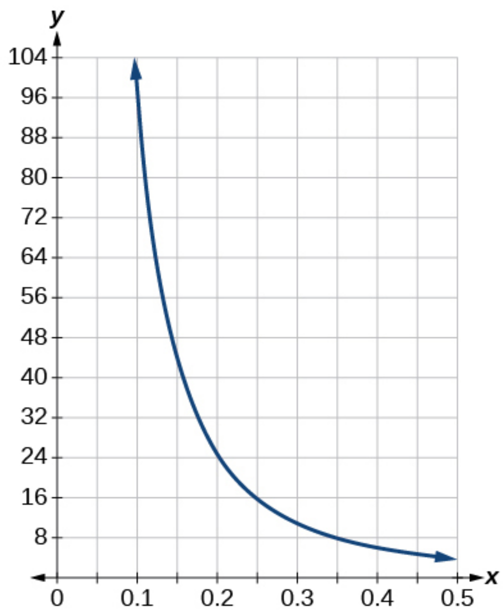
143.
$f(-1) = -5;\ f(0) = 3;\ f(2) = 3;\ f(4) = 16$

145. domain:
$(-\infty, 1) \cup (1, \infty)$

147.



window: $[-0.5,\ -0.1];$ range: $[4,\ 100]$



window: $[0.1,\ 0.5];$ range: $[4,\ 100]$

149. $[0,\ 8]$

151. Many answers. One function is $f(x) = \frac{1}{\sqrt{x-2}}.$

152. The domain is $[0,\ 6];$ it takes 6 seconds for the projectile to leave the ground and return to the ground

154. Yes, the average rate of change of all linear functions is constant.

156. The absolute maximum and minimum relate to the entire graph, whereas the local extrema relate only to a specific region around an open interval.

158.
$4(b+1)$

160. 3
162.
$4x+2h$

164.
$\frac{-1}{13(13+h)}$

166.
$3h^2+9h+9$

168.
$4x+2h-3$

170.
$\frac{4}{3}$

172. increasing on
$(-\infty,\ -2.5)\cup(1,\ \infty),$ decreasing on
$(-2.5,\ 1)$

174. increasing on
$(-\infty,\ 1)\cup(3,\ 4),$ decreasing on
$(1,\ 3)\cup(4,\ \infty)$

176. local maximum:
$(-3,\ 60),$ local minimum:
$(3,\ -60)$

178. absolute maximum at approximately
$(7,\ 150),$ absolute minimum at approximately
$(-7.5,\ -220)$

180. a. –3000; b. –1250
182. -4
184. 27
186. –0.167
188. Local minimum at
$(3,\ -22),$ decreasing on
$(-\infty,\ 3),$ increasing on
$(3,\ \infty)$

190. Local minimum at
$(-2,\ -2),$ decreasing on
$(-3,\ -2),$ increasing on
$(-2,\ \infty)$

192. Local maximum at
$(-0.5,\ 6),$ local minima at
$(-3.25,\ -47)$ and
$(2.1,\ -32),$ decreasing on
$(-\infty,\ -3.25)$ and
$(-0.5,\ 2.1),$ increasing on
$(-3.25,\ -0.5)$ and
$(2.1,\ \infty)$

194. A
196.
$b=5$

198. 2.7 gallons per minute
200. approximately –0.6 milligrams per day

201. Find the numbers that make the function in the denominator g equal to zero, and check for any other domain restrictions on f and $g,$ such as an even-indexed root or zeros in the denominator.

203. Yes. Sample answer: Let $f(x) = x + 1 \text{ and } g(x) = x - 1.$ Then $f(g(x)) = f(x - 1) = (x - 1) + 1 = x$ and $g(f(x)) = g(x + 1) = (x + 1) - 1 = x.$ So $f \circ g = g \circ f.$

205.
$(f + g)(x) = 2x + 6,$ domain: $(-\infty, \infty)$

$(f - g)(x) = 2x^2 + 2x - 6,$ domain: $(-\infty, \infty)$

$(fg)(x) = -x^4 - 2x^3 + 6x^2 + 12x,$ domain: $(-\infty, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x}{6 - x^2},$ domain: $(-\infty, -\sqrt{6}) \cup (-\sqrt{6}, \sqrt{6}) \cup (\sqrt{6}, \infty)$

207.
$(f + g)(x) = \frac{4x^3 + 8x^2 + 1}{2x},$ domain: $(-\infty, 0) \cup (0, \infty)$

$(f - g)(x) = \frac{4x^3 + 8x^2 - 1}{2x},$ domain: $(-\infty, 0) \cup (0, \infty)$

$(fg)(x) = x + 2,$ domain: $(-\infty, 0) \cup (0, \infty)$

$\left(\frac{f}{g}\right)(x) = 4x^3 + 8x^2,$ domain: $(-\infty, 0) \cup (0, \infty)$

209.
$(f + g)(x) = 3x^2 + \sqrt{x - 5},$ domain: $[5, \infty)$

$(f - g)(x) = 3x^2 - \sqrt{x - 5},$ domain: $[5, \infty)$

$(fg)(x) = 3x^2 \sqrt{x - 5},$ domain: $[5, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{3x^2}{\sqrt{x - 5}},$ domain: $(5, \infty)$

211. a. 3; b. $f(g(x)) = 2(3x - 5)^2 + 1;$ c. $f(g(x)) = 6x^2 - 2;$ d. $(g \circ g)(x) = 3(3x - 5) - 5 = 9x - 20;$ e. $(f \circ f)(-2) = 163$

213.
$f(g(x)) = \sqrt{x^2 + 3} + 2,\ g(f(x)) = x + 4\sqrt{x} + 7$

215.
$f(g(x)) = \sqrt[3]{\frac{x+1}{x^3}} = \frac{\sqrt[3]{x+1}}{x},\ g(f(x)) = \frac{\sqrt[3]{x}+1}{x}$

217.
$(f \circ g)(x) = \frac{1}{\frac{2}{x} + 4 - 4} = \frac{x}{2},\ (g \circ f)(x) = 2x - 4$

219.
$f(g(h(x))) = \left(\frac{1}{x+3}\right)^2 + 1$

221. a.
$(g \circ f)(x) = -\frac{3}{\sqrt{2-4x}}$; b.
$\left(-\infty, \frac{1}{2}\right)$

223. a.
$(0, 2) \cup (2, \infty)$; b.
$(-\infty, -2) \cup (2, \infty)$; c.
$(0, \infty)$

225.
$(1, \infty)$

227. sample:
$f(x) = x^3$
$g(x) = x - 5$

229. sample:
$f(x) = \frac{4}{x}$
$g(x) = (x+2)^2$

231. sample:
$f(x) = \sqrt[3]{x}$
$g(x) = \frac{1}{2x - 3}$

233. sample:
$f(x) = \sqrt[4]{x}$
$g(x) = \frac{3x - 2}{x + 5}$

235. sample:
$f(x) = \sqrt{x}$

$g(x) = 2x + 6$

237. sample:
$f(x) = \sqrt[3]{x}$

$g(x) = (x - 1)$

239. sample:
$f(x) = x^3$

$g(x) = \frac{1}{x - 2}$

241. sample:
$f(x) = \sqrt{x}$

$g(x) = \frac{2x - 1}{3x + 4}$

243. 2
245. 5
247. 4
249. 0
251. 2
253. 1
255. 4
257. 4
259. 9
261. 4
263. 2
265. 3
267. 11
269. 0
271. 7
273.
$f(g(0)) = 27,\ g(f(0)) = -94$
275.
$f(g(0)) = \frac{1}{5},\ g(f(0)) = 5$
277.
$18x^2 + 60x + 51$
279.
$g \circ g(x) = 9x + 20$
281. 2
283.
$(-\infty,\ \infty)$
285. False
287.
$(f \circ g)(6) = 6$;
$(g \circ f)(6) = 6$
289.
$(f \circ g)(11) = 11,\ (g \circ f)(11) = 11$
291. c
293.
$A(t) = \pi\left(25\sqrt{t + 2}\right)^2$ and
$A(2) = \pi\left(25\sqrt{4}\right)^2 = 2500\pi$ square inches
295.
$A(5) = \pi(2(5) + 1)^2 = 121\pi$ square units
297. a.
$N(T(t)) = 23(5t + 1.5)^2 - 56(5t + 1.5) + 1;$ b. 3.38 hours
298. A horizontal shift results when a constant is added to or subtracted from the input. A vertical shifts results when a constant is added to or subtracted from the output.
300. A horizontal compression results when a constant greater than 1 is multiplied by the input. A vertical compression results when a constant between 0 and 1 is multiplied by the output.
302. For a function
$f,$ substitute
$(-x)$ for
(x) in

$f(x).$ Simplify. If the resulting function is the same as the original function, $f(-x) = f(x),$ then the function is even. If the resulting function is the opposite of the original function, $f(-x) = -f(x),$ then the original function is odd. If the function is not the same or the opposite, then the function is neither odd nor even.

304.
$g(x) = |x - 1| - 3$

306.
$g(x) = \frac{1}{(x+4)^2} + 2$

308. The graph of $f(x + 43)$ is a horizontal shift to the left 43 units of the graph of $f.$

310. The graph of $f(x - 4)$ is a horizontal shift to the right 4 units of the graph of $f.$

312. The graph of $f(x) + 8$ is a vertical shift up 8 units of the graph of $f.$

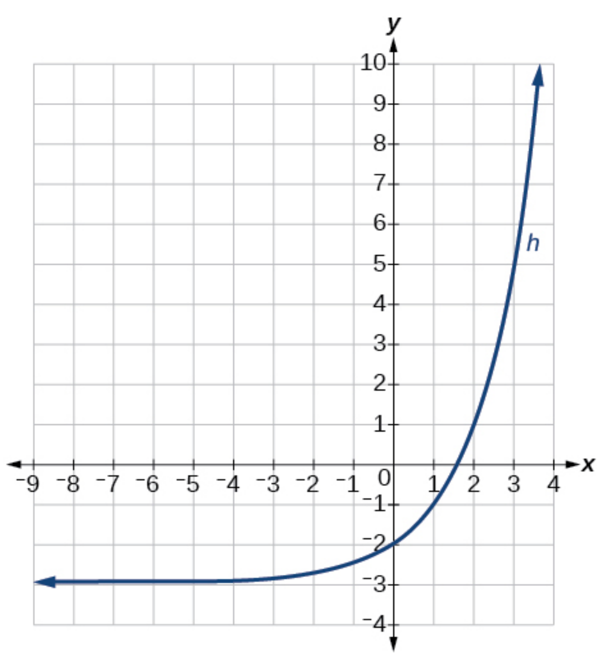
314. The graph of $f(x) - 7$ is a vertical shift down 7 units of the graph of $f.$

316. The graph of $f(x + 4) - 1$ is a horizontal shift to the left 4 units and a vertical shift down 1 unit of the graph of $f.$

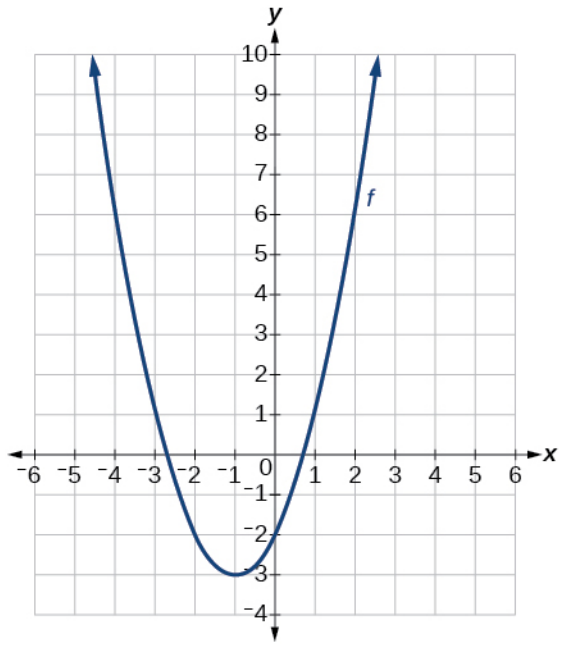
318. decreasing on $(-\infty, -3)$ and increasing on $(-3, \infty)$

320. decreasing on $(0, \infty)$

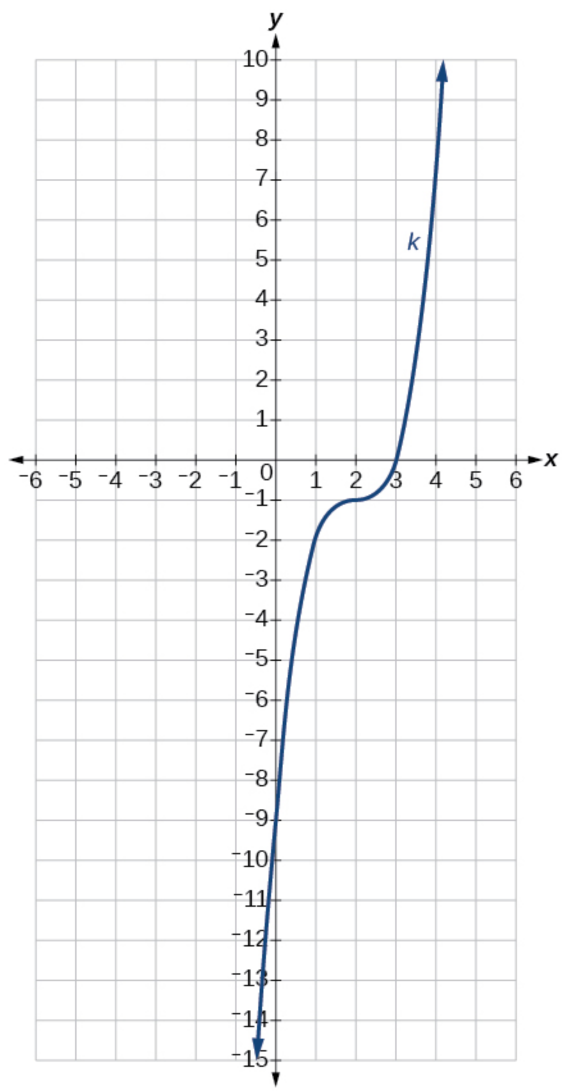
322.



324.


y
x
f
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

326.


y
x
k
10
9
8
7
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4
3
2
1
0
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−6
−7
−8
−9
−10
−11
−12
−13
−14
−15
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

328.
$g(x) = f(x - 1),\ h(x) = f(x) + 1$

330.
$f(x) = |x - 3| - 2$

332.
$f(x) = \sqrt{x + 3} - 1$

334.
$f(x) = (x - 2)^2$

336.
$f(x) = |x + 3| - 2$

338.
$f(x) = -\sqrt{x}$

340.
$f(x) = -(x + 1)^2 + 2$

342.
$f(x) = \sqrt{-x} + 1$

344. even
346. odd
348. even
350. The graph of g is a vertical reflection (across the x-axis) of the graph of f.

352. The graph of g is a vertical stretch by a factor of 4 of the graph of f.

354. The graph of g is a horizontal compression by a factor of $\frac{1}{5}$ of the graph of f.

356. The graph of g is a horizontal stretch by a factor of 3 of the graph of f.

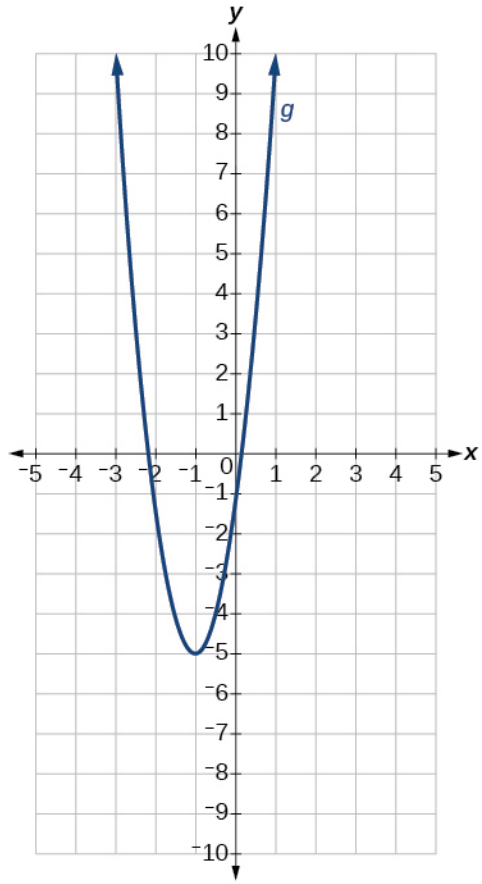
358. The graph of g is a horizontal reflection across the y-axis and a vertical stretch by a factor of 3 of the graph of f.

360.
$g(x) = |-4x|$

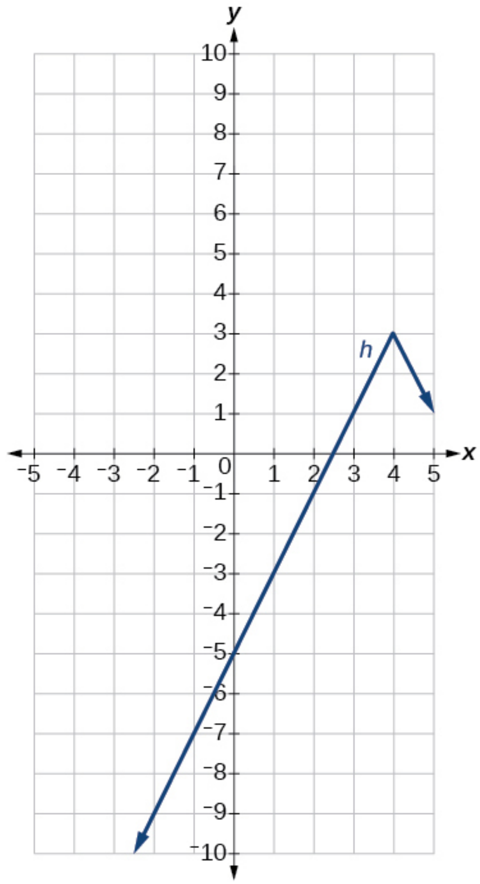
362.
$g(x) = \frac{1}{3(x + 2)^2} - 3$

364.
$g(x) = \frac{1}{2}(x - 5)^2 + 1$

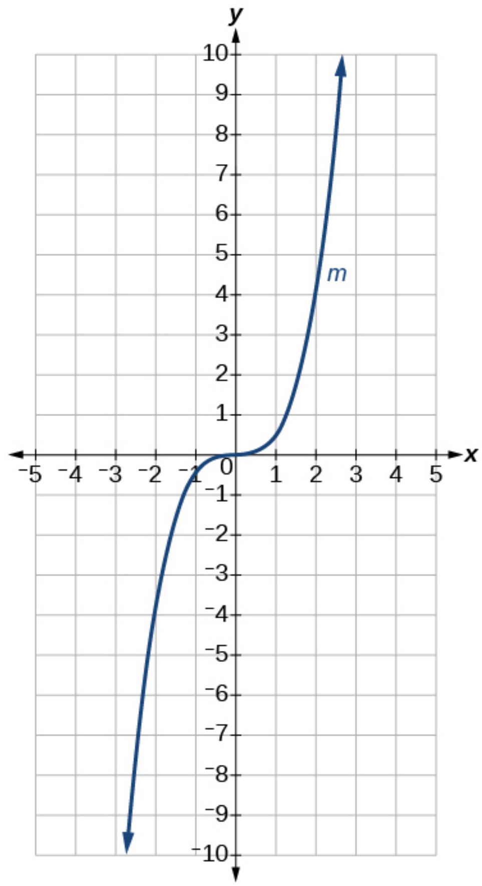
366. The graph of the function $f(x) = x^2$ is shifted to the left 1 unit, stretched vertically by a factor of 4, and shifted down 5 units.



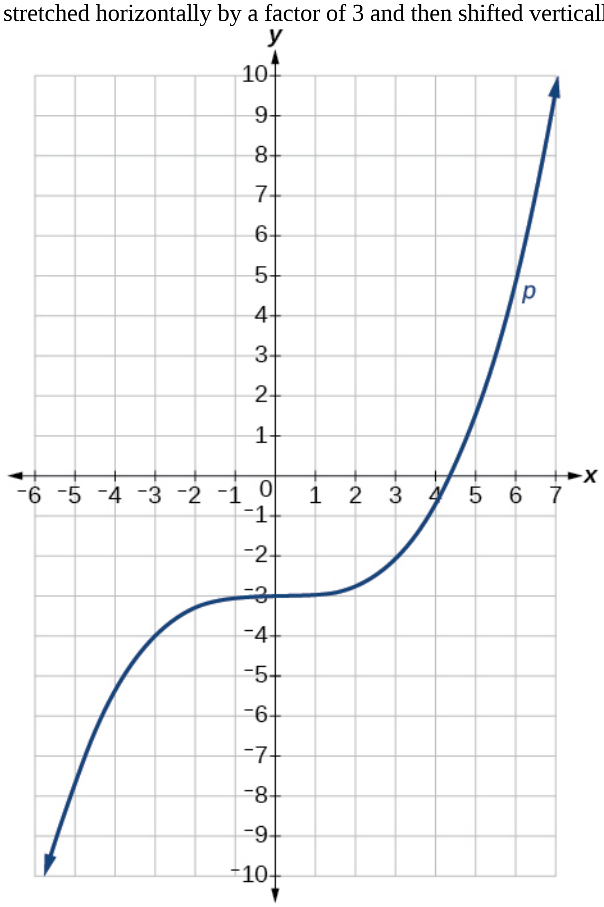
368. The graph of $f(x) = |x|$ is stretched vertically by a factor of 2, shifted horizontally 4 units to the right, reflected across the horizontal axis, and then shifted vertically 3 units up.



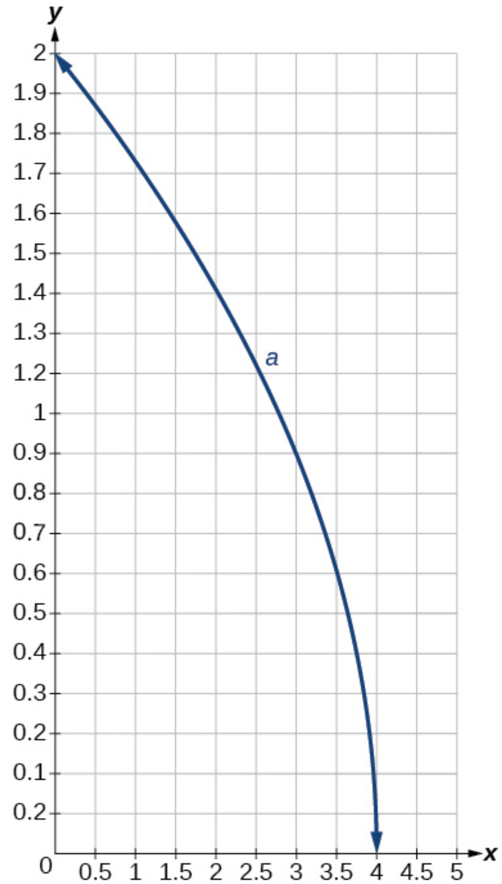
370. The graph of the function $f(x) = x^3$ is compressed vertically by a factor of $\frac{1}{2}$.



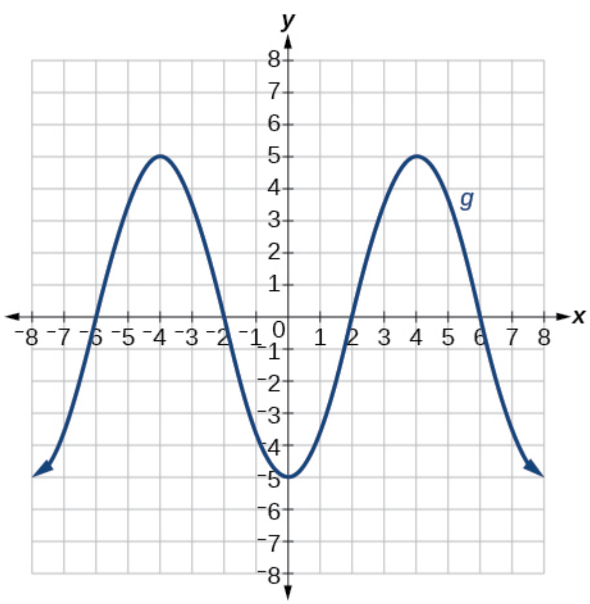
372. The graph of the function is stretched horizontally by a factor of 3 and then shifted vertically downward by 3 units.



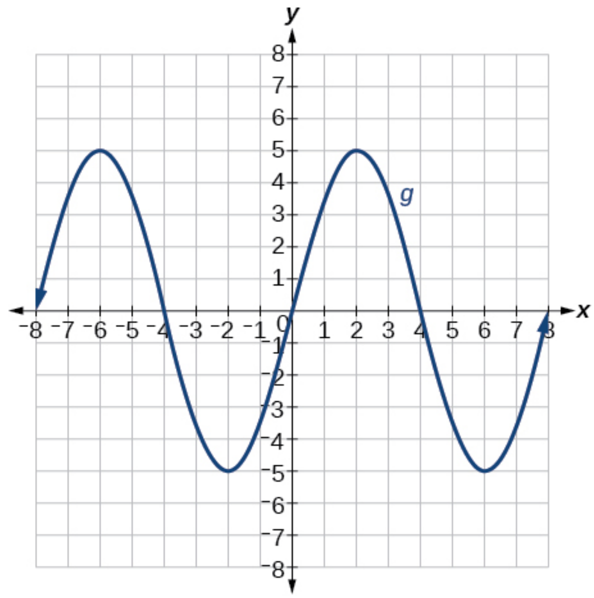
374. The graph of $f(x) = \sqrt{x}$ is shifted right 4 units and then reflected across the vertical line $x = 4$.



376.



378.



379. Isolate the absolute value term so that the equation is of the form $|A| = B.$ Form one equation by setting the expression inside the absolute value symbol, $A,$ equal to the expression on the other side of the equation, $B.$ Form a second equation by setting A equal to the opposite of the expression on the other side of the equation, $-B.$ Solve each equation for the variable.

381. The graph of the absolute value function does not cross the x -axis, so the graph is either completely above or completely below the x -axis.

383. First determine the boundary points by finding the solution(s) of the equation. Use the boundary points to form possible solution intervals. Choose a test value in each interval to determine which values satisfy the inequality.

385.
$|x+4| = \frac{1}{2}$

387.
$|f(x) - 8| < 0.03$

389.
$\{1,\ 11\}$

391.
$\left\{\frac{9}{4},\ \ \frac{13}{4}\right\}$

393.
$\left\{\frac{10}{3},\ \ \frac{20}{3}\right\}$

395.
$\left\{\frac{11}{5},\ \ \frac{29}{5}\right\}$

397.
$\left\{\frac{5}{2},\ \frac{7}{2}\right\}$

399. No solution

401.
$\{-57,\ 27\}$

403.
$(0,\ -8);\ (-6,\ 0),\ (4,\ 0)$

405.
$(0,\ -7);$ no x -intercepts

407.

$(-\infty, \ -8) \cup (12, \ \infty)$

409.

$\frac{-4}{3} \le x \le 4$

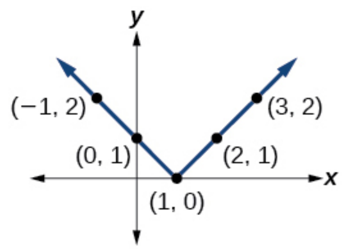
411.

$\left(-\infty, \ -\frac{8}{3}\right] \cup [6, \ \infty)$

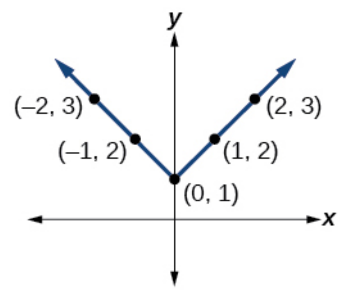
413.

$\left(-\infty, \ -\frac{8}{3}\right] \cup [16, \ \infty)$

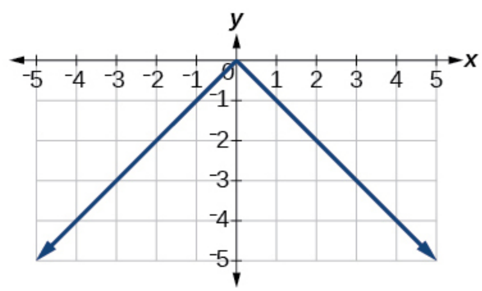
415.



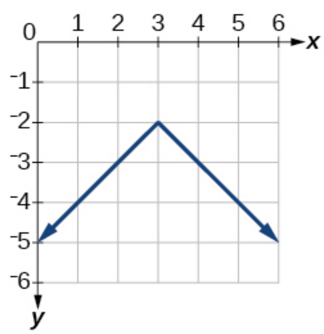
417.



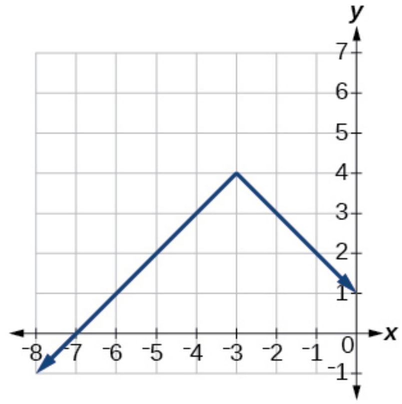
419.



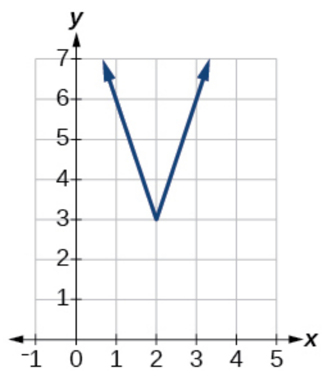
421.



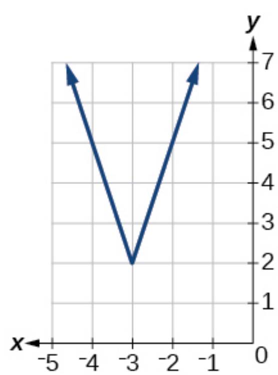
423.



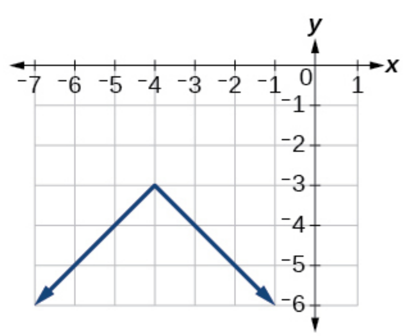
425.



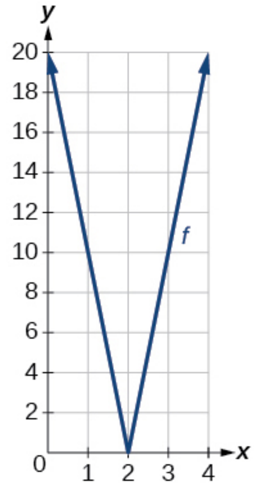
427.



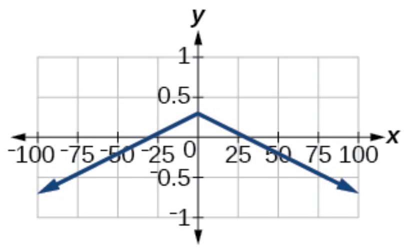
429.



431. range:
$[0,\ 20]$



433.
x- intercepts:



435.
$(-\infty, \infty)$

437. There is no solution for a that will keep the function from having a y-intercept. The absolute value function always crosses the y-intercept when $x = 0.$

439.
$|p - 0.08| \leq 0.015$

441.
$|x - 5.0| \leq 0.01$

443. Each output of a function must have exactly one output for the function to be one-to-one. If any horizontal line crosses the graph of a function more than once, that means that y-values repeat and the function is not one-to-one. If no horizontal line crosses the graph of the function more than once, then no y-values repeat and the function is one-to-one.

445. Yes. For example, $f(x) = \frac{1}{x}$ is its own inverse.

447. Given a function $y = f(x),$ solve for x in terms of $y.$ Interchange the x and $y.$ Solve the new equation for $y.$ The expression for y is the inverse, $y = f^{-1}(x).$

449.
$f^{-1}(x) = x - 3$

451.
$f^{-1}(x) = 2 - x$

453.
$f^{-1}(x) = \frac{-2x}{x-1}$

455. domain of
$f(x):\ [-7, \infty);\ f^{-1}(x) = \sqrt{x} - 7$

457. domain of
$f(x):\ [0, \infty);\ f^{-1}(x) = \sqrt{x+5}$

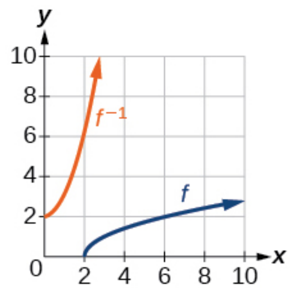
458. a.
$f(g(x)) = x$ and
$g(f(x)) = x.$ b. This tells us that
f and
g are inverse functions

459.
$f(g(x)) = x,\ g(f(x)) = x$

461. one-to-one
463. one-to-one
465. not one-to-one
467.
3

469.
2

471.



473.
$[2, 10]$

475.
6

477.
-4

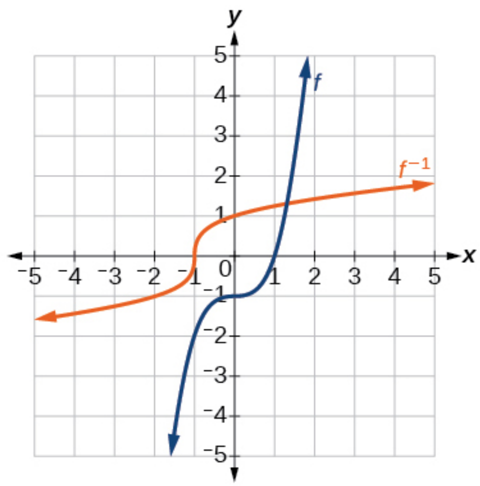
479.
0

481.
1

483.

x	1	4	7	12	16
$f^{-1}(x)$	3	6	9	13	14

485.
$f^{-1}(x) = (1+x)^{1/3}$



487.
$f^{-1}(x) = \frac{5}{9}(x - 32).$ Given the Fahrenheit temperature, $x,$ this formula allows you to calculate the Celsius temperature.

489.
$t(d) = \frac{d}{50},$

$t(180) = \frac{180}{50}.$ The time for the car to travel 180 miles is 3.6 hours.

Review Exercises

490. function
492. not a function
494.
$f(-3) = -27;$

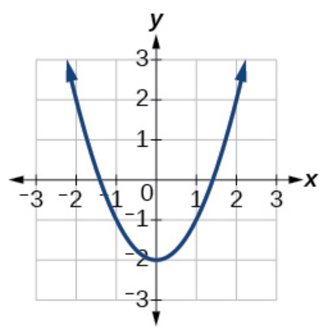
$f(2) = -2;$

$f(-a) = -2a^2 - 3a;$

$-f(a) = 2a^2 - 3a;$

$f(a+h) = -2a^2 + 3a - 4ah + 3h - 2h^2$

496. one-to-one
498. function
500. function
502.



504.
2

506.
$x = -1.8$ or
or $x = 1.8$

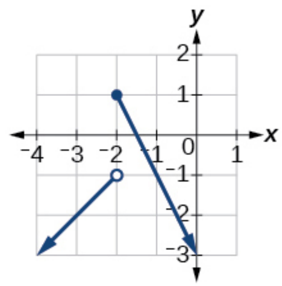
508.

$$\frac{-64 + 80a - 16a^2}{-1 + a} = -16a + 64$$

510.

$(-\infty,\ -2) \cup (-2,\ 6) \cup (6,\ \infty)$

512.



514.

31

516. increasing

$(2,\ \infty)$; decreasing

$(-\infty,\ 2)$

518. increasing

$(-3,\ 1)$; constant

$(-\infty,\ -3) \cup (1,\ \infty)$

520. local minimum

$(-2,\ -3)$; local maximum

$(1,\ 3)$

522.

$(-1.8,\ 10)$

524.

$(f \circ g)(x) = 17 - 18x;\ (g \circ f)(x) = -7 - 18x$

526.

$$(f \circ g)(x) = \sqrt{\frac{1}{x} + 2};\ (g \circ f)(x) = \frac{1}{\sqrt{x + 2}}$$

528.

$$(f \circ g)(x) = \frac{1 + x}{1 + 4x}, \quad x \neq 0, \quad x \neq -\frac{1}{4}$$

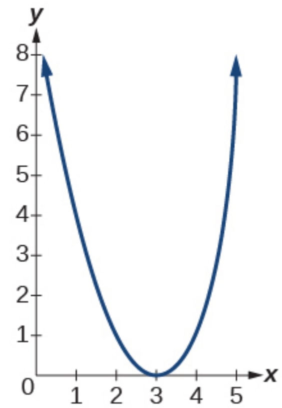
530.

$$(f \circ g)(x) = \frac{1}{\sqrt{x}},\ x > 0$$

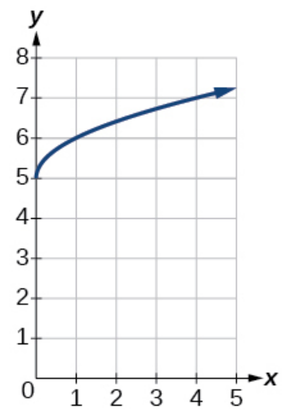
532. sample:

$$g(x) = \frac{2x - 1}{3x + 4};\ f(x) = \sqrt{x}$$

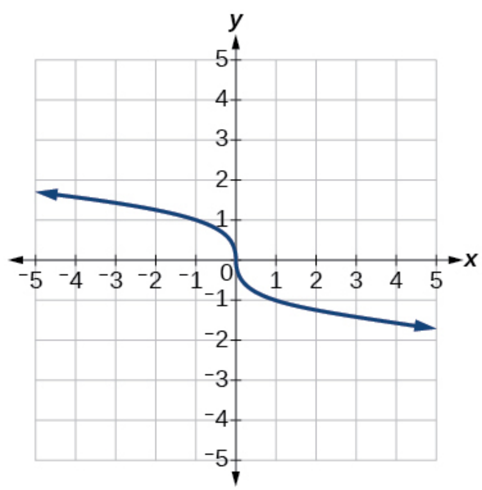
534.


y
8
7
6
5
4
3
2
1
0
1 2 3 4 5
x

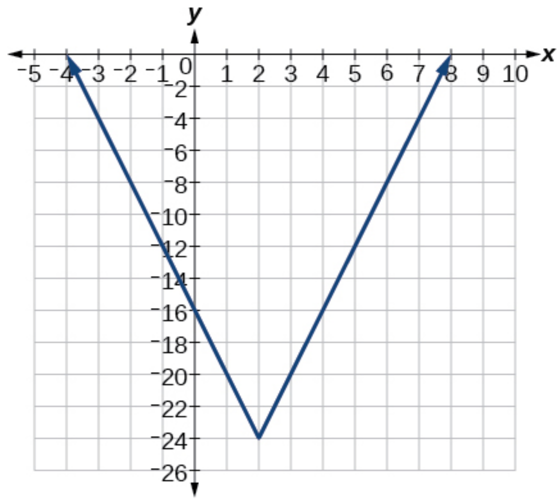
536.


y
8
7
6
5
4
3
2
1
0
1 2 3 4 5
x

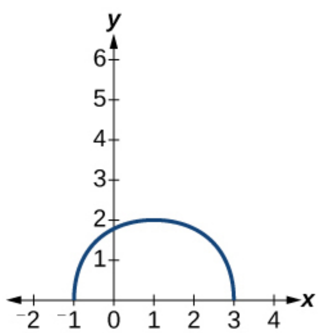
538.


y
5
4
3
2
1
0
−5 −4 −3 −2 −1 1 2 3 4 5
x
−1
−2
−3
−4
−5

540.



542.

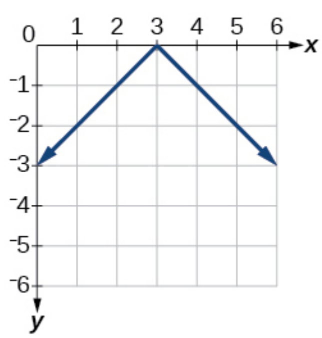


544.
$f(x) = |x - 3|$

546. even
548. odd
550. even
552.
$f(x) = \frac{1}{2}|x + 2| + 1$

554.
$f(x) = -3|x - 3| + 3$

556.



558.
$x = -22, \;\; x = 14$

560.
$\left(-\frac{5}{3}, 3\right)$

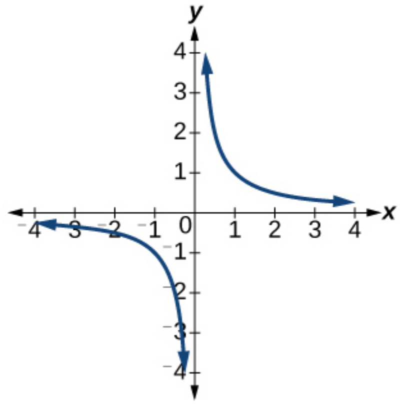
563.
$f^{-1}(x) = \frac{-2x}{x - 1}$

565.

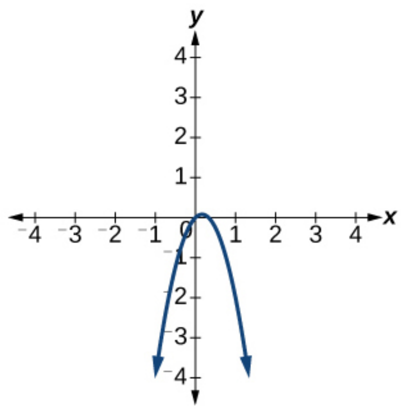
a. $f(g(x)) = x$ and $g(f(x)) = x.$

b. This tells us that f and g are inverse functions

566. The function is one-to-one.



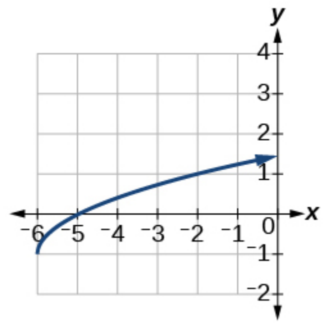
567. The function is not one-to-one.



568.
5

Practice Test

570. The relation is a function.
572. -16
574. The graph is a parabola and the graph fails the horizontal line test.
576.
$2a^2 - a$
578.
$-2(a + b) + 1$
580.
$\sqrt{2}$
582.



584.
even

586.
odd

588.
$x = -7$ and
$x = 10$

590.
$f^{-1}(x) = \frac{x+5}{3}$

592.
$(-\infty, -1.1)$ and $(1.1, \infty)$

594.
$(1.1, -0.9)$

596.
$f(2) = 2$

598.
$f(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$

600.
$x = 2$

602. yes

604.
$f^{-1}(x) = -\frac{x-11}{2}$

Chapter 2

Try It

2.1.
$m = \frac{4-3}{0-2} = \frac{1}{-2} = -\frac{1}{2}$; decreasing because
$m < 0.$

2.2.
$m = \frac{1,868 - 1,442}{2,012 - 2,009} = \frac{426}{3} = 142$ people per year

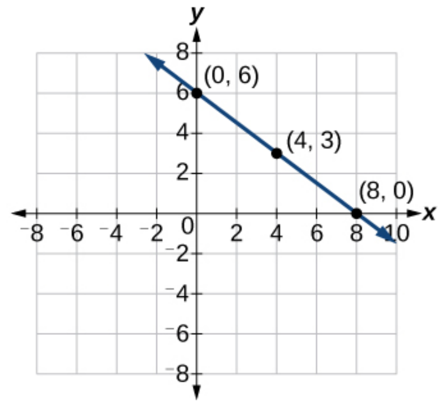
2.3.
$y - 2 = -2(x+2)$;
$y = -2x - 2$

2.4.
$y - 0 = -3(x-0)$;
$y = -3x$

2.5.
$y = -7x + 3$

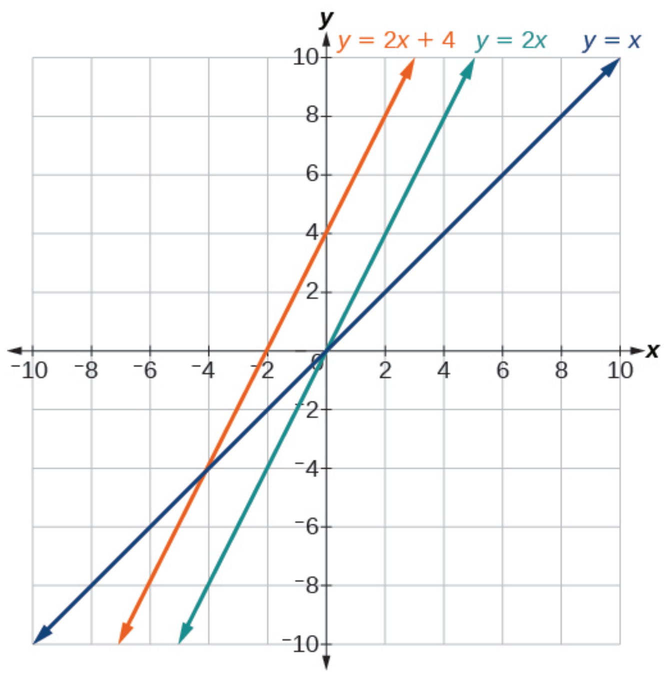
2.6.
$H(x) = 0.5x + 12.5$

2.7.



2.8. Possible answers include
$(-3,\ 7)$,
$(-6,\ 9)$, or
$(-9,\ 11)$.

2.9.



2.10.
$(16,\ \ 0)$

2.11. **a.**
$f(x) = 2x$; **b.**
$g(x) = -\frac{1}{2}x$

2.12.
$y = -\frac{1}{3}x + 6$

2.13.

a.
$(0,\ 5)$

b.
$(5,\ \ 0)$

c. Slope -1
d. Neither parallel nor perpendicular
e. Decreasing function
f. Given the identity function, perform a vertical flip (over the *t*-axis) and shift up 5 units.

2.14.
$C(x) = 0.25x + 25,\ 000$; The *y*-intercept is
$(0,\ 25,\ 000).$ If the company does not produce a single doughnut, they still incur a cost of $25,000.

2.15. 41,100; 2020

2.16. 21.15 miles

2.17.
$54\ °\ \mathrm{F}$

2.18. 150.871 billion gallons; extrapolation

Section Exercises

1. Terry starts at an elevation of 3000 feet and descends 70 feet per second.

3. 3 miles per hour

5.
$d(t) = 100 - 10t$

7. Yes.

9. No.

11. No.

13. No.

15. Increasing.

17. Decreasing.

19. Decreasing.

21. Increasing.

23. Decreasing.

25. 3

27.
$-\frac{1}{3}$

29.
$\frac{4}{5}$

31.
$f(x) = -\frac{1}{2}x + \frac{7}{2}$

33.
$y = 2x + 3$

35.
$y = -\frac{1}{3}x + \frac{22}{3}$

37.
$y = \frac{4}{5}x + 4$

39.
$-\frac{5}{4}$

41.
$y = \frac{2}{3}x + 1$

43.
$y = -2x + 3$

45.
$y = 3$

47. Linear,
$g(x) = -3x + 5$

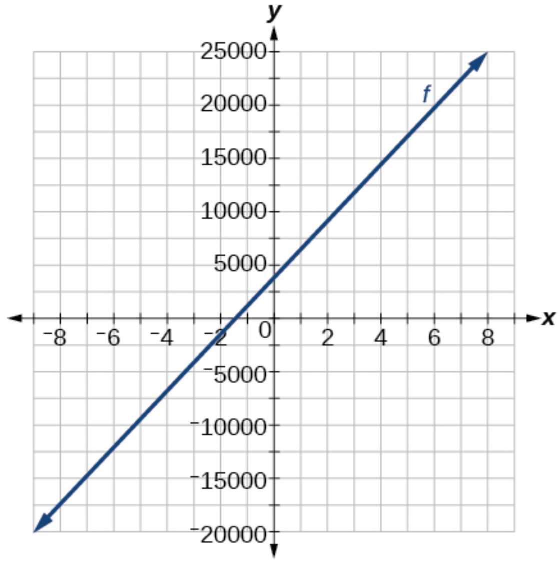
49. Linear,
$f(x) = 5x - 5$

51. Linear,
$g(x) = -\frac{25}{2}x + 6$

53. Linear,
$f(x) = 10x - 24$

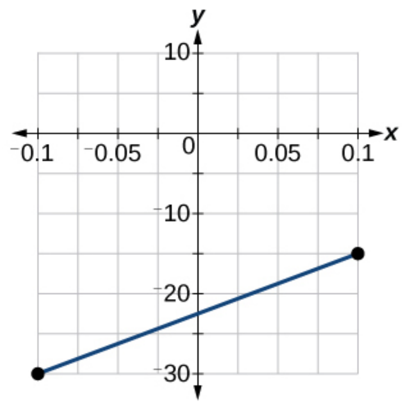
55.
$f(x) = -58x + 17.3$

57.



59. a.
$a = 11,900$;
$b = 1001.1$ b.
$q(p) = 1000p - 100$

61.



63.
$x = -\frac{16}{3}$

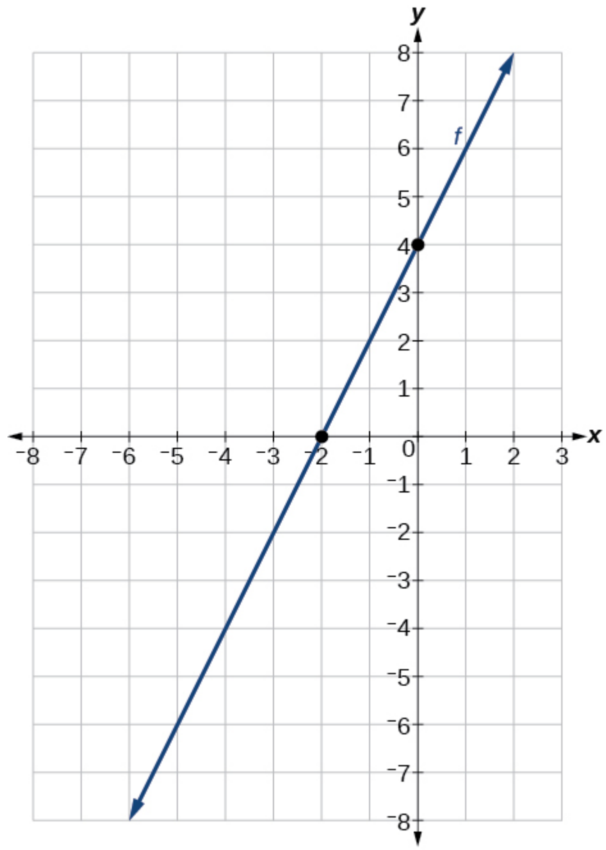
65.
$x = a$

67.
$y = \frac{d}{c-a}x - \frac{ad}{c-a}$

69. $45 per training session.

71. The rate of change is 0.1. For every additional minute talked, the monthly charge increases by $0.1 or 10 cents. The initial value is 24. When there are no minutes talked, initially the charge is $24.

73. The slope is
$-400.$ This means for every year between 1960 and 1989, the population dropped by 400 per year in the city.
75. c.
77. The slopes are equal; *y*-intercepts are not equal.
79. The point of intersection is
$(a,\ a).$ This is because for the horizontal line, all of the
y coordinates are
a and for the vertical line, all of the
x coordinates are
$a.$ The point of intersection will have these two characteristics.
81. First, find the slope of the linear function. Then take the negative reciprocal of the slope; this is the slope of the perpendicular line. Substitute the slope of the perpendicular line and the coordinate of the given point into the equation
$y = mx + b$ and solve for
$b.$ Then write the equation of the line in the form
$y = mx + b$ by substituting in
m and
$b.$
83. neither parallel or perpendicular
85. perpendicular
87. parallel
89.
$(-2, 0)$;
$(0, 4)$
91.
$(\frac{1}{5}, 0)$;
$(0, 1)$
93.
$(8, 0)$;
$(0, 28)$
95.
$\text{Line 1}: \quad m = 8 \qquad \text{Line 2}: \quad m = -6 \qquad \text{Neither}$
97.
$\text{Line 1}: \quad m = -\frac{1}{2} \qquad \text{Line 2}: \quad m = 2 \qquad \text{Perpendicular}$
99.
$\text{Line 1}: \quad m = -2 \qquad \text{Line 2}: \quad m = -2 \qquad \text{Parallel}$
101.
$g(x) = 3x - 3$
103.
$p(t) = -\frac{1}{3}t + 2$
105.
$(-2,\ 1)$
107.
$\left(-\frac{17}{5}, \frac{5}{3}\right)$
109. F
111. C
113. A
115.


y
x
f
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
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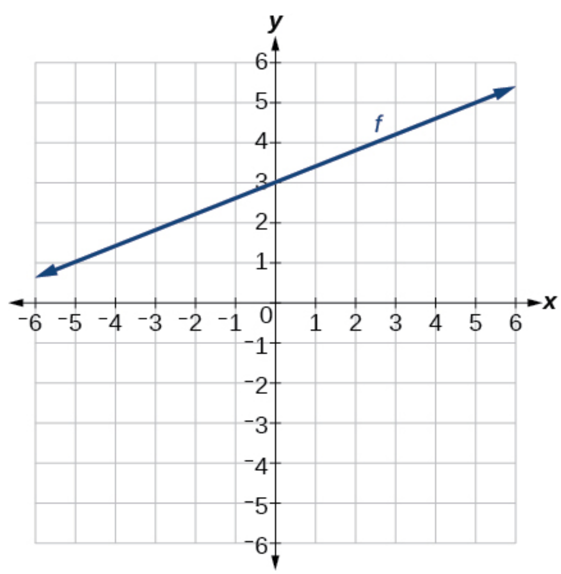
−6
−7
−8
−8 −7 −6 −5 −4 −3 −2 −1 1 2 3

117.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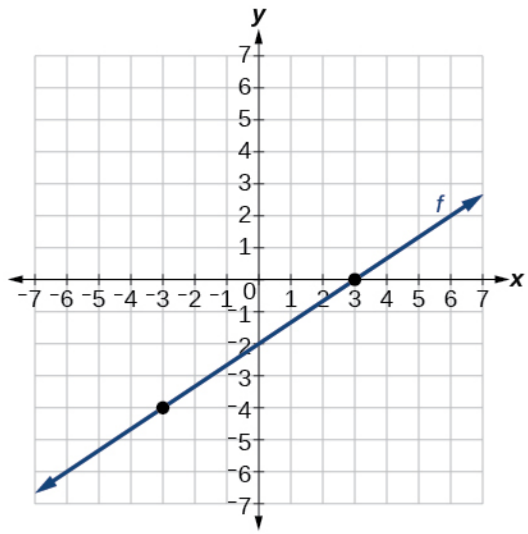
x
f
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

119.


y
x
f
7
6
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1
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−4
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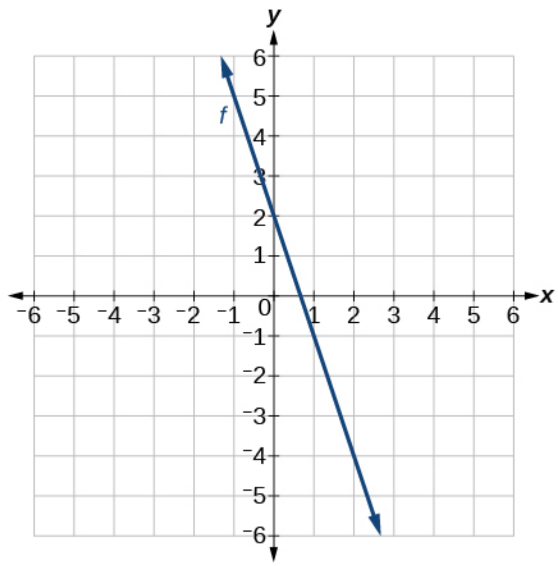
−6
−7
−7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7

121.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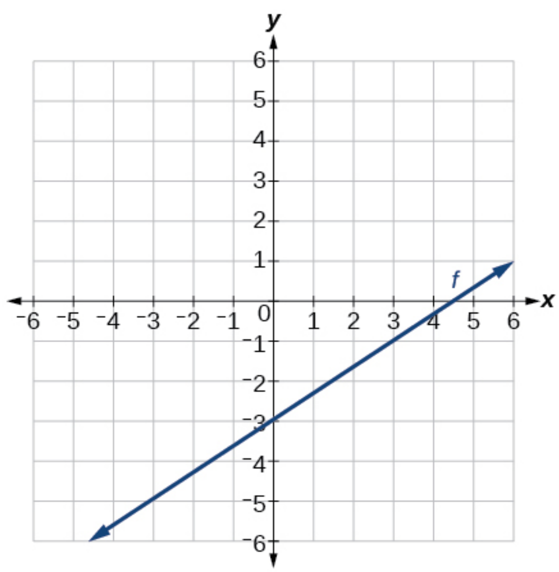
123.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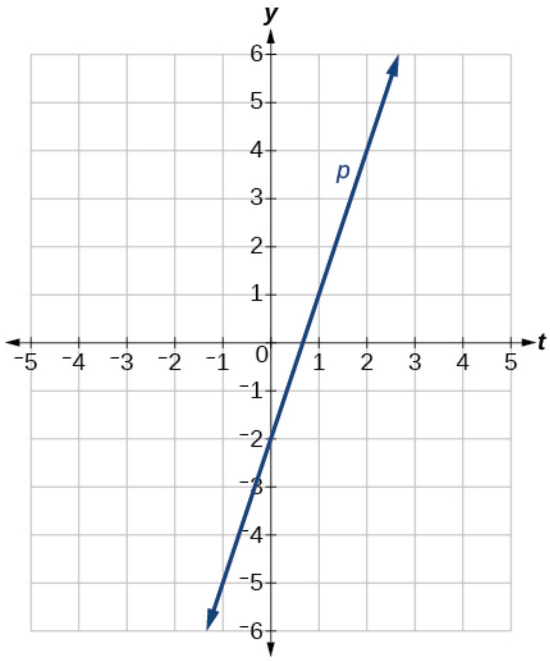
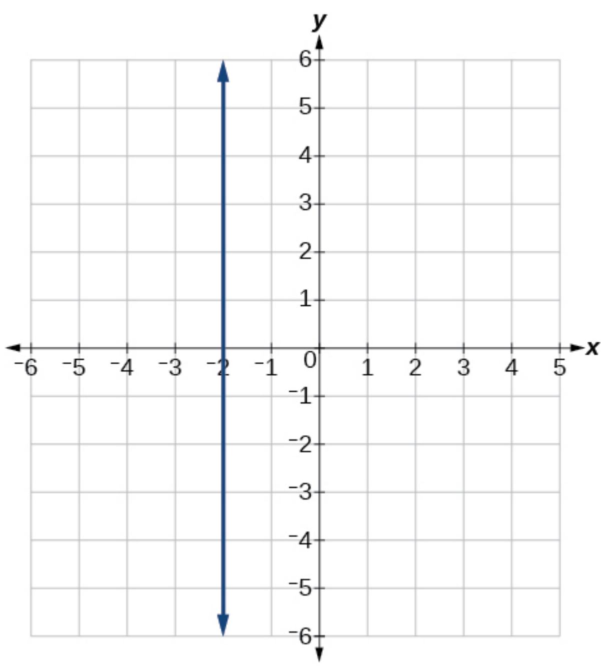
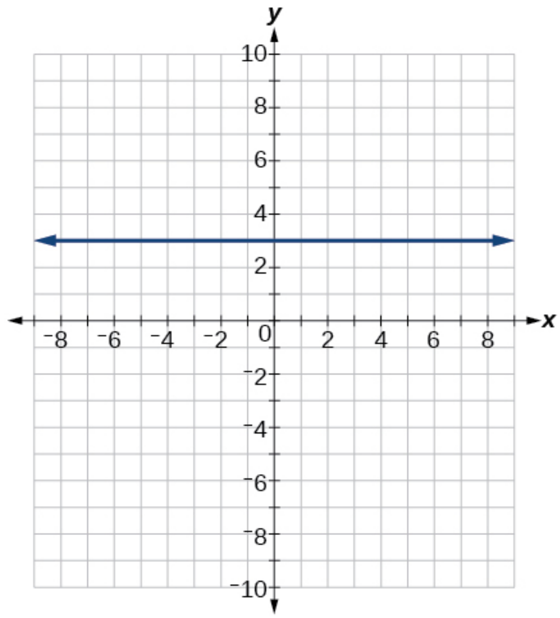
125.



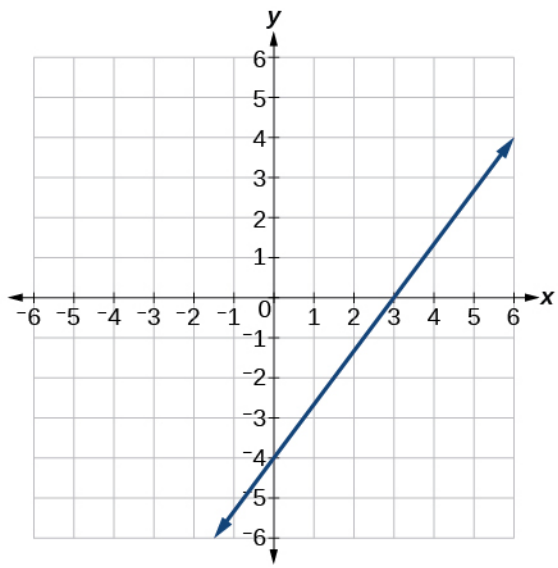
127.



129.


y
x
10
8
6
4
2
0
−2
−4
−6
−8
−10
−8 −6 −4 −2 2 4 6 8

131.


y
x
6
5
4
3
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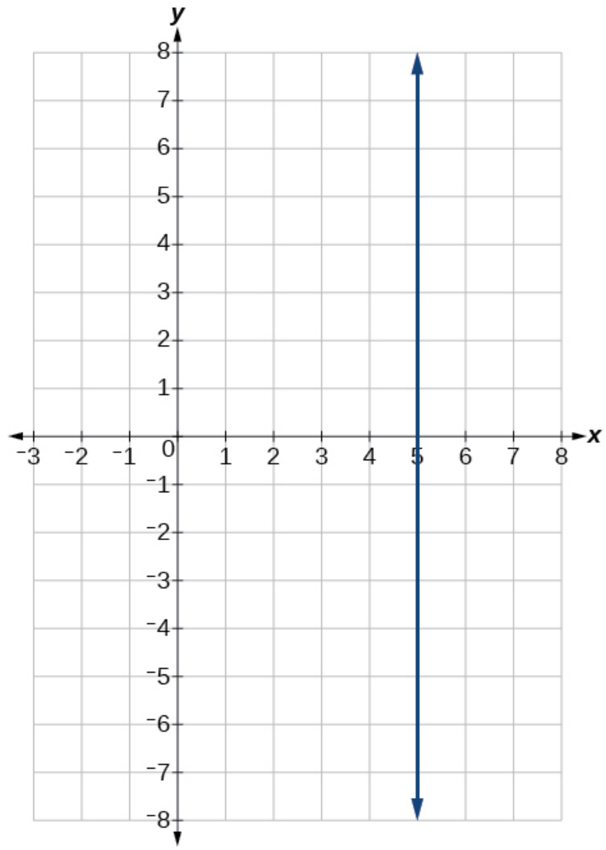
−4
−5
−6
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

133.



135. **a)** $g(x) = 0.75x - 5.5$; **b)** 0.75; **c)** $(0, \ -5.5)$

137. $y = 3$

139. $x = \ -3$

141. no point of intersection

143. $(2, \ 7)$

145. $(-10, \ -5)$

147. $y = 100x - 98$

149. $x < \frac{1999}{201} x > \frac{1999}{201}$

151. Less than 3000 texts

153. Determine the independent variable. This is the variable upon which the output depends.

155. To determine the initial value, find the output when the input is equal to zero.

157. 6 square units

159. 20.012 square units

161. 2,300

163. 64,170

165. $P(t) = 75, \ 000 + 2500t$

167. (–30, 0) Thirty years before the start of this model, the town had no citizens. (0, 75,000) Initially, the town had a population of 75,000.

169. Ten years after the model began.

171.
$W(t) = 7.5t + 0.5$

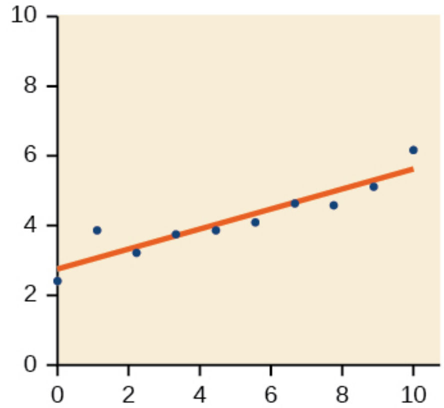
173.
$(-15,\ 0)$: The *x*-intercept is not a plausible set of data for this model because it means the baby weighed 0 pounds 15 months prior to birth.
$(0,\ 7.5)$: The baby weighed 7.5 pounds at birth.
175. At age 5.8 months.
177.
$C(t) = 12,\ 025 - 205t$

179.
$(58.7,\ 0)$: In roughly 59 years, the number of people inflicted with the common cold would be 0.
$(0{,}12{,}025)$: Initially there were 12,025 people afflicted by the common cold.
181. 2064
183.
$y = -2t+180$

185. In 2070, the company's profit will be zero.
187.
$y = 30t - 300$

189. (10, 0) In 1990, the profit earned zero profit.
191. Hawaii
193. During the year 1933
195. $105,620
197. 696 people; 4 years; 174 people per year; 305 people;
$P(t) = 305 + 174t$; 2219 people
199.
$C(x) = 0.15x + 10$; The flat monthly fee is $10 and there is an additional $0.15 fee for each additional minute used; $113.05
201.
$P(t) = 190t + 4360$; 6640 moose
203.
$R(t) = 16 - 2.1t$; 5.5 billion cubic feet; During the year 2017
205. More than 133 minutes
207. More than $42,857.14 worth of jewelry
209. $66,666.67
211. When our model no longer applies, after some value in the domain, the model itself doesn't hold.
213. We predict a value outside the domain and range of the data.
215. The closer the number is to 1, the less scattered the data, the closer the number is to 0, the more scattered the data.
217. 61.966 years
219. No.
221. No.
223. Interpolation. About
$60\,^\circ\ \mathrm{F}.$

225. C
227. B
231.



233. Yes, trend appears linear because $r = 0.985$ and will exceed 12,000 near midyear, 2016, 24.6 years since 1992.

235.
$y = 1.640x + 13.800$,
$r = 0.987$

237.
$y = -0.962x + 26.86, \quad r = -0.965$

239.
$y = -1.981x + 60.197$;
$r = -0.998$

241.
$y = 0.121x - 38.841, \ r = 0.998$

243.
$(-2, -6), (1, -12), (5, -20), (6, -22), (9, -28)$;
$y = -2x - 10$

245.
$(189.8, 0)$ If 18,980 units are sold, the company will have a profit of zero dollars.

247.
$y = 0.00587x + 1985.41$

249.
$y = 20.25x - 671.5$

251.
$y = -10.75x + 742.50$

Review Exercises

253. Yes
255. Increasing.
257.
$y = -3x + 26$

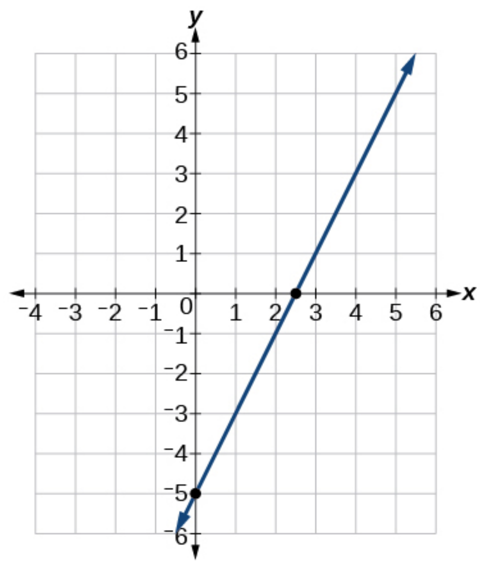
259. 3
261.
$y = 2x - 2$

263. Not linear.
265. parallel
267.
$(-9, 0); (0, -7)$

269. Line 1:
$m = -2;$ Line 2:
$m = -2;$ Parallel

271.
$y = -0.2x + 21$

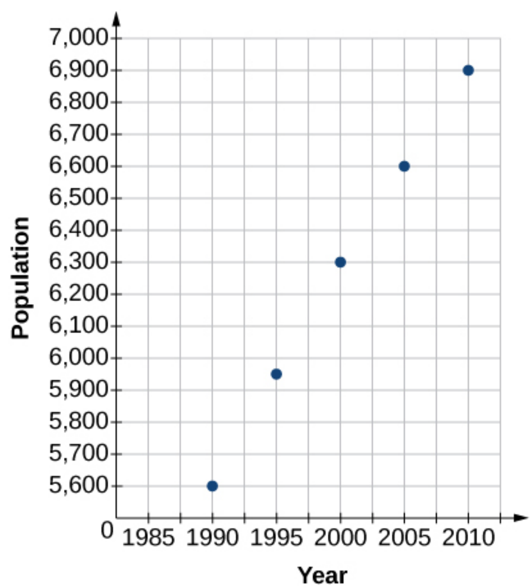
273.

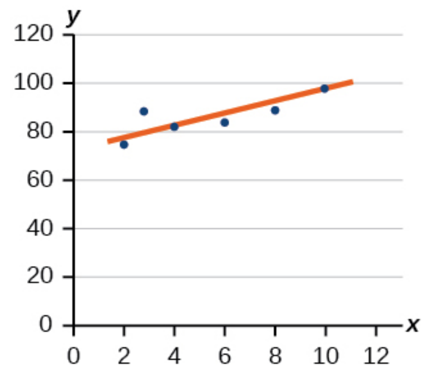


275. 250.
277. 118,000.
279.
$y = -300x + 11,\ 500$

281. a) 800; b) 100 students per year; c)
$P(t) = 100t + 1700$

283. 18,500
285. $91,625
287. Extrapolation.





289.
291. Midway through 2024.
293.
$y = -1.294x + 49.412; \quad r = -0.974$

295. Early in 2022
297. 7,660

Practice Test

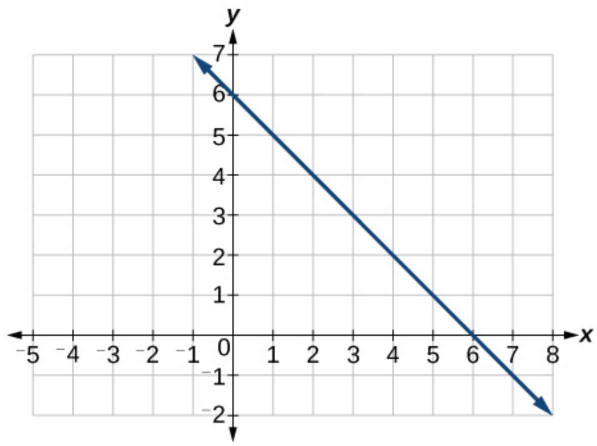
298. Yes.
300. Increasing
302.
$y = -1.5x - 6$

304.
$y = -2x - 1$

306. No.
308. Perpendicular
310.
$(-7, 0)$;
$(0, -2)$

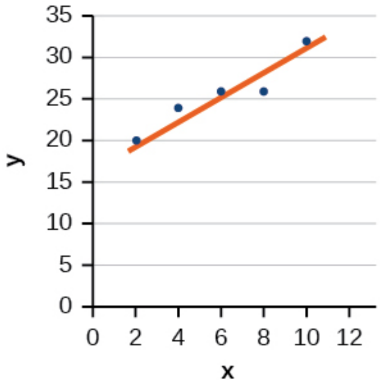
312.
$y = -0.25x + 12$

314.



316. 150
318. 165,000
320.
$y = 875x + 10,675$

322. a) 375; b) dropped an average of 46.875, or about 47 people per year; c)
$y = -46.875t + 1250$



324.
326. Early in 2018
328.
$y = 0.00455x + 1979.5$

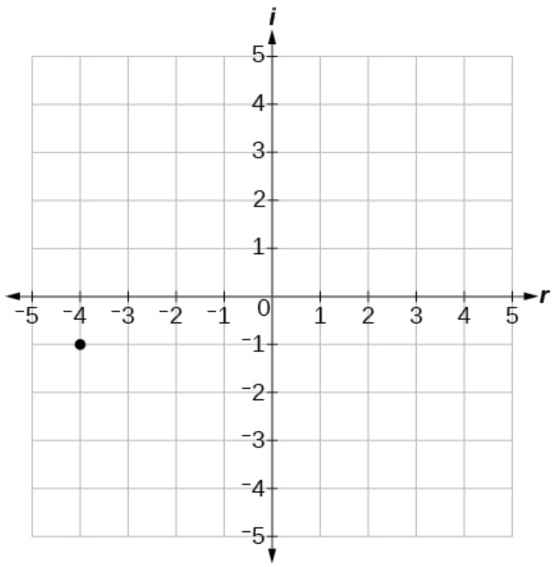
330.
$r = 0.999$

Chapter 3

Try It

3.1.
$\sqrt{-24} = 0 + 2i\sqrt{6}$

3.2.



3.3.
$(3 - 4i) - (2 + 5i) = 1 - 9i$

3.4.
$-8 - 24i$

3.5.
$18 + i$

3.6.
$102 - 29i$

3.7.
$-\frac{3}{17} + \frac{5i}{17}$

3.8. The path passes through the origin and has vertex at
$(-4,\ 7),$ so

$(h)x = -\frac{7}{16}(x+4)^2 + 7.$ To make the shot,
$h(-7.5)$ would need to be about 4 but
$h(-7.5) \approx 1.64;$ he doesn't make it.

3.9.
$g(x) = x^2 - 6x + 13$ in general form;
$g(x) = (x-3)^2 + 4$ in standard form

3.10. The domain is all real numbers. The range is
$f(x) \geq \frac{8}{11},$ or
$\left[\frac{8}{11}, \infty\right).$

3.11. *y*-intercept at (0, 13), No
x- intercepts

3.12. 3 seconds; 256 feet; 7 seconds

3.13.
$f(x)$ is a power function because it can be written as
$f(x) = 8x^5.$ The other functions are not power functions.

3.14. As
x approaches positive or negative infinity,
$f(x)$ decreases without bound: as
$x \to \pm\infty, \quad f(x) \to -\infty$ because of the negative coefficient.

3.15. The degree is 6. The leading term is
$-x^6.$ The leading coefficient is
$-1.$

3.16. As
$x \to \infty, \quad f(x) \to -\infty; \quad as \quad x \to -\infty, \quad f(x) \to -\infty.$ It has the shape of an even degree power function with a negative coefficient.

3.17. The leading term is
$0.2x^3,$ so it is a degree 3 polynomial. As
x approaches positive infinity,
$f(x)$ increases without bound; as
x approaches negative infinity,
$f(x)$ decreases without bound.

3.18. *y*-intercept
$(0, 0);$ *x*-intercepts
$(0, 0), (-2, 0),$ and
$(5, 0)$

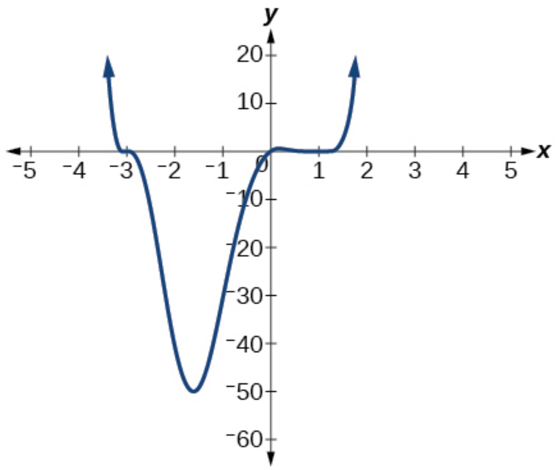
3.19. There are at most 12
x- intercepts and at most 11 turning points.

3.20. The end behavior indicates an odd-degree polynomial function; there are 3
x- intercepts and 2 turning points, so the degree is odd and at least 3. Because of the end behavior, we know that the lead coefficient must be negative.

3.21. The
x- intercepts are
$(2, 0), (-1, 0),$ and
$(5, 0),$ the *y*-intercept is
$(0, 2),$ and the graph has at most 2 turning points.

3.22. *y*-intercept
$(0, 0);$ *x*-intercepts
$(0, 0), (-5, 0), (2, 0),$ and
$(3, 0)$

3.23. The graph has a zero of –5 with multiplicity 1, a zero of –1 with multiplicity 2, and a zero of 3 with even multiplicity.
3.24.



3.25. Because
f is a polynomial function and since
$f(1)$ is negative and
$f(2)$ is positive, there is at least one real zero between
$x = 1$ and
$x = 2.$

3.26.
$f(x) = -\frac{1}{8}(x-2)^3(x+1)^2(x-4)$

3.27. The minimum occurs at approximately the point
$(0, -6.5),$ and the maximum occurs at approximately the point
$(3.5, 7).$

3.28.
$4x^2 - 8x + 15 - \frac{78}{4x+5}$

3.29.
$3x^3 - 3x^2 + 21x - 150 + \frac{1,090}{x+7}$

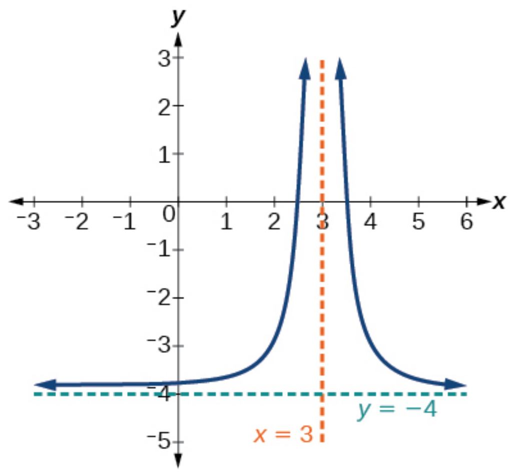
3.30.
$3x^2 - 4x + 1$

3.31.
$f(-3) = -412$

3.32. The zeros are 2, –2, and –4.
3.33. There are no rational zeros.
3.34. The zeros are
$–4, \frac{1}{2},$ and 1.

3.35.
$f(x) = -\frac{1}{2}x^3 + \frac{5}{2}x^2 - 2x + 10$

3.36. There must be 4, 2, or 0 positive real roots and 0 negative real roots. The graph shows that there are 2 positive real zeros and 0 negative real zeros.
3.37. 3 meters by 4 meters by 7 meters
3.38. End behavior: as
$x \to \pm\infty, \quad f(x) \to 0;$ Local behavior: as
$x \to 0, \quad f(x) \to \infty$ (there are no *x*- or *y*-intercepts)
3.39.



The function and the asymptotes are shifted 3 units right and 4 units down. As $x \to 3,\ f(x) \to \infty,$ and as $x \to \pm\infty,\ f(x) \to -4.$ The function is $f(x) = \dfrac{1}{(x-3)^2} - 4.$

3.40. $\frac{12}{11}$

3.41. The domain is all real numbers except $x = 1$ and $x = 5.$

3.42. Removable discontinuity at $x = 5.$ Vertical asymptotes: $x = 0,\ x = 1.$

3.43. Vertical asymptotes at $x = 2$ and $x = -3;$ horizontal asymptote at $y = 4.$

3.44. For the transformed reciprocal squared function, we find the rational form.

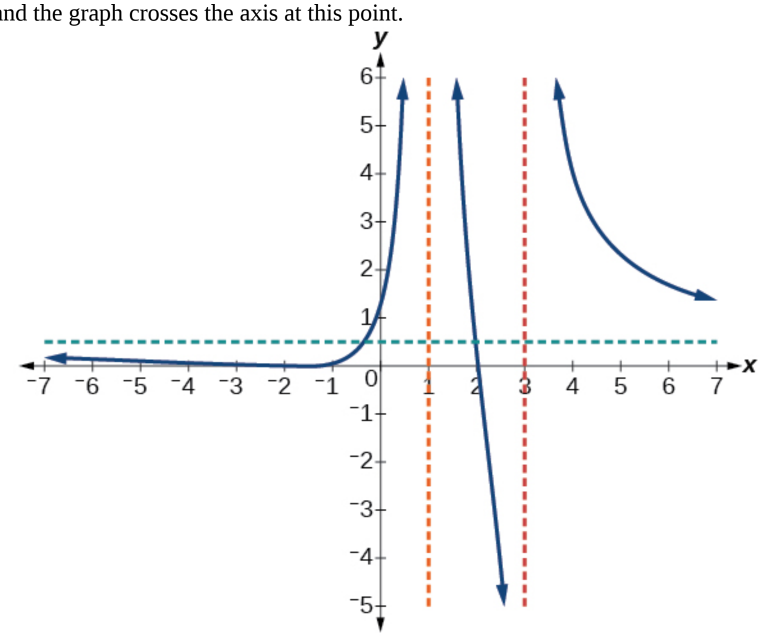
$$f(x) = \frac{1}{(x-3)^2} - 4 = \frac{1-4(x-3)^2}{(x-3)^2} = \frac{1-4(x^2-6x+9)}{(x-3)(x-3)} = \frac{-4x^2+24x-35}{x^2-6x+9}$$

Because the numerator is the same degree as the denominator we know that as $x \to \pm\infty,\quad f(x) \to -4;\quad \text{so}\quad y = -4$ is the horizontal asymptote. Next, we set the denominator equal to zero, and find that the vertical asymptote is $x = 3,$ because as $x \to 3,\ f(x) \to \infty.$ We then set the numerator equal to 0 and find the x-intercepts are at $(2.5,\ 0)$ and $(3.5,\ 0).$ Finally, we evaluate the function at 0 and find the y-intercept to be at $\left(0,\ \frac{-35}{9}\right).$

3.45. Horizontal asymptote at $y = \frac{1}{2}.$ Vertical asymptotes at $x = 1$ and $x = 3.$ y-intercept at $\left(0,\ \frac{4}{3}.\right)$ x-intercepts at $(2,\ 0)$ and $(-2,\ 0).$

$(-2, 0)$ is a zero with multiplicity 2, and the graph bounces off the x-axis at this point.

$(2, 0)$ is a single zero and the graph crosses the axis at this point.



3.46.

$f^{-1}(f(x)) = f^{-1}\left(\frac{x+5}{3}\right) = 3\left(\frac{x+5}{3}\right) - 5 = (x-5)+5 = x$ and

$f\left(f^{-1}(x)\right) = f(3x-5) = \frac{(3x-5)+5}{3} = \frac{3x}{3} = x$

3.47.

$f^{-1}(x) = x^3 - 4$

3.48.

$f^{-1}(x) = \sqrt{x-1}$

3.49.

$f^{-1}(x) = \frac{x^2-3}{2},\ x \geq 0$

3.50.

$f^{-1}(x) = \frac{2x+3}{x-1}$

3.51.

$\frac{128}{3}$

3.52.

$\frac{9}{2}$

3.53.

$x = 20$

Section Exercises

1. Add the real parts together and the imaginary parts together.

3.

i times

i equals –1, which is not imaginary. (answers vary)

5.

$-8 + 2i$

7.

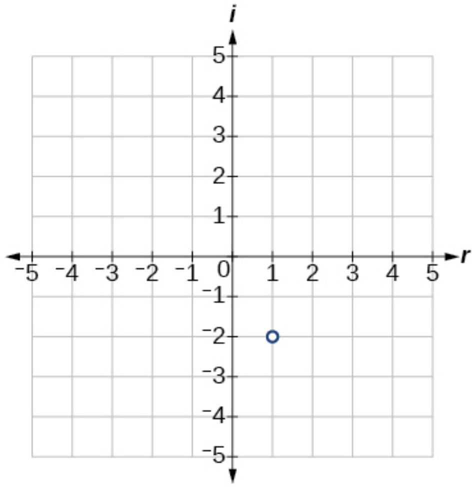
$14 + 7i$

9.

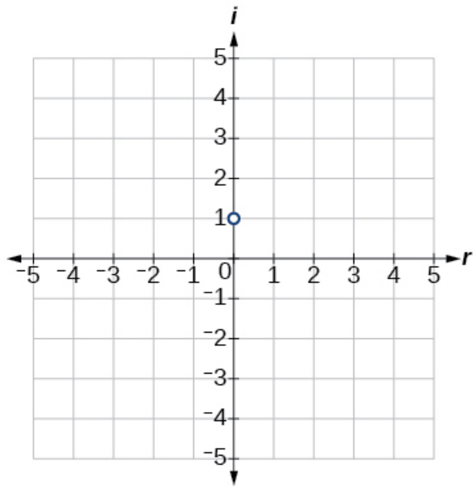
$-\frac{23}{29}+\frac{15}{29}i$

11. 2 real and 0 nonreal

13.



15.



17.

$8-i$

19.

$-11+4i$

21.

$2-5i$

23.

$6+15i$

25.

$-16+32i$

27.

$-4-7i$

29. 25

31.

$2-\frac{2}{3}i$

33.

$4-6i$

35.
$\frac{2}{5}+\frac{11}{5}i$
37.
$15i$
39.
$1+i\sqrt{3}$
41.
1
43.
-1
45. 128i
47.
$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6 = -1$
49.
$3i$
51. 0
53. 5 – 5i
55.
$-2i$
57.
$\frac{9}{2}-\frac{9}{2}i$
59. When written in that form, the vertex can be easily identified.
61. If
$a=0$ then the function becomes a linear function.
63. If possible, we can use factoring. Otherwise, we can use the quadratic formula.
65.
$f(x)=(x+1)^2-2,$ Vertex
$(-1,\ -4)$
67.
$f(x)=\left(x+\frac{5}{2}\right)^2-\frac{33}{4},$ Vertex
$\left(-\frac{5}{2},\ -\frac{33}{4}\right)$
69.
$f(x)=3(x-1)^2-12,$ Vertex
$(1,\ -12)$
71.
$f(x)=3\left(x-\frac{5}{6}\right)^2-\frac{37}{12},$ Vertex
$\left(\frac{5}{6},\ -\frac{37}{12}\right)$
73. Minimum is
$-\frac{17}{2}$ and occurs at
$\frac{5}{2}.$ Axis of symmetry is
$x=\frac{5}{2}.$
75. Minimum is
$-\frac{17}{16}$ and occurs at

$-\frac{1}{8}.$ Axis of symmetry is
$x = -\frac{1}{8}.$

77. Minimum is
$-\frac{7}{2}$ and occurs at
$-3.$ Axis of symmetry is
$x = -3.$

79. Domain is
$(-\infty, \infty).$ Range is
$[2, \infty).$

81. Domain is
$(-\infty, \infty).$ Range is
$[-5, \infty).$

83. Domain is
$(-\infty, \infty).$ Range is
$[-12, \infty).$

85.
$\{2i\sqrt{2}, -2i\sqrt{2}\}$

87.
$\{3i\sqrt{3}, -3i\sqrt{3}\}$

89.
$\{2+i, 2-i\}$

91.
$\{2+3i, 2-3i\}$

93.
$\{5+i, 5-i\}$

95.
$\{2+2\sqrt{6}, \quad 2-2\sqrt{6}\}$

97.
$\left\{-\frac{1}{2}+\frac{3}{2}i, \quad -\frac{1}{2}-\frac{3}{2}i\right\}$

99.
$\left\{-\frac{3}{5}+\frac{1}{5}i, \quad -\frac{3}{5}-\frac{1}{5}i\right\}$

101.
$\left\{-\frac{1}{2}+\frac{1}{2}i\sqrt{7}, \quad -\frac{1}{2}-\frac{1}{2}i\sqrt{7}\right\}$

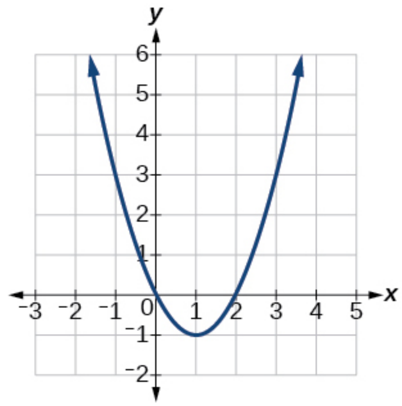
103.
$f(x) = x^2 - 4x + 4$

105.
$f(x) = x^2 + 1$

107.
$f(x) = \frac{6}{49}x^2 + \frac{60}{49}x + \frac{297}{49}$

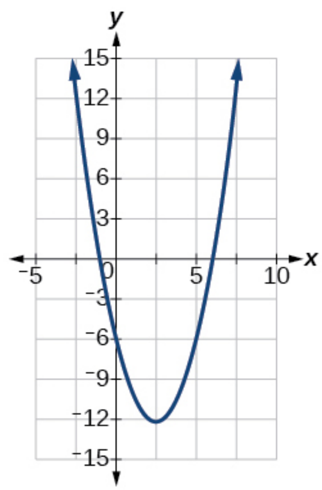
109.
$f(x) = -x^2 + 1$

111.



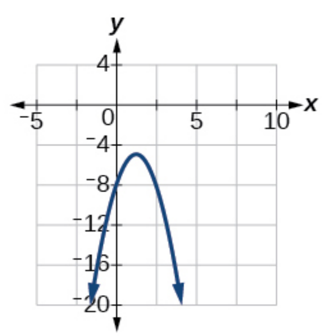
Vertex
$(1,\ -1),$ Axis of symmetry is
$x = 1.$ Intercepts are
$(0,\ 0),\quad (2,\ 0).$

113.



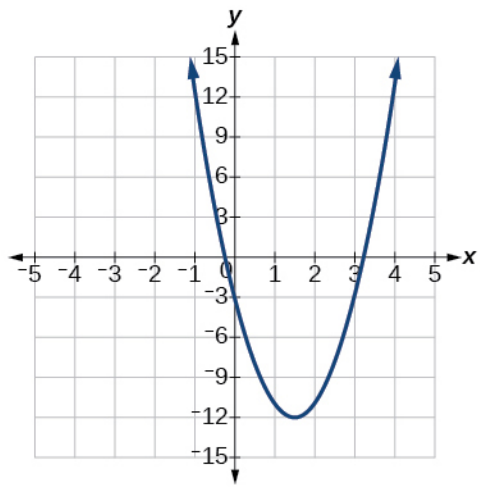
Vertex
$\left(\frac{5}{2},\ \frac{-49}{4}\right),$ Axis of symmetry is
$(0,\ -6),\ (-1,\ 0),\ (6,\ 0).$

115.



Vertex
$\left(\frac{5}{4},\quad -\frac{39}{8}\right),$ Axis of symmetry is
$x = \frac{5}{4}.$ Intercepts are
$(0,\quad -8).$

116.



117.
$f(x) = x^2 - 4x + 1$

119.
$f(x) = -2x^2 + 8x - 1$

121.
$f(x) = \frac{1}{2}x^2 - 3x + \frac{7}{2}$

123.
$f(x) = x^2 + 1$

125.
$f(x) = 2 - x^2$

127.
$f(x) = 2x^2$

129. The graph is shifted up or down (a vertical shift).
131. 50 feet
133. Domain is
$(-\infty, \infty).$ Range is
$[-2, \infty).$

135. Domain is
$(-\infty, \infty)$ Range is
$(-\infty, 11].$

137.
$f(x) = 2x^2 - 1$

139.
$f(x) = 3x^2 - 9$

141.
$f(x) = 5x^2 - 77$

143. 50 feet by 50 feet. Maximize
$f(x) = -x^2 + 100x.$

145. 125 feet by 62.5 feet. Maximize
$f(x) = -2x^2 + 250x.$

147.
6 and
$-6;$ product is –36; maximize
$f(x) = x^2 + 12x.$

149. 2909.56 meters

151. \$10.70
153. The coefficient of the power function is the real number that is multiplied by the variable raised to a power. The degree is the highest power appearing in the function.
155. As x decreases without bound, so does $f(x)$. As x increases without bound, so does $f(x)$.
157. The polynomial function is of even degree and leading coefficient is negative.
159. Power function
161. Neither
163. Neither
165. Degree = 2, Coefficient = –2
167. Degree =4, Coefficient = –2
169. $\text{As } x \to \infty,\ f(x) \to \infty,\ \text{as } x \to -\infty,\ f(x) \to \infty$
171. $\text{As } x \to -\infty,\ f(x) \to -\infty,\ \text{as } x \to \infty,\ f(x) \to -\infty$
173. $\text{As } x \to -\infty,\ f(x) \to -\infty,\ \text{as } x \to \infty,\ f(x) \to -\infty$
175. $\text{As } x \to \infty,\ f(x) \to \infty,\ \text{as } x \to -\infty,\ f(x) \to -\infty$
177. *y*-intercept is $(0,\ 12),$ *t*-intercepts are $(1,\ 0);\ (-2,\ 0);\ \text{and } (3,\ 0).$
179. *y*-intercept is $(0,\ -16).$ *x*-intercepts are $(2,\ 0)$ and $(-2,\ 0).$
181. *y*-intercept is $(0,\ 0).$ *x*-intercepts are $(0,\ 0),\ (4,\ 0),$ and $(-2,\ 0).$
183. 3
185. 5
187. 3
189. 5
191. Yes. Number of turning points is 2. Least possible degree is 3.
193. Yes. Number of turning points is 1. Least possible degree is 2.
195. Yes. Number of turning points is 0. Least possible degree is 1.
196. No.
197. Yes. Number of turning points is 0. Least possible degree is 1.
199.

x	$f(x)$
10	9,500
100	99,950,000
–10	9,500
–100	99,950,000

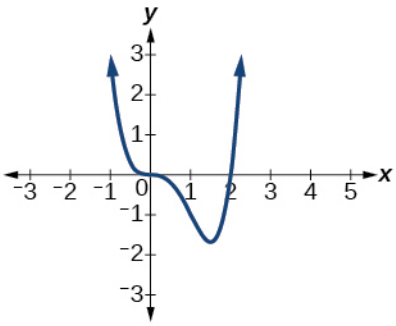
$\text{as}\, x \to -\infty,\ f(x) \to \infty,\ \text{as}\, x \to \infty,\ f(x) \to \infty$

201.

x	$f(x)$
10	–504
100	–941,094
–10	1,716
–100	1,061,106

$\text{as}\, x \to -\infty,\ f(x) \to \infty,\ \text{as}\, x \to \infty,\ f(x) \to -\infty$

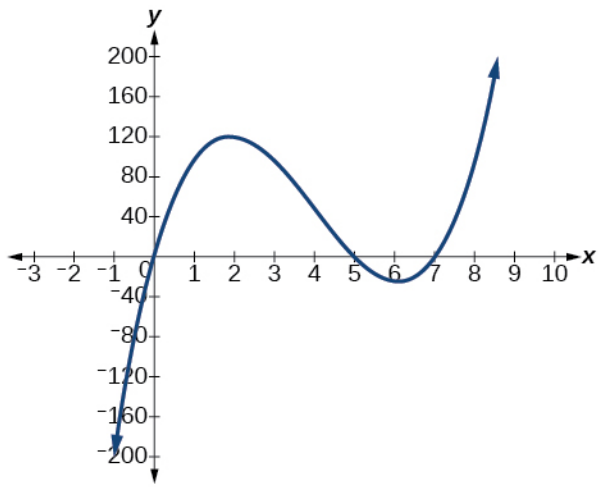
203.



The y- intercept is $(0,\ 0)$. The x- intercepts are $(0,\ 0),\ (2,\ 0)$.

$\text{As}\, x \to -\infty,\ f(x) \to \infty,\ \text{as}\, x \to \infty,\ f(x) \to \infty$

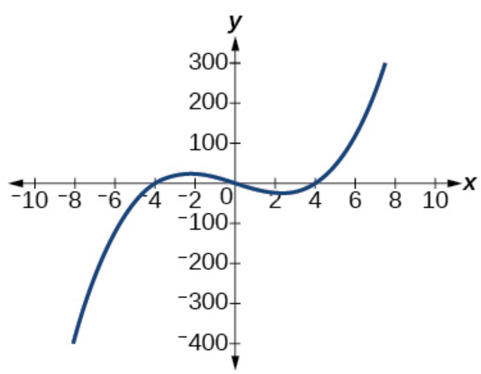
205.



The y- intercept is $(0, 0)$. The x- intercepts are $(0, \quad 0), \ (5, \quad 0), \ (7, \quad 0).$

$\text{As } x \to -\infty, \ f(x) \to -\infty, \ \text{as } x \to \infty, \ f(x) \to \infty$

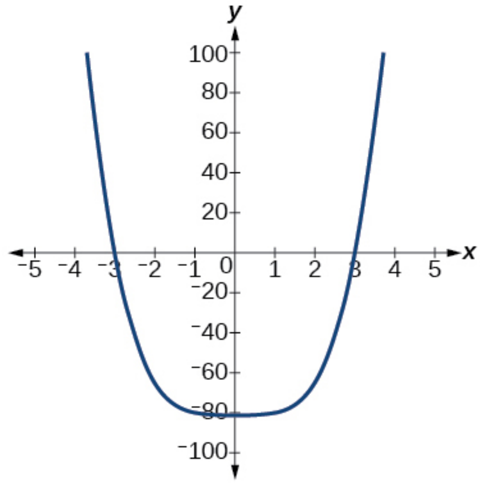
207.



The y- intercept is $(0, \quad 0).$ The x- intercept is $(-4, \quad 0), \ (0, \quad 0), \ (4, \quad 0).$

$As \, x \to -\infty, \ f(x) \to -\infty, \ \text{as } x \to \infty, \ f(x) \to \infty$

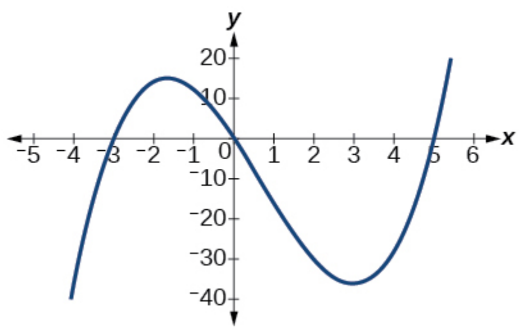
209.



The y- intercept is $(0, \quad -81)$. The x- intercept are $(3, \quad 0), \ (-3, \quad 0)$.

As $x \to -\infty, \ f(x) \to \infty, \ \text{as} \ x \to \infty, \ f(x) \to \infty$

211.



The y- intercept is $(0, \quad 0)$. The x- intercepts are $(-3, \quad 0), \ (0, \quad 0), \ (5, \quad 0)$.

As $x \to -\infty, \ f(x) \to -\infty, \ \text{as} \ x \to \infty, \ f(x) \to \infty$

213.

$f(x) = x^2 - 4$

215.

$f(x) = x^3 - 4x^2 + 4x$

217.

$f(x) = x^4 + 1$

219.

$V(m) = 8m^3 + 36m^2 + 54m + 27$

221.

$V(x) = 4x^3 - 32x^2 + 64x$

223. The x- intercept is where the graph of the function crosses the x- axis, and the zero of the function is the input value for which $f(x) = 0$.

225. If we evaluate the function at
a and at
b and the sign of the function value changes, then we know a zero exists between
a and
$b.$

227. There will be a factor raised to an even power.

229.
$(-2, 0), (3, 0), (-5, 0)$

231.
$(3, 0), (-1, 0), (0, 0)$

233.
$(0, 0),\ (-5, 0),\ (2, 0)$

235.
$(0, 0),\ (-5, 0),\ (4, 0)$

237.
$(2, 0),\ (-2, 0),\ (-1, 0)$

239.
$(-2, 0),\ (2, 0),\ \left(\frac{1}{2}, 0\right)$

241.
$(1, 0),\ (-1, 0)$

243.
$(0, 0),\ (\sqrt{3}, 0),\ (-\sqrt{3}, 0)$

245.
$(0, 0),\ (1, 0), (-1, 0),\ (2, 0),\ (-2, 0)$

247.
$f(2) = -10$ and
$f(4) = 28.$ Sign change confirms.

249.
$f(1) = 3$ and
$f(3) = -77.$ Sign change confirms.

251.
$f(0.01) = 1.000001$ and
$f(0.1) = -7.999.$ Sign change confirms.

253. 0 with multiplicity 2,
$-\frac{3}{2}$ with multiplicity 5, 4 with multiplicity 2

255. 0 with multiplicity 2, –2 with multiplicity 2

257.
$-\frac{2}{3}$ with multiplicity 5, 5 with multiplicity 2

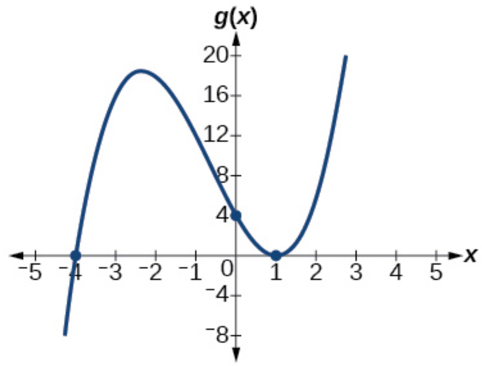
259.
0 with multiplicity 4, 2 with multiplicity 1, -1 with multiplicity 1

261.
$\frac{3}{2}$ with multiplicity 2, 0 with multiplicity 3

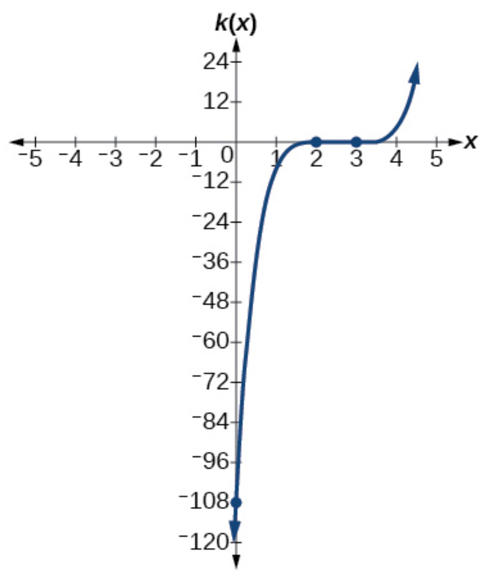
263.
0 with multiplicity 6, $\frac{2}{3}$ with multiplicity 2

265. *x*-intercepts,
$(1, 0)$ with multiplicity 2,
$(-4, 0)$ with multiplicity 1,
y- intercept
$(0, 4)$. As

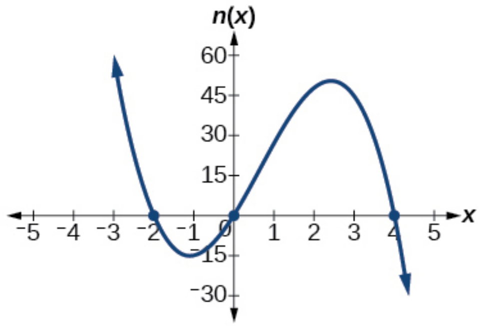
$x \to \infty$,
$f(x) \to \infty$, as
$x \to \infty$,
$f(x) \to \infty$.



267. x-intercepts
$(3, 0)$ with multiplicity 3,
$(2, 0)$ with multiplicity 2,
y- intercept
$(0, -108)$. As
$x \to \infty$,
$f(x) \to -\infty$, as
$x \to \infty$,
$f(x) \to \infty$.



269. x-intercepts
$(0, 0), (-2, 0), (4, 0)$ with multiplicity 1,
y -intercept
$(0, 0).$ As
$x \to \infty$,
$f(x) \to \infty$, as
$x \to \infty$,
$f(x) \to \infty$.



271.

$f(x) = -\frac{2}{9}(x-3)(x+1)(x+3)$

273.

$f(x) = \frac{1}{4}(x+2)^2(x-3)$

275. –4, –2, 1, 3 with multiplicity 1
277. –2, 3 each with multiplicity 2
279.

$f(x) = -\frac{2}{3}(x+2)(x-1)(x-3)$

281.

$f(x) = \frac{1}{3}(x-3)^2(x-1)^2(x+3)$

283.

$f(x) = -15(x-1)^2(x-3)^3$

285.

$f(x) = -2(x+3)(x+2)(x-1)$

287.

$f(x) = -\frac{3}{2}(2x-1)^2(x-6)(x+2)$

289. local max
$(-.58, -.62)$, local min
$(.58, -1.38)$

291. global min
$(-.63, -.47)$

293. global min
$(.75, .89)$

295.

$f(x) = (x-500)^2(x+200)$

297.

$f(x) = 4x^3 - 36x^2 + 80x$

299.

$f(x) = 4x^3 - 36x^2 + 60x + 100$

301.

$f(x) = \pi(9x^3 + 45x^2 + 72x + 36)$

302. The binomial is a factor of the polynomial.
304.

$x + 6 + \frac{5}{x - 1}$, quotient: $x + 6$, remainder: 5

306.

$3x + 2$, quotient: $3x + 2$, remainder: 0

308.

$x - 5$, quotient: $x - 5$, remainder: 0

310.

$2x-7+\frac{16}{x+2}$, quotient: $2x-7$, remainder: 16

312.

$x-2+\frac{6}{3x+1}$, quotient: $x-2$, remainder: 6

314.

$2x^2-3x+5$, quotient: $2x^2-3x+5$, remainder: 0

316.

$2x^2+2x+1+\frac{10}{x-4}$

318.

$2x^2-7x+1-\frac{2}{2x+1}$

320.

$3x^2-11x+34-\frac{106}{x+3}$

322.

x^2+5x+1

324.

$4x^2-21x+84-\frac{323}{x+4}$

326.

$x^2-14x+49$

328.

$3x^2+x+\frac{2}{3x-1}$

330.

x^3-3x+1

332.

x^3-x^2+2

334.

$x^3-6x^2+12x-8$

336.

$x^3-9x^2+27x-27$

338.

$2x^3-2x+2$

340. Yes

$(x-2)(3x^3-5)$

342. Yes

$(x-2)(4x^3+8x^2+x+2)$

344. No

346.

$(x-1)(x^2+2x+4)$

348.

$(x-5)(x^2+x+1)$

350.

Quotient: $4x^2+8x+16$, remainder: -1

352.

Quotient: $3x^2+3x+5$, remainder: 0

354.

Quotient: x^3-2x^2+4x-8, remainder: -6

356.
$x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$
358.
$x^3 - x^2 + x - 1 + \frac{1}{x + 1}$
360.
$1 + \frac{1 + i}{x - i}$
362.
$1 + \frac{1 - i}{x + i}$
364.
$x^2 - ix - 1 + \frac{1 - i}{x - i}$
366.
$2x^2 + 3$
368.
$2x + 3$
370.
$x + 2$
372.
$x - 3$
374.
$3x^2 - 2$
375. The theorem can be used to evaluate a polynomial.
377. Rational zeros can be expressed as fractions whereas real zeros include irrational numbers.
379. Polynomial functions can have repeated zeros, so the fact that number is a zero doesn't preclude it being a zero again.
381.
-106
383.
0
385.
255
387.
-1
389.
$-2, \quad 1, \quad \frac{1}{2}$
391.
-2
393.
-3
395.
$-\frac{5}{2}, \quad \sqrt{6}, \quad -\sqrt{6}$
397.
$2, \quad -4, \quad -\frac{3}{2}$
399.
$4, \quad -4, \quad -5$
401.
$5, \quad -3, \quad -\frac{1}{2}$
403.
$\frac{1}{2}, \quad \frac{1 + \sqrt{5}}{2}, \quad \frac{1 - \sqrt{5}}{2}$

405.
$\frac{3}{2}$

407.
$2, \quad 3, \quad -1, \quad -2$

409.
$\frac{1}{2}, \quad -\frac{1}{2}, \quad 2, \quad -3$

411.
$-1, \quad -1, \quad \sqrt{5}, \quad -\sqrt{5}$

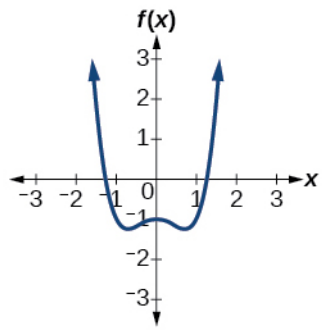
413.
$-\frac{3}{4}, \quad -\frac{1}{2}$

415.
$2, \quad 3+2i, \quad 3-2i$

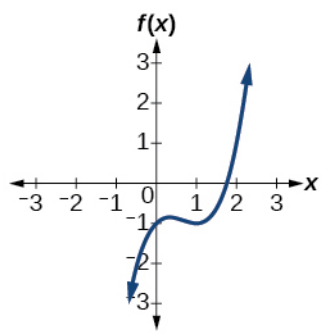
417.
$-\frac{2}{3}, \quad 1+2i, \quad 1-2i$

419.
$-\frac{1}{2}, \quad 1+4i, \quad 1-4i$

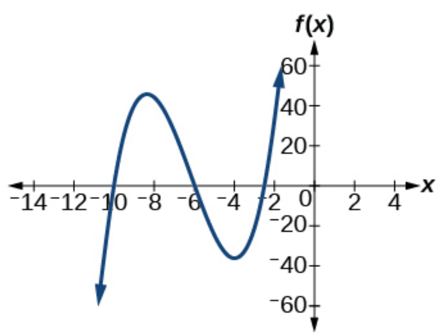
421. 1 positive, 1 negative



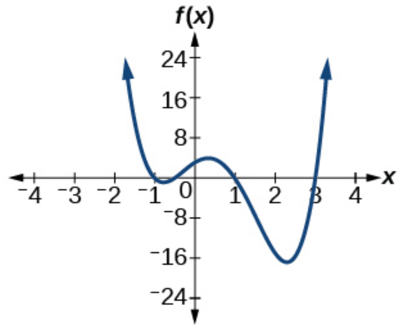
423. 3 or 1 positive, 0 negative



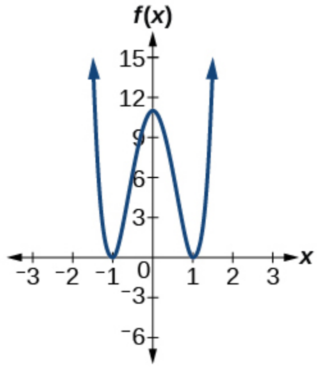
425. 0 positive, 3 or 1 negative



427. 2 or 0 positive, 2 or 0 negative



429. 2 or 0 positive, 2 or 0 negative



431.

$$\pm 5, \quad \pm 1, \quad \pm \frac{5}{2}$$

433.

$$\pm 1, \quad \pm \frac{1}{2}, \quad \pm \frac{1}{3}, \quad \pm \frac{1}{6}$$

435.

$$1, \quad \frac{1}{2}, \quad -\frac{1}{3}$$

437.

$$2, \quad \frac{1}{4}, \quad -\frac{3}{2}$$

439.

$$\frac{5}{4}$$

441.

$$f(x) = \frac{4}{9}\left(x^3 + x^2 - x - 1\right)$$

443.

$$f(x) = -\frac{1}{5}\left(4x^3 - x\right)$$

445. 8 by 4 by 6 inches
447. 5.5 by 4.5 by 3.5 inches
449. 8 by 5 by 3 inches
451. Radius = 6 meters, Height = 2 meters
453. Radius = 2.5 meters, Height = 4.5 meters
455. The rational function will be represented by a quotient of polynomial functions.
457. The numerator and denominator must have a common factor.
459. Yes. The numerator of the formula of the functions would have only complex roots and/or factors common to both the numerator and denominator.
461.

$$\text{All reals } x \neq -1, \quad 1$$

463.

$$\text{All reals } x \neq -1, \quad -2, \quad 1, \quad 2$$

465. V.A. at
$x = -\frac{2}{5};$ H.A. at
$y = 0;$ Domain is all reals
$x \neq -\frac{2}{5}$

467. V.A. at
$x = 4, \quad -9;$ H.A. at
$y = 0;$ Domain is all reals
$x \neq 4, \quad -9$

469. V.A. at
$x = 0, \quad 4, \quad -4;$ H.A. at
$y = 0;$ Domain is all reals
$x \neq 0,\ 4, \quad -4$

471. V.A. at
$x = -5;$ H.A. at
$y = 0;$ Domain is all reals
$x \neq 5,\ -5$

473. V.A. at
$x = \frac{1}{3};$ H.A. at
$y = -\frac{2}{3};$ Domain is all reals
$x \neq \frac{1}{3}.$

475. none

477.
x-intercepts none, y-intercept $\left(0, \frac{1}{4}\right)$

479. Local behavior:
$x \to -\frac{1}{2}^{+},\ f(x) \to -\infty,\ x \to -\frac{1}{2}^{-},\ f(x) \to \infty$ End behavior:
$x \to \pm\infty,\ f(x) \to \frac{1}{2}$

481. Local behavior:
$x \to 6^{+},\ f(x) \to -\infty,\ x \to 6^{-},\ f(x) \to \infty,$ End behavior:
$x \to \pm\infty,\ f(x) \to -2$

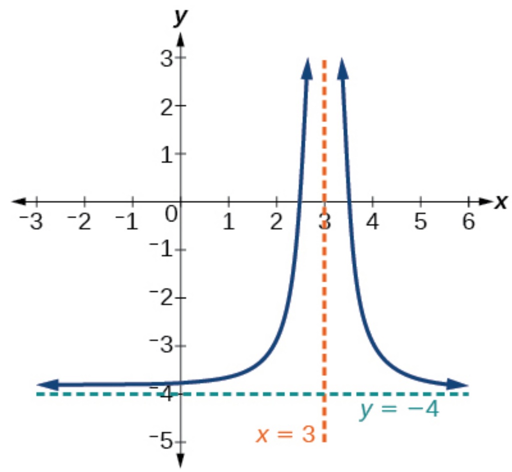
483. Local behavior:
$x \to -\frac{1}{3}^{+},\ f(x) \to \infty,\ x \to -\frac{1}{3}^{-},$
$f(x) \to -\infty,\ x \to \frac{5}{2}^{-},\ f(x) \to \infty,\ x \to \frac{5}{2}^{+},$
$f(x) \to -\infty$

End behavior:
$x \to \pm\infty,$
$f(x) \to \frac{1}{3}$

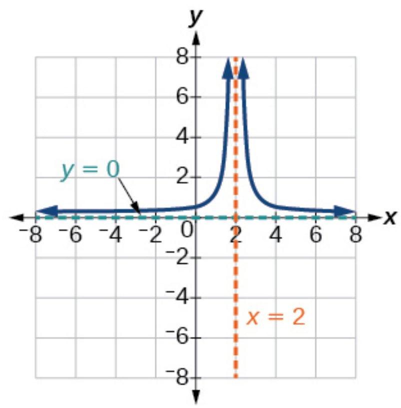
485.
$y = 2x + 4$

487.
$y = 2x$

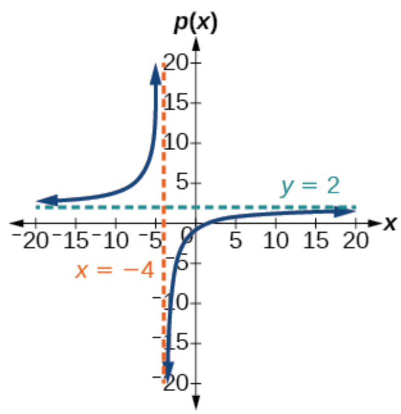
489.
$V.A.\ x = 0,\ H.A.\ y = 2$



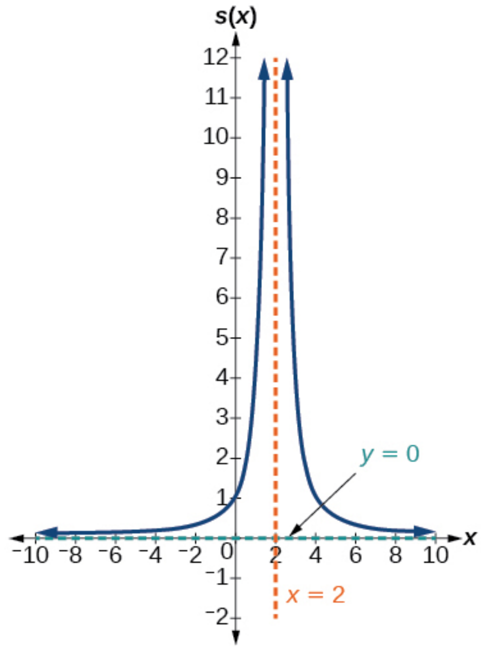
491.
$V.A.\ x = 2,\ \ H.A.\ y = 0$



493.
$V.A.\ x = -4,\ \ H.A.\ y = 2; \left(\frac{3}{2},\ 0\right); \left(0,\ -\frac{3}{4}\right)$

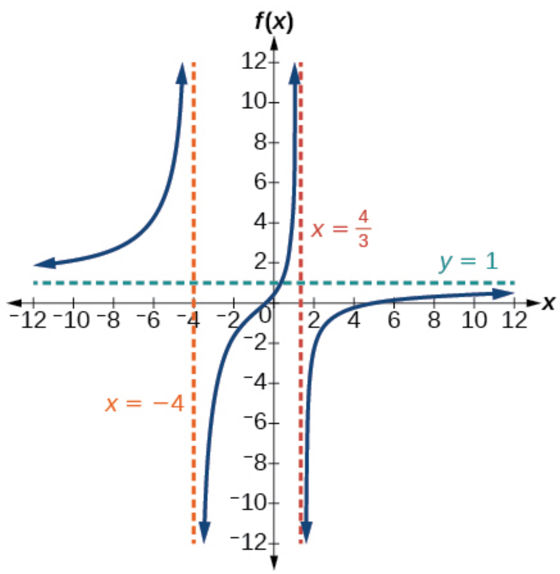


495.
$V.A.\ x = 2,\ \ H.A.\ y = 0,\ \ (0,\ 1)$



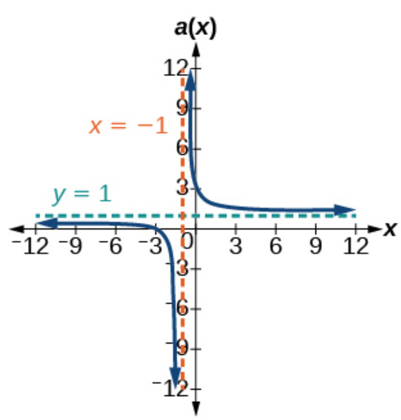
497.

$V.A.\ x = -4,\ \ x = \frac{4}{3},\ \ H.A.\ y = 1;\ (5,\ 0);\ \left(-\frac{1}{3},\ 0\right);\ \left(0,\ \frac{5}{16}\right)$



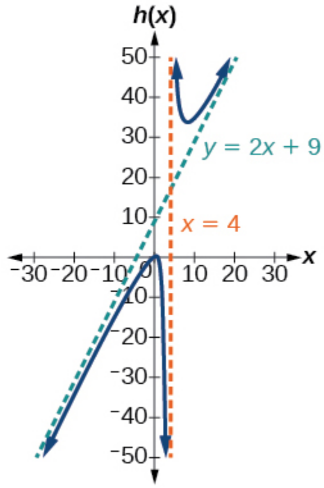
499.

$V.A.\ x = -1,\ \ H.A.\ y = 1;\ (-3,\ 0);\ (0,\ 3)$



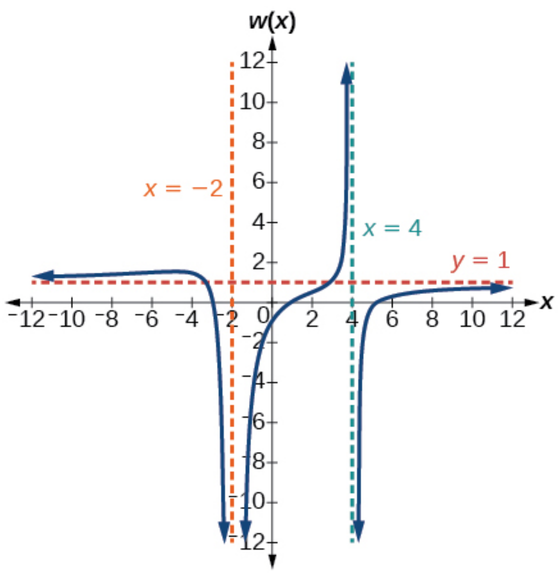
501.

$V.A.\ x = 4,\ \ S.A.\ y = 2x + 9; (-1,\ 0); \left(\frac{1}{2},\ 0\right); \left(0,\ \frac{1}{4}\right)$



503.

$V.A.\ x = \ -2,\ \ x = 4,\ \ H.A.\ y = 1,\ (1,\ 0); (5,\ 0); (-3,\ 0); \left(0,\ \ -\frac{15}{16}\right)$



505.

$y = 50\frac{x^2 - x - 2}{x^2 - 25}$

507.

$y = 7\frac{x^2 + 2x - 24}{x^2 + 9x + 20}$

509.

$y = \frac{1}{2}\frac{x^2 - 4x + 4}{x + 1}$

511.

$y = 4\frac{x - 3}{x^2 - x - 12}$

513.

$y = \ -9\frac{x - 2}{x^2 - 9}$

515.

$y = \frac{1}{3}\frac{x^2 + x - 6}{x - 1}$

517.

$y = -6\frac{(x - 1)^2}{(x + 3)(x - 2)^2}$

519.

x	2.01	2.001	2.0001	1.99	1.999
y	100	1,000	10,000	–100	–1,000

x	10	100	1,000	10,000	100,000
y	.125	.0102	.001	.0001	.00001

Vertical asymptote $x = 2,$ Horizontal asymptote $y = 0$

521.

x	–4.1	–4.01	–4.001	–3.99	–3.999
y	82	802	8,002	–798	–7998

x	10	100	1,000	10,000	100,000
y	1.4286	1.9331	1.992	1.9992	1.999992

Vertical asymptote $x = -4,$ Horizontal asymptote $y = 2$

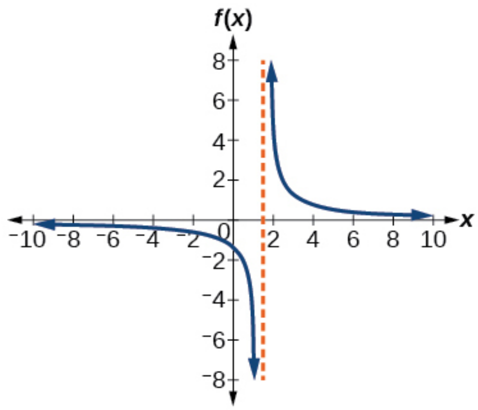
523.

x	–.9	–.99	–.999	–1.1	–1.01
y	81	9,801	998,001	121	10,201

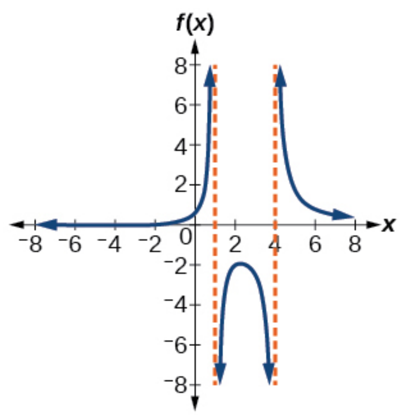
x	10	100	1,000	10,000	100,000
y	.82645	.9803	.998	.9998	

Vertical asymptote $x = -1,$ Horizontal asymptote $y = 1$

525.
$\left(\frac{3}{2}, \infty\right)$



527.
$(-2, 1) \cup (4, \infty)$



529.
$(2, 4)$

531.
$(2, 5)$

533.
$(-1, 1)$

535.
$C(t) = \frac{8 + 2t}{300 + 20t}$

537. After about 6.12 hours.

539.
$A(x) = 50x^2 + \frac{800}{x}.$ 2 by 2 by 5 feet.

541.
$A(x) = \pi x^2 + \frac{100}{x}$. Radius = 2.52 meters.

543. It can be too difficult or impossible to solve for x in terms of y.

545. We will need a restriction on the domain of the answer.

547.
$f^{-1}(x) = \sqrt{x} + 4$

549.
$f^{-1}(x) = \sqrt{x+3} - 1$

551.
$f^{-1}(x) = -\sqrt{\frac{x-5}{3}}$

553.
$f(x) = \sqrt{9-x}$

555.
$f^{-1}(x) = \sqrt[3]{x-5}$

557.
$f^{-1}(x) = \sqrt[3]{4-x}$

559.
$f^{-1}(x) = \frac{x^2-1}{2}, \ [0, \infty)$

561.
$f^{-1}(x) = \frac{(x-9)^2+4}{4}, \ [9, \infty)$

563.
$f^{-1}(x) = \left(\frac{x-9}{2}\right)^3$

565.
$f^{-1}(x) = \frac{2-8x}{x}$

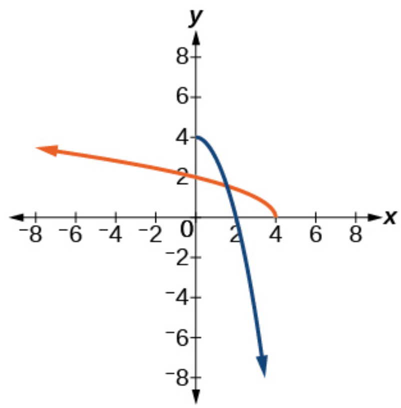
567.
$f^{-1}(x) = \frac{7x-3}{1-x}$

569.
$f^{-1}(x) = \frac{5x-4}{4x+3}$

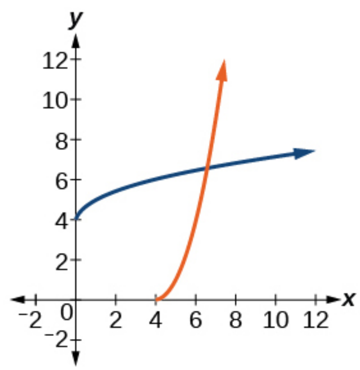
571.
$f^{-1}(x) = \sqrt{x+1} - 1$

573.
$f^{-1}(x) = \sqrt{x+6} + 3$

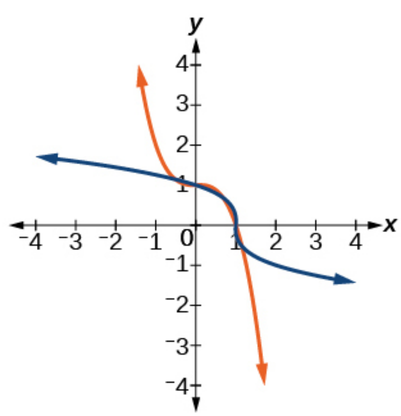
575.
$f^{-1}(x) = \sqrt{4-x}$



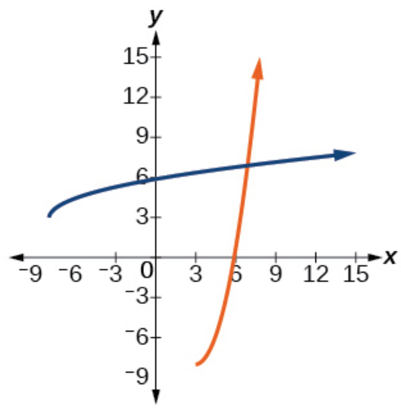
577.
$f^{-1}(x) = \sqrt{x} + 4$



579.
$f^{-1}(x) = \sqrt[3]{1 - x}$

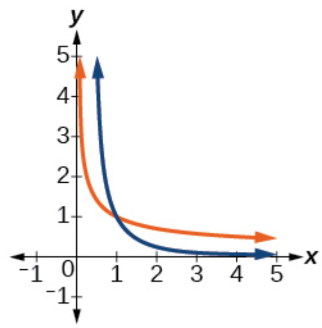


581.
$f^{-1}(x) = \sqrt{x + 8} + 3$



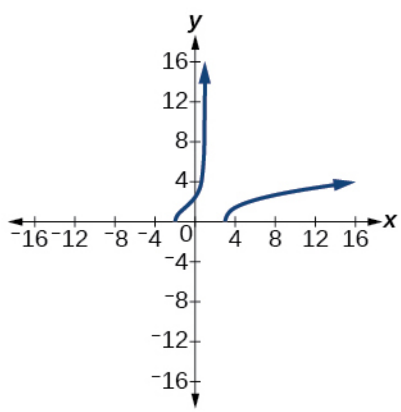
583.

$f^{-1}(x) = \sqrt{\frac{1}{x}}$



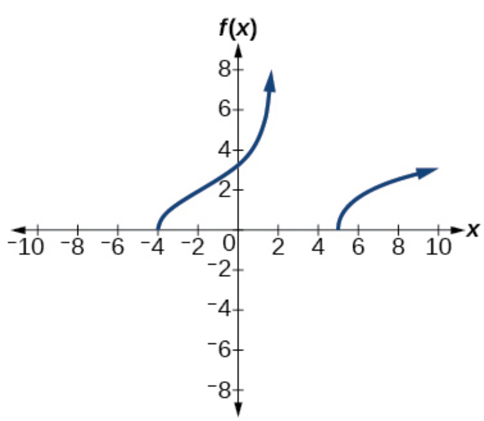
585.

$[-2, 1) \cup [3, \infty)$



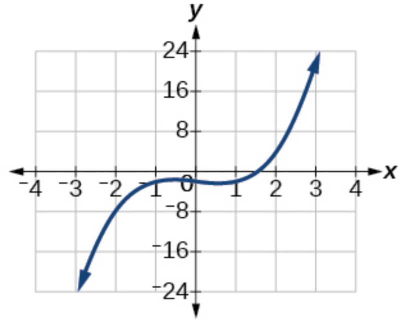
587.

$[-4, 2) \cup [5, \infty)$



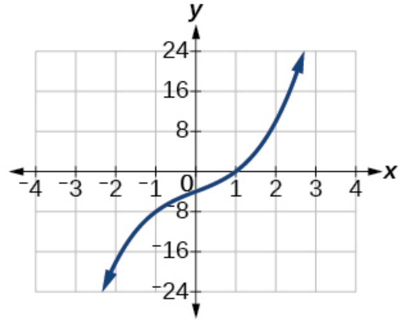
589.

$(-2, \quad 0); \quad (4, \quad 2); \quad (22, \quad 3)$



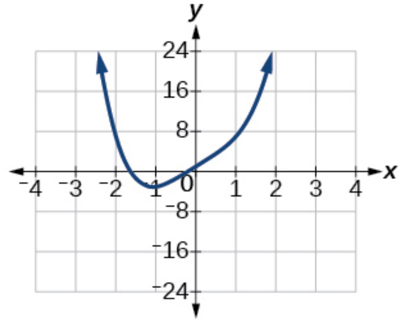
591.

$(-4, \quad 0); \quad (0, \quad 1); \quad (10, \quad 2)$



593.

$(-3, \quad -1); \quad (1, \quad 0); \quad (7, \quad 1)$



595.

$f^{-1}(x) = \sqrt{x + \frac{b^2}{4}} - \frac{b}{2}$

597.

$f^{-1}(x) = \frac{x^3 - b}{a}$

599.

$t(h) = \sqrt{\frac{200 - h}{4.9}}$, 5.53 seconds

601.

$r(V) = \sqrt[3]{\frac{3V}{4\pi}}$, 3.63 feet

603.

$n(C) = \frac{100C - 25}{.6 - C}$, 250 mL

605.

$r(V) = \sqrt{\frac{V}{6\pi}}$, 3.99 meters

607.

$r(V) = \sqrt{\frac{V}{4\pi}},$ 1.99 inches

609. The graph will have the appearance of a power function.

611. No. Multiple variables may jointly vary.

613.

$y = 5x^2$

615.

$y = 10x^3$

617.

$y = 6x^4$

619.

$y = \frac{18}{x^2}$

621.

$y = \frac{81}{x^4}$

623.

$y = \frac{20}{\sqrt[3]{x}}$

625.

$y = 10xzw$

627.

$y = 10x\sqrt{z}$

629.

$y = 4\frac{xz}{w}$

631.

$y = 40\frac{xz}{\sqrt{w}t^2}$

633.

$y = 256$

635.

$y = 6$

637.

$y = 6$

639.

$y = 27$

641.

$y = 3$

643.

$y = 18$

645.

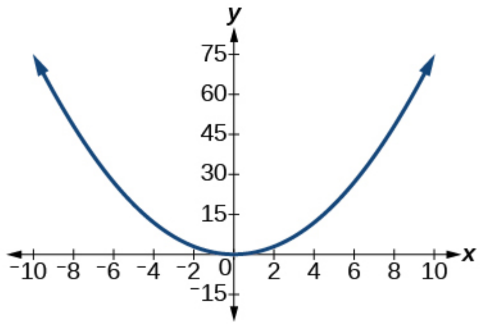
$y = 90$

647.

$y = \frac{81}{2}$

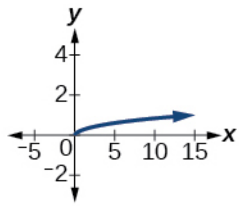
649.

$y = \frac{3}{4}x^2$



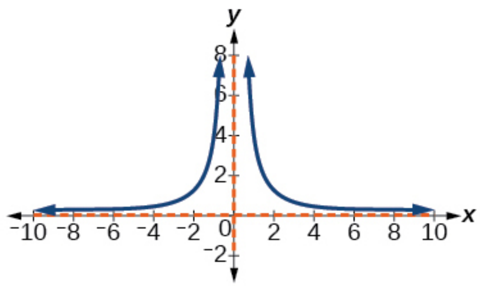
651.

$y = \frac{1}{3}\sqrt{x}$



653.

$y = \frac{4}{x^2}$



655. 1.89 years
657. 0.61 years
659. 3 seconds
661. 48 inches
663. 49.75 pounds
665. 33.33 amperes
667. 2.88 inches

Review Exercises

669.

$2 - 2i$

671.

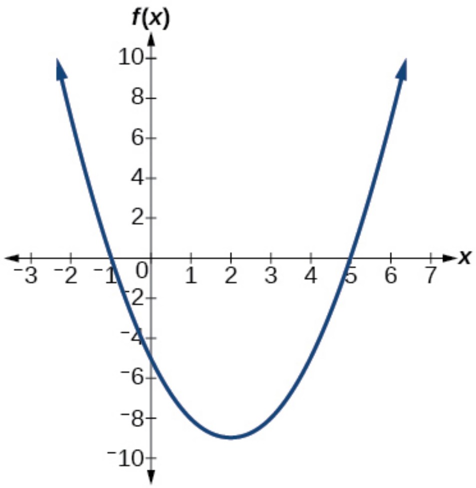
$24 + 3i$

673.

$\{2 + i, \quad 2 - i\}$

675.

$f(x) = (x-2)^2 - 9$ vertex $(2, -9)$, intercepts $(5, 0)$; $(-1, 0)$; $(0, -5)$



677.

$f(x) = \frac{3}{25}(x+2)^2 + 3$

679. 300 meters by 150 meters, the longer side parallel to river.
681. Yes, degree = 5, leading coefficient = 4
683. Yes, degree = 4, leading coefficient = 1
685.

As $x \to -\infty,\ f(x) \to -\infty,$ as $x \to \infty,\ f(x) \to \infty$

687. –3 with multiplicity 2,

$-\frac{1}{2}$ with multiplicity 1, –1 with multiplicity 3

689. 4 with multiplicity 1
691.

$\frac{1}{2}$ with multiplicity 1, 3 with multiplicity 3

693.

$x^2 + 4$ with remainder 12

695.

$x^2 - 5x + 20 - \frac{61}{x+3}$

697.

$2x^2 - 2x - 3$, so factored form is

$(x+4)(2x^2 - 2x - 3)$

699.

$\left\{-2, \quad 4, \quad -\frac{1}{2}\right\}$

701.

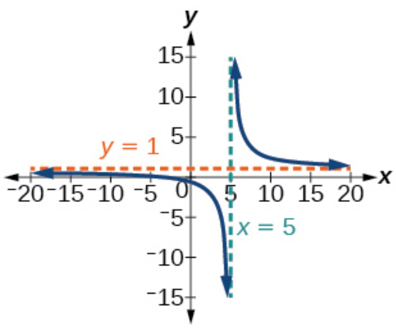
$\left\{1, \quad 3, \quad 4, \quad \frac{1}{2}\right\}$

703. 0 or 2 positive, 1 negative
705. Intercepts

$(-2, 0)$ and $\left(0, -\frac{2}{5}\right)$, Asymptotes

$x = 5$ and

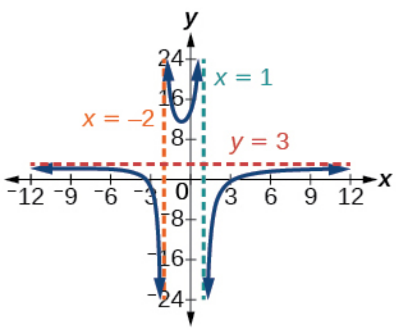
$y = 1.$



707. Intercepts (3, 0), (-3, 0), and

$\left(0, \frac{27}{2}\right)$, Asymptotes

$x = 1, \quad x = -2, \quad y = 3.$



709.

$y = x - 2$

711.

$f^{-1}(x) = \sqrt{x} + 2$

713.

$f^{-1}(x) = \sqrt{x + 11} - 3$

715.

$f^{-1}(x) = \frac{(x+3)^2 - 5}{4}, \ x \geq -3$

717.

$y = 64$

719.

$y = 72$

721. 148.5 pounds

Practice Test

723.

$20 - 10i$

725.

$\{2 + 3i, \quad 2 - 3i\}$

727.

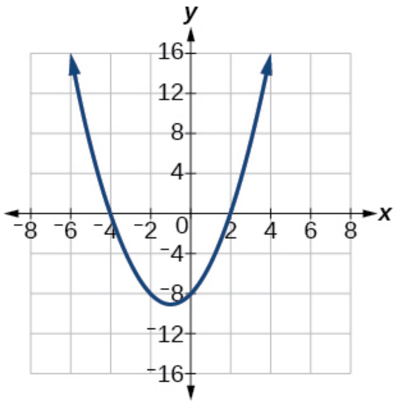
$As\, x \to -\infty, \ f(x) \to -\infty, \ as\, x \to \infty, \ f(x) \to \infty$

729.

$f(x) = (x + 1)^2 - 9$, vertex

$(-1, -9)$, intercepts

$(2,\ 0);\ (-4,\ 0);\ (0,\ -8)$



731. 60,000 square feet
733. 0 with multiplicity 4, 3 with multiplicity 2
735.

$2x^2 - 4x + 11 - \frac{26}{x+2}$

737.

$2x^2 - x - 4$. So factored form is

$(x+3)(2x^2 - x - 4)$

739.

$-\frac{1}{2}$ (has multiplicity 2),

$\frac{-1 \pm i\sqrt{15}}{2}$

741.

-2 (has multiplicity 3),

$\pm i$

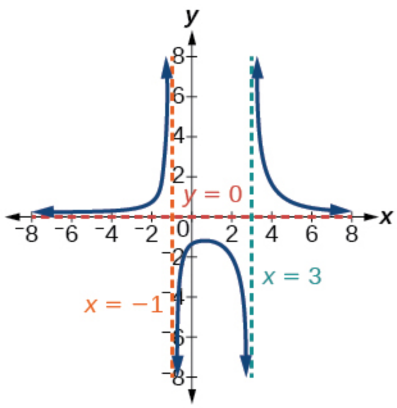
743.

$f(x) = 2(2x-1)^3(x+3)$

745. Intercepts

$(-4,\ 0),\ \left(0,\ -\frac{4}{3}\right)$, Asymptotes

$x = 3,\ \ x = -1,\ \ y = 0$.



747.

$y = x + 4$

749.
$f^{-1}(x) = \sqrt[3]{\frac{x+4}{3}}$

751.
$y = 18$

753. 4 seconds

Chapter 4

Try It

4.1.
$g(x) = 0.875^x$ and
$j(x) = 1095.6^{-2x}$ represent exponential functions.

4.2.
5.5556

4.3. About
1.548 billion people; by the year 2031, India's population will exceed China's by about 0.001 billion, or 1 million people.

4.4.
$(0,\ 129)$ and
$(2,\ 236);\quad N(t) = 129(1.3526)^t$

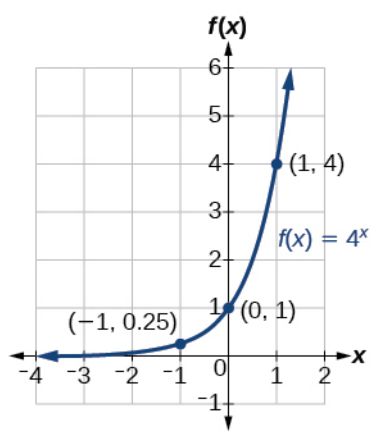
4.5.
$f(x) = 2(1.5)^x$

4.6.
$f(x) = \sqrt{2}(\sqrt{2})^x.$ Answers may vary due to round-off error. The answer should be very close to
$1.4142(1.4142)^x.$

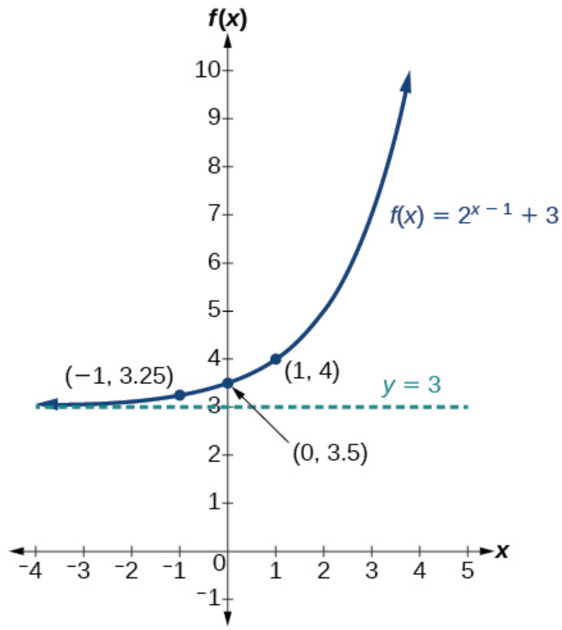
4.7.
$y \approx 12 \cdot 1.85^x$

4.8. about $3,644,675.88
4.9. $13,693
4.10.
$e^{-0.5} \approx 0.60653$

4.11. $3,659,823.44
4.12. 3.77E-26 (This is calculator notation for the number written as
3.77×10^{-26} in scientific notation. While the output of an exponential function is never zero, this number is so close to zero that for all practical purposes we can accept zero as the answer.)
4.13. The domain is
$(-\infty,\ \infty);$ the range is
$(0,\ \infty);$ the horizontal asymptote is
$y = 0.$

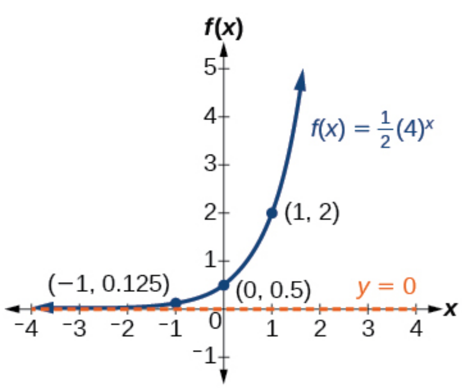


4.14. The domain is
$(-\infty,\ \infty);$ the range is
$(3,\ \infty);$ the horizontal asymptote is
$y = 3.$

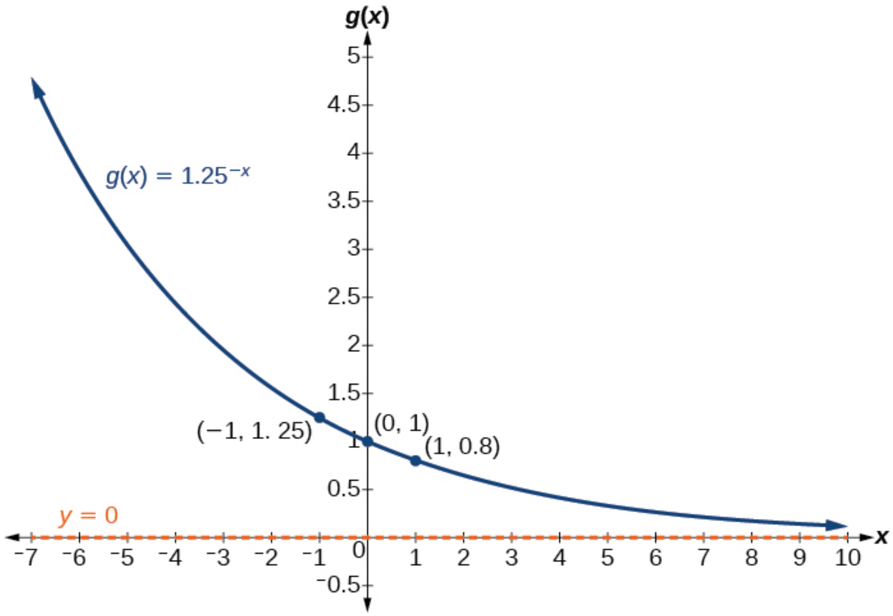


4.15.
$x \approx\ -1.608$

4.16. The domain is
$(-\infty,\ \infty);$ the range is
$(0,\ \infty);$ the horizontal asymptote is
$y = 0.$



4.17. The domain is
$(-\infty,\ \infty);$ the range is
$(0,\ \infty);$ the horizontal asymptote is
$y = 0.$



4.18.

$f(x) = -\frac{1}{3}e^x - 2;$ the domain is $(-\infty, \infty);$ the range is $(-\infty, 2);$ the horizontal asymptote is $y = 2.$

4.19.

a. $\log_{10}(1,000,000) = 6$ is equivalent to $10^6 = 1,000,000$

b. $\log_5(25) = 2$ is equivalent to $5^2 = 25$

4.20.

a. $3^2 = 9$ is equivalent to $\log_3(9) = 2$

b. $5^3 = 125$ is equivalent to $\log_5(125) = 3$

c. $2^{-1} = \frac{1}{2}$ is equivalent to $\log_2(\frac{1}{2}) = -1$

4.21.

$\log_{121}(11) = \frac{1}{2}$ (recalling that $\sqrt{121} = (121)^{\frac{1}{2}} = 11$)

4.22.

$\log_2\left(\frac{1}{32}\right) = -5$

4.23.
$\log(1,000,000) = 6$

4.24.
$\log(123) \approx 2.0899$

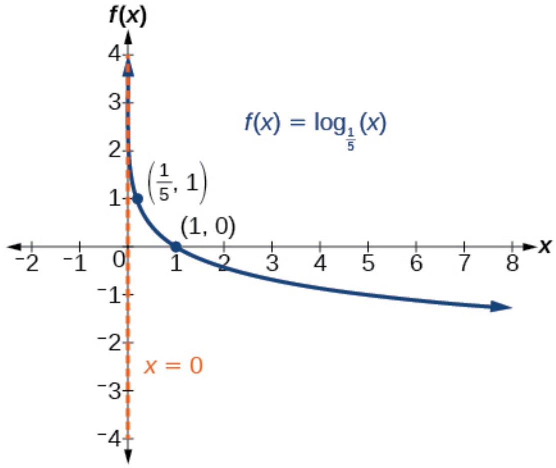
4.25. The difference in magnitudes was about
$3.929.$

4.26. It is not possible to take the logarithm of a negative number in the set of real numbers.

4.27.
$(2, \infty)$

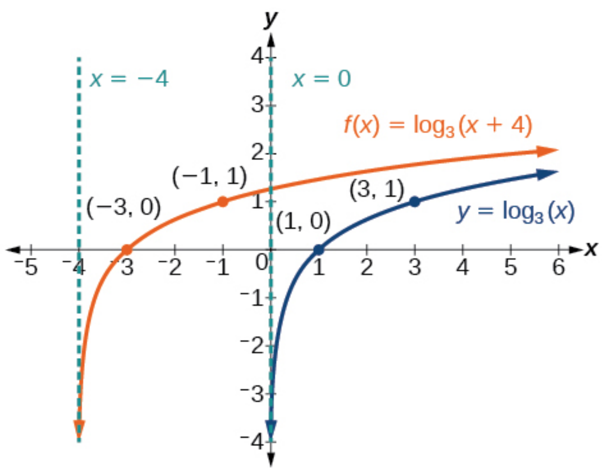
4.28.
$(5, \infty)$

4.29.



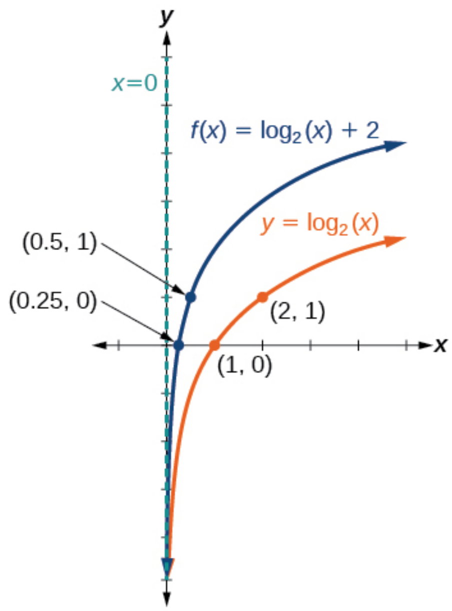
The domain is
$(0, \infty),$ the range is
$(-\infty, \infty),$ and the vertical asymptote is
$x = 0.$

4.30.



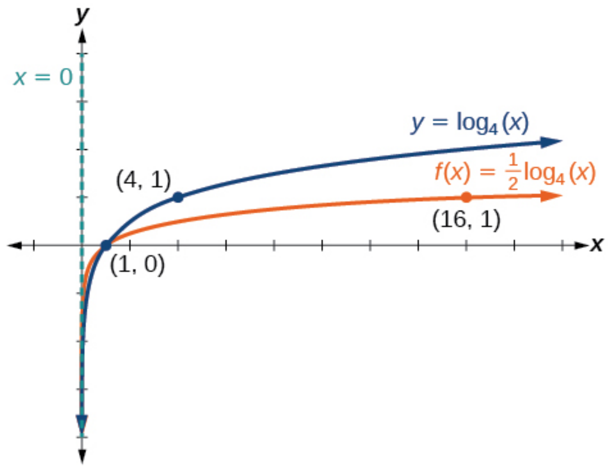
The domain is
$(-4, \infty),$ the range
$(-\infty, \infty),$ and the asymptote
$x = -4.$

4.31.



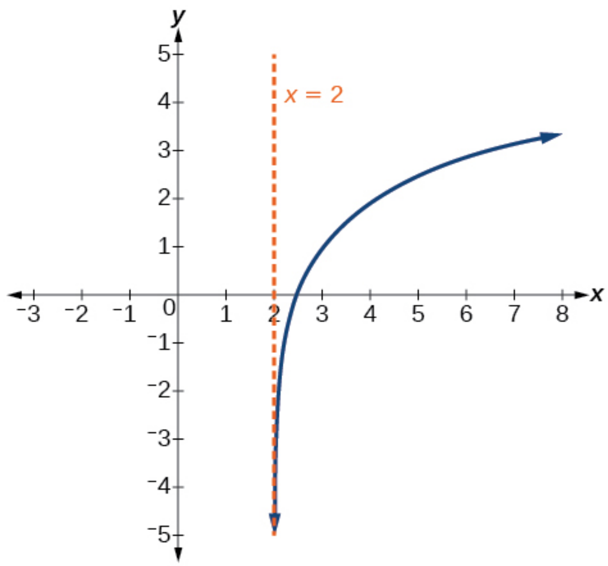
The domain is $(0, \infty),$ the range is $(-\infty, \infty),$ and the vertical asymptote is $x = 0.$

4.32.



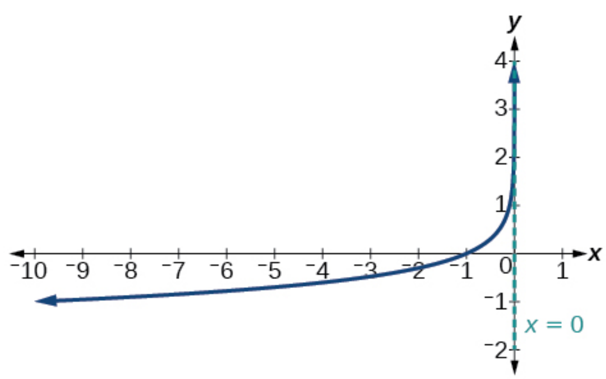
The domain is $(0, \infty),$ the range is $(-\infty, \infty),$ and the vertical asymptote is $x = 0.$

4.33.



The domain is $(2,\ \infty),$ the range is $(-\infty,\ \infty),$ and the vertical asymptote is $x = 2.$

4.34.



The domain is $(-\infty,\ 0),$ the range is $(-\infty,\ \infty),$ and the vertical asymptote is $x = 0.$

4.35.

$x \approx 3.049$

4.36.

$x = 1$

4.37.

$f(x) = 2\ln(x + 3) - 1$

4.38.

$\log_b 2 + \log_b 2 + \log_b 2 + \log_b k = 3\log_b 2 + \log_b k$

4.39.

$\log_3 (x + 3) - \log_3 (x - 1) - \log_3 (x - 2)$

4.40.

$2\ln x$

4.41.

$-2\ln(x)$

4.42.

$\log_3 16$

4.43.
$2\log x + 3\log y - 4\log z$

4.44.
$\frac{2}{3}\ln x$

4.45.
$\frac{1}{2}\ln(x-1) + \ln(2x+1) - \ln(x+3) - \ln(x-3)$

4.46.
$\log\left(\frac{3 \cdot 5}{4 \cdot 6}\right);$ can also be written
$\log\left(\frac{5}{8}\right)$ by reducing the fraction to lowest terms.

4.47.
$\log\left(\frac{5(x-1)^3 \sqrt{x}}{(7x-1)}\right)$

4.48.
$\log\frac{x^{12}(x+5)^4}{(2x+3)^4};$ this answer could also be written
$\log\left(\frac{x^3(x+5)}{(2x+3)}\right)^4.$

4.49. The pH increases by about 0.301.

4.50.
$\frac{\ln 8}{\ln 0.5}$

4.51.
$\frac{\ln 100}{\ln 5} \approx \frac{4.6051}{1.6094} = 2.861$

4.52.
$x = -2$

4.53.
$x = -1$

4.54.
$x = \frac{1}{2}$

4.55. The equation has no solution.

4.56.
$x = \frac{\ln 3}{\ln\left(\frac{2}{3}\right)}$

4.57.
$t = 2\ln\left(\frac{11}{3}\right)$ or
$\ln\left(\frac{11}{3}\right)^2$

4.58.
$t = \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln(2)$

4.59.
$x = \ln 2$

4.60.
$x = e^4$

4.61.
$x = e^5 - 1$

4.62.
$x \approx 9.97$
4.63.
$x = 1$ or
$x = -1$
4.64.
$t = 703,\ 800,\ 000 \times \frac{\ln(0.8)}{\ln(0.5)}$ years $\approx$ 226, 572, 993 years.
4.65.
$f(t) = A_0 e^{-0.0000000087t}$
4.66. less than 230 years, 229.3157 to be exact
4.67.
$f(t) = A_0 e^{\frac{\ln 2}{3}t}$
4.68. 6.026 hours
4.69. 895 cases on day 15
4.70. Exponential.
$y = 2e^{0.5x}$.
4.71.
$y = 3e^{(\ln 0.5)x}$
4.72.

a. The exponential regression model that fits these data is
$y = 522.88585984(1.19645256)^x$.
b. If spending continues at this rate, the graduate's credit card debt will be $4,499.38 after one year.

4.73.

a. The logarithmic regression model that fits these data is
$y = 141.91242949 + 10.45366573\ln(x)$
b. If sales continue at this rate, about 171,000 games will be sold in the year 2015.

4.74.

a. The logistic regression model that fits these data is
$y = \frac{25.65665979}{1 + 6.113686306e^{-0.3852149008x}}$.
b. If the population continues to grow at this rate, there will be about
$25{,}634$ seals in 2020.
c. To the nearest whole number, the carrying capacity is 25,657.

Section Exercises

1. Linear functions have a constant rate of change. Exponential functions increase based on a percent of the original.
3. When interest is compounded, the percentage of interest earned to principal ends up being greater than the annual percentage rate for the investment account. Thus, the annual percentage rate does not necessarily correspond to the real interest earned, which is the very definition of *nominal*.
5. exponential; the population decreases by a proportional rate. .
7. not exponential; the charge decreases by a constant amount each visit, so the statement represents a linear function. .
9. The forest represented by the function
$B(t) = 82(1.029)^t$.
11. After
$t = 20$ years, forest A will have
43 more trees than forest B.
13. Answers will vary. Sample response: For a number of years, the population of forest A will increasingly exceed forest B, but because forest B actually grows at a faster rate, the population will eventually become larger than forest A and will remain that way as long as the population growth models hold. Some factors that might influence the long-term validity of the exponential growth model are drought, an epidemic that culls the population, and other environmental and biological factors.
15. exponential growth; The growth factor,
$1.06,$ is greater than
$1.$

17. exponential decay; The decay factor, $0.97,$ is between 0 and $1.$

19. $f(x) = 2000(0.1)^x$

21. $f(x) = \left(\frac{1}{6}\right)^{-\frac{3}{5}}\left(\frac{1}{6}\right)^{\frac{x}{5}} \approx 2.93(0.699)^x$

23. Linear

25. Neither

27. Linear

29. $\$10,250$

31. $\$13,268.58$

33. $P = A(t) \cdot \left(1 + \frac{r}{n}\right)^{-nt}$

35. $\$4{,}572.56$

37. 4%

39. continuous growth; the growth rate is greater than $0.$

41. continuous decay; the growth rate is less than $0.$

43. $\$669.42$

45. $f(-1) = -4$

47. $f(-1) \approx -0.2707$

49. $f(3) \approx 483.8146$

51. $y = 3 \cdot 5^x$

53. $y \approx 18 \cdot 1.025^x$

55. $y \approx 0.2 \cdot 1.95^x$

57.

$$\text{APY} = \frac{A(t) - a}{a} = \frac{a\left(1 + \frac{r}{365}\right)^{365(1)} - a}{a} = \frac{a\left[\left(1 + \frac{r}{365}\right)^{365} - 1\right]}{a} = \left(1 + \frac{r}{365}\right)^{365} - 1;$$

$$I(n) = \left(1 + \frac{r}{n}\right)^n - 1$$

59. Let f be the exponential decay function $f(x) = a \cdot \left(\frac{1}{b}\right)^x$ such that $b > 1.$ Then for some number

$n > 0,$

$f(x) = a \cdot \left(\frac{1}{b}\right)^x = a\left(b^{-1}\right)^x = a\left((e^n)^{-1}\right)^x = a(e^{-n})^x = a(e)^{-nx}.$

61.
$47,\ 622$ fox

63.
$1.39\%;$

$\$155,\ 368.09$

65.
$\$35,\ 838.76$

67.
$\$82,\ 247.78;$

$\$449.75$

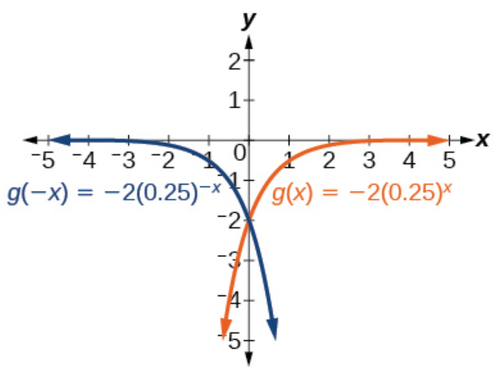
69. An asymptote is a line that the graph of a function approaches, as x either increases or decreases without bound. The horizontal asymptote of an exponential function tells us the limit of the function's values as the independent variable gets either extremely large or extremely small.

71.
$g(x) = 4(3)^{-x};$ y-intercept: $(0,\ 4);$ Domain: all real numbers; Range: all real numbers greater than $0.$

73.
$g(x) = -10^x + 7;$ y-intercept: $(0,\ 6);$ Domain: all real numbers; Range: all real numbers less than $7.$

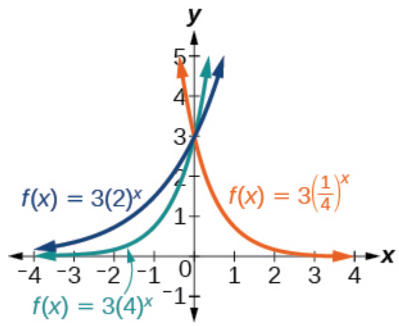
75.
$g(x) = 2\left(\frac{1}{4}\right)^x;$ y-intercept: $(0,\ 2);$ Domain: all real numbers; Range: all real numbers greater than $0.$

77.

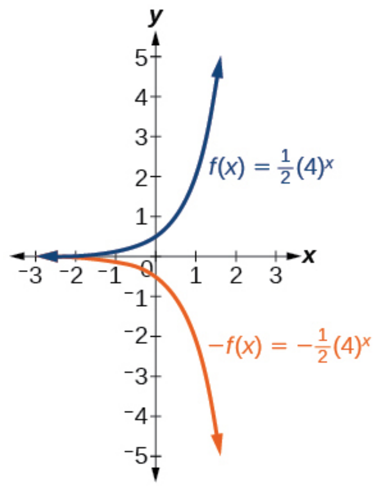


y-intercept: $(0,\ -2)$

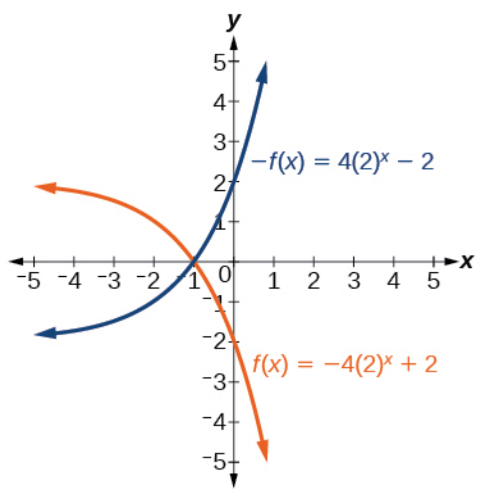
79.



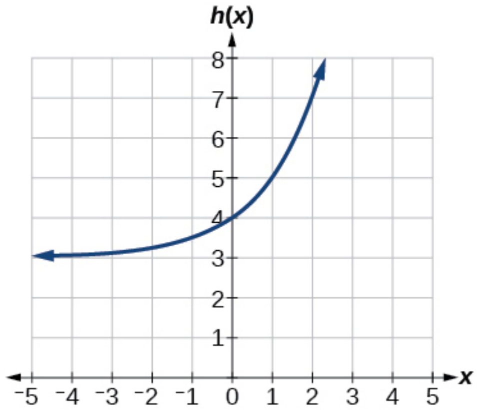
81. B
83. A
85. E
87. D
89. C
91.



93.



95.



Horizontal asymptote:
$h(x) = 3;$ Domain: all real numbers; Range: all real numbers strictly greater than 3.

97. As
$x \to \infty$,
$f(x) \to -\infty$;
As
$x \to -\infty$,
$f(x) \to -1$

99. As
$x \to \infty$,
$f(x) \to 2$;
As
$x \to -\infty$,
$f(x) \to \infty$

101.
$f(x) = 4^x - 3$

103.
$f(x) = 4^{x-5}$

105.
$f(x) = 4^{-x}$

107.
$y = -2^x + 3$

109.
$y = -2(3)^x + 7$

111.
$g(6) = 800 + \frac{1}{3} \approx 800.3333$

113.
$h(-7) = -58$

115.
$x \approx -2.953$

117.
$x \approx -0.222$

119. The graph of
$G(x) = \left(\frac{1}{b}\right)^x$ is the refelction about the y-axis of the graph of
$F(x) = b^x;$ For any real number
$b > 0$ and function
$f(x) = b^x,$ the graph of

$\left(\frac{1}{b}\right)^x$ is the the reflection about the *y*-axis, $F(-x)$.

121. The graphs of $g(x)$ and $h(x)$ are the same and are a horizontal shift to the right of the graph of $f(x);$ For any real number *n*, real number $b > 0,$ and function $f(x) = b^x,$ the graph of $\left(\frac{1}{b^n}\right)b^x$ is the horizontal shift $f(x - n)$.

123. A logarithm is an exponent. Specifically, it is the exponent to which a base b is raised to produce a given value. In the expressions given, the base b has the same value. The exponent, $y,$ in the expression b^y can also be written as the logarithm, $\log_b x,$ and the value of x is the result of raising b to the power of y.

125. Since the equation of a logarithm is equivalent to an exponential equation, the logarithm can be converted to the exponential equation $b^y = x,$ and then properties of exponents can be applied to solve for x.

127. The natural logarithm is a special case of the logarithm with base b in that the natural log always has base $e.$ Rather than notating the natural logarithm as $\log_e(x),$ the notation used is $\ln(x)$.

129.
$a^c = b$

131.
$x^y = 64$

133.
$15^b = a$

135.
$13^a = 142$

137.
$e^n = w$

139.
$\log_c(k) = d$

141.
$\log_{19} y = x$

143.
$\log_n(103) = 4$

145.
$\log_y\left(\frac{39}{100}\right) = x$

147.
$\ln(h) = k$

149.
$x = 2^{-3} = \frac{1}{8}$

151.
$x = 3^3 = 27$

153.
$x = 9^{\frac{1}{2}} = 3$

155.
$x = 6^{-3} = \frac{1}{216}$

157.
$x = e^2$

159.
32

161.
1.06

163.
14.125

165.
$\frac{1}{2}$

167.
4

169.
-3

171.
-12

173.
0

175.
10

177.
2.708

179.
0.151

181. No, the function has no defined value for $x = 0.$ To verify, suppose $x = 0$ is in the domain of the function $f(x) = \log(x).$ Then there is some number n such that $n = \log(0).$ Rewriting as an exponential equation gives: $10^n = 0,$ which is impossible since no such real number n exists. Therefore, $x = 0$ is *not* the domain of the function $f(x) = \log(x).$

183. Yes. Suppose there exists a real number x such that $\ln x = 2.$ Rewriting as an exponential equation gives $x = e^2,$ which is a real number. To verify, let $x = e^2.$ Then, by definition, $\ln(x) = \ln\left(e^2\right) = 2.$

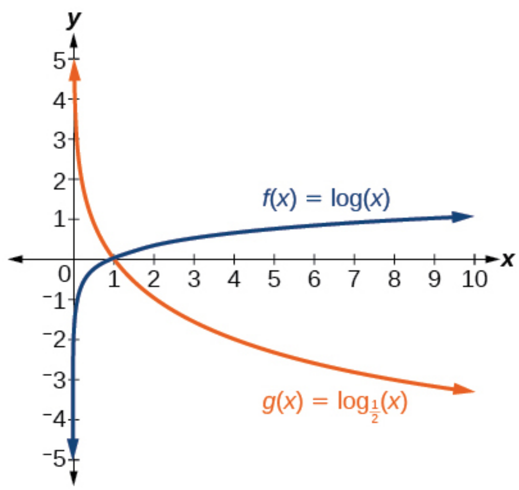
185. No;
$\ln(1) = 0,$ so
$\frac{\ln\left(e^{1.725}\right)}{\ln(1)}$ is undefined.
187.
2
189. Since the functions are inverses, their graphs are mirror images about the line
$y = x.$ So for every point
(a, b) on the graph of a logarithmic function, there is a corresponding point
(b, a) on the graph of its inverse exponential function.
191. Shifting the function right or left and reflecting the function about the y-axis will affect its domain.
193. No. A horizontal asymptote would suggest a limit on the range, and the range of any logarithmic function in general form is all real numbers.
195. Domain:
$\left(-\infty, \frac{1}{2}\right);$ Range:
$(-\infty, \infty)$
197. Domain:
$\left(-\frac{17}{4}, \infty\right);$ Range:
$(-\infty, \infty)$
199. Domain:
$(5, \infty);$ Vertical asymptote:
$x = 5$
201. Domain:
$\left(-\frac{1}{3}, \infty\right);$ Vertical asymptote:
$x = -\frac{1}{3}$
203. Domain:
$(-3, \infty);$ Vertical asymptote:
$x = -3$
205. Domain:
$\left(\frac{3}{7}, \infty\right)$;
Vertical asymptote:
$x = \frac{3}{7}$; End behavior: as
$x \to \left(\frac{3}{7}\right)^{+}, f(x) \to -\infty$ and as
$x \to \infty, f(x) \to \infty$
207. Domain:
$(-3, \infty)$; Vertical asymptote:
$x = -3$;
End behavior: as
$x \to -3^{+}$,
$f(x) \to -\infty$ and as
$x \to \infty$,
$f(x) \to \infty$
209. Domain:
$(1, \infty);$ Range:
$(-\infty, \infty);$ Vertical asymptote:

$x = 1;$ x-intercept:
$\left(\frac{5}{4},\ 0\right);$ y-intercept: DNE

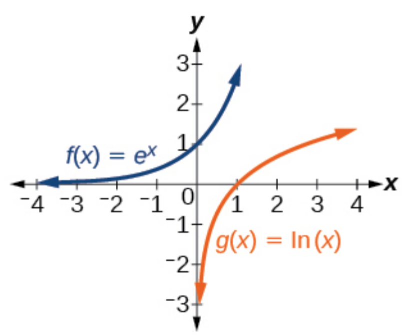
211. Domain:
$(-\infty,\ 0);$ Range:
$(-\infty,\ \infty);$ Vertical asymptote:
$x = 0;$ x-intercept:
$\left(-e^2,\ 0\right);$ y-intercept: DNE

213. Domain:
$(0,\ \infty);$ Range:
$(-\infty,\ \infty);$ Vertical asymptote:
$x = 0;$ x-intercept:
$\left(e^3,\ 0\right);$ y-intercept: DNE

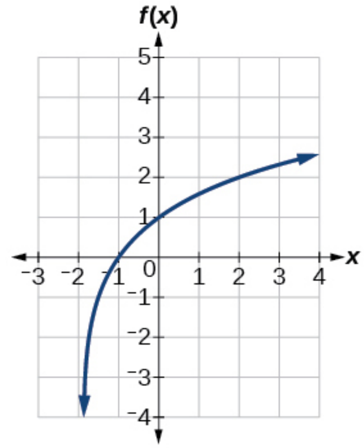
215. B
217. C
219. B
221. C
223.



225.



227. C
229.



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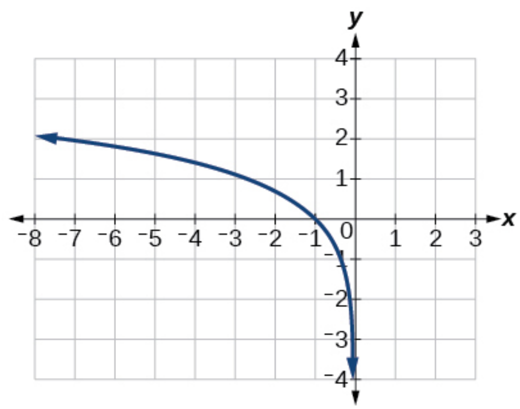
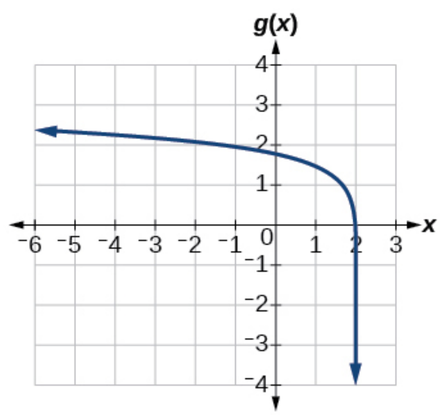


233.



235.
$f(x) = \log_2(-(x-1))$

237.
$f(x) = 3\log_4(x+2)$

239.
$x = 2$

241.
$x \approx 2.303$

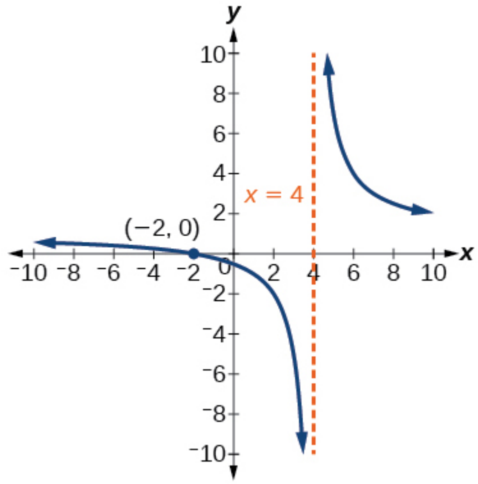
243.
$x \approx -0.472$

245. The graphs of $f(x) = \log_{\frac{1}{2}}(x)$ and $g(x) = -\log_2(x)$ appear to be the same; Conjecture: for any positive base

$b \neq 1,$

$\log_b(x) = -\log_{\frac{1}{b}}(x).$

247. Recall that the argument of a logarithmic function must be positive, so we determine where $\frac{x+2}{x-4} > 0$. From the graph of the function $f(x) = \frac{x+2}{x-4},$ note that the graph lies above the x-axis on the interval $(-\infty, -2)$ and again to the right of the vertical asymptote, that is $(4, \infty).$ Therefore, the domain is $(-\infty, -2) \cup (4, \infty).$



249. Any root expression can be rewritten as an expression with a rational exponent so that the power rule can be applied, making the logarithm easier to calculate. Thus,

$\log_b\left(x^{\frac{1}{n}}\right) = \frac{1}{n}\log_b(x).$

251.

$\log_b(2) + \log_b(7) + \log_b(x) + \log_b(y)$

253.

$\log_b(13) - \log_b(17)$

255.

$-k\ln(4)$

257.

$\ln(7xy)$

259.

$\log_b(4)$

261.

$\log_b(7)$

263.

$15\log(x) + 13\log(y) - 19\log(z)$

265.

$\frac{3}{2}\log(x) - 2\log(y)$

267.

$\frac{8}{3}\log(x) + \frac{14}{3}\log(y)$

269.

$\ln(2x^7)$

271.
$\log\left(\frac{xz^3}{\sqrt{y}}\right)$

273.
$\log_7(15) = \frac{\ln(15)}{\ln(7)}$

275.
$\log_{11}(5) = \frac{\log_5(5)}{\log_5(11)} = \frac{1}{b}$

277.
$\log_{11}\left(\frac{6}{11}\right) = \frac{\log_5\left(\frac{6}{11}\right)}{\log_5(11)} = \frac{\log_5(6) - \log_5(11)}{\log_5(11)} = \frac{a-b}{b} = \frac{a}{b} - 1$

279.
3

281.
2.81359

283.
0.93913

285.
-2.23266

287.
$x = 4;$ By the quotient rule:
$\log_6(x+2) - \log_6(x-3) = \log_6\left(\frac{x+2}{x-3}\right) = 1.$ Rewriting as an exponential equation and solving for $x:$

$$\begin{aligned} 6^1 &= \frac{x+2}{x-3} \\ 0 &= \frac{x+2}{x-3} - 6 \\ 0 &= \frac{x+2}{x-3} - \frac{6(x-3)}{(x-3)} \\ 0 &= \frac{x+2-6x+18}{x-3} \\ 0 &= \frac{x-4}{x-3} \\ x &= 4 \end{aligned}$$

Checking, we find that $\log_6(4+2) - \log_6(4-3) = \log_6(6) - \log_6(1)$ is defined, so $x = 4.$

289. Let b and n be positive integers greater than $1.$ Then, by the change-of-base formula, $\log_b(n) = \frac{\log_n(n)}{\log_n(b)} = \frac{1}{\log_n(b)}.$

291. Determine first if the equation can be rewritten so that each side uses the same base. If so, the exponents can be set equal to each other. If the equation cannot be rewritten so that each side uses the same base, then apply the logarithm to each side and use properties of logarithms to solve.

293. The one-to-one property can be used if both sides of the equation can be rewritten as a single logarithm with the same base. If so, the arguments can be set equal to each other, and the resulting equation can be solved algebraically. The one-to-one property cannot be used when each side of the equation cannot be rewritten as a single logarithm with the same base.

295.
$x = -\frac{1}{3}$

297.
$n = -1$

299.
$b = \frac{6}{5}$

301.
$x = 10$

303. No solution

305.
$p = \log\left(\frac{17}{8}\right) - 7$

307.
$k = -\frac{\ln(38)}{3}$

309.
$x = \frac{\ln\left(\frac{38}{3}\right) - 8}{9}$

311.
$x = \ln 12$

313.
$x = \frac{\ln\left(\frac{3}{5}\right) - 3}{8}$

315. no solution

317.
$x = \ln(3)$

319.
$10^{-2} = \frac{1}{100}$

321.
$n = 49$

323.
$k = \frac{1}{36}$

325.
$x = \frac{9 - e}{8}$

327.
$n = 1$

329. No solution

331. No solution

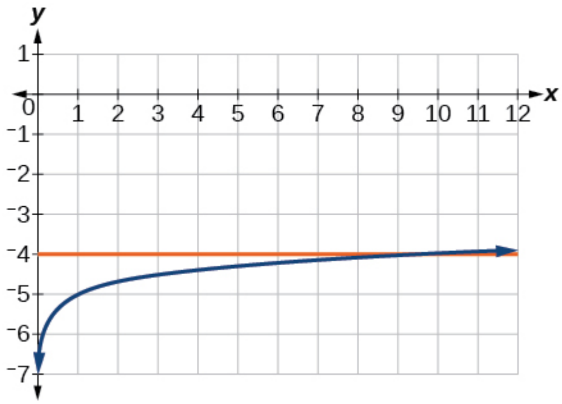
333.
$x = \pm\frac{10}{3}$

335.
$x = 10$

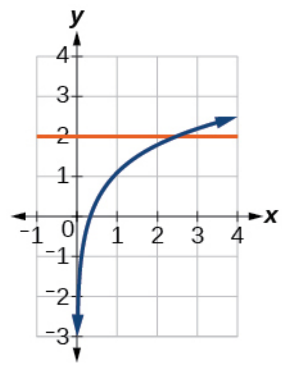
337.
$x = 0$

339.
$x = \frac{3}{4}$

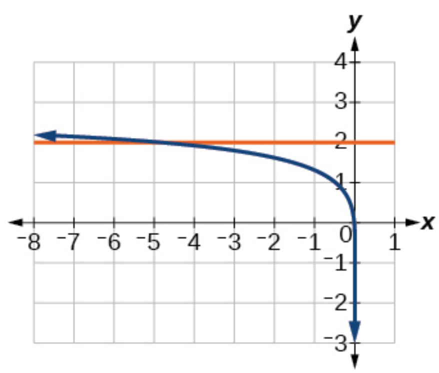
341.
$x = 9$



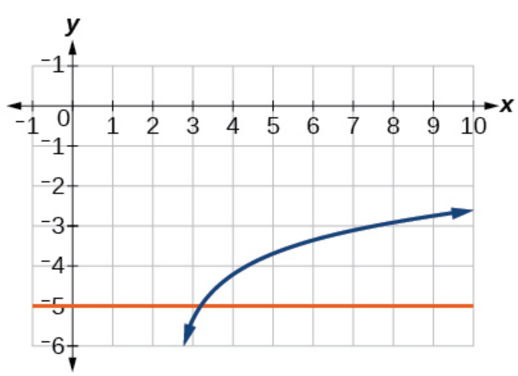
343.
$x = \frac{e^2}{3} \approx 2.5$



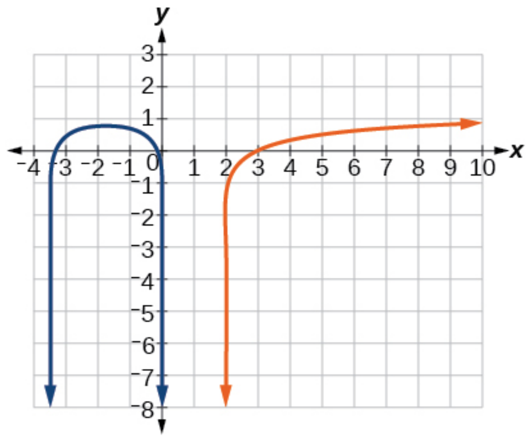
345.
$x = -5$



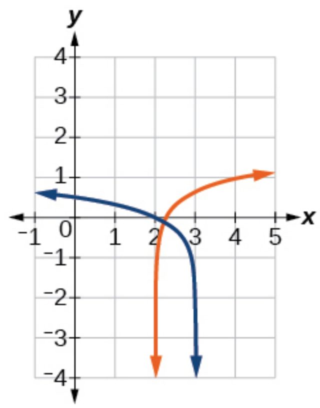
347.
$x = \frac{e + 10}{4} \approx 3.2$



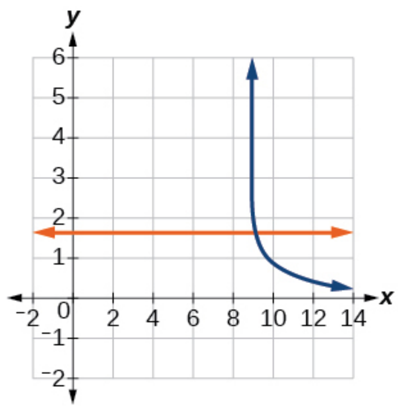
349. No solution



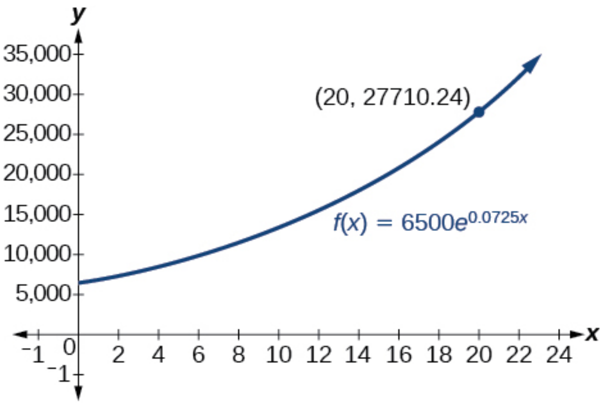
351.
$x = \frac{11}{5} \approx 2.2$



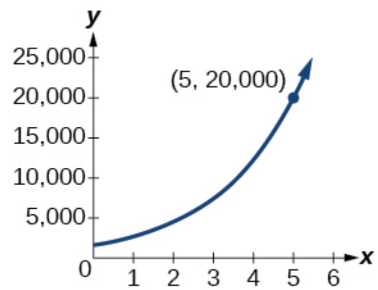
353.
$x = \frac{101}{11} \approx 9.2$



355. about
$\$27,710.24$



357. about 5 years



359.
$\frac{\ln(17)}{5} \approx 0.567$

361.
$x = \frac{\log(38) + 5\log(3)}{4\log(3)} \approx 2.078$

363.
$x \approx 2.2401$

365.
$x \approx -44655.7143$

367. about
5.83

369.
$t = \ln\left(\left(\frac{y}{A}\right)^{\frac{1}{k}}\right)$

371.
$t = \ln\left(\left(\frac{T - T_s}{T_0 - T_s}\right)^{-\frac{1}{k}}\right)$

372. Half-life is a measure of decay and is thus associated with exponential decay models. The half-life of a substance or quantity is the amount of time it takes for half of the initial amount of that substance or quantity to decay.

374. Doubling time is a measure of growth and is thus associated with exponential growth models. The doubling time of a substance or quantity is the amount of time it takes for the initial amount of that substance or quantity to double in size.

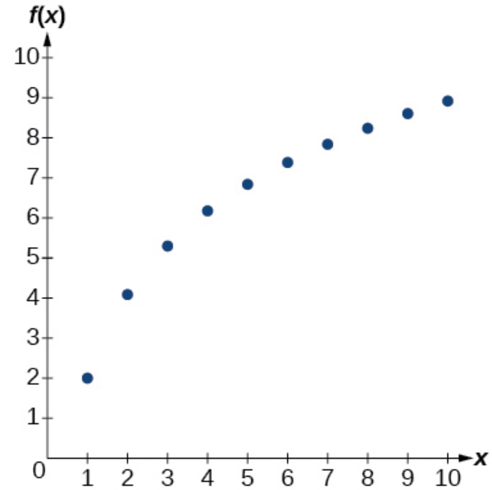
376. An order of magnitude is the nearest power of ten by which a quantity exponentially grows. It is also an approximate position on a logarithmic scale; Sample response: Orders of magnitude are useful when making comparisons between numbers that differ by a great amount. For example, the mass of Saturn is 95 times greater than the mass of Earth. This is the same as saying that the mass of Saturn is about
10^2 times, or *2 orders of magnitude* greater, than the mass of Earth.

378.
$f(0) \approx 16.7;$ The amount initially present is about 16.7 units.

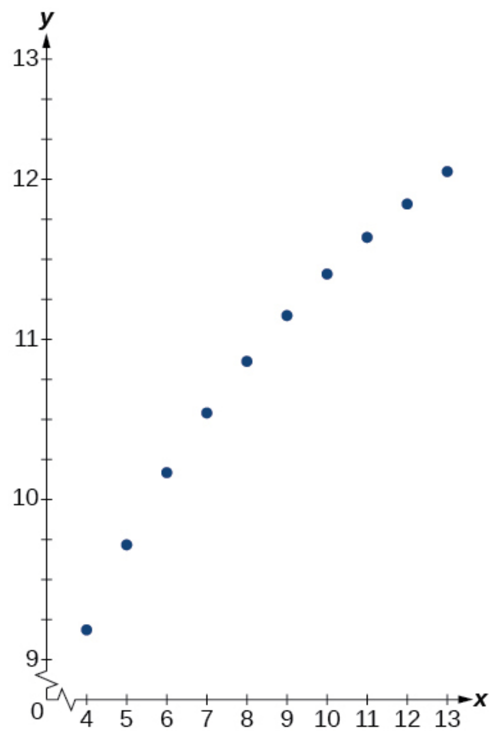
380. 150

383. exponential;
$f(x) = 1.2^x$

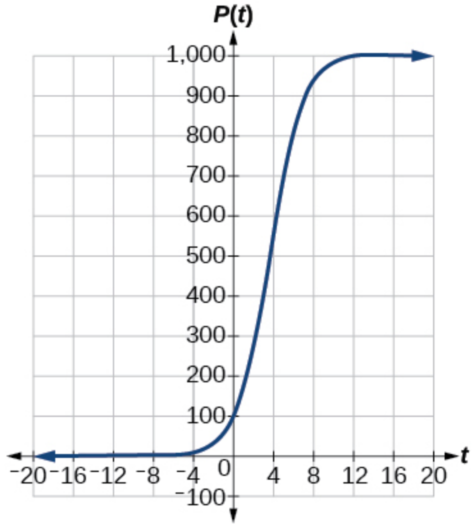
385. logarithmic



387. logarithmic



389.



391. about 1.4 years

393. about 7.3 years

395. 4 half-lives; 8.18 minutes

397.

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$\log\left(\frac{S}{S_0}\right) = \frac{3}{2}M$$

$$\frac{S}{S_0} = 10^{\frac{3M}{2}}$$

$$S = S_0 10^{\frac{3M}{2}}$$

399. Let $y = b^x$ for some non-negative real number b such that $b \neq 1.$ Then,

$$\ln(y) = \ln(b^x)$$

$$\ln(y) = x\ln(b)$$

$$e^{\ln(y)} = e^{x\ln(b)}$$

$$y = e^{x\ln(b)}$$

401. $A = 125e^{(-0.3567t)}; A \approx 43$ mg

403. about 60 days

405. $f(t) = 250e^{(-0.00914t)};$ half-life: about 76 minutes

407. $r \approx -0.0667,$ So the hourly decay rate is about 6.67%

409.
$f(t) = 1350e^{(0.03466t)}$; after 3 hours:
$P(180) \approx 691,\ 200$

411.
$f(t) = 256e^{(0.068110t)}$; doubling time: about
10 minutes

413. about
88 minutes

415.
$T(t) = 90e^{(-0.008377t)} + 75,$ where
t is in minutes.

417. about
113 minutes

419.
$\log(x) = 1.5;\ x \approx 31.623$

421. MMS magnitude:
5.82

423.
$N(3) \approx 71$

425. C

426. Logistic models are best used for situations that have limited values. For example, populations cannot grow indefinitely since resources such as food, water, and space are limited, so a logistic model best describes populations.

428. Regression analysis is the process of finding an equation that best fits a given set of data points. To perform a regression analysis on a graphing utility, first list the given points using the STAT then EDIT menu. Next graph the scatter plot using the STAT PLOT feature. The shape of the data points on the scatter graph can help determine which regression feature to use. Once this is determined, select the appropriate regression analysis command from the STAT then CALC menu.

430. The *y*-intercept on the graph of a logistic equation corresponds to the initial population for the population model.

432. C

434. B

436.
$P(0) = 22$; 175

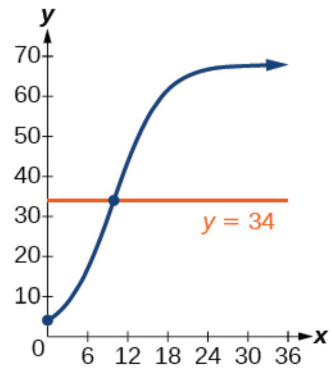
438.
$p \approx 2.67$

440. *y*-intercept:
$(0,\ 15)$

442.
4 koi

444. about
6.8 months.

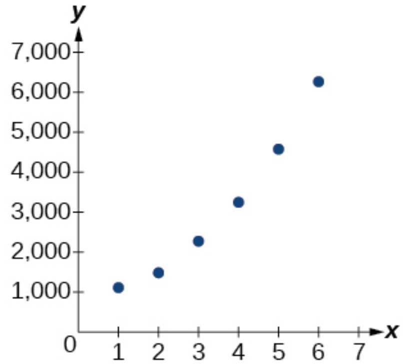
445.



447.
10 wolves

449. about 5.4 years.

451.



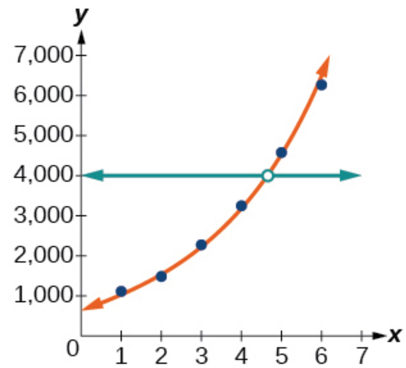
453.

$f(x) = 776.682e^{0.3549x}$

455. When

$f(x) = 4000,$

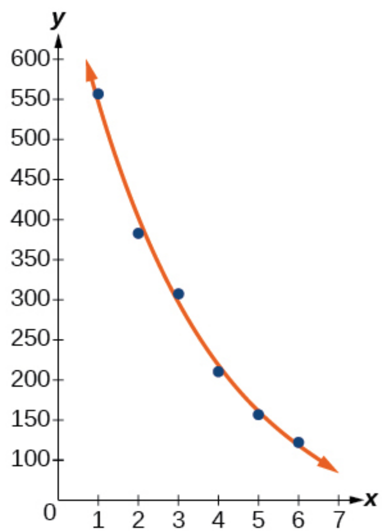
$x \approx 4.6.$



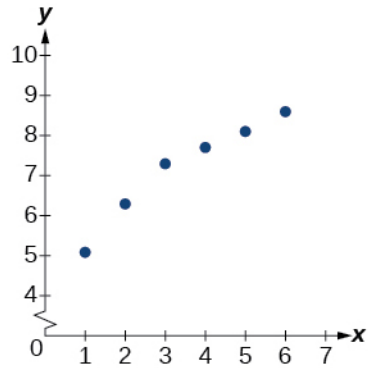
457.

$f(x) = 731.92(0.738)^x$

459.

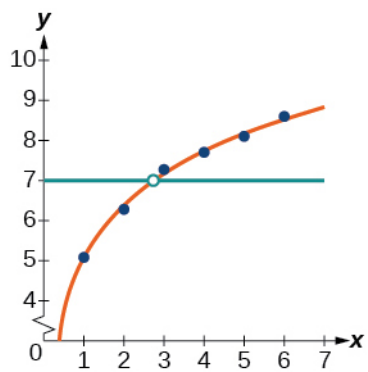


461.



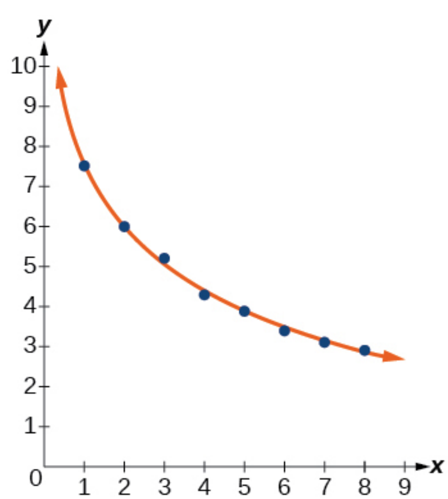
463.
$f(10) \approx 9.5$

465. When
$f(x) = 7,$
$x \approx 2.7.$

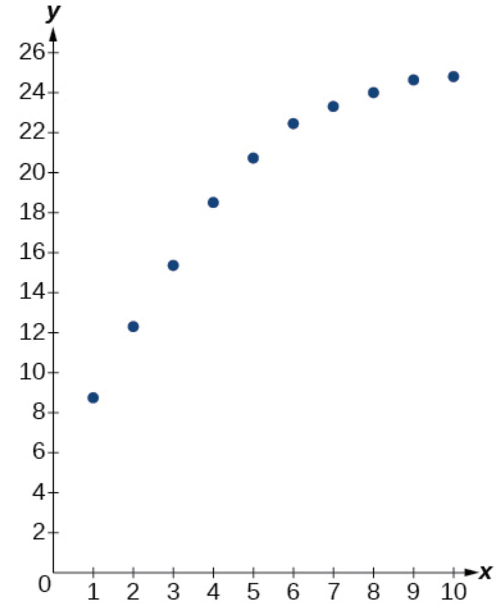


467.
$f(x) = 7.544 - 2.268\ln(x)$

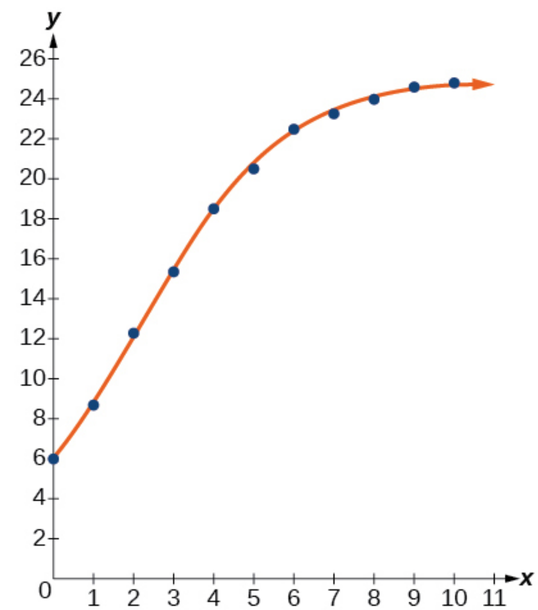
469.



471.



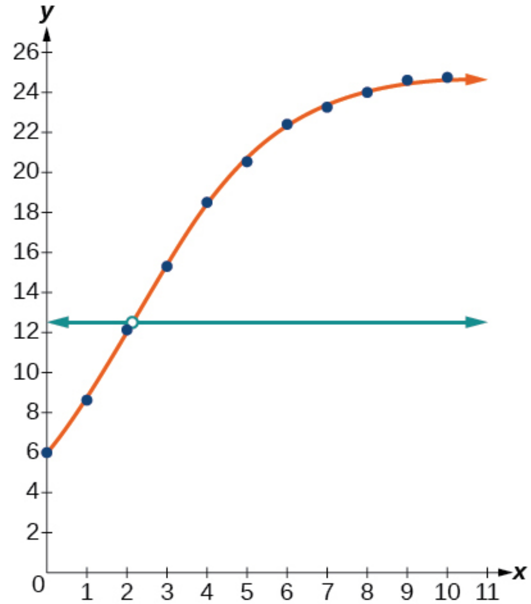
473.



475. When

$f(x) = 12.5,$

$x \approx 2.1.$



477.
$$f(x) = \frac{136.068}{1 + 10.324e^{-0.480x}}$$

479. about
136

481. Working with the left side of the equation, we see that it can be rewritten as
ae^{-bt} :

$$\frac{c - P(t)}{P(t)} = \frac{c - \frac{c}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = \frac{\frac{c\left(1 + ae^{-bt}\right) - c}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = \frac{\frac{c\left(1 + ae^{-bt} - 1\right)}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = 1 + ae^{-bt} - 1 = ae^{-bt}$$

Working with the right side of the equation we show that it can also be rewritten as
ae^{-bt}. But first note that when
$t = 0,$
$P_0 = \frac{c}{1 + ae^{-b(0)}} = \frac{c}{1 + a}.$ Therefore,

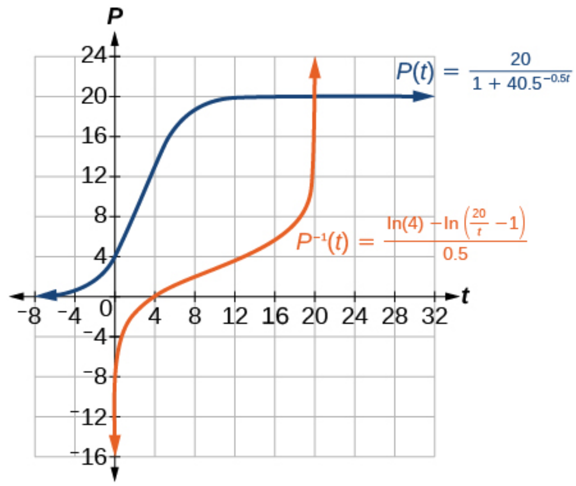
$$\frac{c - P_0}{P_0}e^{-bt} = \frac{c - \frac{c}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = \frac{\frac{c(1 + a) - c}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = \frac{\frac{c(1 + a - 1)}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = (1 + a - 1)e^{-bt} = ae^{-bt}$$

Thus,
$$\frac{c - P(t)}{P(t)} = \frac{c - P_0}{P_0}e^{-bt}.$$

483. First rewrite the exponential with base e:
$f(x) = 1.034341e^{0.247800x}.$ Then test to verify that
$f(g(x)) = x,$ taking rounding error into consideration:

$$\begin{aligned}
g(f(x)) &= 4.035510\ln\left(1.034341e^{0.247800x}\right) - 0.136259 \\
&= 4.03551\left(\ln(1.034341) + \ln\left(e^{0.2478x}\right)\right) - 0.136259 \\
&= 4.03551(\ln(1.034341) + 0.2478x) - 0.136259 \\
&= 0.136257 + 0.999999x - 0.136259 \\
&= -0.000002 + 0.999999x \\
&\approx 0 + x \\
&= x
\end{aligned}$$

485.



The graph of $P(t)$ has a y-intercept at (0, 4) and horizontal asymptotes at y = 0 and y = 20. The graph of $P^{-1}(t)$ has an x- intercept at (4, 0) and vertical asymptotes at x = 0 and x = 20.

Review Exercises

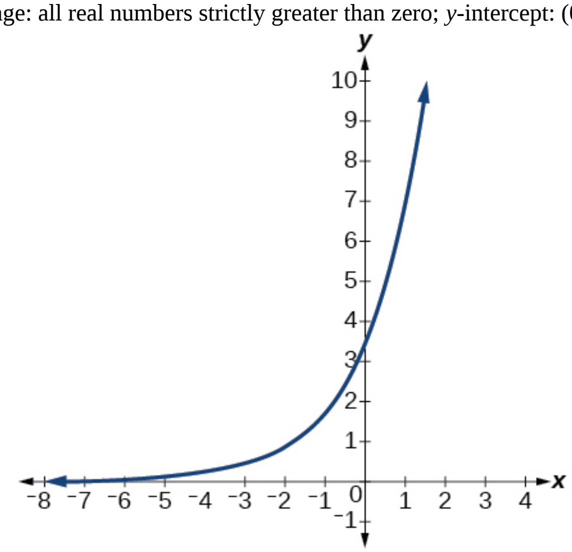
486. exponential decay; The growth factor, $0.825,$ is between 0 and $1.$

488. $y = 0.25(3)^x$

490. $\$42,\ 888.18$

492. continuous decay; the growth rate is negative.

494. domain: all real numbers; range: all real numbers strictly greater than zero; y-intercept: (0, 3.5);



496. $g(x) = 7(6.5)^{-x};$ y-intercept: $(0,\ 7);$ Domain: all real numbers; Range: all real numbers greater than $0.$

498. $17^x = 4913$

500.

$\log_a b = -\frac{2}{5}$

502.

$x = 64^{\frac{1}{3}} = 4$

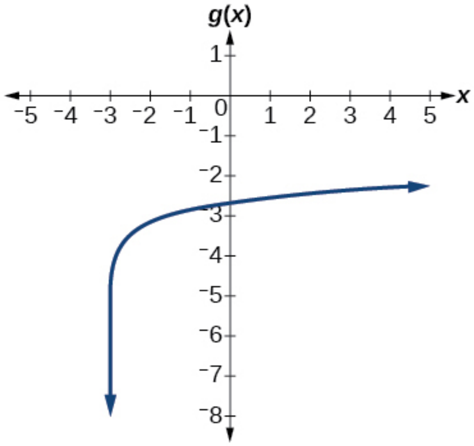
504.

$\log(0.000001) = -6$

506.

$\ln\left(e^{-0.8648}\right) = -0.8648$

508.



510. Domain:
$x > -5$; Vertical asymptote:
$x = -5$; End behavior: as
$x \to -5^+, f(x) \to -\infty$ and as
$x \to \infty, f(x) \to \infty$.

512.

$\log_8(65xy)$

514.

$\ln\left(\frac{z}{xy}\right)$

516.

$\log_y(12)$

518.

$\ln(2) + \ln(b) + \frac{\ln(b+1) - \ln(b-1)}{2}$

520.

$\log_7\left(\frac{v^3 w^6}{\sqrt[3]{u}}\right)$

522.

$x = \frac{\frac{\log(125)}{\log(5)} + 17}{12} = \frac{5}{3}$

524.

$x = -3$

526. no solution
528. no solution

530.
$x = \ln(11)$

532.
$a = e^4 - 3$

534.
$x = \pm \frac{9}{5}$

536. about
5.45 years

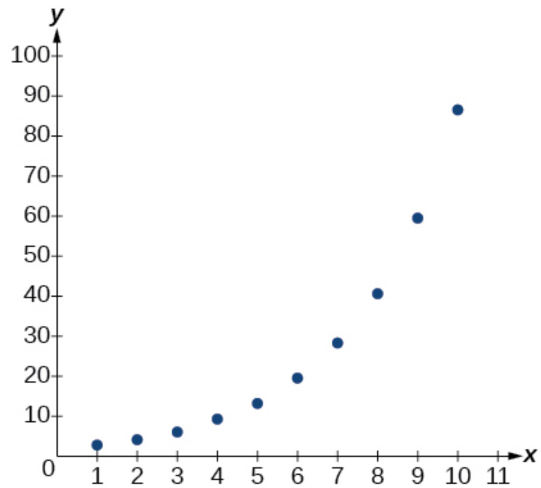
538.
$f^{-1}(x) = \sqrt[3]{2^{4x} - 1}$

540.
$f(t) = 300(0.83)^t$; $f(24) \approx 3.43\,g$

542. about
45 minutes

544. about
8.5 days

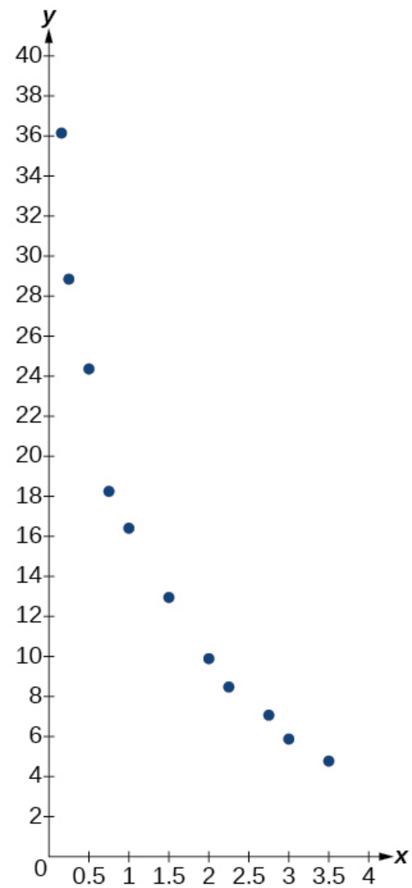
546. exponential



548.
$y = 4(0.2)^x$;
$y = 4e^{-1.609438x}$

550. about
7.2 days

552. logarithmic;
$y = 16.68718 - 9.71860\ln(x)$

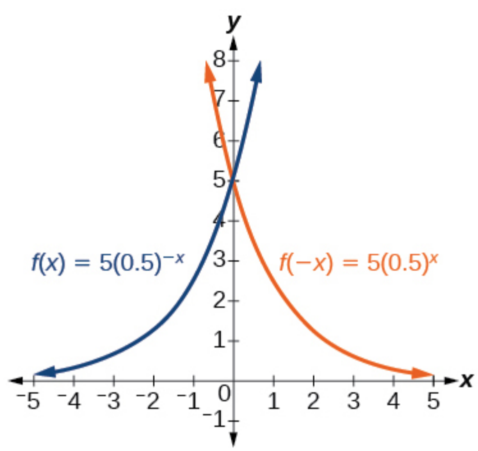


Practice Test

554. About
13 dolphins.

556.
$\$1,947$

558. *y*-intercept:
$(0,\ \ 5)$



560.
$8.5^a = 614.125$

562.
$x = \left(\frac{1}{7}\right)^2 = \frac{1}{49}$

564.
$\ln(0.716) \approx -0.334$

566. Domain:
$x < 3;$ Vertical asymptote:
$x = 3;$ End behavior:
$x \to 3^-, f(x) \to -\infty$ and
$x \to -\infty, f(x) \to \infty$

568.
$\log_t(12)$

570.
$$3\ \ln(y) + 2\ln(z) + \frac{\ln(x-4)}{3}$$

572.
$$x = \frac{\frac{\ln(1000)}{\ln(16)} + 5}{3} \approx 2.497$$

574.
$$a = \frac{\ln(4) + 8}{10}$$

576. no solution

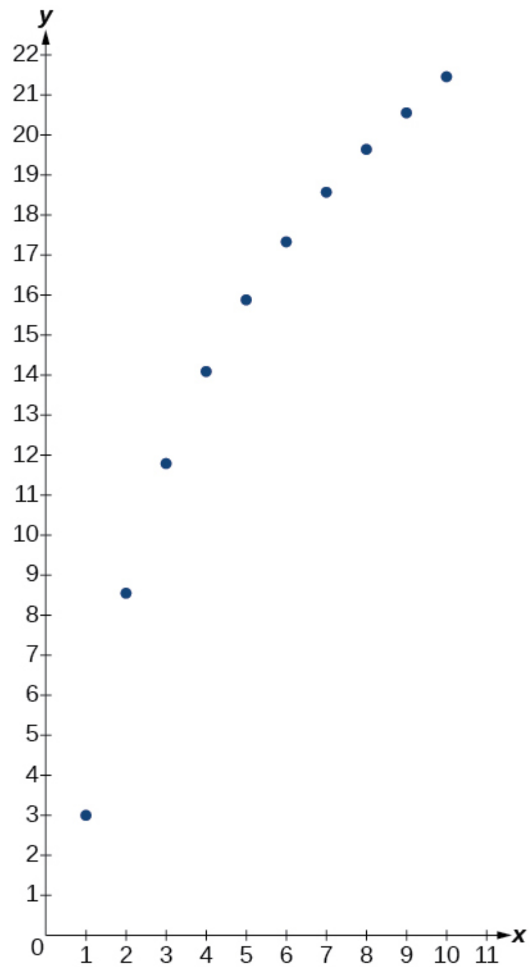
578.
$x = \ln(9)$

580.
$$x = \pm\frac{3\sqrt{3}}{2}$$

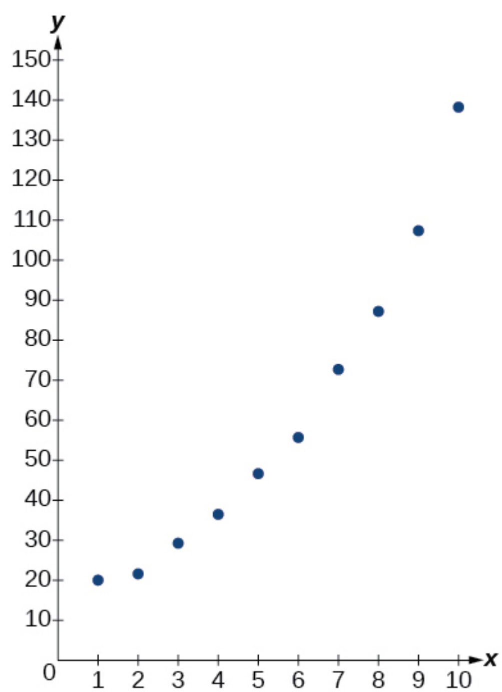
582.
$f(t) = 112e^{-.019792t};$ half-life: about 35 days

584.
$T(t) = 36e^{-0.025131t} + 35;\ T(60) \approx 43^{\text{o}}\text{F}$

586. logarithmic

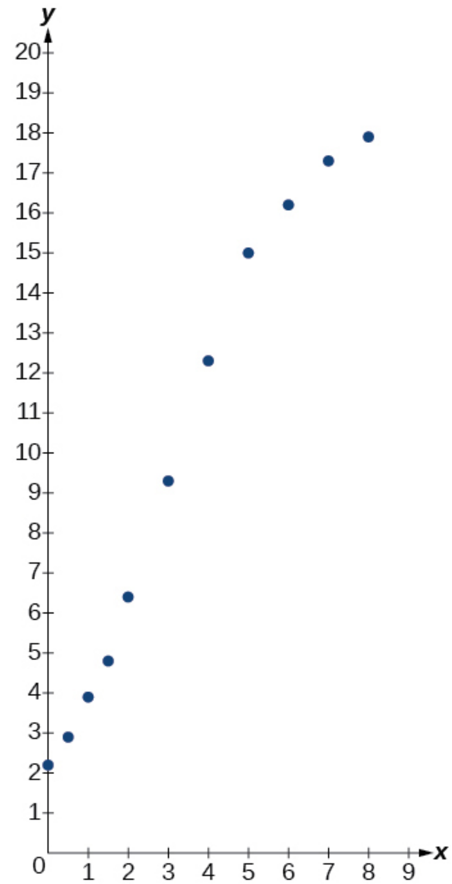


588. exponential;
$y = 15.10062(1.24621)^x$



590. logistic;

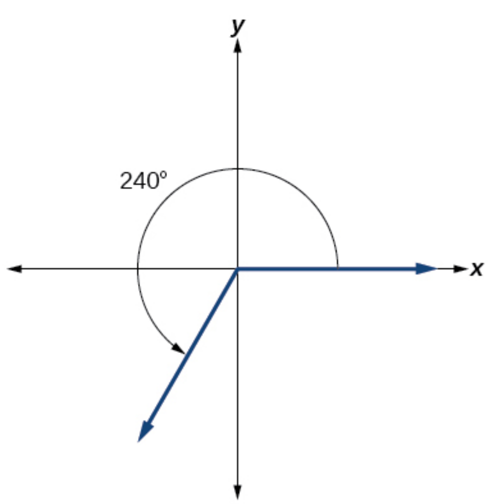
$$y = \frac{18.41659}{1 + 7.54644e^{-0.68375x}}$$



Chapter 5

Try It

5.1.



5.2.

$$\frac{3\pi}{2}$$

5.3. $-135°$

5.4.
$\frac{7\pi}{10}$

5.5.
$\alpha = 150\,°$

5.6.
$\beta = 60\,°$

5.7.
$\frac{7\pi}{6}$

5.8.
$\frac{215\pi}{18} = 37.525$ units

5.9. 1.88

5.10.
$\frac{-3\pi}{2}$ rad/s

5.11. 1655 kilometers per hour

5.12.
$\cos(t) = -\frac{\sqrt{2}}{2}, \sin(t) = \frac{\sqrt{2}}{2}$

5.13.
$\cos(\pi) = -1,$

$\sin(\pi) = 0$

5.14.
$\sin(t) = -\frac{7}{25}$

5.15. approximately 0.866025403

5.16.
$\frac{\pi}{3}$

5.17.

a.

$\cos(315\,°) = \frac{\sqrt{2}}{2}, \sin(315\,°) = \frac{-\sqrt{2}}{2}$

b.

$\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}, \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

5.18.
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

5.19.
$\sin t = -\frac{\sqrt{2}}{2}, \cos t = \frac{\sqrt{2}}{2}, \tan t = -1, \sec t = \sqrt{2}, \csc t = -\sqrt{2}, \cot t = -1$

5.20.
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

$\sec\frac{\pi}{3} = 2$

$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$

$\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

5.21.
$\sin\left(\frac{-7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{-7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \tan\left(\frac{-7\pi}{4}\right) = 1,$

$\sec\left(\frac{-7\pi}{4}\right) = \sqrt{2}, \csc\left(\frac{-7\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{-7\pi}{4}\right) = 1$

5.22.
$-\sqrt{3}$

5.23.
-2

5.24.
$\sin t$

5.25.
$\cos t = -\frac{8}{17}, \sin t = \frac{15}{17}, \tan t = -\frac{15}{8}$

$\csc t = \frac{17}{15}, \cot t = -\frac{8}{15}$

5.26.
$\sin t = -1, \cos t = 0, \tan t = \text{Undefine}$
$\sec t = \text{Undefined} \csc t = -1, \cot t = 0$

5.27.
$\sec t = \sqrt{2}, \csc t = \sqrt{2}, \tan t = 1, \cot t = 1$

5.28.
≈ -2.414

5.29.
$\frac{7}{25}$

5.30.
$sin \quad t = \frac{33}{65}, \cos \quad t = \frac{56}{65}, tan \quad t = \frac{33}{56},$
$\sec \quad t = \frac{65}{56}, \csc \quad t = \frac{65}{33}, \cot \quad t = \frac{56}{33}$

5.31.
$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \tan\left(\frac{\pi}{4}\right) = 1,$

$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}, csc\left(\frac{\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{\pi}{4}\right) = 1$

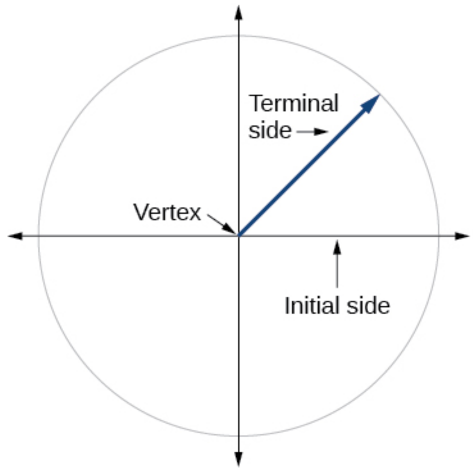
5.32. 2

5.33.
$\text{adjacent} = 10;$
$\text{opposite} = 10\sqrt{3}$; missing angle is
$\frac{\pi}{6}$

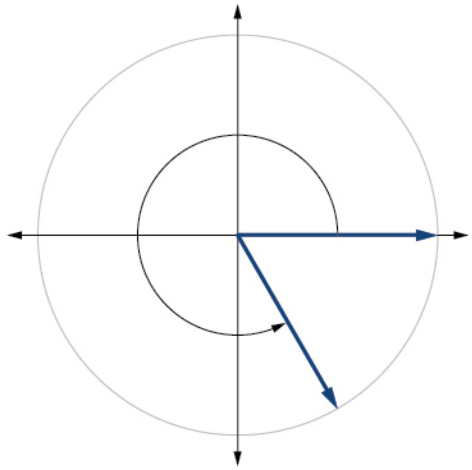
5.34. About 52 ft

Section Exercises

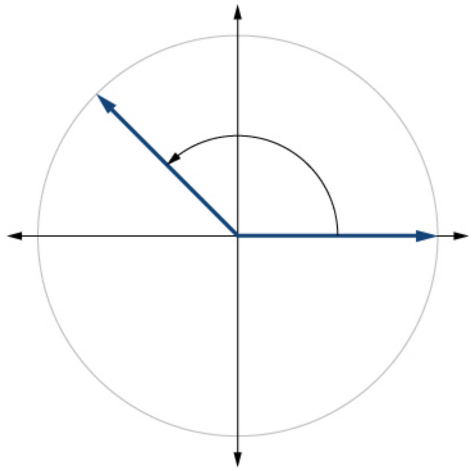
1.



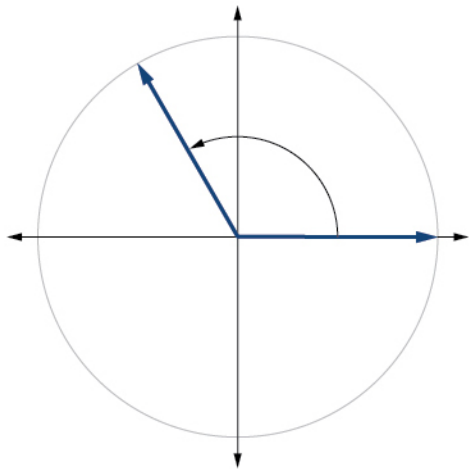
3. Whether the angle is positive or negative determines the direction. A positive angle is drawn in the counterclockwise direction, and a negative angle is drawn in the clockwise direction.
5. Linear speed is a measurement found by calculating distance of an arc compared to time. Angular speed is a measurement found by calculating the angle of an arc compared to time.
7.



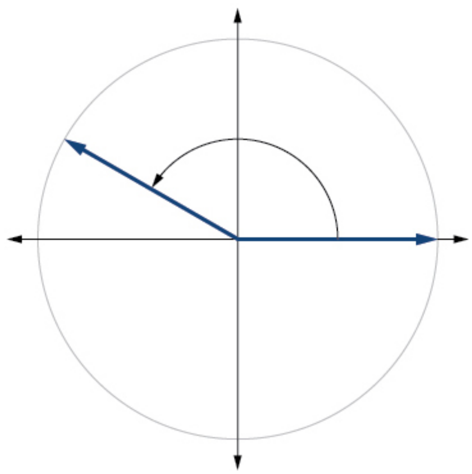
9.



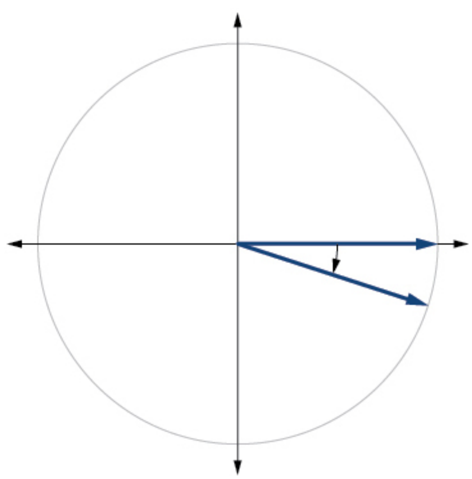
11.



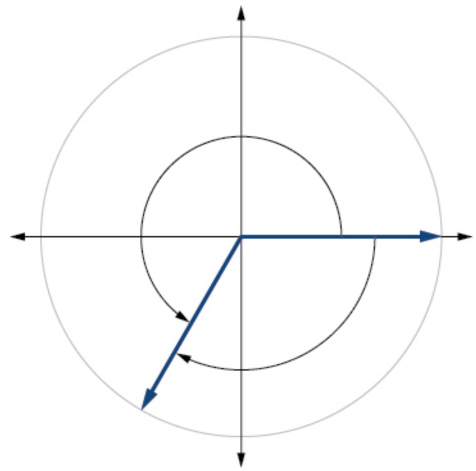
13.



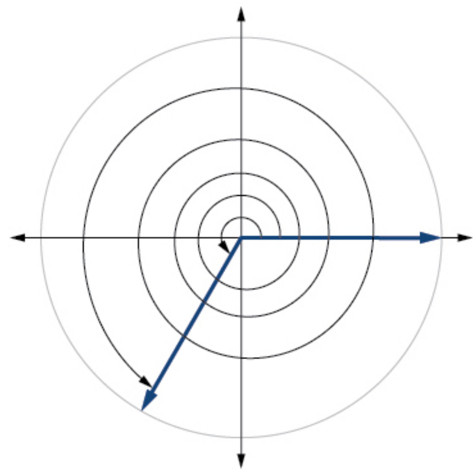
15.



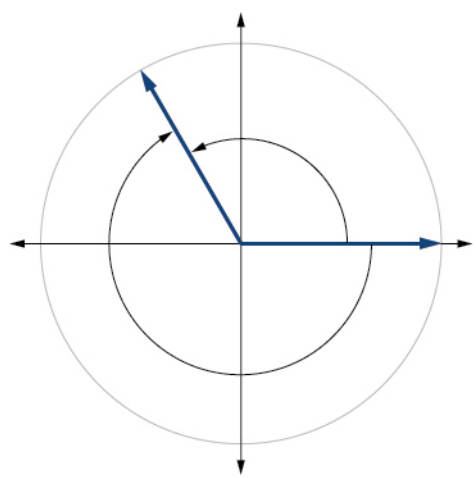
17. 240°



19.
$\frac{4\pi}{3}$



21.
$\frac{2\pi}{3}$



23.
$\frac{7\pi}{2} \approx 11.00 \text{ in}^2$

25.
$\frac{81\pi}{20} \approx 12.72 \text{ cm}^2$

27. $20°$

29. 60°
31. −75°
33.
$\frac{\pi}{2}$ radians
35.
-3π radians
37.
π radians
39.
$\frac{5\pi}{6}$ radians
41.
$\frac{5.02\pi}{3} \approx 5.26$ miles
43.
$\frac{25\pi}{9} \approx 8.73$ centimeters
45.
$\frac{21\pi}{10} \approx 6.60$ meters
47. 104.7198 cm^2
49. 0.7697 in^2
51. 250°
53. 320°
55.
$\frac{4\pi}{3}$
57.
$\frac{8\pi}{9}$
59. 1320 rad 210.085 RPM
61. 7 in./s, 4.77 RPM, 28.65 deg/s
63.
$1,\ 809,\ 557.37\ \text{mm/min} = 30.16\ \text{m/s}$
65.
5.76 miles
67.
$120\ °$
69. 794 miles per hour
71. 2,234 miles per hour
73. 11.5 inches
75. The unit circle is a circle of radius 1 centered at the origin.
77. Coterminal angles are angles that share the same terminal side. A reference angle is the size of the smallest acute angle, $t,$ formed by the terminal side of the angle t and the horizontal axis.
79. The sine values are equal.
81. I
83. IV
85.
$\frac{\sqrt{3}}{2}$
87.
$\frac{1}{2}$
89.
$\frac{\sqrt{2}}{2}$
91. 0

93. -1

95.

$\frac{\sqrt{3}}{2}$

97.

$60°$

99.

$80°$

101.

$45°$

103.

$\frac{\pi}{3}$

105.

$\frac{\pi}{3}$

107.

$\frac{\pi}{8}$

109.

$60°$, Quadrant IV,

$\sin(300°) = -\frac{\sqrt{3}}{2}, \cos(300°) = \frac{1}{2}$

111.

$45°$, Quadrant II,

$\sin(135°) = \frac{\sqrt{2}}{2},$

$\cos(135°) = -\frac{\sqrt{2}}{2}$

113.

$60°$, Quadrant II,

$\sin(120°) = \frac{\sqrt{3}}{2},$

$\cos(120°) = -\frac{1}{2}$

115.

$30°$, Quadrant II,

$\sin(150°) = \frac{1}{2},$

$\cos(150°) = -\frac{\sqrt{3}}{2}$

117.

$\frac{\pi}{6}$, Quadrant III,

$\sin(\frac{7\pi}{6}) = -\frac{1}{2},$

$\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$

119.

$\frac{\pi}{4}$, Quadrant II,

$\sin(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2},$

$\cos\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{2}}{2}$

121.

$\frac{\pi}{3}$, Quadrant II,

$\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2},$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

123.
$\frac{\pi}{4},$ Quadrant IV,

$\sin(\frac{7\pi}{4}) = -\frac{\sqrt{2}}{2},$

$\cos(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$

125.
$\frac{\sqrt{77}}{9}$

127.
$-\frac{\sqrt{15}}{4}$

129.
$\left(-10,\ 10\sqrt{3}\right)$

131.
$(-2.778,\ 15.757)$

133.
$[-1,\ 1]$

135.
$\sin t = \frac{1}{2}, \cos t = -\frac{\sqrt{3}}{2}$

137.
$\sin t = -\frac{\sqrt{2}}{2}, \cos t = -\frac{\sqrt{2}}{2}$

139.
$\sin t = \frac{\sqrt{3}}{2}, \cos t = -\frac{1}{2}$

141.
$\sin t = -\frac{\sqrt{2}}{2}, \cos t = \frac{\sqrt{2}}{2}$

143.
$\sin t = 0,\ \cos t = -1$

145.
$\sin t = -0.596,\ \cos t = 0.803$

147.
$\sin t = \frac{1}{2}, \cos t = \frac{\sqrt{3}}{2}$

149.
$\sin t = -\frac{1}{2}, \cos t = \frac{\sqrt{3}}{2}$

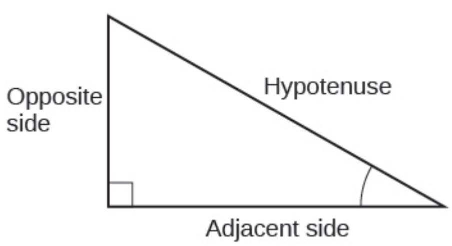
151.
$\sin t = 0.761,\ \cos t = -0.649$

153.
$\sin t = 1,\ \cos t = 0$

155. −0.1736
157. 0.9511
159. −0.7071
161. −0.1392
163. −0.7660

165.
$\frac{\sqrt{2}}{4}$
167.
$-\frac{\sqrt{6}}{4}$
169.
$\frac{\sqrt{2}}{4}$
171.
$\frac{\sqrt{2}}{4}$
173. 0
175.
$(0,\ -1)$
177. 37.5 seconds, 97.5 seconds, 157.5 seconds, 217.5 seconds, 277.5 seconds, 337.5 seconds
179. Yes, when the reference angle is
$\frac{\pi}{4}$ and the terminal side of the angle is in quadrants I and III. Thus, at
$x = \frac{\pi}{4}, \frac{5\pi}{4},$ the sine and cosine values are equal.
181. Substitute the sine of the angle in for
y in the Pythagorean Theorem
$x^2 + y^2 = 1.$ Solve for
x and take the negative solution.
183. The outputs of tangent and cotangent will repeat every
π units.
185.
$\frac{2\sqrt{3}}{3}$
187.
$\sqrt{3}$
189.
$\sqrt{2}$
191. 1
193. 2
195.
$\frac{\sqrt{3}}{3}$
197.
$-\frac{2\sqrt{3}}{3}$
199.
$\sqrt{3}$
201.
$-\sqrt{2}$
203. −1
205. −2
207.
$-\frac{\sqrt{3}}{3}$
209. 2
211.
$\frac{\sqrt{3}}{3}$
213. −2
215. −1

217. If
$$\sin t = -\frac{2\sqrt{2}}{3}, \sec t = -3, \csc t = -\frac{3\sqrt{2}}{4}, \tan t = 2\sqrt{2}, \cot t = \frac{\sqrt{2}}{4}$$
219.
$$\sec t = 2, \csc t = \frac{2\sqrt{3}}{3}, \ \tan t = \sqrt{3}, \ \cot t = \frac{\sqrt{3}}{3}$$
221.
$$-\frac{\sqrt{2}}{2}$$
223. 3.1
225. 1.4
227.
$$\sin t = \frac{\sqrt{2}}{2}, \cos t = \frac{\sqrt{2}}{2}, \tan t = 1, \cot t = 1, \sec t = \sqrt{2}, \csc t = \sqrt{2}$$
229.
$$\sin t = -\frac{\sqrt{3}}{2}, \cos t = -\frac{1}{2}, \tan t = \sqrt{3}, \cot t = \frac{\sqrt{3}}{3}, \sec t = -2, \csc t = -\frac{2\sqrt{3}}{3}$$
231. –0.228
233. –2.414
235. 1.414
237. 1.540
239. 1.556
241.
$$\sin(t) \approx 0.79$$
243.
$$\csc t \approx 1.16$$
245. even
247. even
249.
$$\frac{\sin t}{\cos t} = \tan t$$
251. 13.77 hours, period:
$$1000\pi$$
253. 7.73 inches



255.
257. The tangent of an angle is the ratio of the opposite side to the adjacent side.
259. For example, the sine of an angle is equal to the cosine of its complement; the cosine of an angle is equal to the sine of its complement.
261.
$$\frac{\pi}{6}$$
263.
$$\frac{\pi}{4}$$
265.
$$b = \frac{20\sqrt{3}}{3}, \ c = \frac{40\sqrt{3}}{3}$$
267.
$$a = 10,\ 000,\ c = 10,\ 000.5$$

269.
$b = \frac{5\sqrt{3}}{3}, c = \frac{10\sqrt{3}}{3}$

271.
$\frac{5\sqrt{29}}{29}$

273.
$\frac{5}{2}$

275.
$\frac{\sqrt{29}}{2}$

277.
$\frac{5\sqrt{41}}{41}$

279.
$\frac{5}{4}$

281.
$\frac{\sqrt{41}}{4}$

283.
$c = 14, \quad b = 7\sqrt{3}$

285.
$a = 15, \quad b = 15$

287.
$b = 9.9970, \quad c = 12.2041$

289.
$a = 2.0838, \quad b = 11.8177$

291.
$a = 55.9808, c = 57.9555$

293.
$a = 46.6790, b = 17.9184$

295.
$a = 16.4662, c = 16.8341$

297. 188.3159
299. 200.6737
301. 498.3471 ft
303. 1060.09 ft
305. 27.372 ft
307. 22.6506 ft
309. 368.7633 ft

Review Exercises

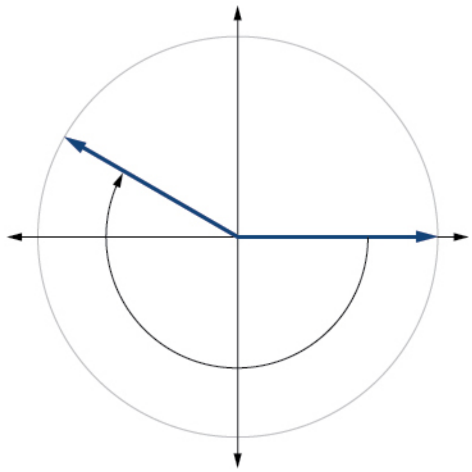
311.
$45°$

313.
$-\frac{7\pi}{6}$

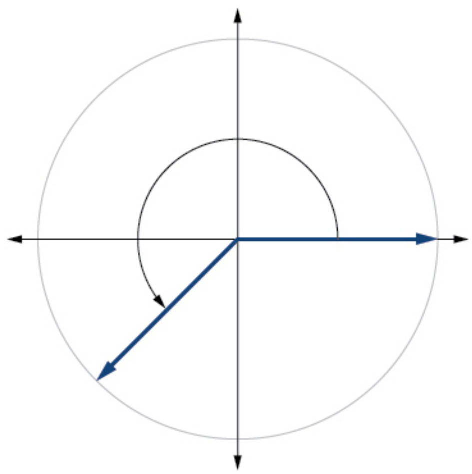
315. 10.385 meters
317.
$60°$

319.
$\frac{2\pi}{11}$

321.



323.



325. 1036.73 miles per hour

327. $\frac{\sqrt{3}}{2}$

329. –1

331. $\frac{\pi}{4}$

333. $-\frac{\sqrt{2}}{2}$

335. $[-1,\ 1]$

337. 1

339. $\sqrt{2}$

341. $\sqrt{2}$

343. 0.6

345. $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$

347. sine, cosecant, tangent, cotangent

349.
$\frac{\sqrt{3}}{3}$

351. 0
353.
$b = 8,\ c = 10$

355.
$\frac{11\sqrt{157}}{157}$

357.
$a = 4, \quad b = 4$

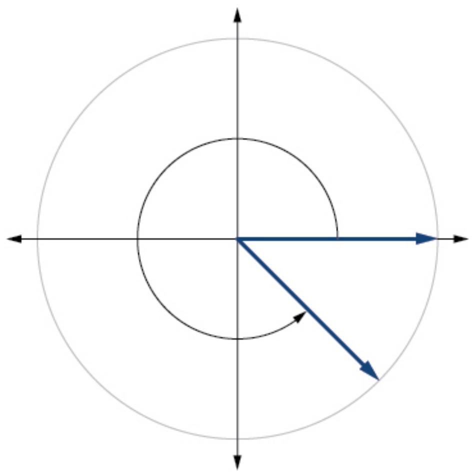
359. 14.0954 ft

Practice Test

361.
$150°$

363. 6.283 centimeters
365.
$15°$

367.



369. 3.351 feet per second,
$\frac{2\pi}{75}$ radians per second
371.
$-\frac{\sqrt{3}}{2}$

373.
$[-1,\ 1]$

375.
$\sqrt{3}$

377.
$\frac{\sqrt{3}}{3}$

379.
$\frac{\sqrt{3}}{2}$

381.
$\frac{\pi}{3}$

383.
$a = \frac{9}{2},\ b = \frac{9\sqrt{3}}{2}$

Chapter 6

Try It

6.1.
6π

6.2.
$\frac{1}{2}$ compressed

6.3.
$\frac{\pi}{2};$ right

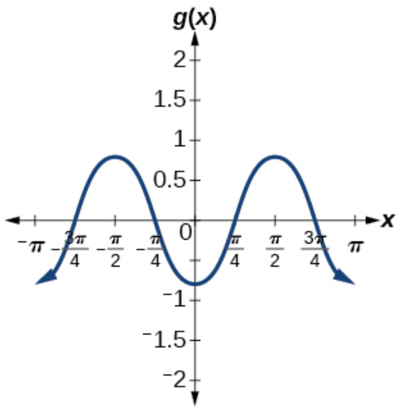
6.4. 2 units up

6.5. midline:
$y = 0;$ amplitude:
$|A| = \frac{1}{2};$ period:
$P = \frac{2\pi}{|B|} = 6\pi;$ phase shift:
$\frac{C}{B} = \pi$

6.6.
$f(x) = \sin(x) + 2$

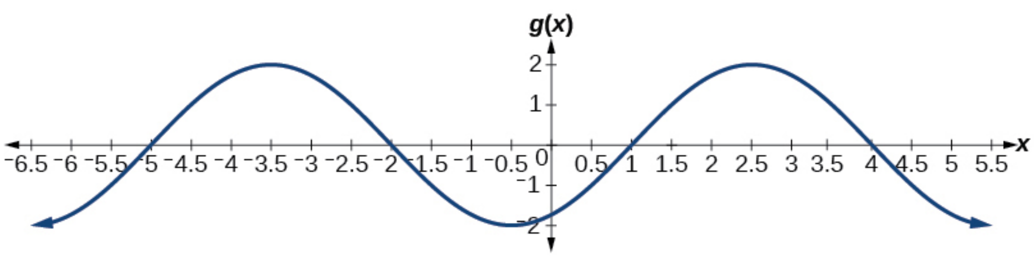
6.7. two possibilities:
$y = 4\sin\left(\frac{\pi}{5}x - \frac{\pi}{5}\right) + 4$ or
$y = -4\sin\left(\frac{\pi}{5}x + \frac{4\pi}{5}\right) + 4$

6.8.



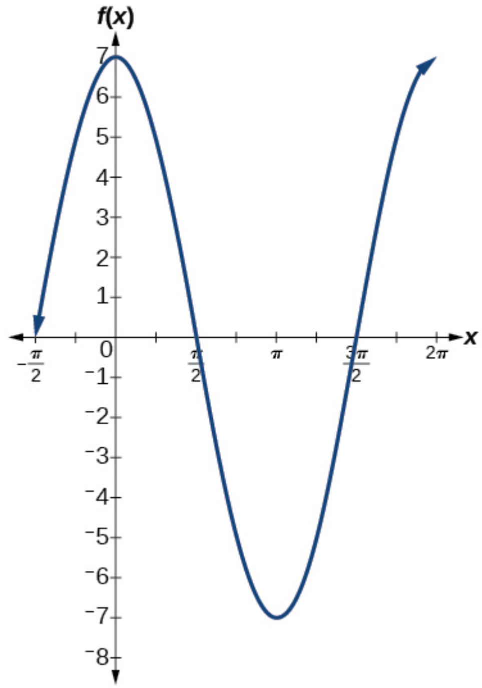
midline:
$y = 0;$ amplitude:
$|A| = 0.8;$ period:
$P = \frac{2\pi}{|B|} = \pi;$ phase shift:
$\frac{C}{B} = 0$ or none

6.9.

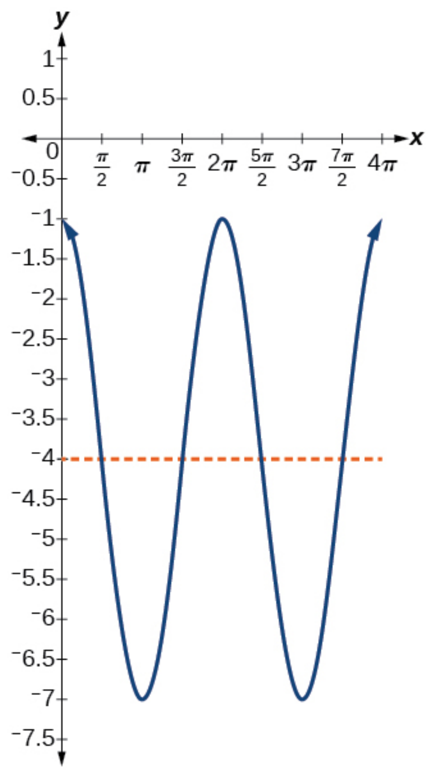


midline:
$y = 0;$ amplitude:
$|A| = 2;$ period:
$P = \frac{2\pi}{|B|} = 6;$ phase shift:
$\frac{C}{B} = -\frac{1}{2}$

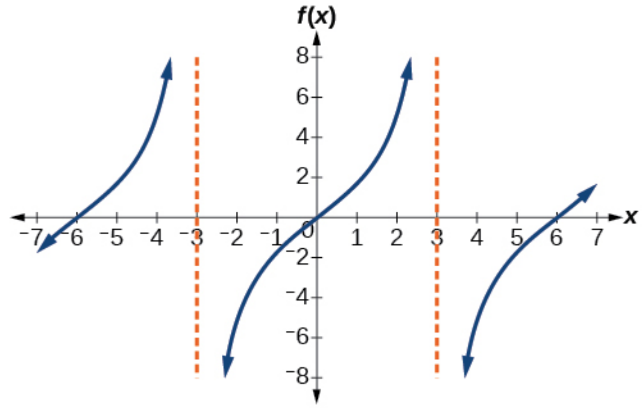
6.10. 7



6.11.
$y = 3\cos(x) - 4$



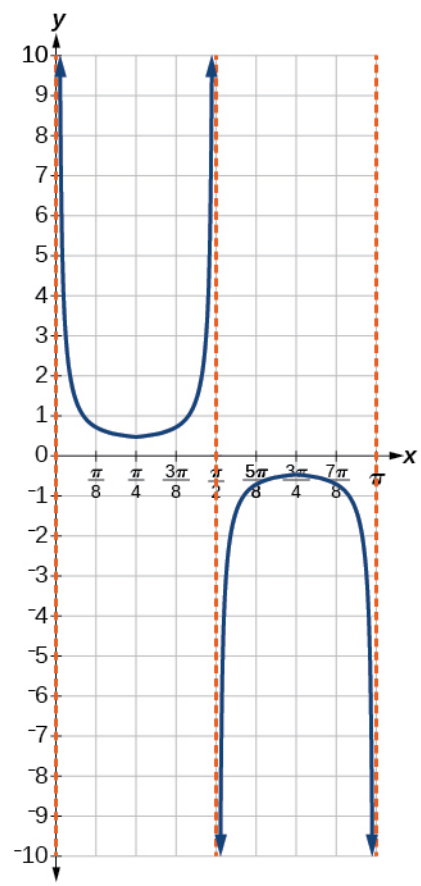
6.12.



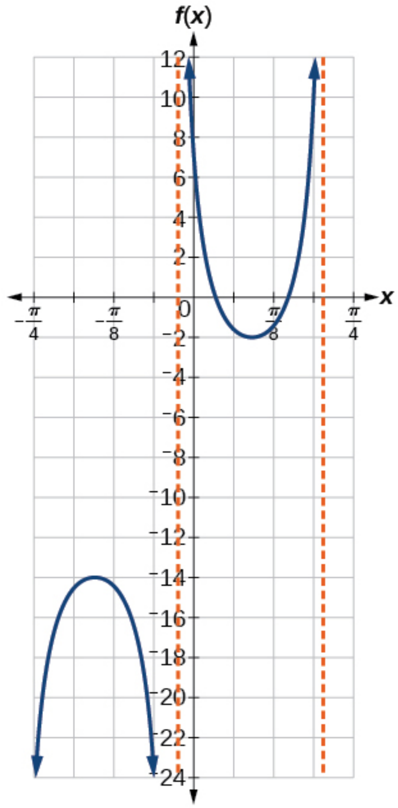
6.13. It would be reflected across the line $y = -1,$ becoming an increasing function.

6.14.
$g(x) = 4\tan(2x)$

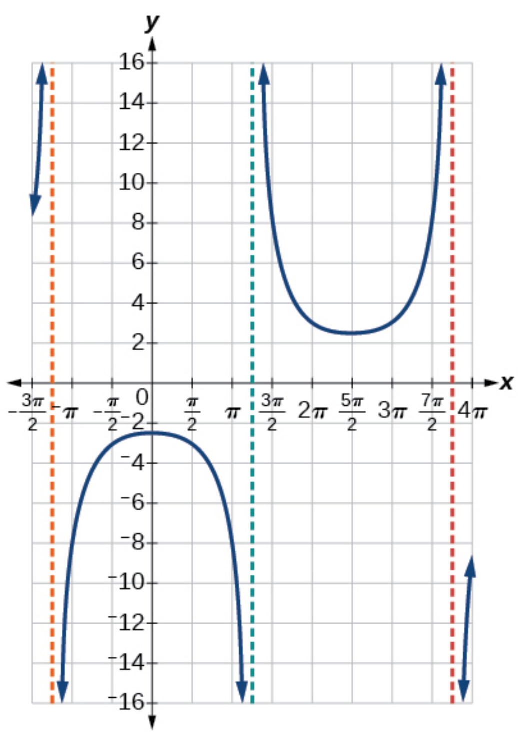
6.15. This is a vertical reflection of the preceding graph because A is negative.



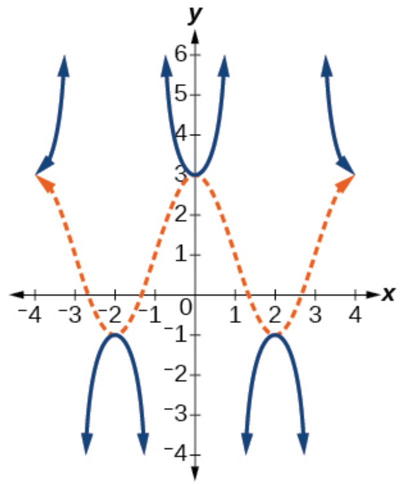
6.16.


f(x)
x
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
−22
−24
−π/4
−π/8
π/8
π/4

6.17.


y
x
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−3π/2
−π
−π/2
π/2
π
3π/2
2π
5π/2
3π
7π/2
4π

6.18.



6.19.

$\arccos(0.8776) \approx 0.5$

6.20. a.

$-\frac{\pi}{2};$ b.

$-\frac{\pi}{4};$ c.

$\pi;$ d.

$\frac{\pi}{3}$

6.21. 1.9823 or 113.578°

6.22.

$\sin^{-1}(0.6) = 36.87° = 0.6435$ radians

6.23.

$\frac{\pi}{8}; \frac{2\pi}{9}$

6.24.

$\frac{3\pi}{4}$

6.25.

$\frac{12}{13}$

6.26.

$\frac{4\sqrt{2}}{9}$

6.27.

$\frac{4x}{\sqrt{16x^2 + 1}}$

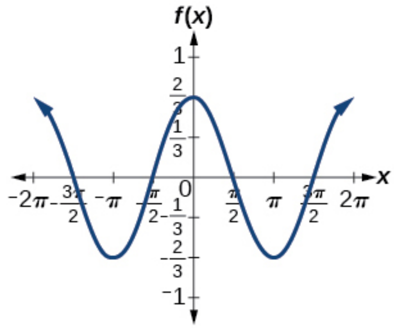
Section Exercises

1. The sine and cosine functions have the property that $f(x + P) = f(x)$ for a certain $P.$ This means that the function values repeat for every P units on the *x*-axis.

3. The absolute value of the constant A (amplitude) increases the total range and the constant D (vertical shift) shifts the graph vertically.

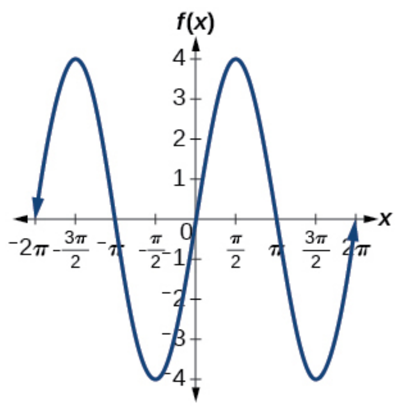
5. At the point where the terminal side of t intersects the unit circle, you can determine that the $\sin t$ equals the *y*-coordinate of the point.

7.



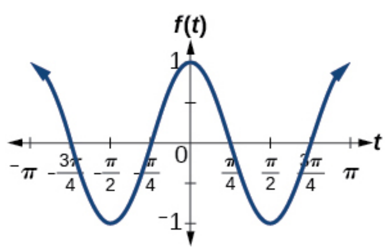
amplitude: $\frac{2}{3}$; period: 2π; midline: $y = 0$; maximum: $y = \frac{2}{3}$ occurs at $x = 0$; minimum: $y = -\frac{2}{3}$ occurs at $x = \pi$; for one period, the graph starts at 0 and ends at 2π

9.



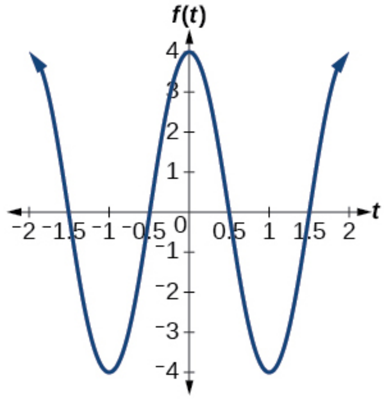
amplitude: 4; period: 2π; midline: $y = 0$; maximum $y = 4$ occurs at $x = \frac{\pi}{2}$; minimum: $y = -4$ occurs at $x = \frac{3\pi}{2}$; one full period occurs from $x = 0$ to $x = 2\pi$

11.



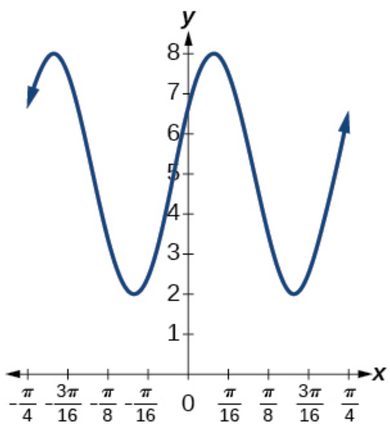
amplitude: 1; period: $\pi;$ midline: $y=0;$ maximum: $y=1$ occurs at $x=\pi;$ minimum: $y=-1$ occurs at $x=\frac{\pi}{2};$ one full period is graphed from $x=0$ to $x=\pi$

13.



amplitude: 4; period: 2; midline: $y=0;$ maximum: $y=4$ occurs at $x=0;$ minimum: $y=-4$ occurs at $x=1$

15.

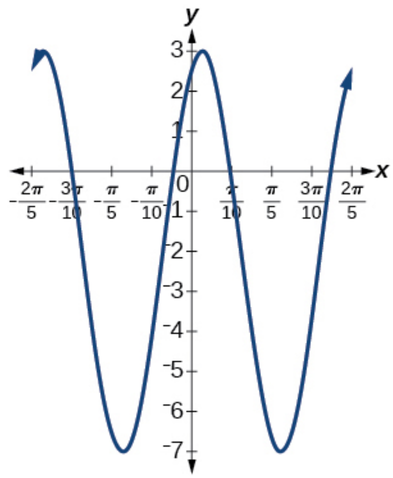


amplitude: 3; period: $\frac{\pi}{4};$ midline: $y=5;$ maximum: $y=8$ occurs at $x=0.12;$ minimum: $y=2$ occurs at $x=0.516;$ horizontal shift: $-4;$ vertical translation 5; one period occurs from

$x = 0$ to

$x = \frac{\pi}{4}$

17.



amplitude: 5; period:

$\frac{2\pi}{5};$ midline:

$y = -2;$ maximum:

$y = 3$ occurs at

$x = 0.08;$ minimum:

$y = -7$ occurs at

$x = 0.71;$ phase shift:

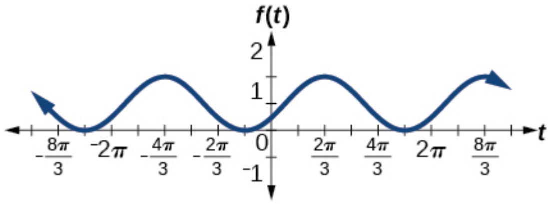
$-4;$ vertical translation:

$-2;$ one full period can be graphed on

$x = 0$ to

$x = \frac{2\pi}{5}$

19.



amplitude: 1 ; period:

$2\pi;$ midline:

$y = 1;$ maximum:

$y = 2$ occurs at

$x = 2.09;$ maximum:

$y = 2$ occurs at

$t = 2.09;$ minimum:

$y = 0$ occurs at

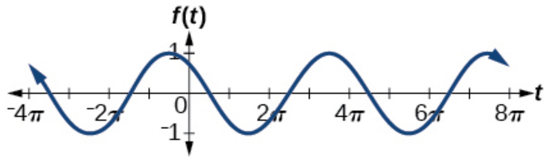
$t = 5.24;$ phase shift:

$-\frac{\pi}{3};$ vertical translation: 1; one full period is from

$t = 0$ to

$t = 2\pi$

21.



amplitude: 1; period: $4\pi;$ midline: $y = 0;$ maximum: $y = 1$ occurs at $t = 11.52;$ minimum: $y = -1$ occurs at $t = 5.24;$ phase shift: $-\frac{10\pi}{3};$ vertical shift: 0

23. amplitude: 2; midline: $y = -3;$ period: 4; equation: $f(x) = 2\sin\left(\frac{\pi}{2}x\right) - 3$

25. amplitude: 2; period: 5; midline: $y = 3;$ equation: $f(x) = -2\cos\left(\frac{2\pi}{5}x\right) + 3$

27. amplitude: 4; period: 2; midline: $y = 0;$ equation: $f(x) = -4\cos\left(\pi\left(x - \frac{\pi}{2}\right)\right)$

29. amplitude: 2; period: 2; midline $y = 1;$ equation: $f(x) = 2\cos(\pi x) + 1$

32.
$\frac{\pi}{6}, \frac{5\pi}{6}$

34.
$\frac{\pi}{4}, \frac{3\pi}{4}$

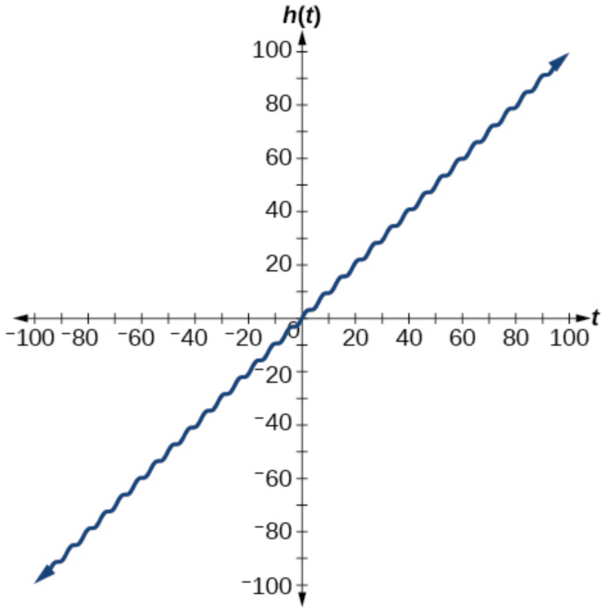
36.
$\frac{3\pi}{2}$

38.
$\frac{\pi}{2}, \frac{3\pi}{2}$

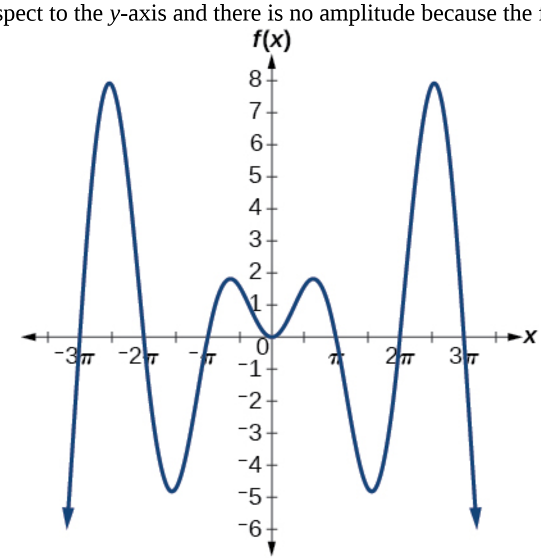
40.
$\frac{\pi}{2}, \frac{3\pi}{2}$

42.
$\frac{\pi}{6}, \frac{11\pi}{6}$

44. The graph appears linear. The linear functions dominate the shape of the graph for large values of x.



46. The graph is symmetric with respect to the *y*-axis and there is no amplitude because the function is not periodic.



48.

a. Amplitude: 12.5; period: 10; midline: $y = 13.5;$

b. $$h(t) = 12.5\sin\left(\frac{\pi}{5}(t - 2.5)\right) + 13.5;$$

c. 26 ft

49. Since $y = \csc x$ is the reciprocal function of $y = \sin x,$ you can plot the reciprocal of the coordinates on the graph of $y = \sin x$ to obtain the *y*-coordinates of $y = \csc x.$ The *x*-intercepts of the graph $y = \sin x$ are the vertical asymptotes for the graph of $y = \csc x.$

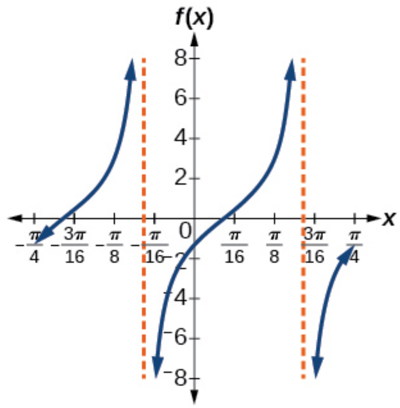
51. Answers will vary. Using the unit circle, one can show that $\tan(x + \pi) = \tan x.$

53. The period is the same: $2\pi.$

55. IV

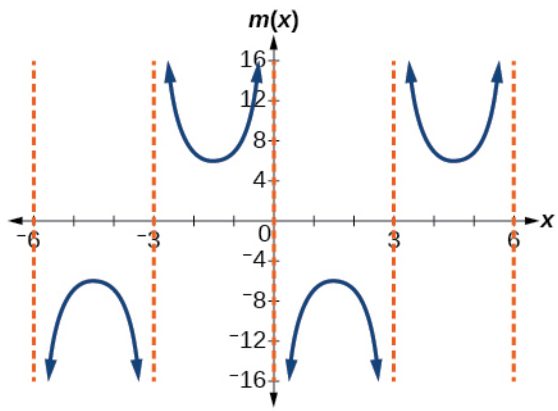
57. III
59. period: 8; horizontal shift: 1 unit to left
61. 1.5
63. 5
65.
$-\cot x \cos x - \sin x$

67.



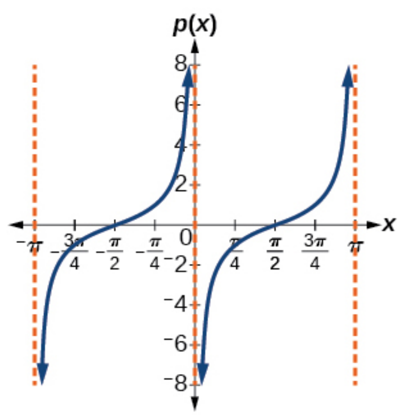
stretching factor: 2; period: $\frac{\pi}{4}$; asymptotes: $x = \frac{1}{4}\left(\frac{\pi}{2} + \pi k\right) + 8$, where k is an integer

69.



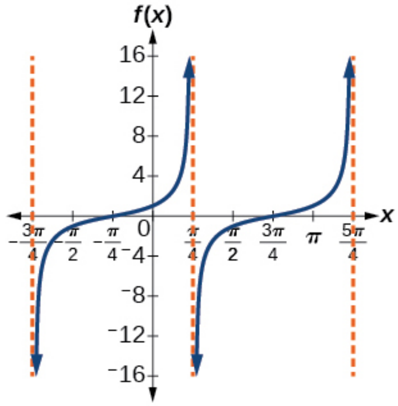
stretching factor: 6; period: 6; asymptotes: $x = 3k$, where k is an integer

71.



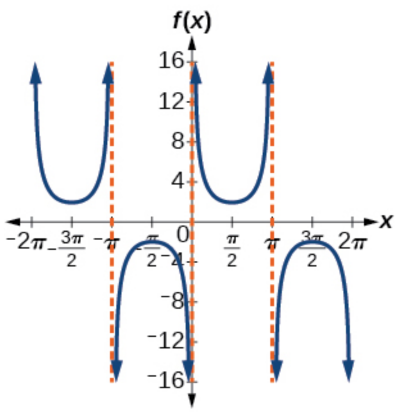
stretching factor: 1; period: $\pi;$ asymptotes: $x = \pi k,$ where k is an integer

73.



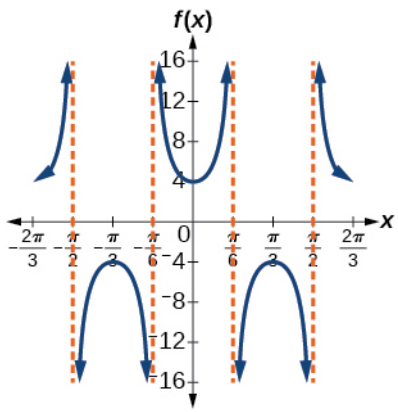
Stretching factor: 1; period: $\pi;$ asymptotes: $x = \frac{\pi}{4} + \pi k,$ where k is an integer

75.



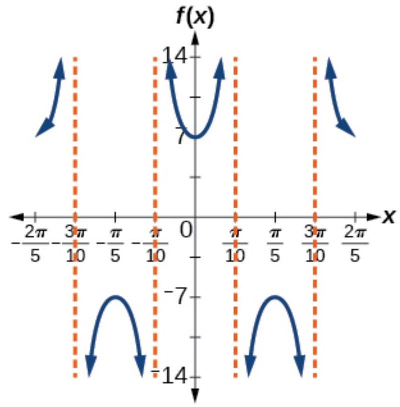
stretching factor: 2; period: $2\pi;$ asymptotes: $x = \pi k,$ where k is an integer

77.



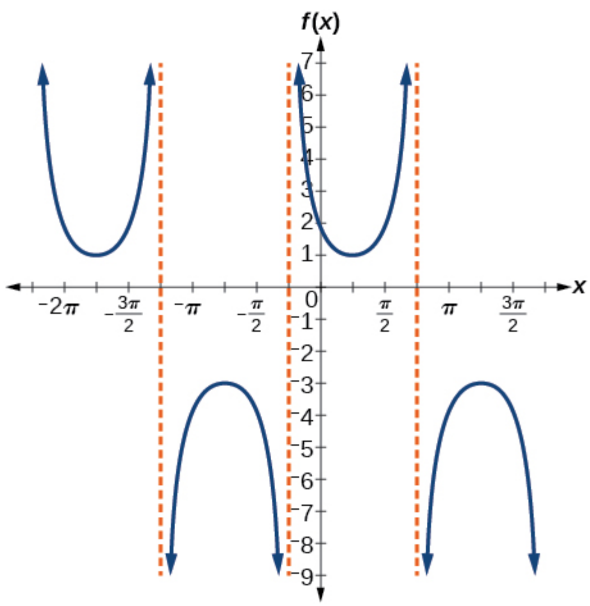
stretching factor: 4; period: $\frac{2\pi}{3};$ asymptotes: $x = \frac{\pi}{6}k,$ where k is an odd integer

79.



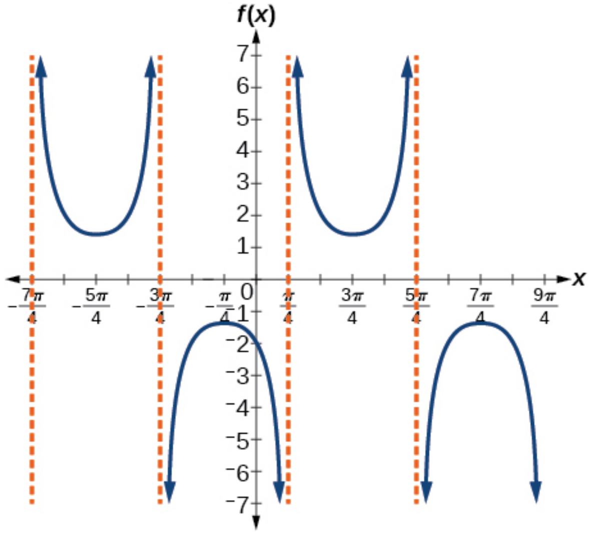
stretching factor: 7; period: $\frac{2\pi}{5};$ asymptotes: $x = \frac{\pi}{10}k,$ where k is an odd integer

81.



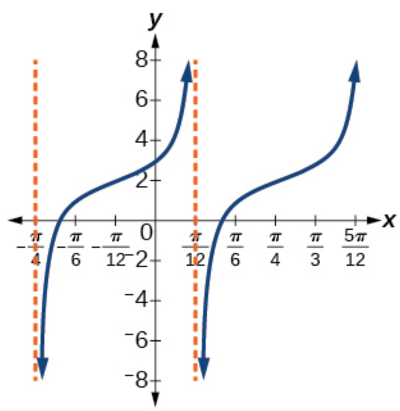
stretching factor: 2; period: $2\pi;$ asymptotes: $x = -\frac{\pi}{4} + \pi k,$ where k is an integer

83.



stretching factor:
$\frac{7}{5}$; period:
2π; asymptotes:
$x = \frac{\pi}{4} + \pi k$, where k is an integer

85.
$y = \tan\left(3\left(x - \frac{\pi}{4}\right)\right) + 2$



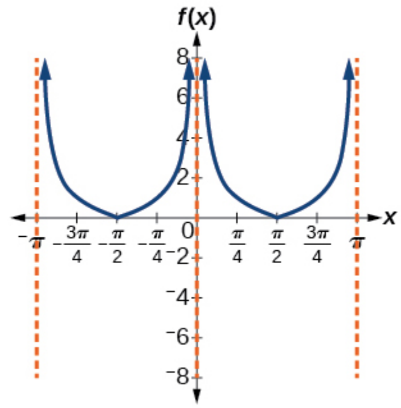
87.
$f(x) = \csc(2x)$

89.
$f(x) = \csc(4x)$

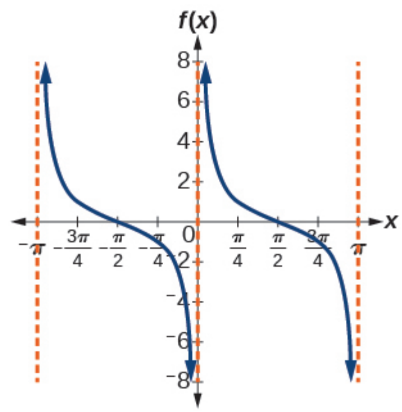
91.
$f(x) = 2\csc x$

93.
$f(x) = \frac{1}{2}\tan(100\pi x)$

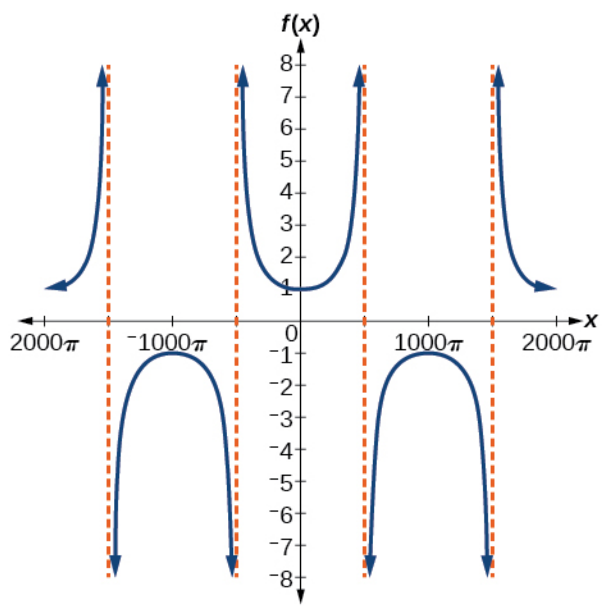
95.


f(x)
x
8
6
4
2
0
−2
−4
−6
−8
−π
−3π/4
−π/2
−π/4
π/4
π/2
3π/4
π

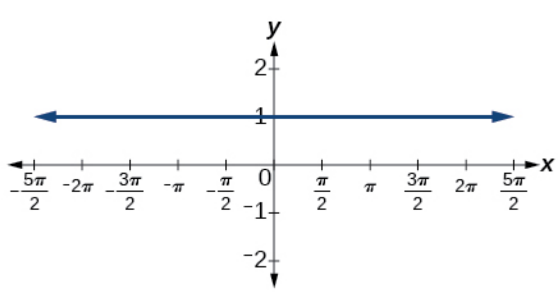
97.


f(x)
x
8
6
4
2
0
−2
−4
−6
−8
−π
−3π/4
−π/2
−π/4
π/4
π/2
3π/4
π

99.

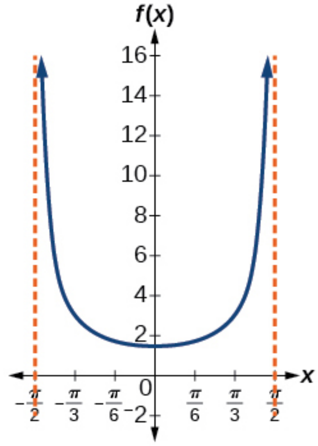

f(x)
x
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
2000π
−1000π
1000π
2000π

101.


y
x
2
1
0
−1
−2
−5π/2
-2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
5π/2

103.

a. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;

b. 

c. $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$; the distance grows without bound as $|x|$ approaches $\frac{\pi}{2}$ —i.e., at right angles to the line representing due north, the boat would be so far away, the fisherman could not see it;

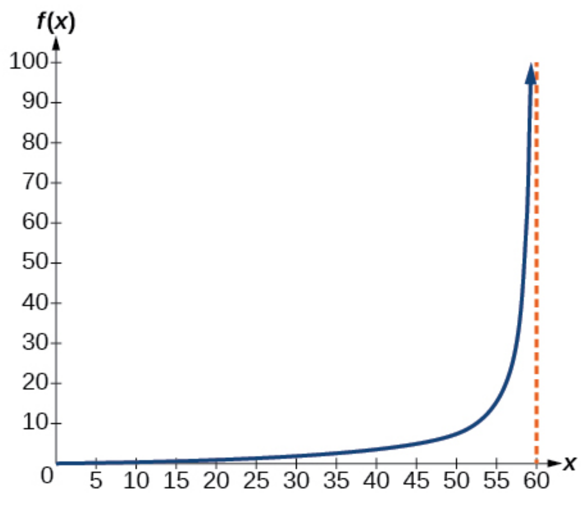
d. 3; when $x = -\frac{\pi}{3}$, the boat is 3 km away;

e. 1.73; when $x = \frac{\pi}{6}$, the boat is about 1.73 km away;

f. 1.5 km; when $x = 0$

105.

a. $h(x) = 2\tan\left(\frac{\pi}{120}x\right)$;

b. 

c. $h(0) = 0$: after 0 seconds, the rocket is 0 mi above the ground;

$h(30) = 2$: after 30 seconds, the rockets is 2 mi high;

d. As x approaches 60 seconds, the values of $h(x)$ grow increasingly large. The distance to the rocket is growing so large that the camera can no longer track it.

106. The function $y = \sin x$ is one-to-one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right];$ thus, this interval is the range of the inverse function of $y = \sin x,$ $f(x) = \sin^{-1} x.$ The function $y = \cos x$ is one-to-one on $[0, \pi];$ thus, this interval is the range of the inverse function of $y = \cos x,\ f(x) = \cos^{-1} x.$

108. $\frac{\pi}{6}$ is the radian measure of an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is 0.5.

110. In order for any function to have an inverse, the function must be one-to-one and must pass the horizontal line test. The regular sine function is not one-to-one unless its domain is restricted in some way. Mathematicians have agreed to restrict the sine function to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so that it is one-to-one and possesses an inverse.

112. True . The angle, θ_1 that equals $\arccos(-x)$, $x > 0$, will be a second quadrant angle with reference angle, θ_2 , where θ_2 equals $\arccos x$, $x > 0$. Since θ_2 is the reference angle for θ_1 , $\theta_2 = \pi - \theta_1$ and $\arccos(-x)$ = $\pi - \arccos x$.

114. $-\frac{\pi}{6}$

116. $\frac{3\pi}{4}$

118. $-\frac{\pi}{3}$

120. $\frac{\pi}{3}$

122. 1.98

124. 0.93

126. 1.41

128. 0.56 radians

130. 0

132. 0.71

134. -0.71

136.

$-\frac{\pi}{4}$

138. 0.8

140.

$\frac{5}{13}$

142.

$\frac{x-1}{\sqrt{-x^2+2x}}$

144.

$\frac{\sqrt{x^2-1}}{x}$

146.

$\frac{x+0.5}{\sqrt{-x^2-x+\frac{3}{4}}}$

148.

$\frac{\sqrt{2x+1}}{x+1}$

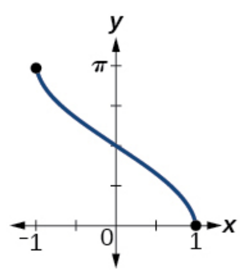
150.

$\frac{\sqrt{2x+1}}{x}$

152.

t

154.



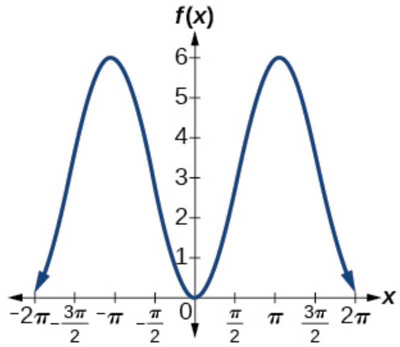
domain $[-1,\ 1];$ range $[0,\ \pi]$

156. approximately $x = 0.00$

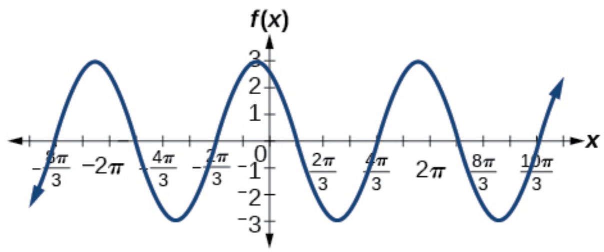
158. 0.395 radians
160. 1.11 radians
162. 1.25 radians
164. 0.405 radians
166. No. The angle the ladder makes with the horizontal is 60 degrees.

Review Exercises

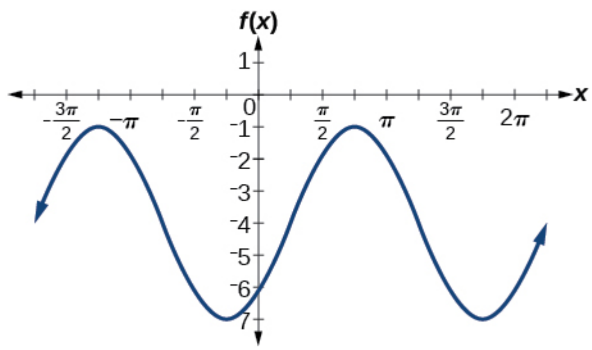
168. amplitude: 3; period: $2\pi;$ midline: $y = 3;$ no asymptotes



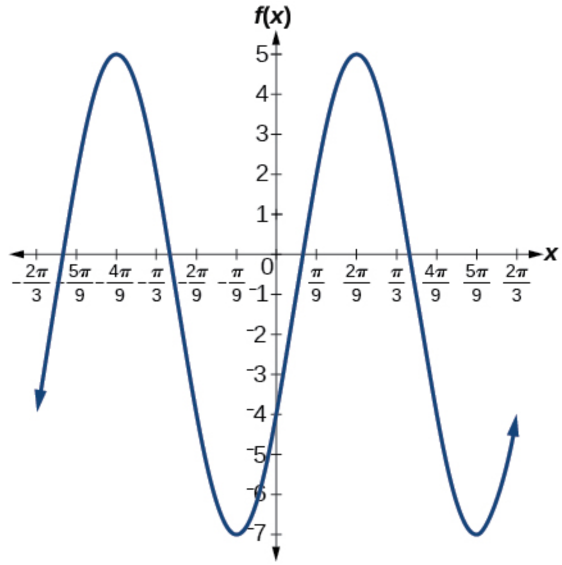
170. amplitude: 3; period: $2\pi;$ midline: $y = 0;$ no asymptotes



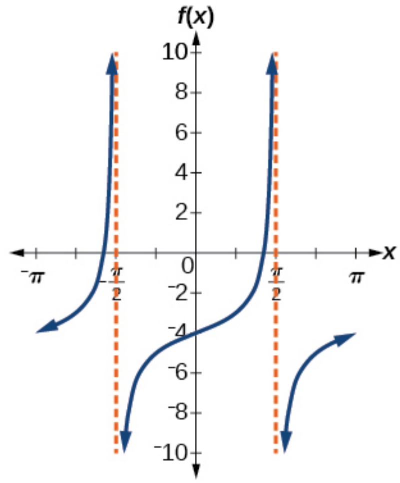
172. amplitude: 3; period: $2\pi;$ midline: $y = -4;$ no asymptotes



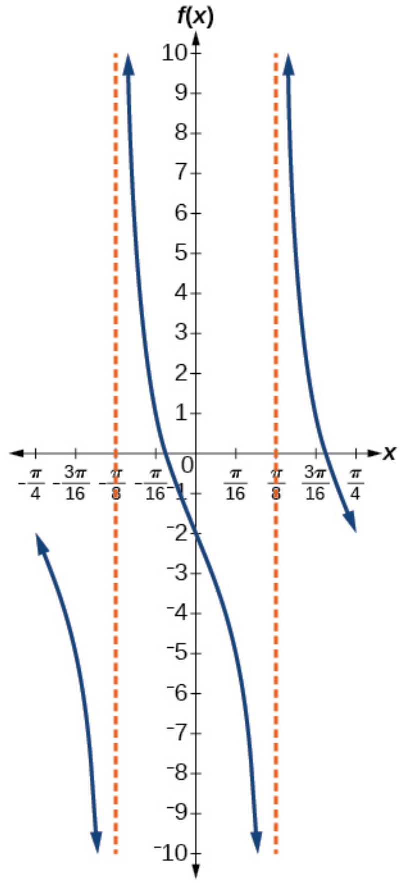
174. amplitude: 6; period: $\frac{2\pi}{3};$ midline: $y = -1;$ no asymptotes



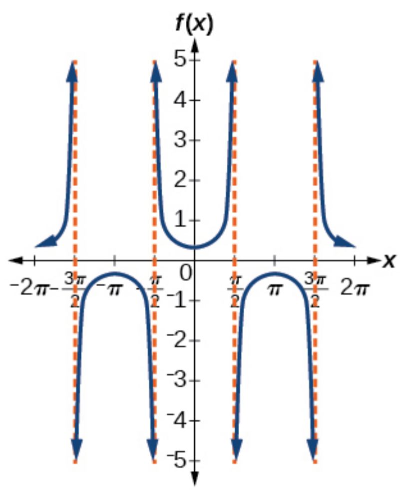
176. stretching factor: none; period: $\pi;$ midline: $y = -4;$ asymptotes: $x = \frac{\pi}{2} + \pi k,$ where k is an integer



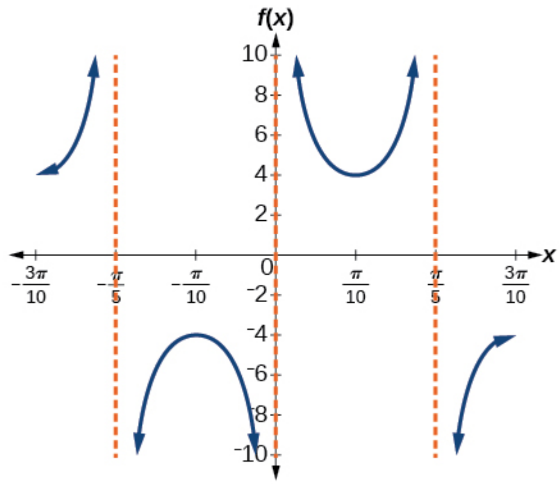
178. stretching factor: 3; period: $\frac{\pi}{4};$ midline: $y = -2;$ asymptotes: $x = \frac{\pi}{8} + \frac{\pi}{4}k,$ where k is an integer



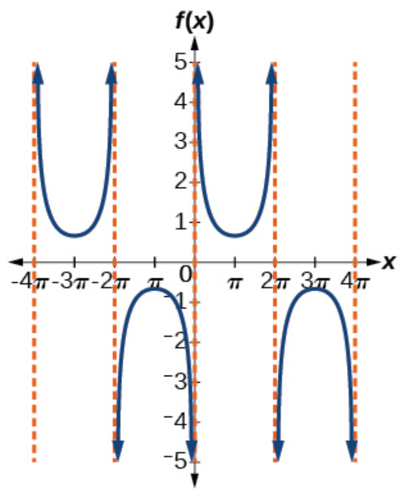
180. amplitude: none; period: $2\pi;$ no phase shift; asymptotes: $x = \frac{\pi}{2}k,$ where k is an odd integer



182. amplitude: none; period: $\frac{2\pi}{5};$ no phase shift; asymptotes: $x = \frac{\pi}{5}k,$ where k is an integer



184. amplitude: none; period: $4\pi;$ no phase shift; asymptotes: $x = 2\pi k,$ where k is an integer



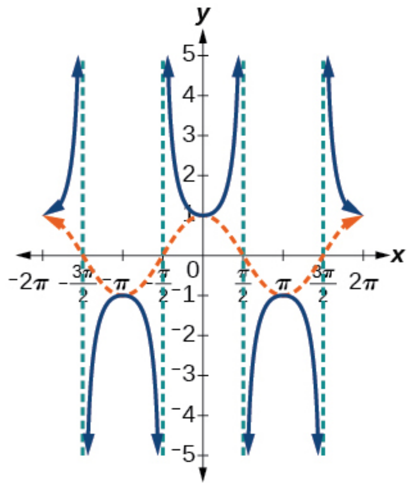
186. largest: 20,000; smallest: 4,000
188. amplitude: 8,000; period: 10; phase shift: 0
190. In 2007, the predicted population is 4,413. In 2010, the population will be 11,924.
192. 5 in.
194. 10 seconds
196. $\frac{\pi}{6}$

198. $\frac{\pi}{4}$

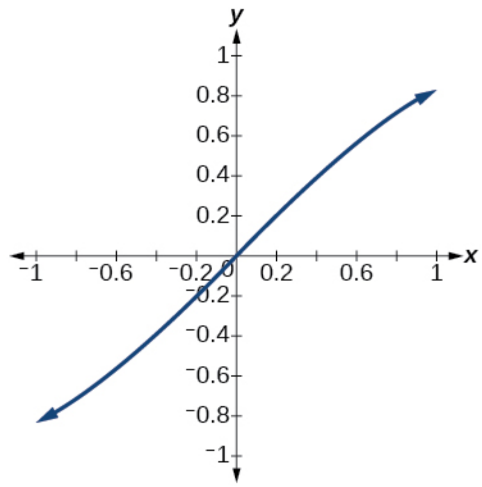
200. $\frac{\pi}{3}$

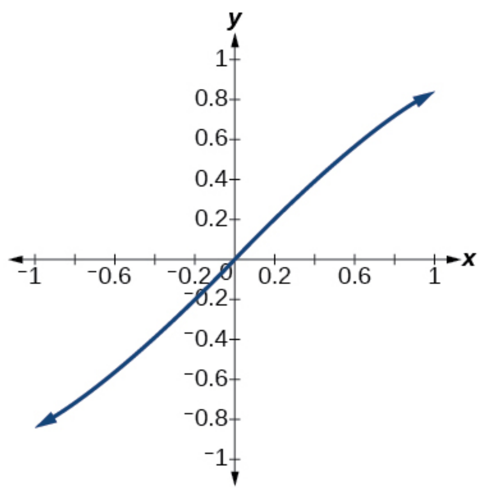
202. No solution
204. $\frac{12}{5}$

206. The graphs are not symmetrical with respect to the line $y = x.$ They are symmetrical with respect to the y -axis.



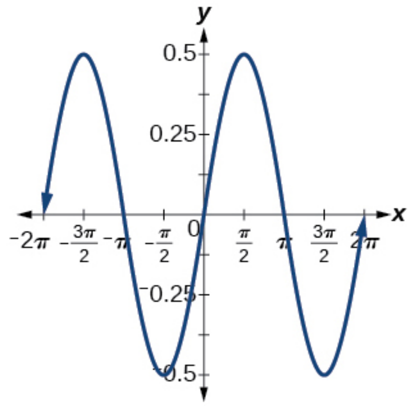
208. The graphs appear to be identical.



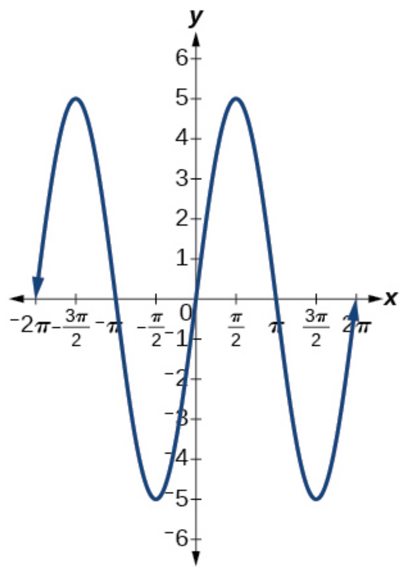


Practice Test

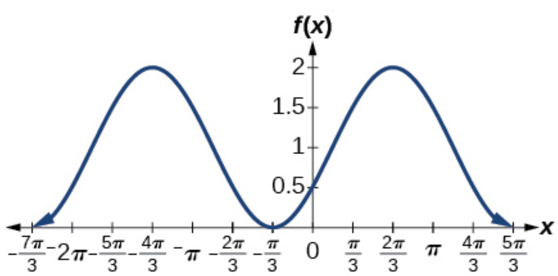
209. amplitude: 0.5; period:
$2\pi;$ midline
$y = 0$



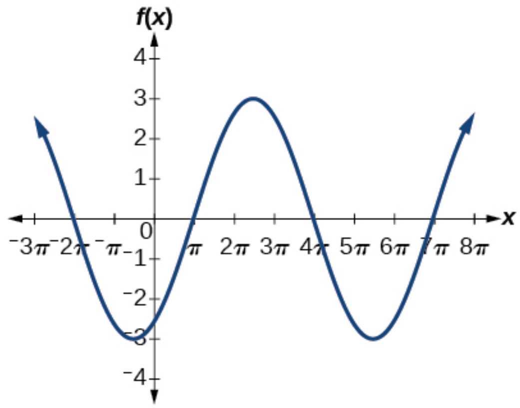
211. amplitude: 5; period: $2\pi;$ midline: $y = 0$



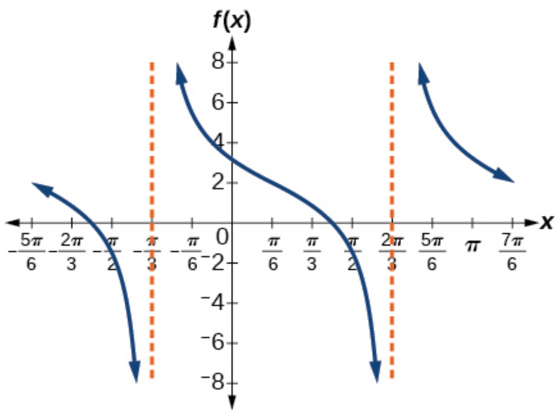
213. amplitude: 1; period: $2\pi;$ midline: $y = 1$



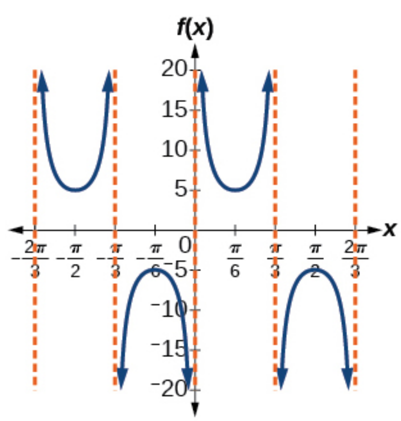
215. amplitude: 3; period: $6\pi;$ midline: $y = 0$



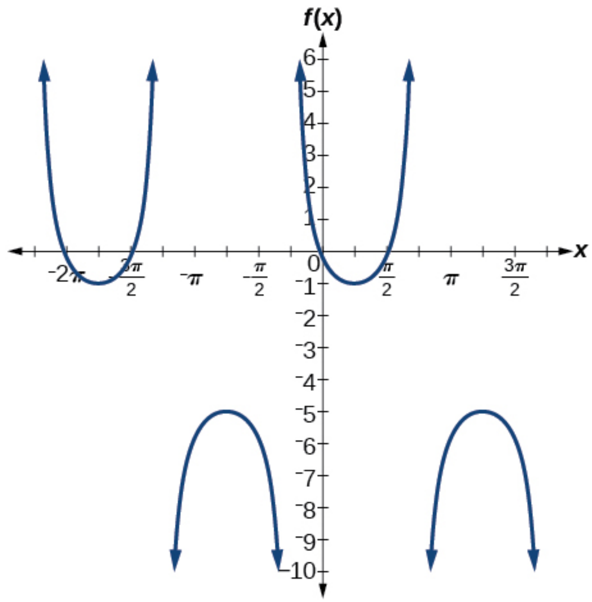
217. amplitude: none; period: $\pi;$ midline: $y = 0,$ asymptotes: $x = \frac{2\pi}{3} + \pi k,$ where k is an integer



219. amplitude: none; period: $\frac{2\pi}{3},$ midline: $y = 0,$ asymptotes: $x = \frac{\pi}{3}k,$ where k is an integer



221. amplitude: none; period: $2\pi;$ midline: $y = -3$



223. amplitude: 2; period: 2; midline:
$y = 0;$
$f(x) = 2\sin(\pi(x - 1))$

225. amplitude: 1; period: 12; phase shift:
$-6;$ midline
$y = -3$

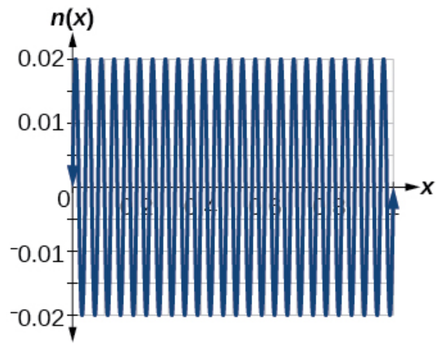
227.
$D(t) = 68 - 12\sin\left(\frac{\pi}{12}x\right)$

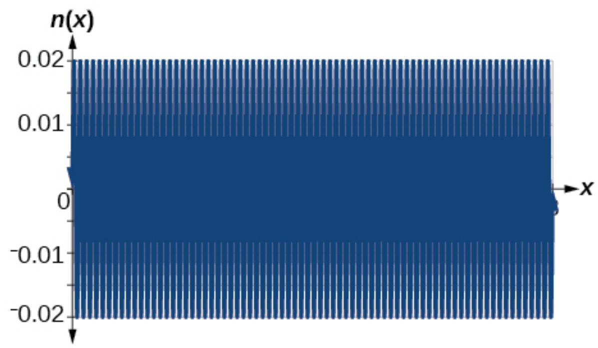
229. period:
$\frac{\pi}{6};$ horizontal shift:
-7

231.
$f(x) = \sec(\pi x);$ period: 2; phase shift: 0

233.
4

235. The views are different because the period of the wave is
$\frac{1}{25}.$ Over a bigger domain, there will be more cycles of the graph.

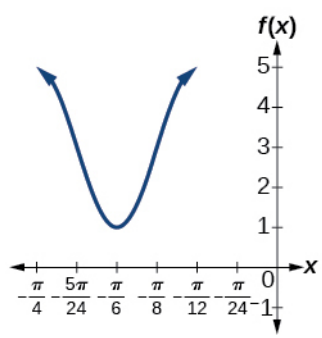




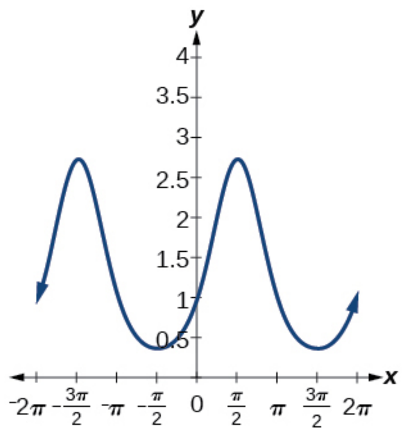
237.
$\frac{3}{5}$

239. On the approximate intervals
$(0.5,\ 1), (1.6,\ 2.1), (2.6,\ 3.1), (3.7,\ 4.2), (4.7,\ 5.2), (5.6,\ 6.28)$

241.
$f(x) = 2\cos\left(12\left(x + \frac{\pi}{4}\right)\right) + 3$



243. This graph is periodic with a period of
2π.



245. $\frac{\pi}{3}$

247. $\frac{\pi}{2}$

249. $\sqrt{1-(1-2x)^2}$

251. $\frac{1}{\sqrt{1+x^4}}$

253. $\frac{x+1}{x}$

255. False

257. approximately 0.07 radians

Chapter 7

Try It

7.1. (7.24)

$$\begin{aligned}\csc\theta\cos\theta\tan\theta &= \left(\frac{1}{\sin\theta}\right)\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)\\ &= \frac{\cos\theta}{\sin\theta}\left(\frac{\sin\theta}{\cos\theta}\right)\\ &= \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}\\ &= 1\end{aligned}$$

7.2. (7.29)

$$\begin{aligned}\frac{\cot\theta}{\csc\theta} &= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\\ &= \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1}\\ &= \cos\theta\end{aligned}$$

7.3.

$$\begin{aligned}\frac{\sin^2\theta-1}{\tan\theta\sin\theta-\tan\theta} &= \frac{(\sin\theta+1)(\sin\theta-1)}{\tan\theta(\sin\theta-1)}\\ &= \frac{\sin\theta+1}{\tan\theta}\end{aligned}$$

7.4. This is a difference of squares formula: $25-9\sin^2\theta=(5-3\sin\theta)(5+3\sin\theta)$.

7.5. (7.40)

$$\begin{aligned}\frac{\cos\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) &= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\\ &= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\\ &= \frac{1-\sin\theta}{\cos\theta}\end{aligned}$$

7.6.
$\frac{\sqrt{2}+\sqrt{6}}{4}$

7.7.
$\frac{\sqrt{2}-\sqrt{6}}{4}$

7.8.
$\frac{1-\sqrt{3}}{1+\sqrt{3}}$

7.9.
$\cos\left(\frac{5\pi}{14}\right)$

7.10. (7.84)

$$\begin{aligned}\tan(\pi-\theta) &= \frac{\tan(\pi)-\tan\theta}{1+\tan(\pi)\tan\theta}\\ &= \frac{0-\tan\theta}{1+0\cdot\tan\theta}\\ &= -\tan\theta\end{aligned}$$

7.11.
$\cos(2\alpha) = \frac{7}{32}$

7.12.
$\cos^4\theta - \sin^4\theta = \left(\cos^2\theta + \sin^2\theta\right)\left(\cos^2\theta - \sin^2\theta\right) = \cos(2\theta)$

7.13.
$\cos(2\theta)\cos\theta = \left(\cos^2\theta - \sin^2\theta\right)\cos\theta = \cos^3\theta - \cos\theta\sin^2\theta$

7.14.

$$\begin{aligned}10\cos^4 x = 10\cos^4 x &= 10(\cos^2 x)^2\\ &= 10\left[\frac{1+\cos(2x)}{2}\right]^2 && \text{Substitute reduction formula for } \cos^2 x.\\ &= \frac{10}{4}[1+2\cos(2x)+\cos^2(2x)]\\ &= \frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{4}\left(\frac{1+\cos2(2x)}{2}\right) && \text{Substitute reduction formula for } \cos^2 x.\\ &= \frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{8}+\frac{10}{8}\cos(4x)\\ &= \frac{30}{8}+5\cos(2x)+\frac{10}{8}\cos(4x)\\ &= \frac{15}{4}+5\cos(2x)+\frac{5}{4}\cos(4x)\end{aligned}$$

7.15.
$-\frac{2}{\sqrt{5}}$

7.16.
$\frac{1}{2}(\cos6\theta + \cos2\theta)$

7.17.
$\frac{1}{2}(\sin2x + \sin2y)$

7.18.

$\frac{-2-\sqrt{3}}{4}$

7.19.

$2\sin(2\theta)\cos(\theta)$

7.20.

$$\begin{aligned}\tan\theta\cot\theta-\cos^2\theta &= \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)-\cos^2\theta\\ &= 1-\cos^2\theta\\ &= \sin^2\theta\end{aligned}$$

7.21.

$x=\frac{7\pi}{6},\ \frac{11\pi}{6}$

7.22.

$\frac{\pi}{3}\pm\pi k$

7.23.

$\theta\approx 1.7722\pm 2\pi k$ and
$\theta\approx 4.5110\pm 2\pi k$

7.24.

$\cos\theta=-1,\ \theta=\pi$

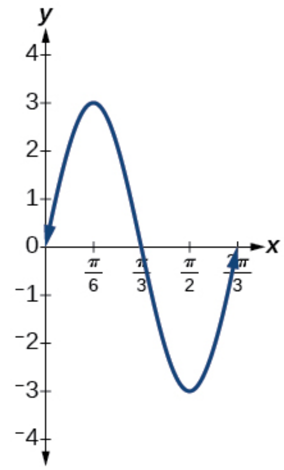
7.25.

$\frac{\pi}{2},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{3\pi}{2}$

7.26. The amplitude is $3,$ and the period is $\frac{2}{3}.$

7.27.

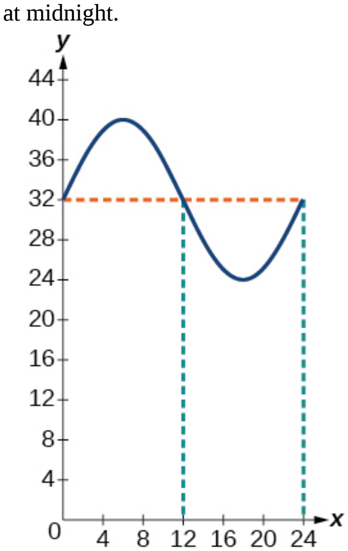
x	**3sin(3*x*)**
0	0
$\frac{\pi}{6}$	3
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	-3
$\frac{2\pi}{3}$	0



7.28.

$y = 8\sin\left(\frac{\pi}{12}t\right) + 32$

The temperature reaches freezing at noon and at midnight.



7.29. initial displacement =6, damping constant = -6, frequency =

$\frac{2}{\pi}$

7.30.

$y = 10e^{-0.5t}\cos(\pi t)$

7.31.

$y = 5\cos(6\pi t)$

Section Exercises

1. All three functions,

$F, G,$ and

$H,$ are even. This is because

$F(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x = F(x),\ G(-x) = \cos(-x)\cos(-x) = \cos x\cos x = \cos^2 x = G(x)$

and

$H(-x) = \tan(-x)\tan(-x) = (-\tan x)(-\tan x) = \tan^2 x = H(x).$

3. When

$\cos t = 0,$ then

$\sec t = \frac{1}{0},$ which is undefined.

5.
$\sin x$
7.
$\sec x$
9.
$\csc t$
11.
-1
13.
$\sec^2 x$
15.
$\sin^2 x + 1$
17.
$\frac{1}{\sin x}$
19.
$\frac{1}{\cot x}$
21.
$\tan x$
23.
$-4\sec x \tan x$
25.
$\pm\sqrt{\frac{1}{\cot^2 x} + 1}$
27.
$\frac{\pm\sqrt{1 - \sin^2 x}}{\sin x}$
29. Answers will vary. Sample proof:
$\cos x - \cos^3 x = \cos x\left(1 - \cos^2 x\right)$

$= \cos x \sin^2 x$
31. Answers will vary. Sample proof:

$$\frac{1 + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x + \tan^2 x = \tan^2 x + 1 + \tan^2 x = 1 + 2\tan^2 x$$
33. Answers will vary. Sample proof:

$$\cos^2 x - \tan^2 x = 1 - \sin^2 x - \left(\sec^2 x - 1\right) = 1 - \sin^2 x - \sec^2 x + 1 = 2 - \sin^2 x - \sec^2 x$$

35. False
37. False
39. Proved with negative and Pythagorean identities
41. True
$$3\sin^2\theta + 4\cos^2\theta = 3\sin^2\theta + 3\cos^2\theta + \cos^2\theta = 3\left(\sin^2\theta + \cos^2\theta\right) + \cos^2\theta = 3 + \cos^2\theta$$

43. The cofunction identities apply to complementary angles. Viewing the two acute angles of a right triangle, if one of those angles measures $x,$ the second angle measures $\frac{\pi}{2} - x.$ Then $\sin x = \cos\left(\frac{\pi}{2} - x\right).$ The same holds for the other cofunction identities. The key is that the angles are complementary.
45.
$\sin(-x) = -\sin x,$ so $\sin x$ is odd.

$\cos(-x) = \cos(0 - x) = \cos x,$ so
$\cos x$ is even.

47.
$\frac{\sqrt{2} + \sqrt{6}}{4}$

49.
$\frac{\sqrt{6} - \sqrt{2}}{4}$

51.
$-2 - \sqrt{3}$

53.
$-\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x$

55.
$-\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

57.
$\csc\theta$

59.
$\cot x$

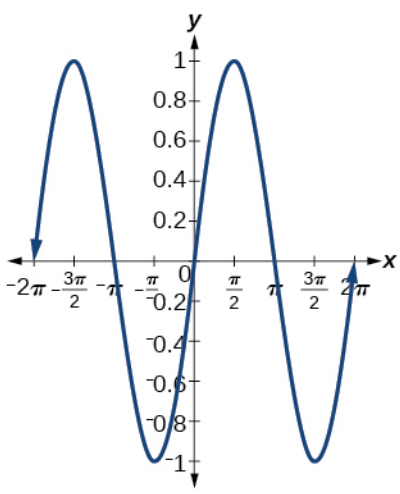
61.
$\tan\left(\frac{x}{10}\right)$

63.
$\sin(a - b) = \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{4 - 6\sqrt{2}}{15}$

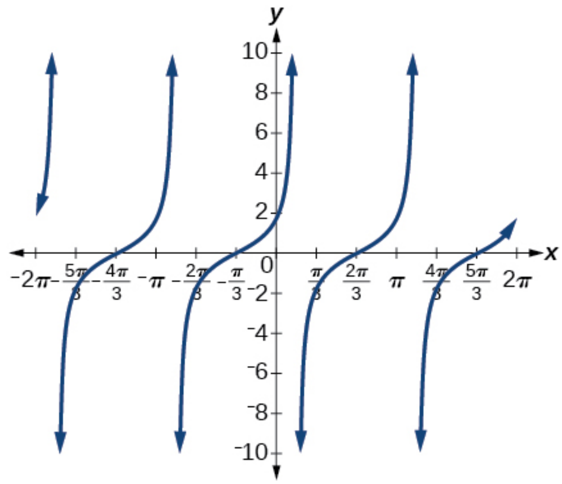
$\cos(a + b) = \left(\frac{3}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{4}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{3 - 8\sqrt{2}}{15}$

65.
$\frac{\sqrt{2} - \sqrt{6}}{4}$

67.
$\sin x$

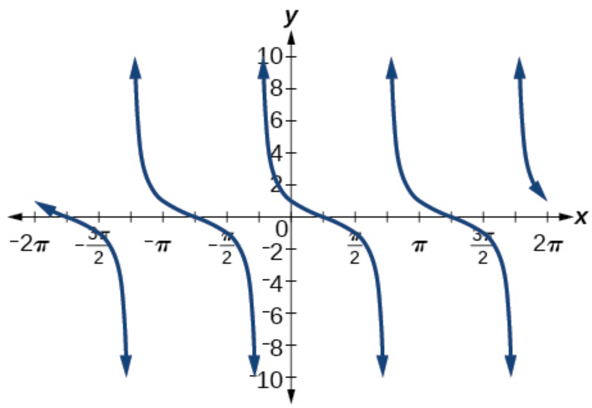


69.
$\cot\left(\frac{\pi}{6} - x\right)$



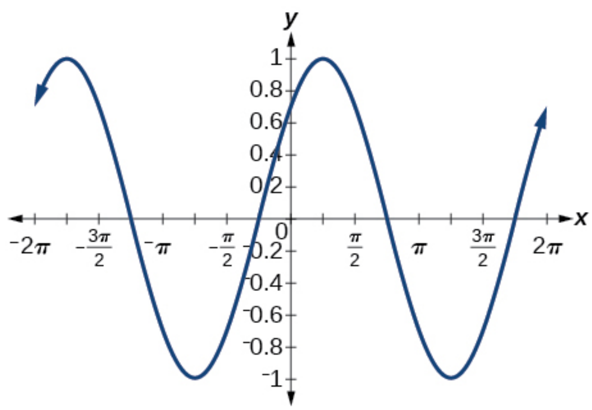
71.

$\cot\left(\frac{\pi}{4}+x\right)$



73.

$\frac{\sin x}{\sqrt{2}}+\frac{\cos x}{\sqrt{2}}$



75. They are the same.

77. They are the different, try

$g(x)=\sin(9x)-\cos(3x)\sin(6x)$.

79. They are the same.

81. They are the different, try

$g(\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$.

83. They are different, try

$g(x)=\frac{\tan x-\tan(2x)}{1+\tan x\tan(2x)}$.

85.
$-\frac{\sqrt{3}-1}{2\sqrt{2}},\ \text{or}\ -0.2588$

87.
$\frac{1+\sqrt{3}}{2\sqrt{2}},$ or 0.9659

89.
$\tan\left(x+\frac{\pi}{4}\right)=$

$\frac{\tan x+\tan\left(\frac{\pi}{4}\right)}{1-\tan x\tan\left(\frac{\pi}{4}\right)}=$

$\frac{\tan x+1}{1-\tan x(1)}=\frac{\tan x+1}{1-\tan x}$

91.
$\frac{\cos(a+b)}{\cos a\cos b}=$

$\frac{\cos a\cos b}{\cos a\cos b}-\frac{\sin a\sin b}{\cos a\cos b}=1-\tan a\tan b$

93.
$\frac{\cos(x+h)-\cos x}{h}=$

$\frac{\cos x\cosh-\sin x\sinh-\cos x}{h}=$

$\frac{\cos x(\cosh-1)-\sin x\sinh}{h}=\cos x\frac{\cos h-1}{h}-\sin x\frac{\sin h}{h}$

95. True

97. True. Note that
$\sin(\alpha+\beta)=\sin(\pi-\gamma)$ and expand the right hand side.

99. Use the Pythagorean identities and isolate the squared term.

101.
$\frac{1-\cos x}{\sin x},\ \frac{\sin x}{1+\cos x},$ multiplying the top and bottom by
$\sqrt{1-\cos x}$ and
$\sqrt{1+\cos x},$ respectively.

103. a)
$\frac{3\sqrt{7}}{32}$ b)
$\frac{31}{32}$ c)
$\frac{3\sqrt{7}}{31}$

105. a)
$\frac{\sqrt{3}}{2}$ b)
$-\frac{1}{2}$ c)
$-\sqrt{3}$

107.
$\cos\theta=-\frac{2\sqrt{5}}{5},\ \sin\theta=\frac{\sqrt{5}}{5},\ \tan\theta=-\frac{1}{2},\ \csc\theta=\sqrt{5},\ \sec\theta=-\frac{\sqrt{5}}{2},\ \cot\theta=-2$

109.
$2\sin\left(\frac{\pi}{2}\right)$

111.
$\frac{\sqrt{2-\sqrt{2}}}{2}$

113.
$\frac{\sqrt{2-\sqrt{3}}}{2}$

115.
$2+\sqrt{3}$

117.
$-1-\sqrt{2}$

119. a)
$\frac{3\sqrt{13}}{13}$ b)
$-\frac{2\sqrt{13}}{13}$ c)
$-\frac{3}{2}$

121. a)
$\frac{\sqrt{10}}{4}$ b)
$\frac{\sqrt{6}}{4}$ c)
$\frac{\sqrt{15}}{3}$

123.
$\frac{120}{169}, -\frac{119}{169}, -\frac{120}{119}$

125.
$\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13}, \frac{2}{3}$

127.
$\cos(74°)$

129.
$\cos(18x)$

131.
$3\sin(10x)$

133.
$-2\sin(-x)\cos(-x) = -2(-\sin(x)\cos(x)) = \sin(2x)$

135.
$\frac{\sin(2\theta)}{1+\cos(2\theta)}\tan^2\theta = \frac{2\sin(\theta)\cos(\theta)}{1+\cos^2\theta-\sin^2\theta}\tan^2\theta =$
$\frac{2\sin(\theta)\cos(\theta)}{2\cos^2\theta}\tan^2\theta = \frac{\sin(\theta)}{\cos\theta}\tan^2\theta =$
$\cot(\theta)\tan^2\theta = \tan\theta$

137.
$\frac{1+\cos(12x)}{2}$

139.
$\frac{3+\cos(12x)-4\cos(6x)}{8}$

141.
$\frac{2+\cos(2x)-2\cos(4x)-\cos(6x)}{32}$

143.
$$\frac{3+\cos(4x)-4\cos(2x)}{3+\cos(4x)+4\cos(2x)}$$
145.
$$\frac{1-\cos(4x)}{8}$$
147.
$$\frac{3+\cos(4x)-4\cos(2x)}{4(\cos(2x)+1)}$$
149.
$$\frac{(1+\cos(4x))\sin x}{2}$$
151.
$$4\sin x\cos x\left(\cos^2 x-\sin^2 x\right)$$
153.
$$\frac{2\tan x}{1+\tan^2 x}=\frac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}=\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}}=$$
$$\frac{2\sin x}{\cos x}.\frac{\cos^2 x}{1}=2\sin x\cos x=\sin(2x)$$
155.
$$\frac{2\sin x\cos x}{2\cos^2 x-1}=\frac{\sin(2x)}{\cos(2x)}=\tan(2x)$$
157.
$$\begin{aligned}\sin(x+2x)&=\sin x\cos(2x)+\sin(2x)\cos x\\&=\sin x(\cos^2 x-\sin^2 x)+2\sin x\cos x\cos x\\&=\sin x\cos^2 x-\sin^3 x+2\sin x\cos^2 x\\&=3\sin x\cos^2 x-\sin^3 x\end{aligned}$$
159.
$$\begin{aligned}\frac{1+\cos(2t)}{\sin(2t)-\cos t}&=\frac{1+2\cos^2 t-1}{2\sin t\cos t-\cos t}\\&=\frac{2\cos^2 t}{\cos t(2\sin t-1)}\\&=\frac{2\cos t}{2\sin t-1}\end{aligned}$$
161.
$$\begin{aligned}&\left(\cos^2(4x)-\sin^2(4x)-\sin(8x))(\cos^2(4x)-\sin^2(4x)+\sin(8x)\right)=\\&=(\cos(8x)-\sin(8x))(\cos(8x)+\sin(8x))\\&=\cos^2(8x)-\sin^2(8x)\\&=\cos(16x)\end{aligned}$$

162. Substitute
α into cosine and
β into sine and evaluate.
164. Answers will vary. There are some equations that involve a sum of two trig expressions where when converted to a product are easier to solve. For example:
$\frac{\sin(3x)+\sin x}{\cos x}=1.$ When converting the numerator to a product the equation becomes:
$$\frac{2\sin(2x)\cos x}{\cos x}=1$$
166.
$$8(\cos(5x)-\cos(27x))$$

168.
$\sin(2x) + \sin(8x)$

170.
$\frac{1}{2}(\cos(6x) - \cos(4x))$

172.
$2\cos(5t)\cos t$

174.
$2\cos(7x)$

176.
$2\cos(6x)\cos(3x)$

178.
$\frac{1}{4}(1 + \sqrt{3})$

180.
$\frac{1}{4}(\sqrt{3} - 2)$

182.
$\frac{1}{4}(\sqrt{3} - 1)$

184.
$\cos(80°) - \cos(120°)$

186.
$\frac{1}{2}(\sin(221°) + \sin(205°))$

188.
$\sqrt{2}\cos(31°)$

190.
$2\cos(66.5°)\sin(34.5°)$

192.
$2\sin(-1.5°)\cos(0.5°)$

194.
$$2\sin(7x) - 2\sin x = 2\sin(4x + 3x) - 2\sin(4x - 3x) =$$
$$2(\sin(4x)\cos(3x) + \sin(3x)\cos(4x)) - 2(\sin(4x)\cos(3x) - \sin(3x)\cos(4x)) =$$
$$2\sin(4x)\cos(3x) + 2\sin(3x)\cos(4x)) - 2\sin(4x)\cos(3x) + 2\sin(3x)\cos(4x)) =$$
$$4\sin(3x)\cos(4x)$$

196.
$$\sin x + \sin(3x) = 2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right) =$$

$$2\sin(2x)\cos x = 2(2\sin x\cos x)\cos x =$$

$$4\sin x\cos^2 x$$

198.
$$2\tan x\cos(3x) = \frac{2\sin x\cos(3x)}{\cos x} = \frac{2(.5(\sin(4x) - \sin(2x)))}{\cos x}$$

$$= \frac{1}{\cos x}(\sin(4x) - \sin(2x)) = \sec x(\sin(4x) - \sin(2x))$$

200.
$2\cos(35°)\cos(23°), \quad 1.5081$

202.
$-2\sin(33°)\sin(11°), \quad -0.2078$

204.
$\frac{1}{2}(\cos(99^\circ) - \cos(71^\circ)), \quad -0.2410$

206. It is and identity.
208. It is not an identity, but
$2\cos^3 x$ is.
210.
$\tan(3t)$

212.
$2\cos(2x)$

214.
$-\sin(14x)$

216. Start with
$\cos x + \cos y.$ Make a substitution and let
$x = \alpha + \beta$ and let
$y = \alpha - \beta,$ so
$\cos x + \cos y$ becomes

$\cos(\alpha + \beta) + \cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta = 2\cos\alpha\cos\beta$ Since
$x = \alpha + \beta$ and
$y = \alpha - \beta,$ we can solve for
α and
β in terms of *x* and *y* and substitute in for
$2\cos\alpha\cos\beta$ and get
$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right).$

218.
$\frac{\cos(3x) + \cos x}{\cos(3x) - \cos x} = \frac{2\cos(2x)\cos x}{-2\sin(2x)\sin x} = -\cot(2x)\cot x$

220.
$\frac{\cos(2y) - \cos(4y)}{\sin(2y) + \sin(4y)} = \frac{-2\sin(3y)\sin(-y)}{2\sin(3y)\cos y} =$
$\frac{2\sin(3y)\sin(y)}{2\sin(3y)\cos y} = \tan y$

222.
$\cos x - \cos(3x) = -2\sin(2x)\sin(-x) =$
$2(2\sin x\cos x)\sin x = 4\sin^2 x\cos x$

224.

$\tan\left(\frac{\pi}{4} - t\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan t}{1 + \tan\left(\frac{\pi}{4}\right)\tan(t)} = \frac{1 - \tan t}{1 + \tan t}$

225. There will not always be solutions to trigonometric function equations. For a basic example,
$\cos(x) = -5.$

227. If the sine or cosine function has a coefficient of one, isolate the term on one side of the equals sign. If the number it is set equal to has an absolute value less than or equal to one, the equation has solutions, otherwise it does not. If the sine or cosine does not have a coefficient equal to one, still isolate the term but then divide both sides of the equation by the leading coefficient. Then, if the number it is set equal to has an absolute value greater than one, the equation has no solution.
229.
$\frac{\pi}{3}, \frac{2\pi}{3}$

231.
$\frac{3\pi}{4}, \frac{5\pi}{4}$

233.
$\frac{\pi}{4}, \frac{5\pi}{4}$

235.
$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

237.
$\frac{\pi}{4}, \frac{7\pi}{4}$

239.
$\frac{7\pi}{6}, \frac{11\pi}{6}$

241.
$\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

243.
$\frac{3\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}$

245.
$\frac{1}{6}, \frac{5}{6}, \frac{13}{6}, \frac{17}{6}, \frac{25}{6}, \frac{29}{6}, \frac{37}{6}$

247.
$0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

249.
$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

251.
$\frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

253.
$0, \pi$

255.
$\pi - \sin^{-1}\left(-\frac{1}{4}\right), \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi + \sin^{-1}\left(-\frac{1}{4}\right)$

257.
$\frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{\pi}{3} - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{2\pi}{3} + \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \pi - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{4\pi}{3} + \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{5\pi}{3} - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right)$

259.
0

261.
$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

263.
$\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

265.
$0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$

267. There are no solutions.

269.
$\cos^{-1}\left(\frac{1}{3}(1 - \sqrt{7})\right), 2\pi - \cos^{-1}\left(\frac{1}{3}(1 - \sqrt{7})\right)$

271.
$\tan^{-1}\left(\frac{1}{2}(\sqrt{29} - 5)\right), \pi + \tan^{-1}\left(\frac{1}{2}(-\sqrt{29} - 5)\right), \pi + \tan^{-1}\left(\frac{1}{2}(\sqrt{29} - 5)\right), 2\pi + \tan^{-1}\left(\frac{1}{2}(-\sqrt{29} - 5)\right)$

273. There are no solutions.

275. There are no solutions.

277.
$0, \frac{2\pi}{3}, \frac{4\pi}{3}$

279.
$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

281.
$\sin^{-1}\left(\frac{3}{5}\right), \frac{\pi}{2}, \pi - \sin^{-1}\left(\frac{3}{5}\right), \frac{3\pi}{2}$

283.
$\cos^{-1}\left(-\frac{1}{4}\right), 2\pi - \cos^{-1}\left(-\frac{1}{4}\right)$

285.
$\frac{\pi}{3}, \cos^{-1}\left(-\frac{3}{4}\right), 2\pi - \cos^{-1}\left(-\frac{3}{4}\right), \frac{5\pi}{3}$

287.
$\cos^{-1}\left(\frac{3}{4}\right), \cos^{-1}\left(-\frac{2}{3}\right), 2\pi - \cos^{-1}\left(-\frac{2}{3}\right), 2\pi - \cos^{-1}\left(\frac{3}{4}\right)$

289.
$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

291.
$\frac{\pi}{3}, \cos^{-1}\left(-\frac{1}{4}\right), 2\pi - \cos^{-1}\left(-\frac{1}{4}\right), \frac{5\pi}{3}$

293. There are no solutions.

295.
$\pi + \tan^{-1}(-2), \pi + \tan^{-1}\left(-\frac{3}{2}\right), 2\pi + \tan^{-1}(-2), 2\pi + \tan^{-1}\left(-\frac{3}{2}\right)$

297.
$2\pi k + 0.2734, 2\pi k + 2.8682$

299.
$\pi k - 0.3277$

301.
$0.6694, 1.8287, 3.8110, 4.9703$

303.
$1.0472, 3.1416, 5.2360$

305.
$0.5326, 1.7648, 3.6742, 4.9064$

307.
$\sin^{-1}\left(\frac{1}{4}\right), \pi - \sin^{-1}\left(\frac{1}{4}\right), \frac{3\pi}{2}$

309.
$\frac{\pi}{2}, \frac{3\pi}{2}$

311. There are no solutions.

313.
$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

315. There are no solutions.

317.
7.2°

319.
5.7°

321.
82.4°

323.
31.0°

325.
$88.7°$

327.
$59.0°$

329.
$36.9°$

331. Physical behavior should be periodic, or cyclical.
333. Since cumulative rainfall is always increasing, a sinusoidal function would not be ideal here.
335.
$y = -3\cos\left(\frac{\pi}{6}x\right) - 1$

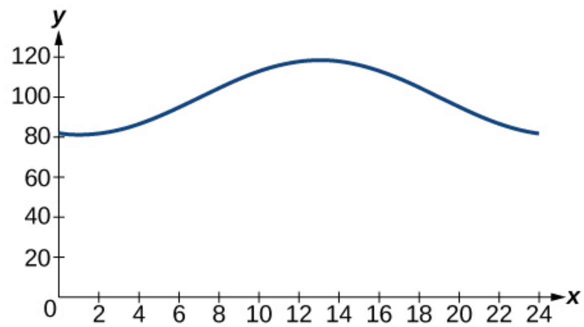
337.
$5\sin(2x) + 2$

339.
$4\cos\left(\frac{x\pi}{2}\right) - 3$

341.
$5 - 8\sin\left(\frac{x\pi}{2}\right)$

343.
$\tan\left(\frac{x\pi}{12}\right)$

345.



Answers will vary. Sample answer: This function could model temperature changes over the course of one very hot day in Phoenix, Arizona.
347. 9 years from now
349.
$56\ °F$

351.
1.8024 hours
353. 4:30
355. From July 8 to October 23
357. From day 19 through day 40
359. Floods: July 24 through October 7. Droughts: February 4 through March 27
361. Amplitude: 11, period:
$\frac{1}{6},$ frequency: 6 Hz
363. Amplitude: 5, period:
$\frac{1}{30},$ frequency: 30 Hz
365.
$P(t) = -15\cos\left(\frac{\pi}{6}t\right) + 650 + \frac{55}{6}t$

367.
$P(t) = -40\cos\left(\frac{\pi}{6}t\right) + 800(1.04)^t$

369.
$D(t) = 7(0.89)^t \cos(40\pi t)$

371.
$D(t) = 19(0.9265)^t \cos(26\pi t)$

373.
20.1 years

375. 17.8 seconds

377. Spring 2 comes to rest first after 8.0 seconds.

379. 500 miles, at
$90°$

381.
$y = 6(5)^x + 4\sin\left(\frac{\pi}{2}x\right)$

383.
$y = 8\left(\frac{1}{2}\right)^x \cos\left(\frac{\pi}{2}x\right) + 3$

Review Exercises

385.
$\sin^{-1}\left(\frac{\sqrt{3}}{3}\right),\ \pi - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right),\ \pi + \sin^{-1}\left(\frac{\sqrt{3}}{3}\right),\ 2\pi - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right)$

387.
$\frac{7\pi}{6},\ \frac{11\pi}{6}$

389.
$\sin^{-1}\left(\frac{1}{4}\right),\ \pi - \sin^{-1}\left(\frac{1}{4}\right)$

391.
1

393. Yes

395.
$-2 - \sqrt{3}$

397.
$\frac{\sqrt{2}}{2}$

399.

$$\begin{aligned}\cos(4x) - \cos(3x)\cos x &= \cos(2x + 2x) - \cos(x + 2x)\cos x \\ &= \cos(2x)\cos(2x) - \sin(2x)\sin(2x) - \cos x\cos(2x)\cos x + \sin x\sin(2x)\cos x \\ &= \left(\cos^2 x - \sin^2 x\right)^2 - 4\cos^2 x\sin^2 x - \cos^2 x\left(\cos^2 x - \sin^2 x\right) + \sin x(2)\sin x\cos x\cos x \\ &= \left(\cos^2 x - \sin^2 x\right)^2 - 4\cos^2 x\sin^2 x - \cos^2 x\left(\cos^2 x - \sin^2 x\right) + 2\sin^2 x\cos^2 x \\ &= \cos^4 x - 2\cos^2 x\sin^2 x + \sin^4 x - 4\cos^2 x\sin^2 x - \cos^4 x + \cos^2 x\sin^2 x + 2\sin^2 x\cos^2 x \\ &= \sin^4 x - 4\cos^2 x\sin^2 x + \cos^2 x\sin^2 x \\ &= \sin^2 x\left(\sin^2 x + \cos^2 x\right) - 4\cos^2 x\sin^2 x \\ &= \sin^2 x - 4\cos^2 x\sin^2 x\end{aligned}$$

401.
$\tan\left(\frac{5}{8}x\right)$

403.
$\frac{\sqrt{3}}{3}$

405.
$-\frac{24}{25},\ -\frac{7}{25},\ \frac{24}{7}$

407.
$\sqrt{2(2+\sqrt{2})}$

409.
$\frac{\sqrt{2}}{10}, \frac{7\sqrt{2}}{10}, \frac{1}{7}, \frac{3}{5}, \frac{4}{5}, \frac{3}{4}$

411.

$$\begin{aligned}\cot x\cos(2x) &= \cot x(1-2\sin^2 x)\\ &= \cot x - \frac{\cos x}{\sin x}(2)\sin^2 x\\ &= -2\sin x\cos x + \cot x\\ &= -\sin(2x) + \cot x\end{aligned}$$

413.
$\frac{10\sin x - 5\sin(3x) + \sin(5x)}{8(\cos(2x)+1)}$

415.
$\frac{\sqrt{3}}{2}$

417.
$-\frac{\sqrt{2}}{2}$

419.
$\frac{1}{2}(\sin(6x) + \sin(12x))$

421.
$2\sin\left(\frac{13}{2}x\right)\cos\left(\frac{9}{2}x\right)$

423.
$\frac{3\pi}{4}, \frac{7\pi}{4}$

425.
$0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

427.
$\frac{3\pi}{2}$

429. No solution

431.
$0.2527, 2.8889, 4.7124$

433.
$1.3694, \ 1.9106, \ 4.3726, \ 4.9137$

435.
$3\sin\left(\frac{x\pi}{2}\right) - 2$

437.
71.6°

439.
$P(t) = 950 - 450\sin\left(\frac{\pi}{6}t\right)$

441. Amplitude: 3, period: 2, frequency: $\frac{1}{2}$ Hz

443.
$C(t) = 20\sin(2\pi t) + 100(1.4427)^t$

Practice Test

445. 1

447.
$\frac{\sqrt{2}-\sqrt{6}}{4}$

449.
$-\sqrt{2}-\sqrt{3}$

451.
$0,\ \pi$

452.
$\sin^{-1}\left(\frac{1}{4}(\sqrt{13}-1)\right),\ \pi-\sin^{-1}\left(\frac{1}{4}(\sqrt{13}-1)\right)$

454.
$0,\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \pi$

456.
$\frac{\pi}{3}+k\pi$

458.
$-\frac{24}{25},\ -\frac{7}{25},\ \frac{24}{7}$

460.
$\frac{1}{8}(3+\cos(4x)-4\cos(2x))$

462.

$$\begin{aligned}\sin(3x)-\cos x\sin(2x)&=\\ \sin(x+2x)-\cos x(2\sin x\cos x)&=\\ \sin x\cos(2x)+\sin(2x)\cos x-2\sin x\cos^2 x&=\\ \sin x(\cos^2 x-\sin^2 x)+2\sin x\cos x\cos x-2\sin x\cos^2 x&=\\ \sin x\cos^2 x-\sin^3+0&=\\ \cos^2 x\sin x-\sin^3 x&=\cos^2 x\sin x-\sin^3 x\end{aligned}$$

464.
$y=2\cos(\pi x+\pi)$

466.
$81.5^\circ,\ 78.7^\circ$

468.
$6+5\cos\left(\frac{\pi}{6}(1-x)\right)$. From November 23 to February 6.

470.
$D(t)=2\cos\left(\frac{\pi}{6}t\right)+108+\frac{1}{4}t,$ 93.5855 months (or 7.8 years) from now

Chapter 8

Try It

8.1.

$$\begin{aligned}\alpha&=98^\circ & a&=34.6\\ \beta&=39^\circ & b&=22\\ \gamma&=43^\circ & c&=23.8\end{aligned}$$

8.2. Solution 1 (8.16)

$$\begin{aligned}\alpha&=80^\circ & a&=120\\ \beta&\approx 83.2^\circ & b&=121\\ \gamma&\approx 16.8^\circ & c&\approx 35.2\end{aligned}$$

Solution 2 (8.17)

$$\begin{array}{ll} \alpha' = 80° & a' = 120 \\ \beta' \approx 96.8° & b' = 121 \\ \gamma' \approx 3.2° & c' \approx 6.8 \end{array}$$

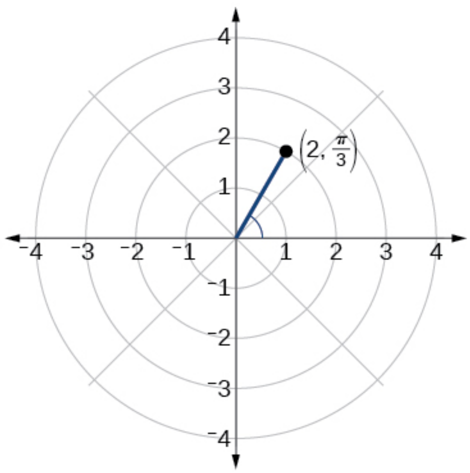
8.3.
$\beta \approx 5.7°,\ \gamma \approx 94.3°,\ c \approx 101.3$

8.4. two
8.5. about
8.2 square feet

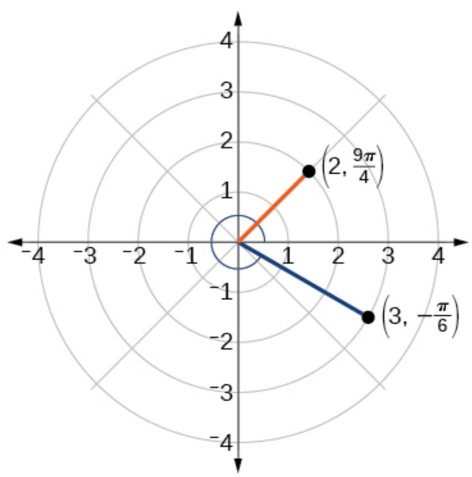
8.6. 161.9 yd.
8.7.
$a \approx 14.9, \quad \beta \approx 23.8°, \quad \gamma \approx 126.2°.$

8.8.
$\alpha \approx 27.7°, \quad \beta \approx 40.5°, \quad \gamma \approx 111.8°$

8.9. Area = 552 square feet
8.10. about 8.15 square feet



8.11.
8.12.



8.13.
$(x,\ y) = \left(\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$

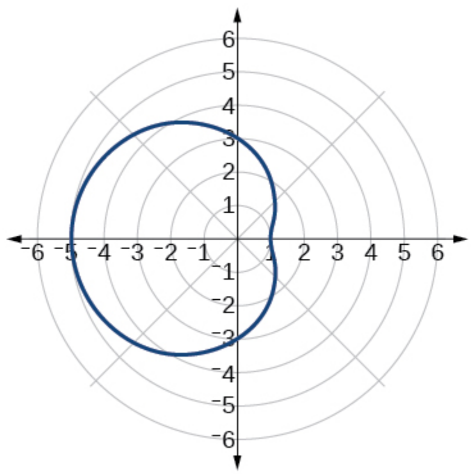
8.14.
$r = \sqrt{3}$

8.15.
$x^2 + y^2 = 2y$ or, in the standard form for a circle,
$x^2 + (y - 1)^2 = 1$

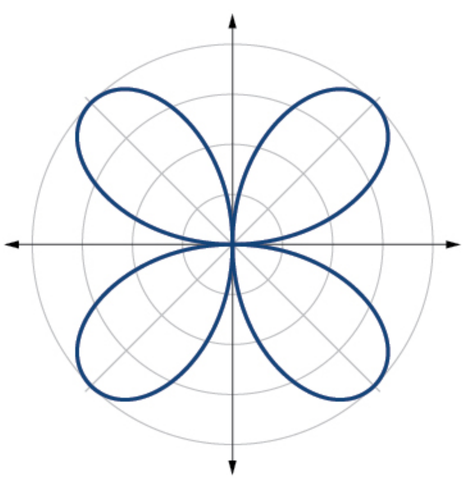
8.16. The equation fails the symmetry test with respect to the line $\theta = \frac{\pi}{2}$ and with respect to the pole. It passes the polar axis symmetry test.

8.17. Tests will reveal symmetry about the polar axis. The zero is $\left(0, \frac{\pi}{2}\right),$ and the maximum value is $(3, 0).$

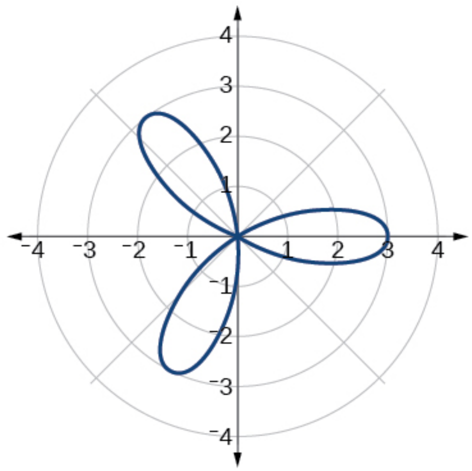
8.18.



8.19. The graph is a rose curve, n even

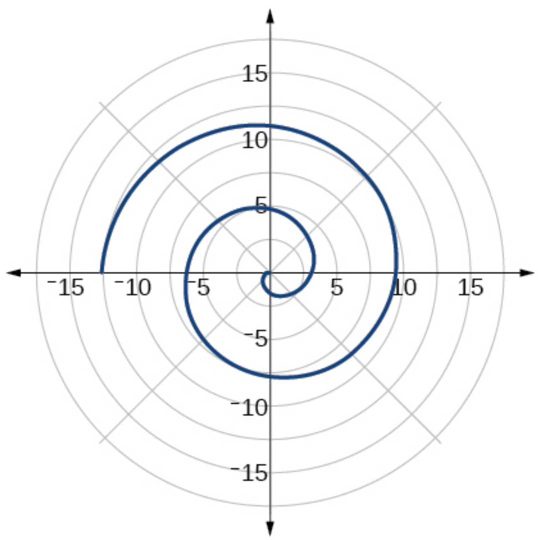


8.20.

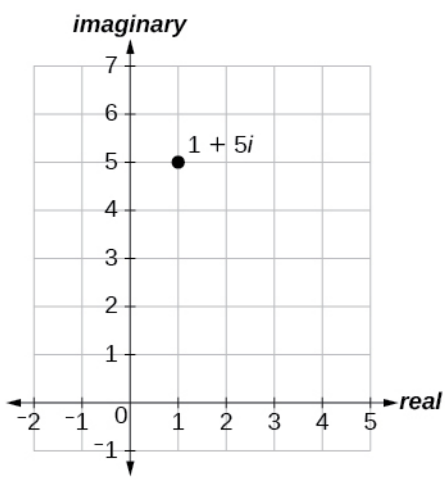


Rose curve,
n odd

8.21.



8.22.



8.23. 13

8.24.

$|z| = \sqrt{50} = 5\sqrt{2}$

8.25.

$z = 3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

8.26.

$z=2\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$

8.27.

$z=2\sqrt{3}-2i$

8.28.

$z_1z_2=-4\sqrt{3};\ \frac{z_1}{z_2}=-\frac{\sqrt{3}}{2}+\frac{3}{2}i$

8.29.

$z_0=2(\cos(30°)+i\sin(30°))$

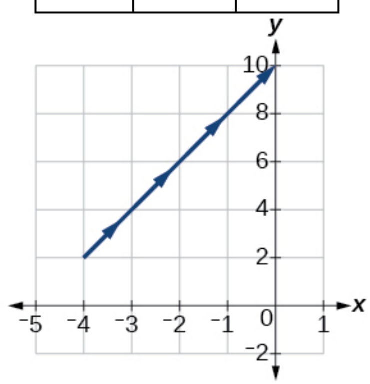
$z_1=2(\cos(120°)+i\sin(120°))$

$z_2=2(\cos(210°)+i\sin(210°))$

$z_3=2(\cos(300°)+i\sin(300°))$

8.30.

t	$x(t)$	$y(t)$
-1	-4	2
0	-3	4
1	-2	6
2	-1	8



8.31.

$x(t)=t^3-2t$

$y(t)=t$

8.32.

$y=5-\sqrt{\frac{1}{2}x-3}$

8.33.

$y=\ln\sqrt{x}$

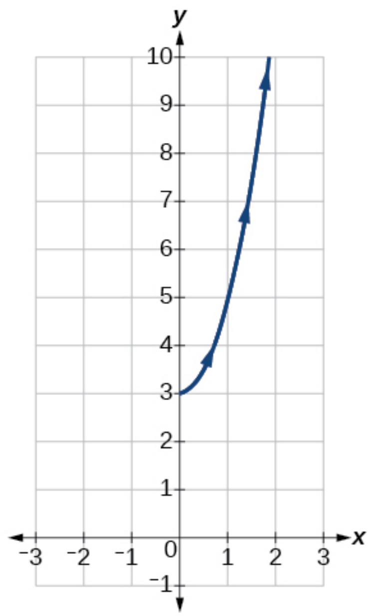
8.34.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

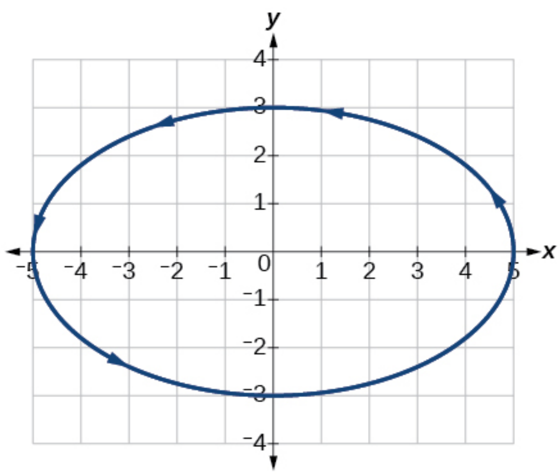
8.35.

$$y = x^2$$

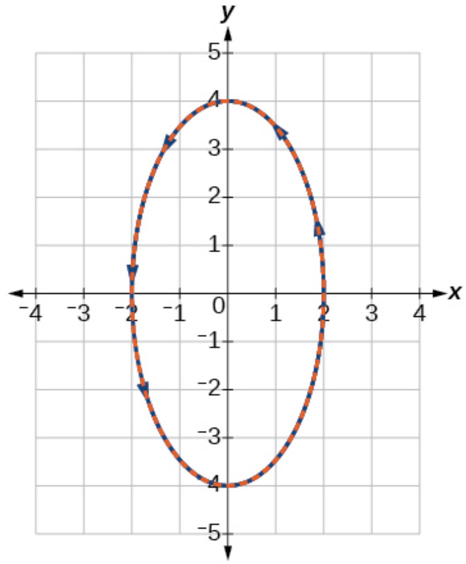
8.36.



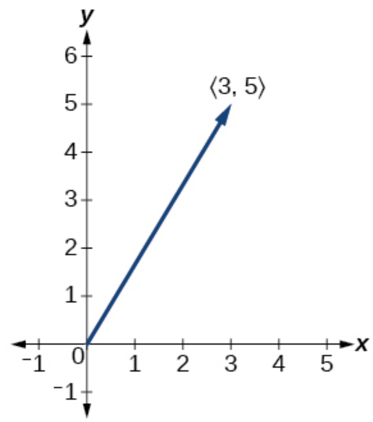
8.37.



8.38. The graph of the parametric equations is in red and the graph of the rectangular equation is drawn in blue dots on top of the parametric equations.



8.39.



8.40.
$3u = \langle 15, 12 \rangle$

8.41.
$u = 8i - 11j$

8.42.
$v = \sqrt{34}\cos(59°)i + \sqrt{34}\sin(59°)j$ Magnitude =
$\sqrt{34}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

Section Exercises

1. The altitude extends from any vertex to the opposite side or to the line containing the opposite side at a 90° angle.
3. When the known values are the side opposite the missing angle and another side and its opposite angle.
5. A triangle with two given sides and a non-included angle.
7.
$\beta = 72°,\ a \approx 12.0,\ b \approx 19.9$

9.
$\gamma = 20°,\ b \approx 4.5,\ c \approx 1.6$

11.
$b \approx 3.78$

13.
$c \approx 13.70$

15. one triangle,
$\alpha \approx 50.3°,\ \beta \approx 16.7°,\ a \approx 26.7$

17. two triangles,
$\gamma \approx 54.3°,\ \beta \approx 90.7°,\ b \approx 20.9$ or
$\gamma' \approx 125.7°,\ \beta' \approx 19.3°,\ b' \approx 6.9$

19. two triangles,
$\beta \approx 75.7°,\quad \gamma \approx 61.3°,\ b \approx 9.9$ or
$\beta' \approx 18.3°,\ \gamma' \approx 118.7°,\ b' \approx 3.2$

21. two triangles,
$\alpha \approx 143.2°,\ \beta \approx 26.8°,\ a \approx 17.3$ or
$\alpha' \approx 16.8°,\ \beta' \approx 153.2°,\ a' \approx 8.3$

23. no triangle possible

25.
$A \approx 47.8°$ or
$A' \approx 132.2°$

27.
8.6

29.
370.9

31.
12.3

33.
12.2

35.
16.0

37.
$29.7°$

39.
$x = 76.9°$ or $x = 103.1°$

41.
$110.6°$

43.
$A \approx 39.4,\ \ C \approx 47.6,\ \ BC \approx 20.7$

45.
57.1

47.
42.0

49.
430.2

51.
10.1

53.
$AD \approx\ 13.8$

55.
$AB \approx 2.8$

57.
$L \approx 49.7,\ \ N \approx 56.3,\ \ LN \approx 5.8$

59. 51.4 feet

61. The distance from the satellite to station
A is approximately 1716 miles. The satellite is approximately 1706 miles above the ground.

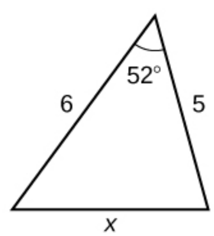
63. 2.6 ft

65. 5.6 km

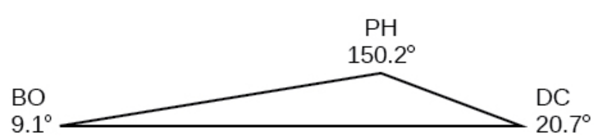
67. 371 ft

69. 5936 ft

71. 24.1 ft
73. 19,056 ft^2
75. 445,624 square miles
77. 8.65 ft^2
78. two sides and the angle opposite the missing side.
80.
s is the semi-perimeter, which is half the perimeter of the triangle.
82. The Law of Cosines must be used for any oblique (non-right) triangle.
84. 11.3
86. 34.7
88. 26.7
90. 257.4
92. not possible
94. 95.5°
96. 26.9°
98.
$B \approx 45.9°, C \approx 99.1°, a \approx 6.4$
100.
$A \approx 20.6°, B \approx 38.4°, c \approx 51.1$
102.
$A \approx 37.8°, B \approx 43.8, C \approx 98.4°$
104. 177.56 in^2
106. 0.04 m^2
108. 0.91 yd^2
110. 3.0
112. 29.1
114. 0.5
116. 70.7°
118. 77.4°
120. 25.0
122. 9.3
124. 43.52
126. 1.41
128. 0.14
130. 18.3
132. 48.98
134.



136. 7.62
138. 85.1
140. 24.0 km
142. 99.9 ft
144. 37.3 miles
146. 2371 miles
148.



150. 599.8 miles
152. 65.4 cm^2
154. 468 ft^2

156. For polar coordinates, the point in the plane depends on the angle from the positive *x*-axis and distance from the origin, while in Cartesian coordinates, the point represents the horizontal and vertical distances from the origin. For each point in the coordinate plane, there is one representation, but for each point in the polar plane, there are infinite representations.

158. Determine θ for the point, then move r units from the pole to plot the point. If r is negative, move r units from the pole in the opposite direction but along the same angle. The point is a distance of r away from the origin at an angle of θ from the polar axis.

160. The point $\left(-3, \frac{\pi}{2}\right)$ has a positive angle but a negative radius and is plotted by moving to an angle of $\frac{\pi}{2}$ and then moving 3 units in the negative direction. This places the point 3 units down the negative *y*-axis. The point $\left(3, -\frac{\pi}{2}\right)$ has a negative angle and a positive radius and is plotted by first moving to an angle of $-\frac{\pi}{2}$ and then moving 3 units down, which is the positive direction for a negative angle. The point is also 3 units down the negative *y*-axis.

162.
$(-5, 0)$

164.
$\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

166.
$\left(2\sqrt{5}, \quad 0.464\right)$

168.
$\left(\sqrt{34}, 5.253\right)$

170.
$\left(8\sqrt{2}, \frac{\pi}{4}\right)$

172.
$r = 4\csc\theta$

174.
$r = \sqrt[3]{\frac{sin\theta}{2cos^4\theta}}$

176.
$r = 3\cos\theta$

178.
$r = \frac{3\sin\theta}{\cos(2\theta)}$

180.
$r = \frac{9\sin\theta}{\cos^2\theta}$

182.
$r = \sqrt{\frac{1}{9\cos\theta\sin\theta}}$

184.
$x^2 + y^2 = 4x$ or
$\frac{(x-2)^2}{4} + \frac{y^2}{4} = 1;$ circle

186.
$3y + x = 6;$ line

188.
$y = 3;$ line

190.
$xy = 4;$ hyperbola

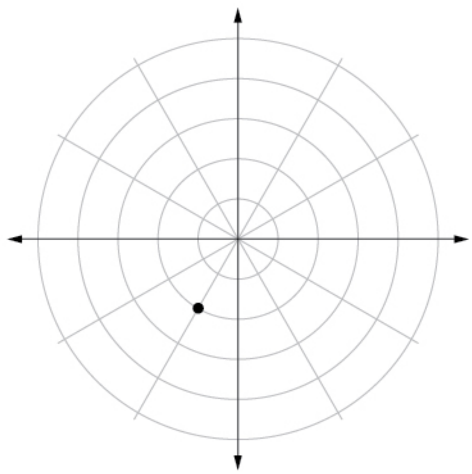
192.
$x^2 + y^2 = 4;$ circle

194.
$x - 5y = 3;$ line

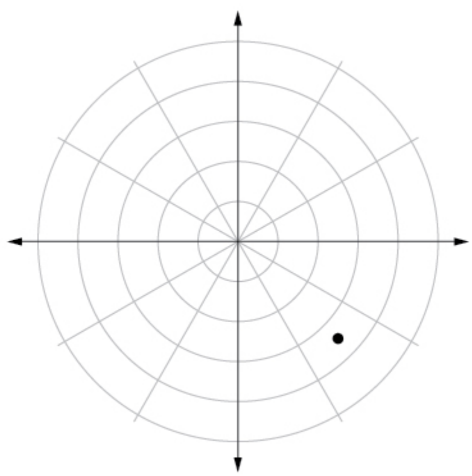
196.
$\left(3, \frac{3\pi}{4}\right)$

198.
$(5, \pi)$

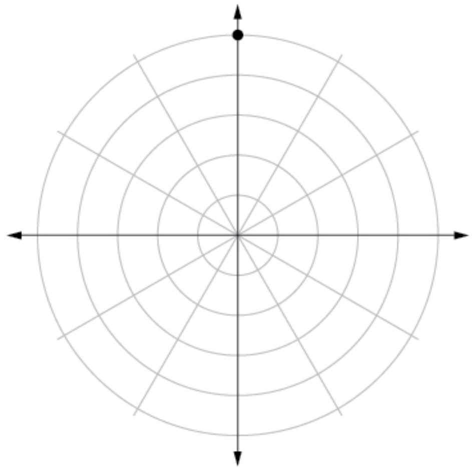
200.



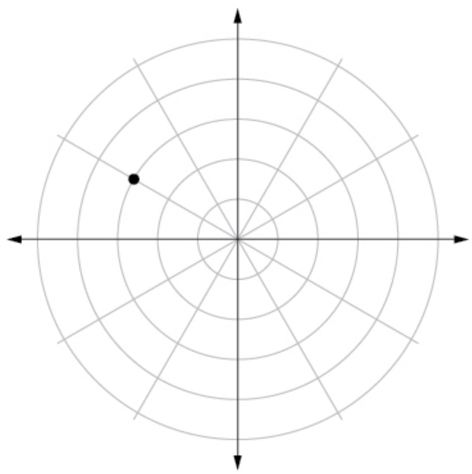
202.



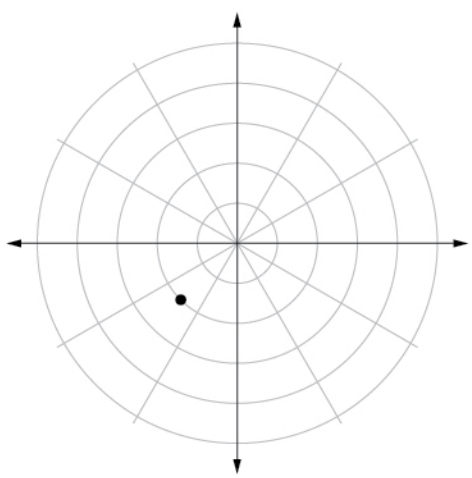
204.



206.

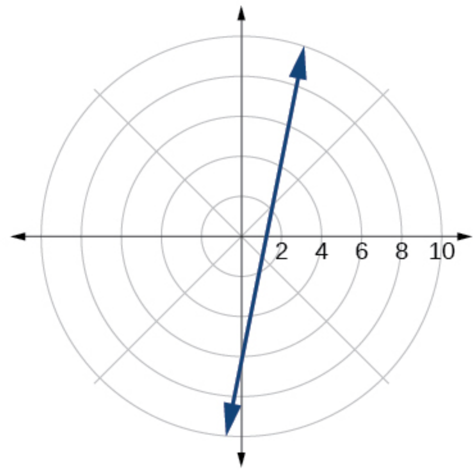


208.

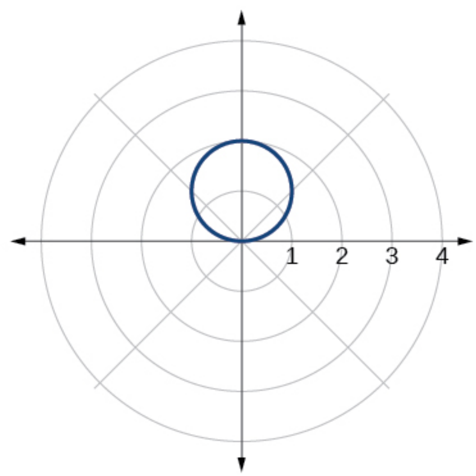


210.

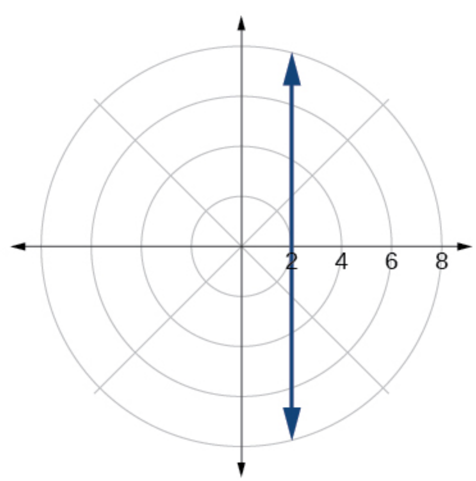
$$r = \frac{6}{5\cos\theta - \sin\theta}$$



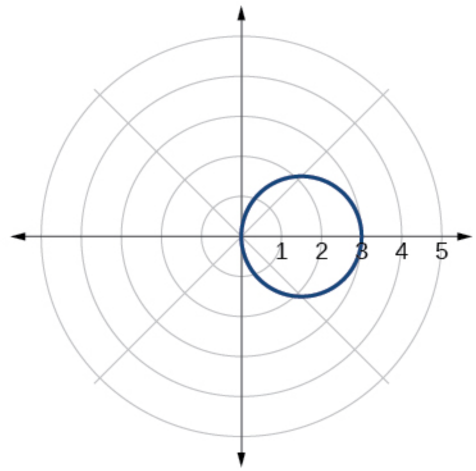
212.
$r = 2\sin\theta$



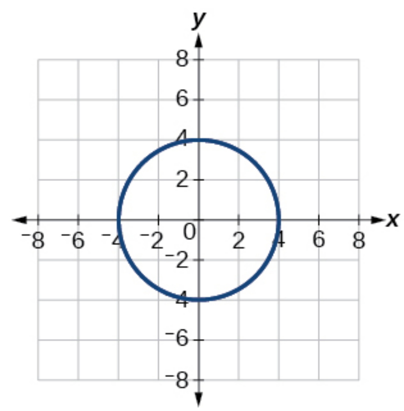
214.
$r = \frac{2}{\cos\theta}$



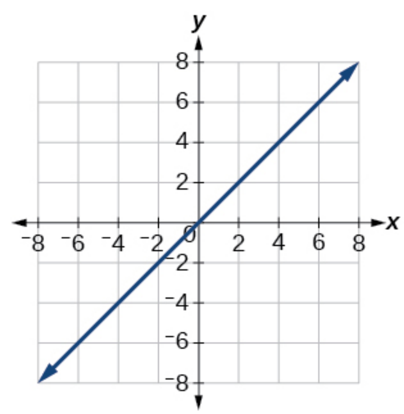
216.
$r = 3\cos\theta$



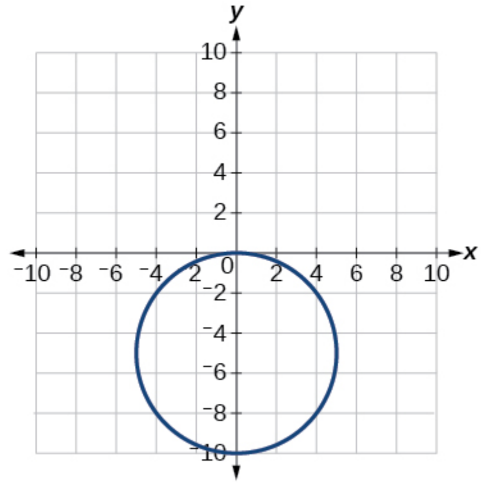
218.
$x^2 + y^2 = 16$



220.
$y = x$



222.
$x^2 + (y + 5)^2 = 25$



224.
$(1.618, \ -1.176)$

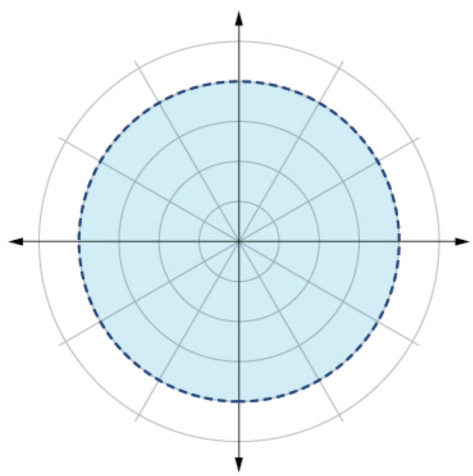
226.
$(10.630, \ 131.186°)$

228.
$(2, \ 3.14) or (2, \ \pi)$

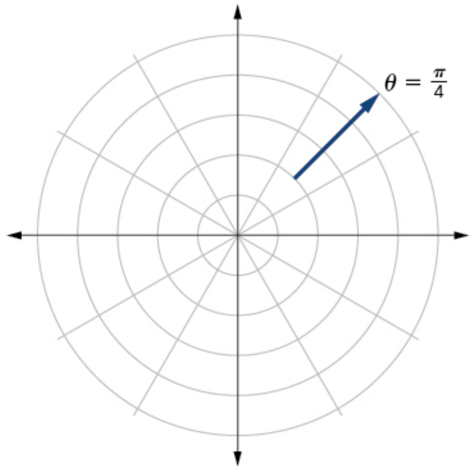
230. A vertical line with
a units left of the *y*-axis.

232. A horizontal line with
a units below the *x*-axis.

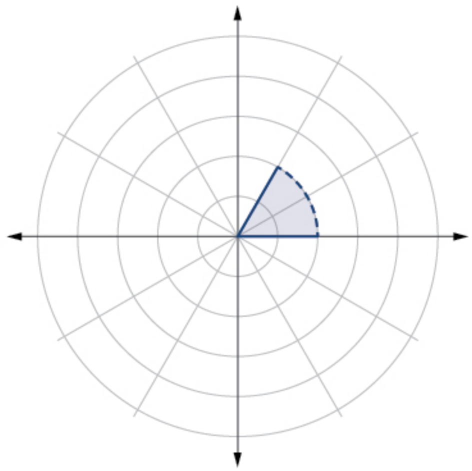
234.



236.



238.



240. Symmetry with respect to the polar axis is similar to symmetry about the x -axis, symmetry with respect to the pole is similar to symmetry about the origin, and symmetric with respect to the line $\theta = \frac{\pi}{2}$ is similar to symmetry about the y -axis.

242. Test for symmetry; find zeros, intercepts, and maxima; make a table of values. Decide the general type of graph, cardioid, limaçon, lemniscate, etc., then plot points at $\theta = 0,\ \frac{\pi}{2},\ \ \pi \text{ and } \frac{3\pi}{2},$ and sketch the graph.

244. The shape of the polar graph is determined by whether or not it includes a sine, a cosine, and constants in the equation.

246. symmetric with respect to the polar axis

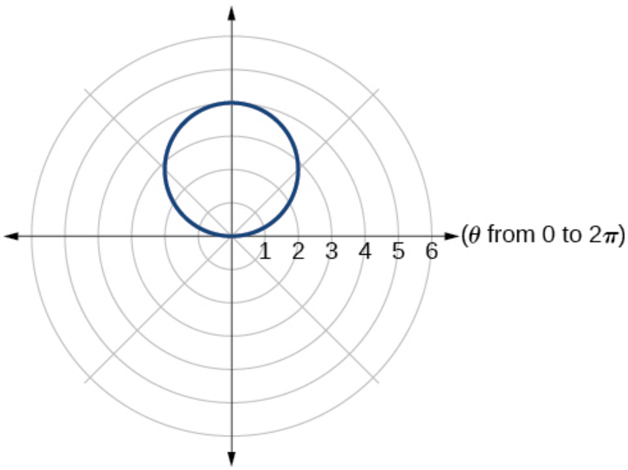
248. symmetric with respect to the polar axis, symmetric with respect to the line $\theta = \frac{\pi}{2},$ symmetric with respect to the pole

250. no symmetry

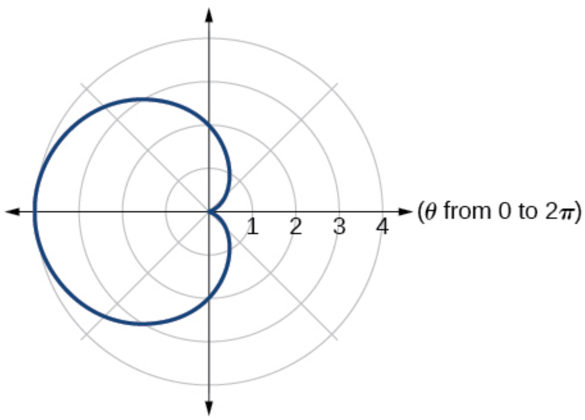
252. no symmetry

254. symmetric with respect to the pole

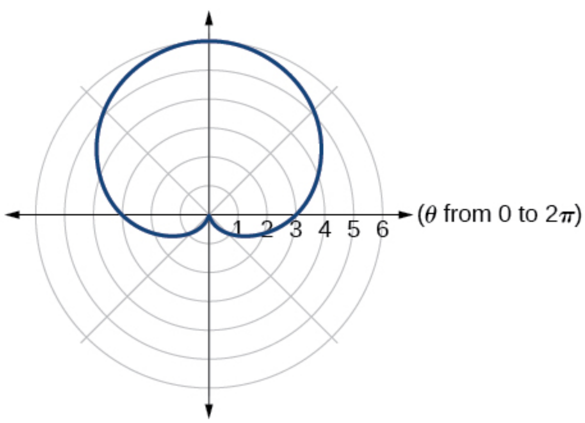
256. circle



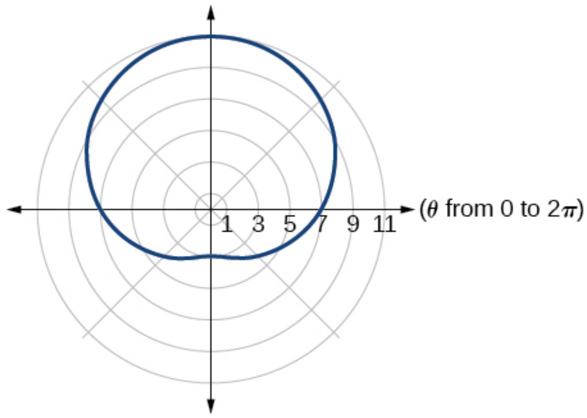
258. cardioid



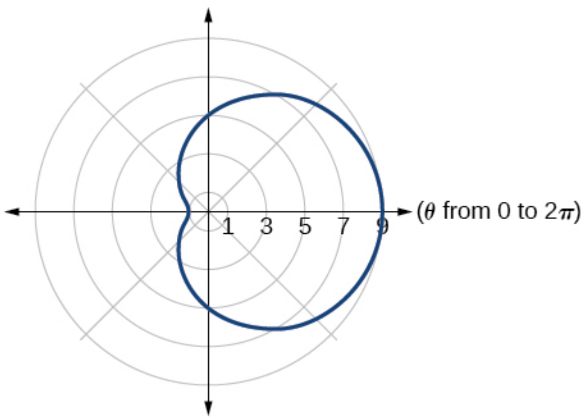
260. cardioid



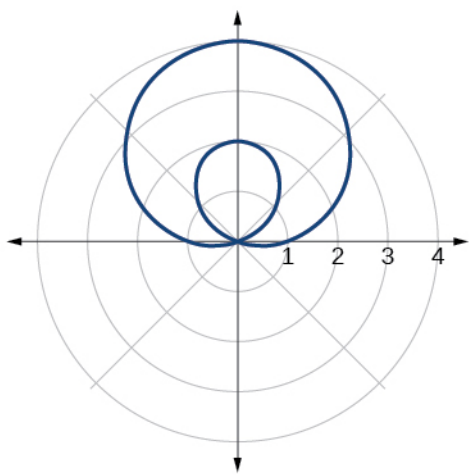
262. one-loop/dimpled limaçon



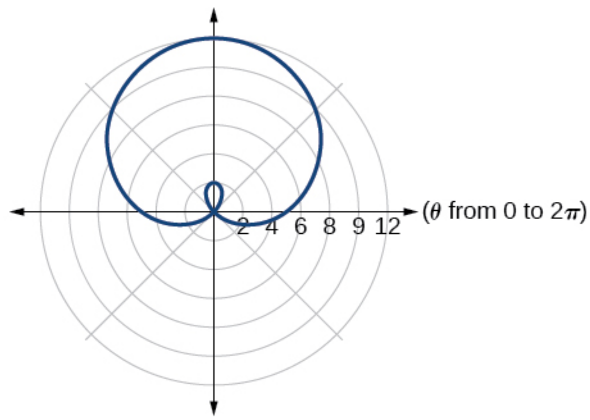
264. one-loop/dimpled limaçon



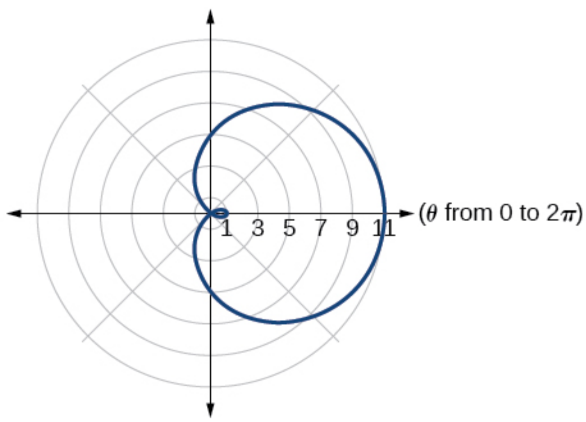
266. inner loop/two-loop limaçon



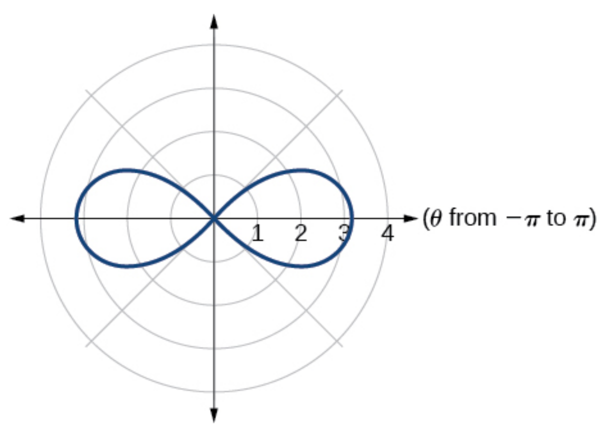
268. inner loop/two-loop limaçon



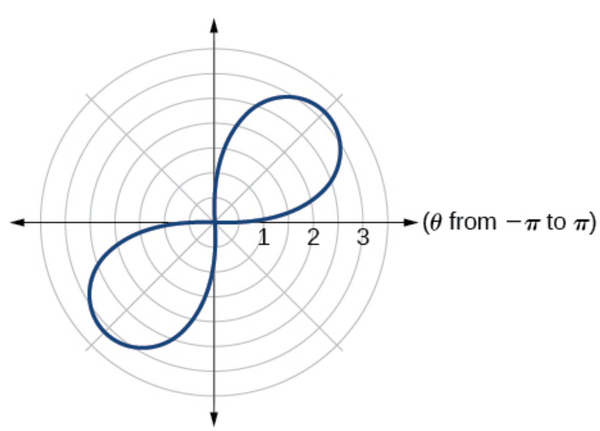
270. inner loop/two-loop limaçon



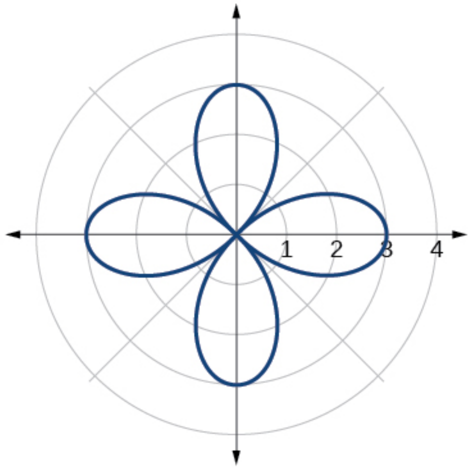
272. lemniscate



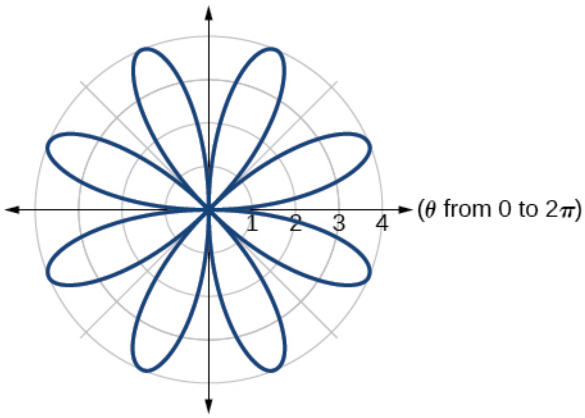
274. lemniscate



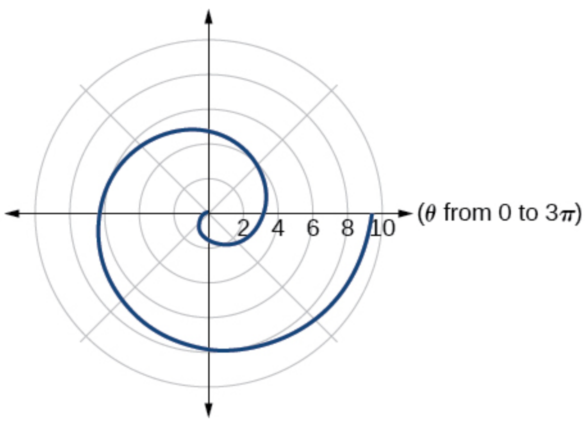
276. rose curve



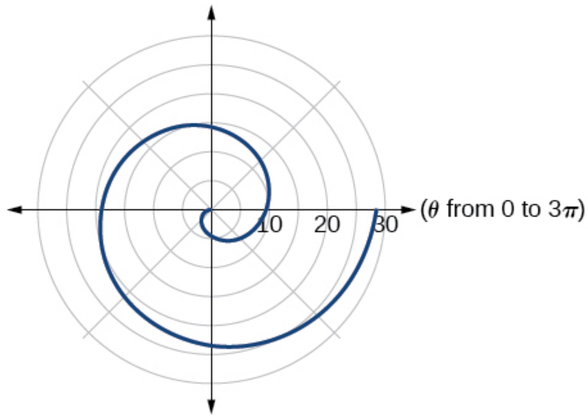
278. rose curve



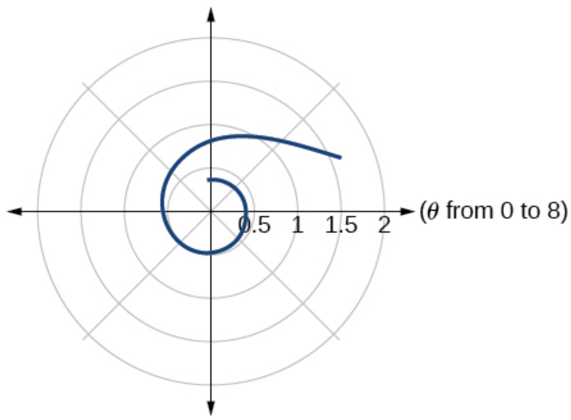
280. Archimedes’ spiral



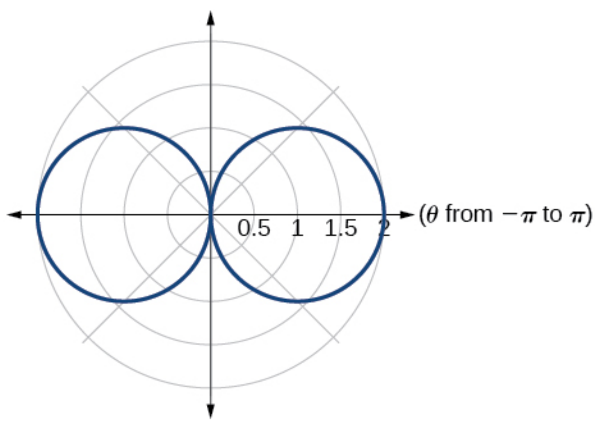
282. Archimedes’ spiral



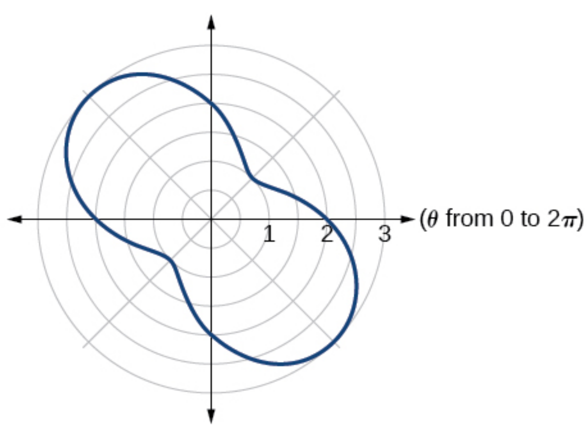
284.



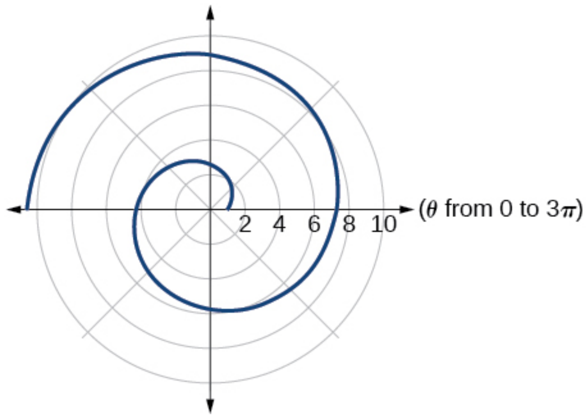
286.



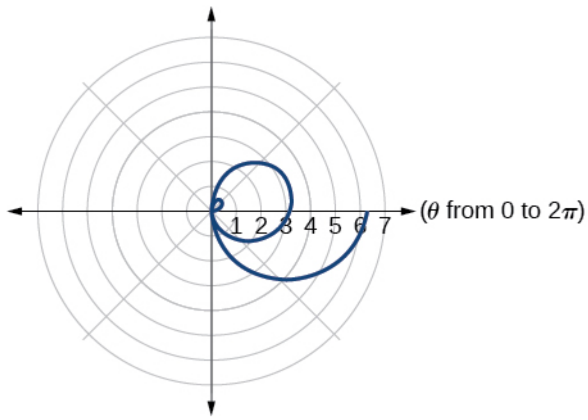
288.



290.



292.



294. They are both spirals, but not quite the same.
296. Both graphs are curves with 2 loops. The equation with a coefficient of θ has two loops on the left, the equation with a coefficient of 2 has two loops side by side. Graph these from 0 to 4π to get a better picture.
298. When the width of the domain is increased, more petals of the flower are visible.
300. The graphs are three-petal, rose curves. The larger the coefficient, the greater the curve's distance from the pole.
302. The graphs are spirals. The smaller the coefficient, the tighter the spiral.
304.
$\left(4, \frac{\pi}{3}\right), \left(4, \frac{5\pi}{3}\right)$

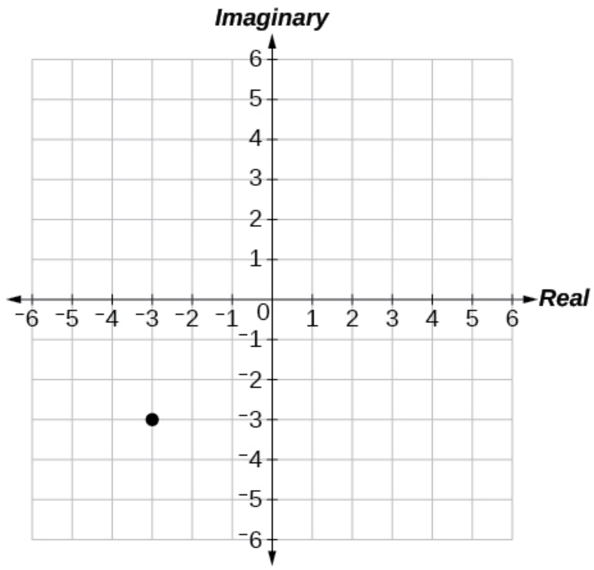
306.
$\left(\frac{3}{2}, \frac{\pi}{3}\right), \left(\frac{3}{2}, \frac{5\pi}{3}\right)$

308.
$\left(0, \frac{\pi}{2}\right),\ (0, \pi),\ \left(0, \frac{3\pi}{2}\right),\ (0, 2\pi)$

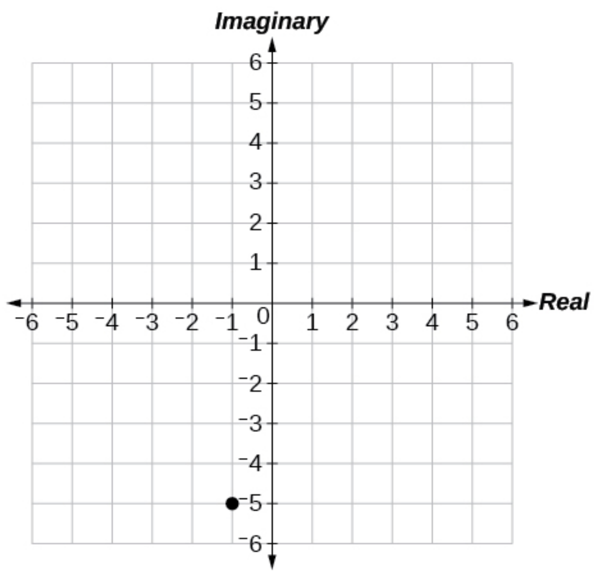
310.
$\left(\frac{\sqrt[4]{8}}{2}, \frac{\pi}{4}\right), \left(\frac{\sqrt[4]{8}}{2}, \frac{5\pi}{4}\right)$ and at $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ since r is squared
312. *a* is the real part, *b* is the imaginary part, and $i = \sqrt{-1}$
314. Polar form converts the real and imaginary part of the complex number in polar form using $x = r\cos\theta$ and $y = r\sin\theta.$

316.
$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$ It is used to simplify polar form when a number has been raised to a power.

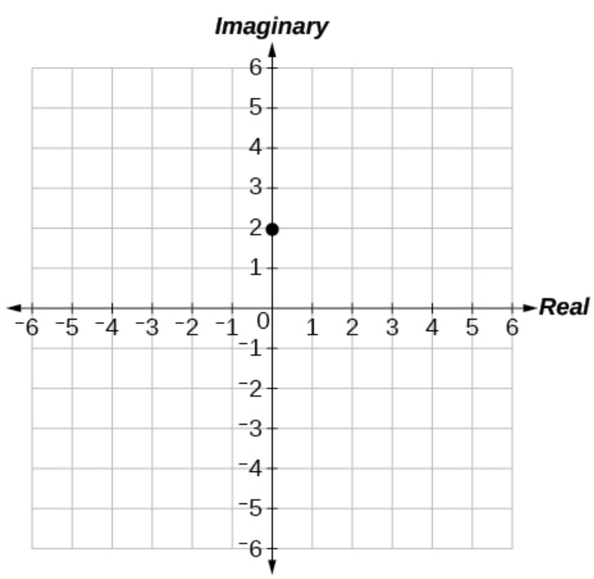
318.
$5\sqrt{2}$
320.
$\sqrt{38}$
322.
$\sqrt{14.45}$
324.
$4\sqrt{5}\text{cis}(333.4°)$
326.
$2\text{cis}\left(\frac{\pi}{6}\right)$
328.
$\frac{7\sqrt{3}}{2} + i\frac{7}{2}$
330.
$-2\sqrt{3} - 2i$
332.
$-1.5 - i\frac{3\sqrt{3}}{2}$
334.
$4\sqrt{3}\text{cis}(198°)$
336.
$\frac{3}{4}\text{cis}(180°)$
338.
$5\sqrt{3}\text{cis}\left(\frac{17\pi}{24}\right)$
340.
$7\text{cis}(70°)$
342.
$5\text{cis}(80°)$
344.
$5\text{cis}\left(\frac{\pi}{3}\right)$
346.
$125\text{cis}(135°)$
348.
$9\text{cis}(240°)$
350.
$\text{cis}\left(\frac{3\pi}{4}\right)$
352.
$3\text{cis}(80°),\ 3\text{cis}(200°),\ 3\text{cis}(320°)$
354.
$2\sqrt[3]{4}\text{cis}\left(\frac{2\pi}{9}\right),\ 2\sqrt[3]{4}\text{cis}\left(\frac{8\pi}{9}\right),\ 2\sqrt[3]{4}\text{cis}\left(\frac{14\pi}{9}\right)$
356.
$2\sqrt{2}\text{cis}\left(\frac{7\pi}{8}\right),\ 2\sqrt{2}\text{cis}\left(\frac{15\pi}{8}\right)$
358.



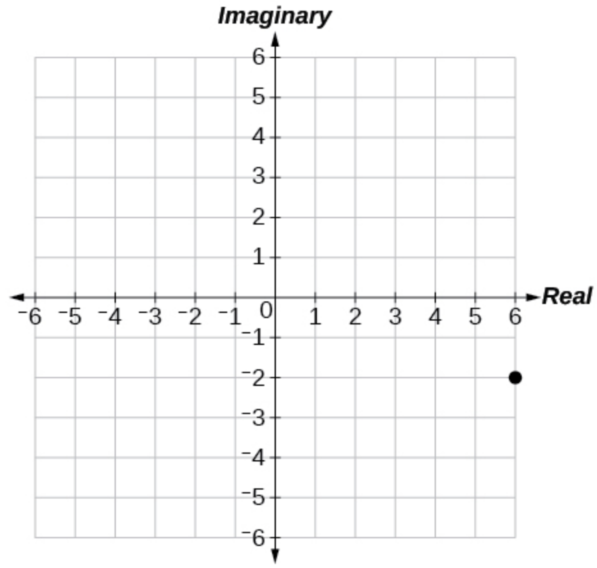
360.



362.



364.



366.

Plot of 1-4i in the complex plane (1 along the real axis, -4 along the imaginary axis).

368.

$3.61e^{-0.59i}$

370.

$-2+3.46i$

372.

$-4.33-2.50i$

373. A pair of functions that is dependent on an external factor. The two functions are written in terms of the same parameter. For example, $x=f(t)$ and $y=f(t)$.

375. Choose one equation to solve for $t,$ substitute into the other equation and simplify.

377. Some equations cannot be written as functions, like a circle. However, when written as two parametric equations, separately the equations are functions.

379.

$y=-2+2x$

381.

$y=3\sqrt{\frac{x-1}{2}}$

383.

$x=2e^{\frac{1-y}{5}}$ or $y=1-5ln\left(\frac{x}{2}\right)$

385.

$x=4\log\left(\frac{y-3}{2}\right)$

387.

$x=\left(\frac{y}{2}\right)^3-\frac{y}{2}$

389.

$y=x^3$

391.

$\left(\frac{x}{4}\right)^2+\left(\frac{y}{5}\right)^2=1$

393.
$y^2 = 1 - \frac{1}{2}x$

395.
$y = x^2 + 2x + 1$

397.
$y = \left(\frac{x+1}{2}\right)^3 - 2$

399.
$y = -3x + 14$

401.
$y = x + 3$

403.
$\begin{cases} x(t) = t \\ y(t) = 2\sin t + 1 \end{cases}$

405.
$\begin{cases} x(t) = \sqrt{t} + 2t \\ y(t) = t \end{cases}$

407.
$\begin{cases} x(t) = 4\cos t \\ y(t) = 6\sin t \end{cases}$; Ellipse

409.
$\begin{cases} x(t) = \sqrt{10}\cos t \\ y(t) = \sqrt{10}\sin t \end{cases}$; Circle

411.
$\begin{cases} x(t) = -1 + 4t \\ y(t) = -2t \end{cases}$

413.
$\begin{cases} x(t) = 4 + 2t \\ y(t) = 1 - 3t \end{cases}$

415. yes, at
$t = 2$

417.

t	*x*	*y*
1	-3	1
2	0	7
3	5	17

419. answers may vary:
$\begin{cases} x(t) = t - 1 \\ y(t) = t^2 \end{cases}$ and $\begin{cases} x(t) = t + 1 \\ y(t) = (t+2)^2 \end{cases}$

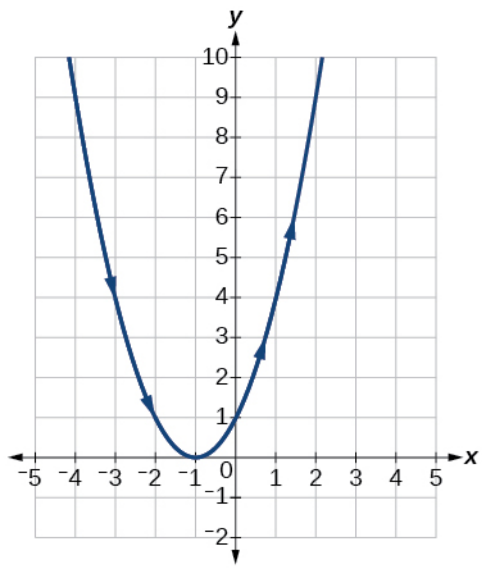
421. answers may vary: ,
$\begin{cases} x(t) = t \\ y(t) = t^2 - 4t + 4 \end{cases}$ and $\begin{cases} x(t) = t + 2 \\ y(t) = t^2 \end{cases}$

422. plotting points with the orientation arrow and a graphing calculator

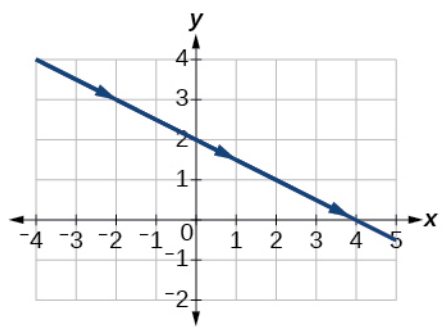
424. The arrows show the orientation, the direction of motion according to increasing values of t.

426. The parametric equations show the different vertical and horizontal motions over time.

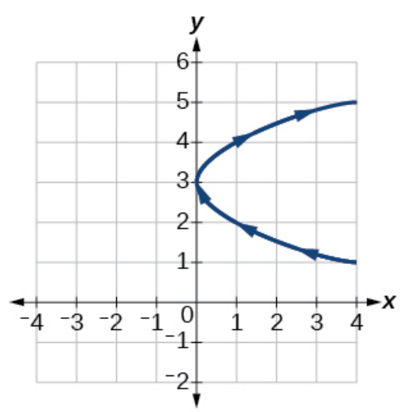
428.



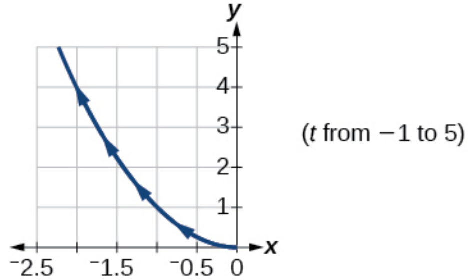
430.



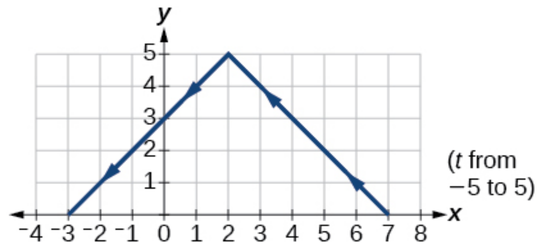
432.



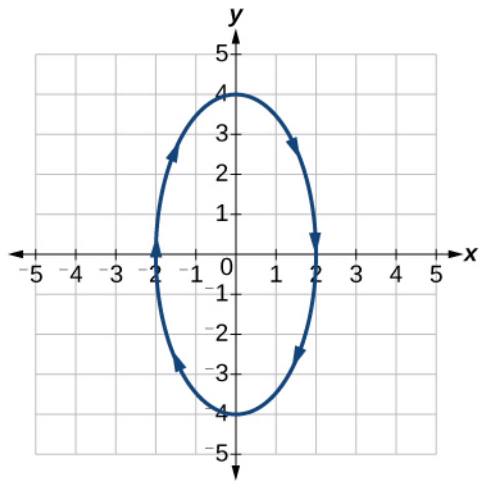
434.



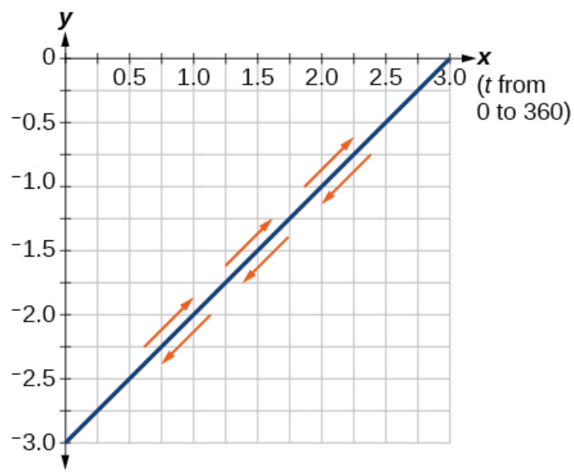
436.



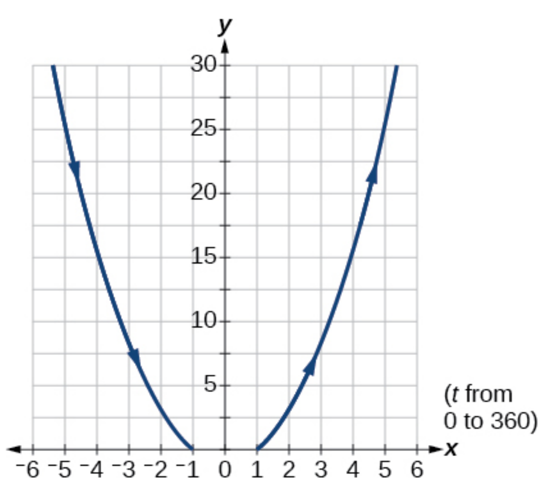
438.



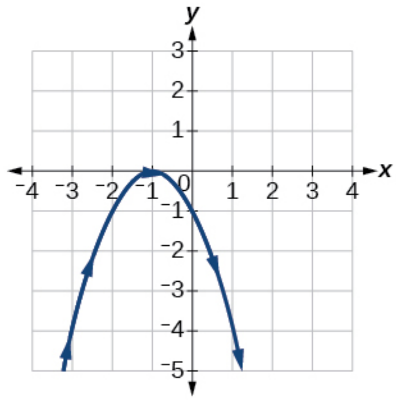
440.

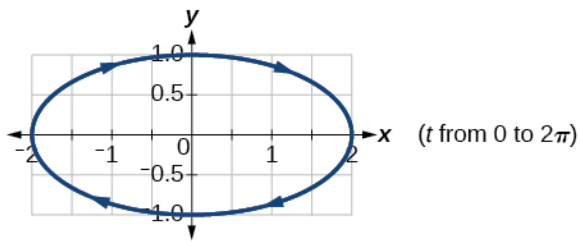


442.

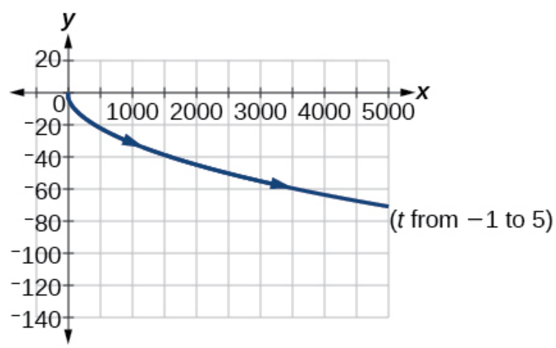


444.

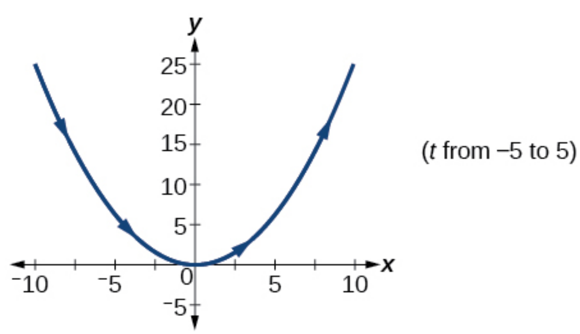




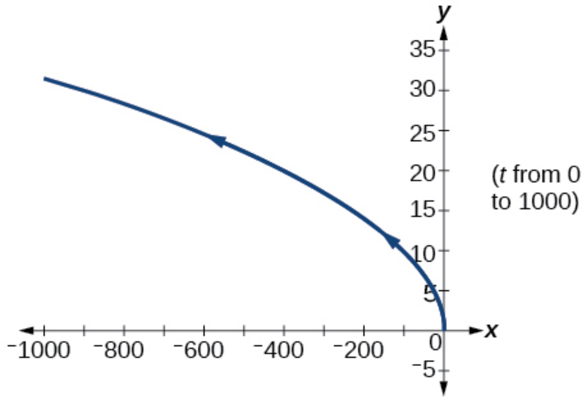
446.
448.



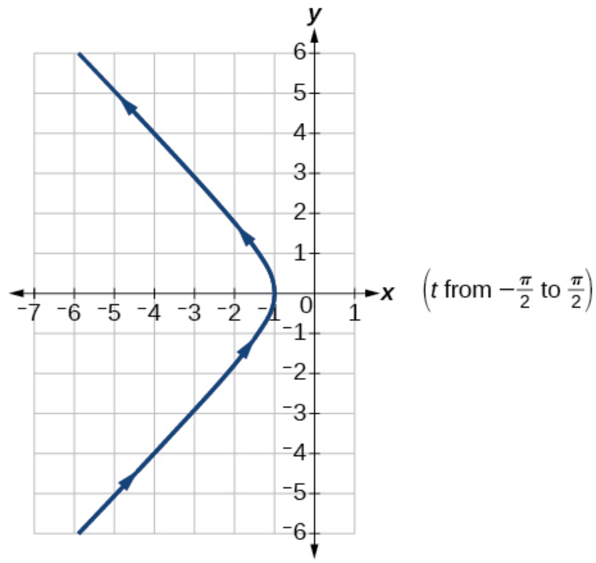
450.



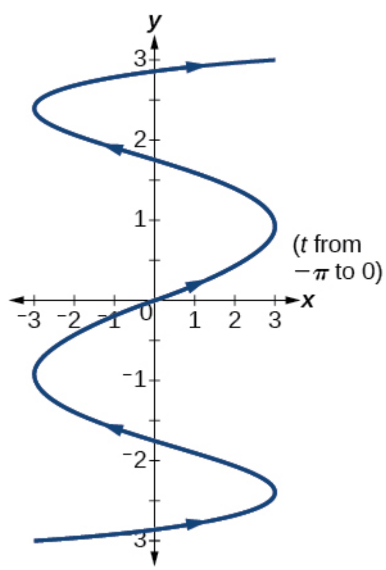
452.


y
35
30
25
20
15
10
5
0
−5
x
−1000
−800
−600
−400
−200
(t from 0 to 1000)

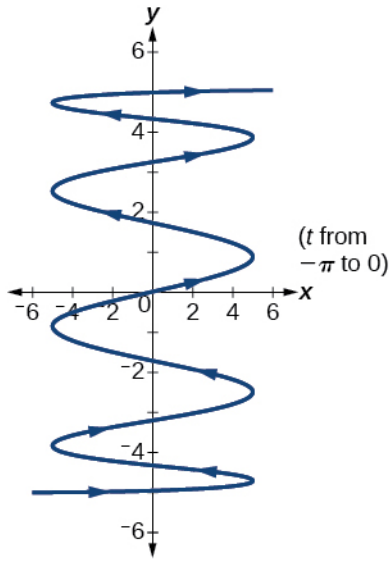
454.


y
x
(t from −π/2 to π/2)

456.


y
x
(t from −π to 0)

458.



460. There will be 100 back-and-forth motions.

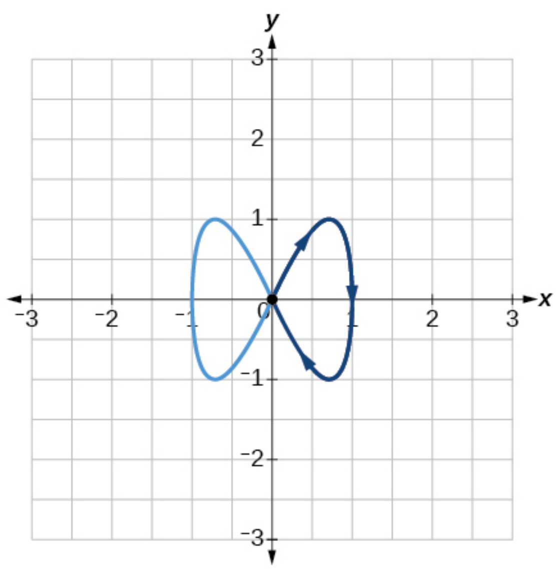
462. Take the opposite of the $x(t)$ equation.

464. The parabola opens up.

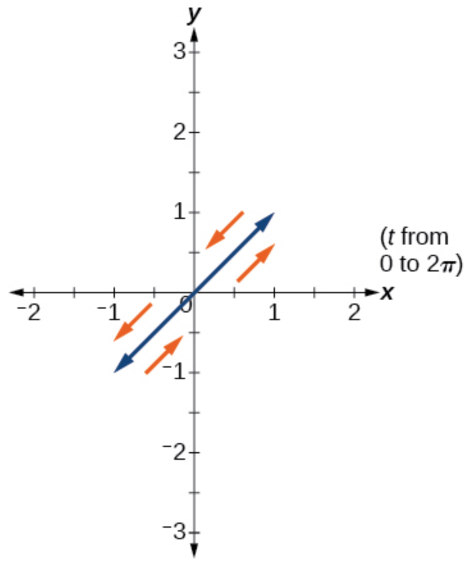
466.

$$\begin{cases} x(t) = 5\cos t \\ y(t) = 5\sin t \end{cases}$$

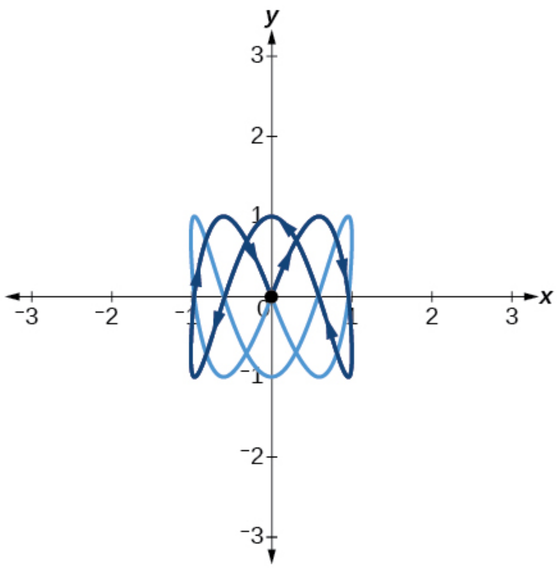
468.



470.

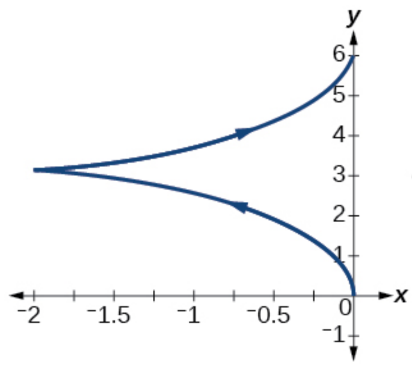


472.



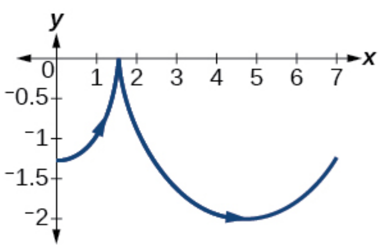
474.
$a = 4, \ b = 3, \ c = 6, \ d = 1$

476.
$a = 4, \ b = 2, \ c = 3, \ d = 3$

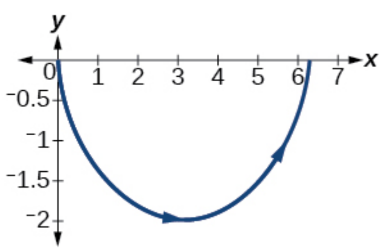


(*t* from 0 to 2π)

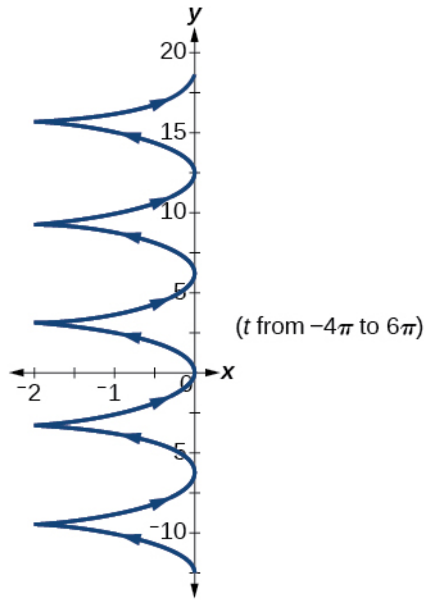
478.

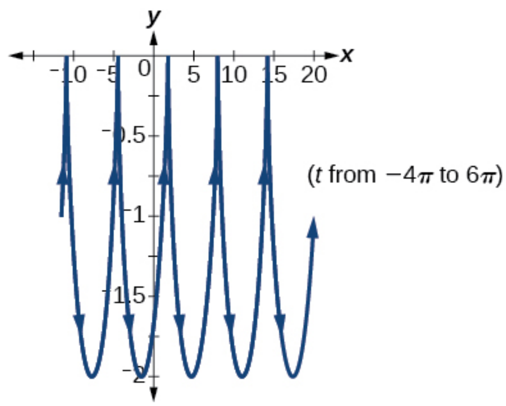


(*t* from 0 to 2π)

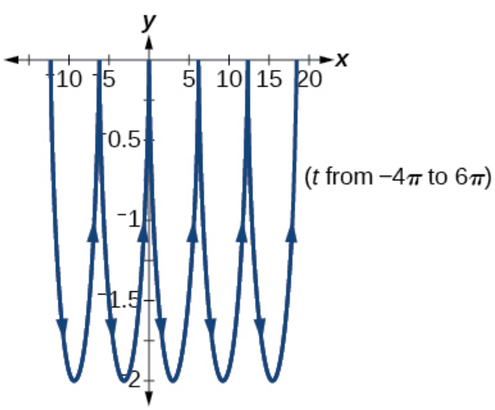


(*t* from 0 to 2π)





480.



482. The y-intercept changes.

484.

$$y(x) = -16\left(\frac{x}{15}\right)^2 + 20\left(\frac{x}{15}\right)$$

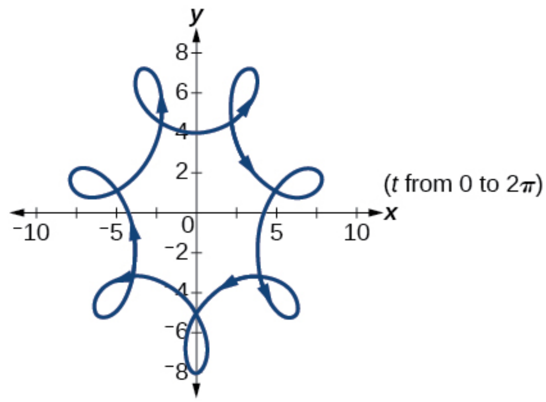
486.

$$\begin{cases} x(t) = 64t\cos(52°) \\ y(t) = -16t^2 + 64t\sin(52°) \end{cases}$$

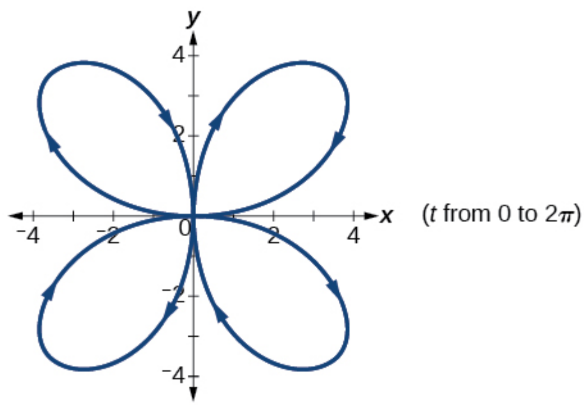
488. approximately 3.2 seconds

490. 1.6 seconds

492.



494.



495. lowercase, bold letter, usually u, v, w

497. They are unit vectors. They are used to represent the horizontal and vertical components of a vector. They each have a magnitude of 1.

499. The first number always represents the coefficient of the $i,$ and the second represents the $j.$

501.
$\langle 7, -5 \rangle$

503. not equal

505. equal

507. equal

509.
$7i - 3j$

511.
$-6i - 2j$

513.
$u + v = \langle -5, 5 \rangle, u - v = \langle -1, 3 \rangle, 2u - 3v = \langle 0, 5 \rangle$

515.
$-10i - 4j$

517.
$-\frac{2\sqrt{29}}{29}i + \frac{5\sqrt{29}}{29}j$

519.
$-\frac{2\sqrt{229}}{229}i + \frac{15\sqrt{229}}{229}j$

521.
$-\frac{7\sqrt{2}}{10}i + \frac{\sqrt{2}}{10}j$

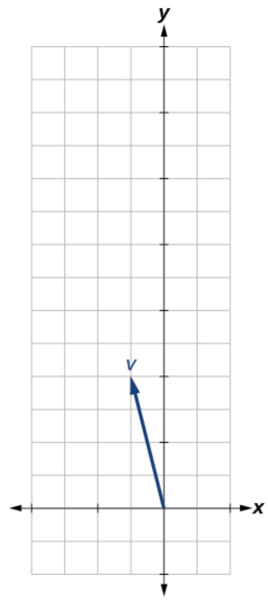
523.
$|v| = 7.810,\ \theta = 39.806\ °$

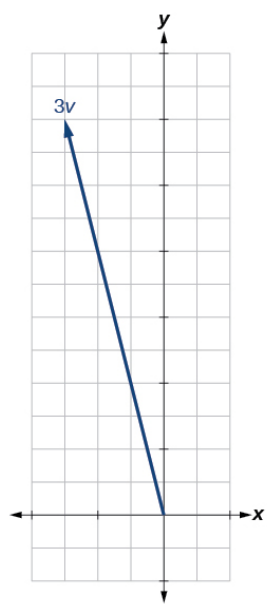
525.
$|v| = 7.211,\ \theta = 236.310°$

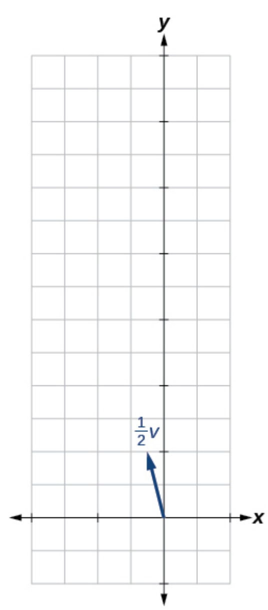
527.
-6

529.
-12

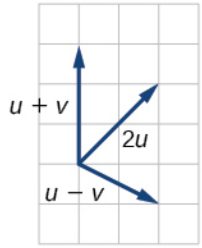
531.


y
x
v

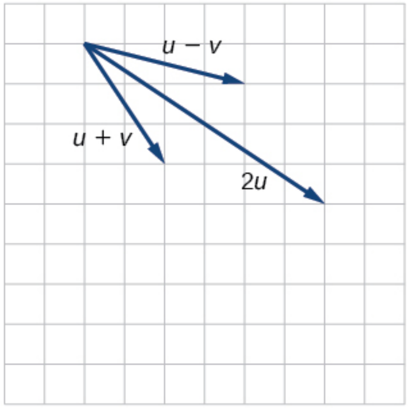

y
x
3v


y
x
$\frac{1}{2}v$

533.

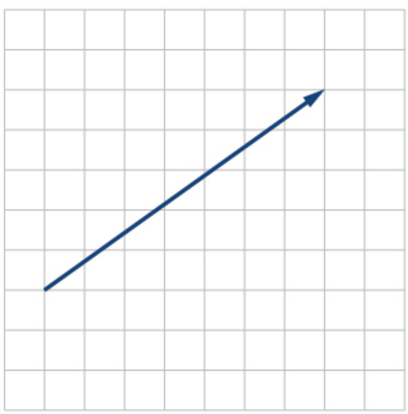


535.



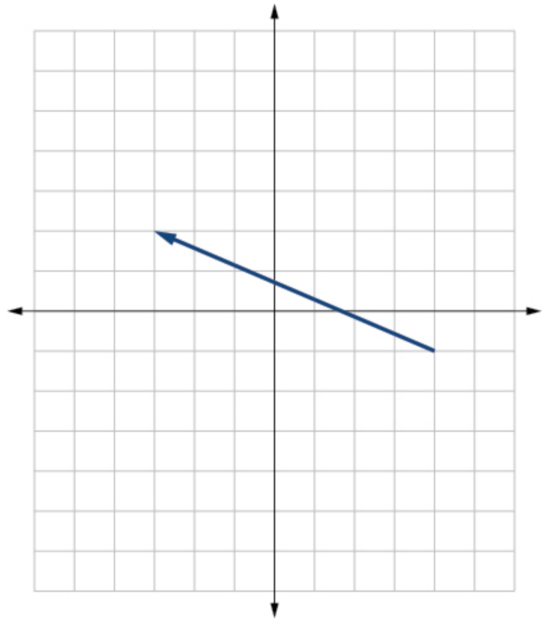
537.

539.



541.
$\langle 4,\ 1\ \rangle$

543.
$v = -7i + 3j$



545.
$3\sqrt{2}i + 3\sqrt{2}j$

547.
$i - \sqrt{3}j$

549. a. 58.7; b. 12.5
551.
$x = 7.13$ pounds,
$y = 3.63$ pounds
553.
$x = 2.87$ pounds,
$y = 4.10$ pounds
555. 4.635 miles, 17.764° N of E
557. 17 miles. 10.318 miles
559. Distance: 2.868. Direction: 86.474° North of West, or 3.526° West of North
561. 4.924°. 659 km/hr
563. 4.424°
565.
$(0.081,\ 8.602)$

567. 21.801°, relative to the car’s forward direction
569. parallel: 16.28, perpendicular: 47.28 pounds
571. 19.35 pounds, 231.54° from the horizontal
573. 5.1583 pounds, 75.8° from the horizontal

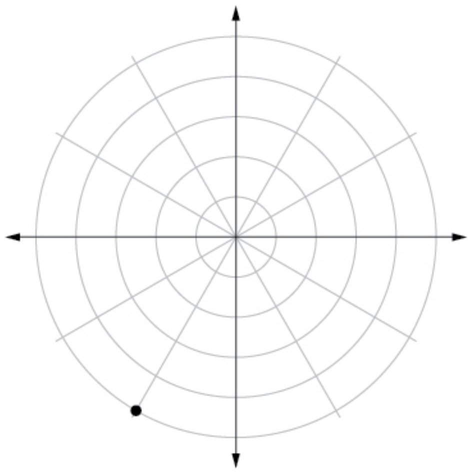
Review Exercises

574. Not possible
576.
$C = 120°,\ a = 23.1,\ c = 34.1$

578. distance of the plane from point
$A:$ 2.2 km, elevation of the plane: 1.6 km
580.
$B = 71.0°,\ C = 55.0°,\ a = 12.8$

582. 40.6 km

584.



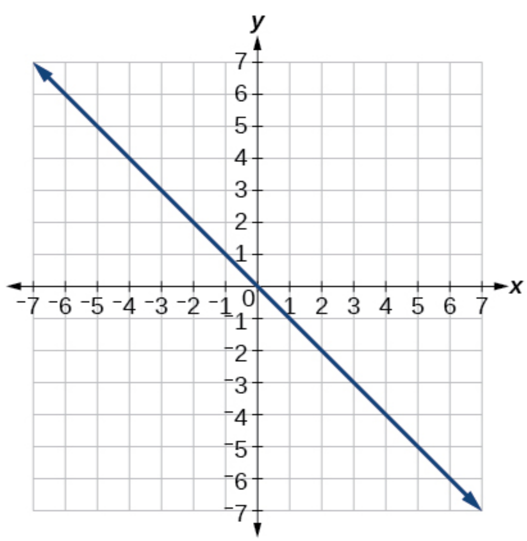
586.
$(0,\ 2)$

588.
$(9.8489,\ 203.96°)$

590.
$r = 8$

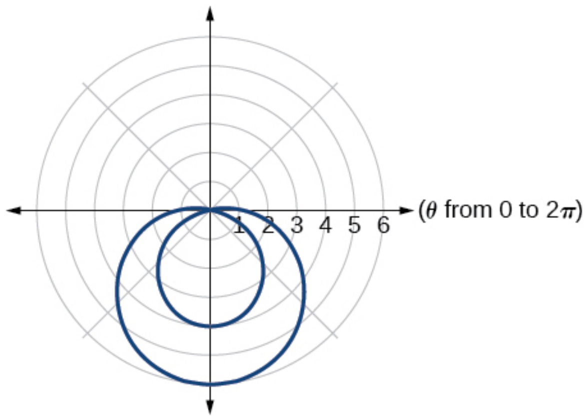
592.
$x^2 + y^2 = 7x$

594.
$y = \ -x$

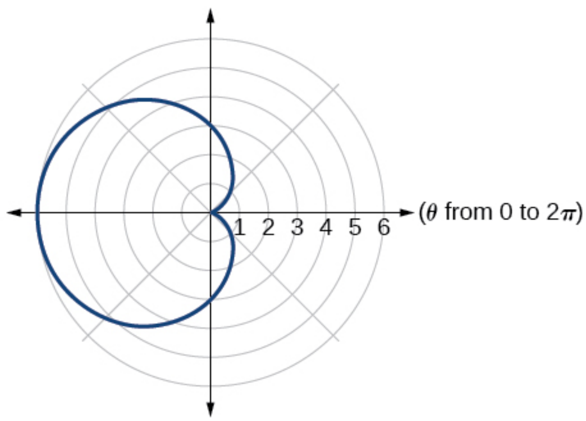


596. symmetric with respect to the line
$\theta = \frac{\pi}{2}$

598.



600.



602. 5

604.

$\text{cis}\left(-\frac{\pi}{3}\right)$

606.

$2.3 + 1.9i$

608.

$60\text{cis}\left(\frac{\pi}{2}\right)$

610.

$3\text{cis}\left(\frac{4\pi}{3}\right)$

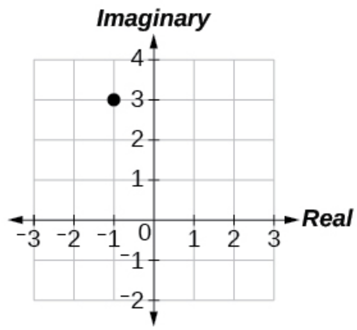
612.

$25\text{cis}\left(\frac{3\pi}{2}\right)$

614.

$5\text{cis}\left(\frac{3\pi}{4}\right),\ 5\text{cis}\left(\frac{7\pi}{4}\right)$

616.



618.

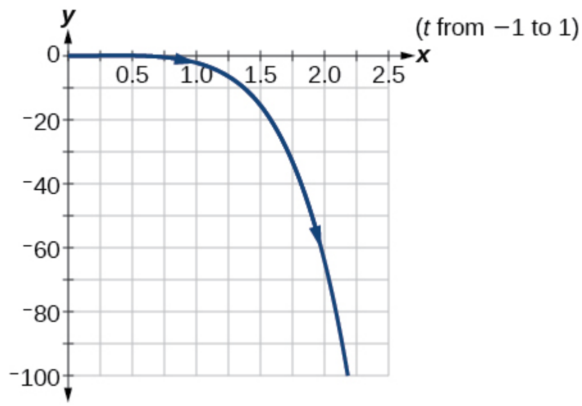
$x^2 + \frac{1}{2}y = 1$

620.

$\begin{cases} x(t) = -2 + 6t \\ y(t) = 3 + 4t \end{cases}$

622.

$y = -2x^5$



624.

a. $\begin{cases} x(t) = (80\cos(40°))t \\ y(t) = -16t^2 + (80\sin(40°))t + 4 \end{cases}$

b. The ball is 14 feet high and 184 feet from where it was launched.

c. 3.3 seconds

626. not equal

628. $4\boldsymbol{i}$

630.

$-\frac{3\sqrt{10}}{10}\boldsymbol{i}$

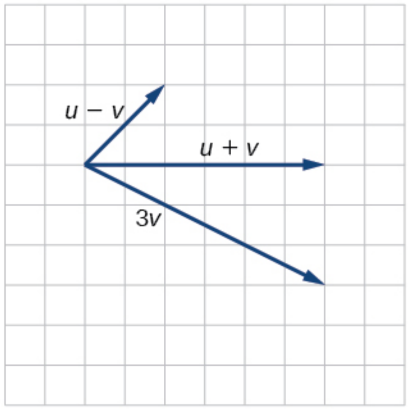
$-\frac{\sqrt{10}}{10}\boldsymbol{j}$

632. Magnitude: $3\sqrt{2}$, Direction: $225°$

634.

16

636.



Practice Test

638.

$\alpha = 67.1°, \gamma = 44.9°, a = 20.9$

640.

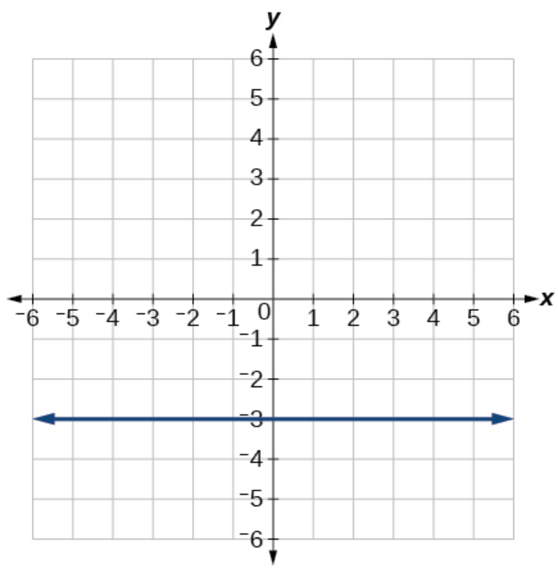
1712 miles

642.

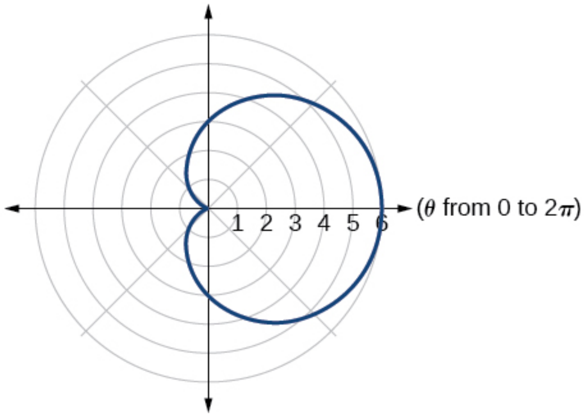
$\left(1, \sqrt{3}\right)$

644.

$y = -3$



646.



648.
$\sqrt{106}$

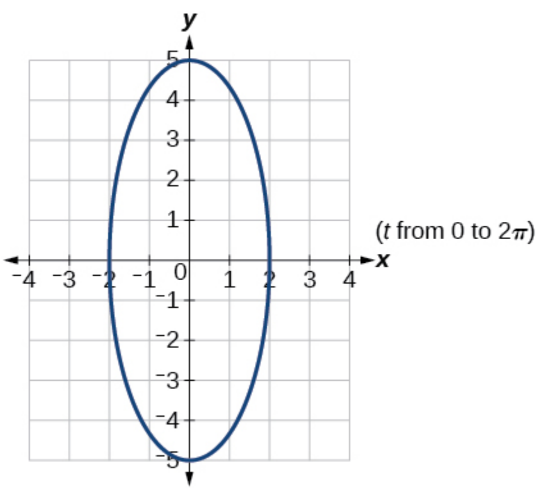
650.
$\frac{-5}{2} + i\frac{5\sqrt{3}}{2}$

652.
$4\text{cis}(21°)$

654.
$2\sqrt{2}\text{cis}(18°),\ 2\sqrt{2}\text{cis}(198°)$

656.
$y = 2(x-1)^2$

658.

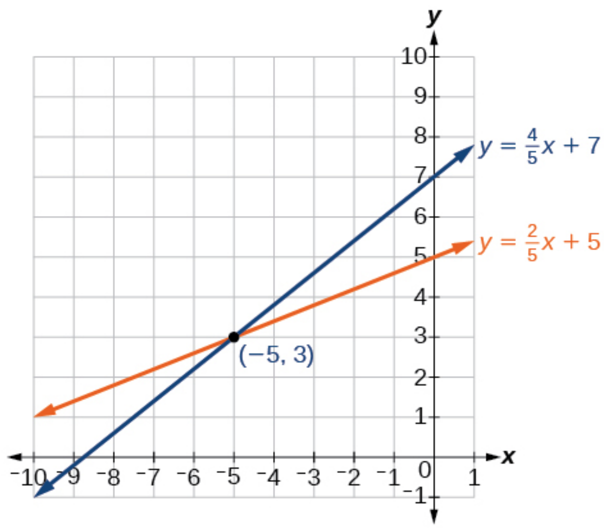


660. $-4\boldsymbol{i} - 15\boldsymbol{j}$
662.
$\frac{2\sqrt{13}}{13}i + \frac{3\sqrt{13}}{13}j$

Chapter 9

Try It

9.1. Not a solution.
9.2. The solution to the system is the ordered pair $(-5,\ 3)$.



9.3.
$(-2, -5)$

9.4.
$(-6, -2)$

9.5.
$(10, -4)$

9.6. No solution. It is an inconsistent system.

9.7. The system is dependent so there are infinite solutions of the form $(x, 2x + 5)$.

9.8. 700 children, 950 adults

9.9.
$(1, -1, 1)$

9.10. No solution.

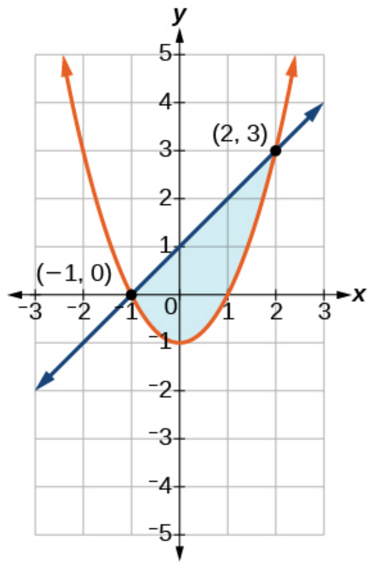
9.11. Infinite number of solutions of the form $(x, 4x - 11, -5x + 18)$.

9.12.
$\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(2, 8)$

9.13.
$(-1, 3)$

9.14.
$\{(1, 3), (1, -3), (-1, 3), (-1, -3)\}$

9.15. Shade the area bounded by the two curves, above the quadratic and below the line.



9.16.
$$\frac{3}{x-3} - \frac{2}{x-2}$$

9.17.
$$\frac{6}{x-1} - \frac{5}{(x-1)^2}$$

9.18.
$$\frac{3}{x-1} + \frac{2x-4}{x^2+1}$$

9.19.
$$\frac{x-2}{x^2-2x+3} + \frac{2x+1}{\left(x^2-2x+3\right)^2}$$

9.20. (9.192)

$$A + B = \begin{bmatrix} 2 & 6 \\ 1 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 2+3 & 6+(-2) \\ 1+1 & 0+5 \\ 1+(-4) & -3+3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 2 & 5 \\ -3 & 0 \end{bmatrix}$$

9.21.
$$-2B = \begin{bmatrix} -8 & -2 \\ -6 & -4 \end{bmatrix}$$

9.22.
$$\left[\begin{array}{cc|c} 4 & -3 & 11 \\ 3 & 2 & 4 \end{array}\right]$$

9.23.
$$\begin{aligned} x - y + z &= 5 \\ 2x - y + 3z &= 1 \\ y + z &= -9 \end{aligned}$$

9.24.
$(2,\ 1)$

9.25.
$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{2} & \frac{5}{2} & \frac{17}{2} \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

9.26.
$(1,\ 1,\ 1)$

9.27. \$150,000 at 7%, \$750,000 at 8%, \$600,000 at 10%

9.28. (9.308)

$$AB = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(-3) + 4(1) & 1(-4) + 4(1) \\ -1(-3) + -3(1) & -1(-4) + -3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$BA = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} -3(1) + -4(-1) & -3(4) + -4(-3) \\ 1(1) + 1(-1) & 1(4) + 1(-3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9.29.
$$A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

9.30.
$$A^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

9.31.
$$X = \begin{bmatrix} 4 \\ 38 \\ 58 \end{bmatrix}$$

9.32.
$(3,\ -7)$

9.33.
-10

9.34.
$\left(-2, \frac{3}{5}, \frac{12}{5}\right)$

Section Exercises

1. No, you can either have zero, one, or infinitely many. Examine graphs.
3. This means there is no realistic break-even point. By the time the company produces one unit they are already making profit.
5. You can solve by substitution (isolating x or y), graphically, or by addition.
7. Yes
9. Yes
11.
$(-1,\ 2)$

13.
$(-3,\ 1)$

15.
$\left(-\frac{3}{5}, 0\right)$

17. No solutions exist.
19.
$\left(\frac{72}{5}, \frac{132}{5}\right)$

21.
$(6,\ -6)$

23.
$\left(-\frac{1}{2}, \frac{1}{10}\right)$

25. No solutions exist.
27.
$\left(-\frac{1}{5}, \frac{2}{3}\right)$

29.
$\left(x,\ \frac{x+3}{2}\right)$

31.
$(-4,\ 4)$

33.
$\left(\frac{1}{2},\ \frac{1}{8}\right)$

35.
$\left(\frac{1}{6},\ 0\right)$

37.
$(x,\ 2(7x-6))$

39.
$\left(-\frac{5}{6},\ \frac{4}{3}\right)$

41. Consistent with one solution
43. Consistent with one solution
45. Dependent with infinitely many solutions
47.
$(-3.08,\ 4.91)$

49.
$(-1.52,\ 2.29)$

51.
$\left(\frac{A+B}{2},\ \frac{A-B}{2}\right)$

53.
$\left(\frac{-1}{A-B},\ \frac{A}{A-B}\right)$

55.
$\left(\frac{CE-BF}{BD-AE},\ \frac{AF-CD}{BD-AE}\right)$

57. They never turn a profit.
59.
$(1,\ 250,\ 100,\ 000)$

61. The numbers are 7.5 and 20.5.
63. 24,000
65. 790 sophomores, 805 freshman
67. 56 men, 74 women
69. 10 gallons of 10% solution, 15 gallons of 60% solution
71. Swan Peak: \$750,000, Riverside: \$350,000
73. \$12,500 in the first account, \$10,500 in the second account.
75. High-tops: 45, Low-tops: 15
77. Infinitely many solutions. We need more information.
78. No, there can be only one, zero, or infinitely many solutions.
80. Not necessarily. There could be zero, one, or infinitely many solutions. For example, $(0,\ 0,\ 0)$ is not a solution to the system below, but that does not mean that it has no solution.

$$\begin{aligned} 2x+3y-6z&=1\\ -4x-6y+12z&=-2\\ x+2y+5z&=10 \end{aligned}$$

82. Every system of equations can be solved graphically, by substitution, and by addition. However, systems of three equations become very complex to solve graphically so other methods are usually preferable.
84. No
86. Yes
88.
$(-1,\ 4,\ 2)$

90.
$\left(-\frac{85}{107}, \frac{312}{107}, \frac{191}{107}\right)$
92.
$\left(1, \frac{1}{2}, 0\right)$
94.
$(4, -6, 1)$
96.
$\left(x, \frac{1}{27}(65-16x), \frac{x+28}{27}\right)$
98.
$\left(-\frac{45}{13}, \frac{17}{13}, -2\right)$
100. No solutions exist
102.
$(0, 0, 0)$
104.
$\left(\frac{4}{7}, -\frac{1}{7}, -\frac{3}{7}\right)$
106.
$(7, 20, 16)$
108.
$(-6, 2, 1)$
110.
$(5, 12, 15)$
112.
$(-5, -5, -5)$
114.
$(10, 10, 10)$
116.
$\left(\frac{1}{2}, \frac{1}{5}, \frac{4}{5}\right)$
118.
$\left(\frac{1}{2}, \frac{2}{5}, \frac{4}{5}\right)$
120.
$(2, 0, 0)$
122.
$(1, 1, 1)$
124.
$\left(\frac{128}{557}, \frac{23}{557}, \frac{28}{557}\right)$
126.
$(6, -1, 0)$
128. 24, 36, 48
130. 70 grandparents, 140 parents, 190 children
132. Your share was $19.95, Sarah's share was $40, and your other roommate's share was $22.05.
134. There are infinitely many solutions; we need more information
136. 500 students, 225 children, and 450 adults
138. The BMW was $49,636, the Jeep was $42,636, and the Toyota was $47,727.
140. $400,000 in the account that pays 3% interest, $500,000 in the account that pays 4% interest, and $100,000 in the account that pays 2% interest.
142. The United States consumed 26.3%, Japan 7.1%, and China 6.4% of the world's oil.
144. Saudi Arabia imported 16.8%, Canada imported 15.1%, and Mexico 15.0%
146. Birds were 19.3%, fish were 18.6%, and mammals were 17.1% of endangered species

148. A nonlinear system could be representative of two circles that overlap and intersect in two locations, hence two solutions. A nonlinear system could be representative of a parabola and a circle, where the vertex of the parabola meets the circle and the branches also intersect the circle, hence three solutions.
150. No. There does not need to be a feasible region. Consider a system that is bounded by two parallel lines. One inequality represents the region above the upper line; the other represents the region below the lower line. In this case, no points in the plane are located in both regions; hence there is no feasible region.
152. Choose any number between each solution and plug into $C(x)$ and $R(x).$ If $C(x) < R(x),$ then there is profit.
154.
$(0, -3), (3, 0)$

156.
$\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right), \left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

158.
$(-3, 0), (3, 0)$

160.
$\left(\frac{1}{4}, -\frac{\sqrt{62}}{8}\right), \left(\frac{1}{4}, \frac{\sqrt{62}}{8}\right)$

162.
$\left(-\frac{\sqrt{398}}{4}, \frac{199}{4}\right), \left(\frac{\sqrt{398}}{4}, \frac{199}{4}\right)$

164.
$(0, 2), (1, 3)$

166.
$\left(-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}, \frac{1}{2}\left(1-\sqrt{5}\right)\right), \left(\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}, \frac{1}{2}\left(1-\sqrt{5}\right)\right)$

168.
$(5, 0)$

170.
$(0, 0)$

172.
$(3, 0)$

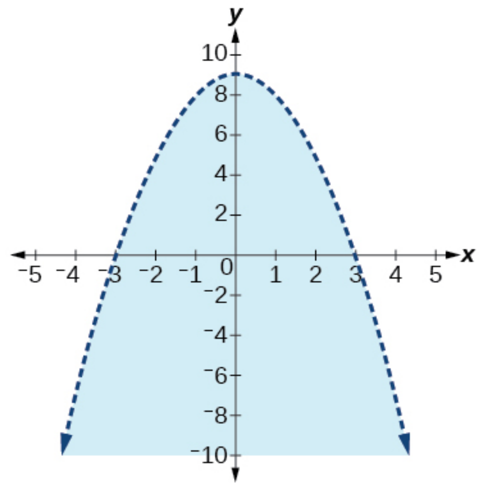
174. No Solutions Exist
176. No Solutions Exist
178.
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

180.
$(2, 0)$

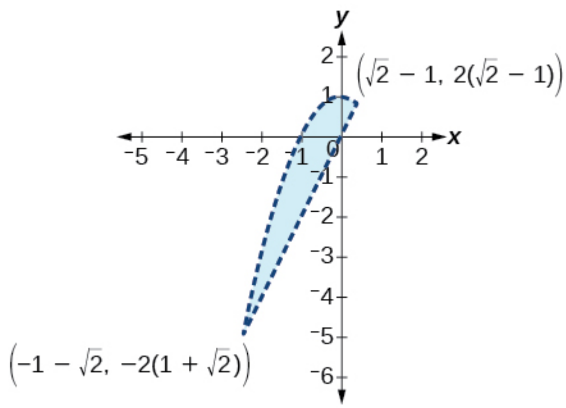
182.
$\left(-\sqrt{7}, -3\right), \left(-\sqrt{7}, 3\right), \left(\sqrt{7}, -3\right), \left(\sqrt{7}, 3\right)$

184.
$\left(-\sqrt{\frac{1}{2}\left(\sqrt{73}-5\right)}, \frac{1}{2}\left(7-\sqrt{73}\right)\right), \left(\sqrt{\frac{1}{2}\left(\sqrt{73}-5\right)}, \frac{1}{2}\left(7-\sqrt{73}\right)\right)$

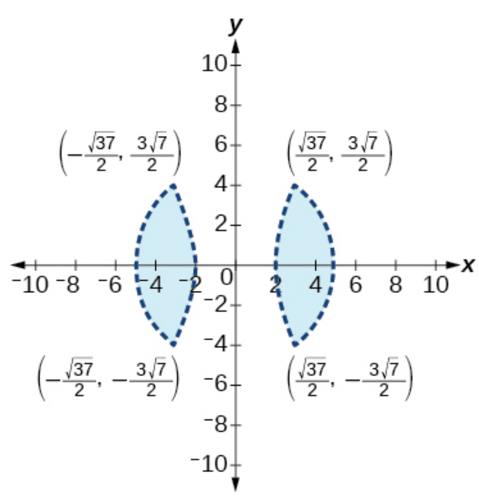
186.


y
x

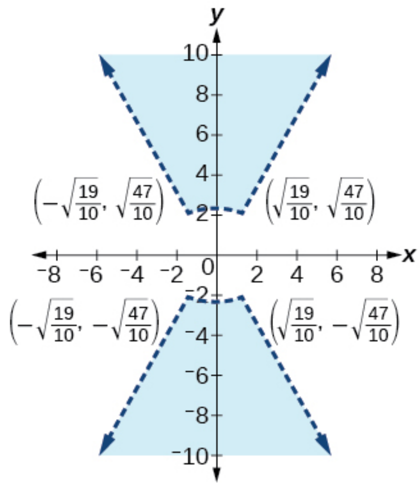
188.


y
x
$(\sqrt{2}-1,\ 2(\sqrt{2}-1))$
$(-1-\sqrt{2},\ -2(1+\sqrt{2}))$

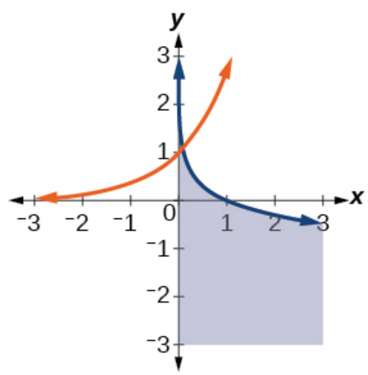
190.


y
x
$\left(-\frac{\sqrt{37}}{2}, \frac{3\sqrt{7}}{2}\right)$
$\left(\frac{\sqrt{37}}{2}, \frac{3\sqrt{7}}{2}\right)$
$\left(-\frac{\sqrt{37}}{2}, -\frac{3\sqrt{7}}{2}\right)$
$\left(\frac{\sqrt{37}}{2}, -\frac{3\sqrt{7}}{2}\right)$

192.



194.



196.

$\left(-2\sqrt{\frac{70}{383}}, -2\sqrt{\frac{35}{29}}\right), \left(-2\sqrt{\frac{70}{383}}, 2\sqrt{\frac{35}{29}}\right), \left(2\sqrt{\frac{70}{383}}, -2\sqrt{\frac{35}{29}}\right), \left(2\sqrt{\frac{70}{383}}, 2\sqrt{\frac{35}{29}}\right)$

198. No Solution Exists

200.

$x = 0,\ y > 0$ and

$0 < x < 1,\ \sqrt{x} < y < \frac{1}{x}$

202. 12, 288

204. 2–20 computers

206. No, a quotient of polynomials can only be decomposed if the denominator can be factored. For example, $\frac{1}{x^2 + 1}$ cannot be decomposed because the denominator cannot be factored.

208. Graph both sides and ensure they are equal.

210. If we choose $x = -1,$ then the *B*-term disappears, letting us immediately know that $A = 3.$ We could alternatively plug in $x = -\frac{5}{3},$ giving us a *B*-value of $-2.$

212.

$\frac{8}{x+3} - \frac{5}{x-8}$

214.

$\frac{1}{x+5} + \frac{9}{x+2}$

216.

$\frac{3}{5x-2} + \frac{4}{4x-1}$

218.

$$\frac{5}{2(x+3)}+\frac{5}{2(x-3)}$$

220.

$$\frac{3}{x+2}+\frac{3}{x-2}$$

222.

$$\frac{9}{5(x+2)}+\frac{11}{5(x-3)}$$

224.

$$\frac{8}{x-3}-\frac{5}{x-2}$$

226.

$$\frac{1}{x-2}+\frac{2}{(x-2)^2}$$

228.

$$-\frac{6}{4x+5}+\frac{3}{(4x+5)^2}$$

230.

$$-\frac{1}{x-7}-\frac{2}{(x-7)^2}$$

232.

$$\frac{4}{x}-\frac{3}{2(x+1)}+\frac{7}{2(x+1)^2}$$

234.

$$\frac{4}{x}+\frac{2}{x^2}-\frac{3}{3x+2}+\frac{7}{2(3x+2)^2}$$

236.

$$\frac{x+1}{x^2+x+3}+\frac{3}{x+2}$$

238.

$$\frac{4-3x}{x^2+3x+8}+\frac{1}{x-1}$$

240.

$$\frac{2x-1}{x^2+6x+1}+\frac{2}{x+3}$$

242.

$$\frac{1}{x^2+x+1}+\frac{4}{x-1}$$

244.

$$\frac{2}{x^2-3x+9}+\frac{3}{x+3}$$

246.

$$-\frac{1}{4x^2+6x+9}+\frac{1}{2x-3}$$

248.

$$\frac{1}{x}+\frac{1}{x+6}-\frac{4x}{x^2-6x+36}$$

250.

$$\frac{x+6}{x^2+1}+\frac{4x+3}{\left(x^2+1\right)^2}$$

252.

$$\frac{x+1}{x+2}+\frac{2x+3}{(x+2)^2}$$

254.

$$\frac{1}{x^2+3x+25}-\frac{3x}{\left(x^2+3x+25\right)^2}$$

256.

$$\frac{1}{8x}-\frac{x}{8\left(x^2+4\right)}+\frac{10-x}{2\left(x^2+4\right)^2}$$

258.

$$-\frac{16}{x}-\frac{9}{x^2}+\frac{16}{x-1}-\frac{7}{(x-1)^2}$$

260.

$$\frac{1}{x+1}-\frac{2}{(x+1)^2}+\frac{5}{(x+1)^3}$$

262.

$$\frac{5}{x-2}-\frac{3}{10(x+2)}+\frac{7}{x+8}-\frac{7}{10(x-8)}$$

264.

$$-\frac{5}{4x}-\frac{5}{2(x+2)}+\frac{11}{2(x+4)}+\frac{5}{4(x+4)}$$

265. No, they must have the same dimensions. An example would include two matrices of different dimensions. One cannot add the following two matrices because the first is a 2×2 matrix and the second is a 2×3 matrix. $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}6 & 5 & 4\\3 & 2 & 1\end{bmatrix}$ has no sum.

267. Yes, if the dimensions of A are $m\times n$ and the dimensions of B are $n\times m,$ both products will be defined.

269. Not necessarily. To find $AB,$ we multiply the first row of A by the first column of B to get the first entry of $AB.$ To find $BA,$ we multiply the first row of B by the first column of A to get the first entry of $BA.$ Thus, if those are unequal, then the matrix multiplication does not commute.

271.

$$\begin{bmatrix}11 & 19\\15 & 94\\17 & 67\end{bmatrix}$$

273.

$$\begin{bmatrix}-4 & 2\\8 & 1\end{bmatrix}$$

275. Undidentified; dimensions do not match

277.

$$\begin{bmatrix}9 & 27\\63 & 36\\0 & 192\end{bmatrix}$$

279.

$$\begin{bmatrix}-64 & -12 & -28 & -72\\-360 & -20 & -12 & -116\end{bmatrix}$$

281.
$$\begin{bmatrix} 1,800 & 1,200 & 1,300 \\ 800 & 1,400 & 600 \\ 700 & 400 & 2,100 \end{bmatrix}$$
283.
$$\begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}$$
285.
$$\begin{bmatrix} 60 & 41 & 2 \\ -16 & 120 & -216 \end{bmatrix}$$
287.
$$\begin{bmatrix} -68 & 24 & 136 \\ -54 & -12 & 64 \\ -57 & 30 & 128 \end{bmatrix}$$
289. Undefined; dimensions do not match.
291.
$$\begin{bmatrix} -8 & 41 & -3 \\ 40 & -15 & -14 \\ 4 & 27 & 42 \end{bmatrix}$$
293.
$$\begin{bmatrix} -840 & 650 & -530 \\ 330 & 360 & 250 \\ -10 & 900 & 110 \end{bmatrix}$$
295.
$$\begin{bmatrix} -350 & 1,050 \\ 350 & 350 \end{bmatrix}$$
297. Undefined; inner dimensions do not match.
299.
$$\begin{bmatrix} 1,400 & 700 \\ -1,400 & 700 \end{bmatrix}$$
301.
$$\begin{bmatrix} 332,500 & 927,500 \\ -227,500 & 87,500 \end{bmatrix}$$
303.
$$\begin{bmatrix} 490,000 & 0 \\ 0 & 490,000 \end{bmatrix}$$
305.
$$\begin{bmatrix} -2 & 3 & 4 \\ -7 & 9 & -7 \end{bmatrix}$$
307.
$$\begin{bmatrix} -4 & 29 & 21 \\ -27 & -3 & 1 \end{bmatrix}$$
309.
$$\begin{bmatrix} -3 & -2 & -2 \\ -28 & 59 & 46 \\ -4 & 16 & 7 \end{bmatrix}$$
311.
$$\begin{bmatrix} 1 & -18 & -9 \\ -198 & 505 & 369 \\ -72 & 126 & 91 \end{bmatrix}$$
313.
$$\begin{bmatrix} 0 & 1.6 \\ 9 & -1 \end{bmatrix}$$

315.

$$\begin{bmatrix} 2 & 24 & -4.5 \\ 12 & 32 & -9 \\ -8 & 64 & 61 \end{bmatrix}$$

317.

$$\begin{bmatrix} 0.5 & 3 & 0.5 \\ 2 & 1 & 2 \\ 10 & 7 & 10 \end{bmatrix}$$

319.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

321.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

323.

$$B^n = \begin{cases} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, & n \text{ even}, \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, & n \text{ odd}. \end{cases}$$

324. Yes. For each row, the coefficients of the variables are written across the corresponding row, and a vertical bar is placed; then the constants are placed to the right of the vertical bar.

326. No, there are numerous correct methods of using row operations on a matrix. Two possible ways are the following: (1) Interchange rows 1 and 2. Then $R_2 = R_2 - 9R_1.$ (2) $R_2 = R_1 - 9R_2.$ Then divide row 1 by 9.

328. No. A matrix with 0 entries for an entire row would have either zero or infinitely many solutions.

330.

$$\left[\begin{array}{cc|c} 0 & 16 & 4 \\ 9 & -1 & 2 \end{array}\right]$$

332.

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 16 \\ 12 & 3 & 0 & 4 \\ 3 & 4 & 9 & -7 \end{array}\right]$$

334.

$$-2x + 5y = 5$$
$$6x - 18y = 26$$

336.
$$\begin{aligned} 3x + 2y &= 13 \\ -x - 9y + 4z &= 53 \\ 8x + 5y + 7z &= 80 \end{aligned}$$

338.
$$\begin{aligned} 4x + 5y - 2z &= 12 \\ y + 58z &= 2 \\ 8x + 7y - 3z &= -5 \end{aligned}$$

340. No solutions

342.
$(-1, -2)$

344.
$(6, 7)$

346.
$(3, 2)$

348.
$\left(\frac{1}{5}, \frac{1}{2}\right)$

350.
$\left(x, \frac{4}{15}(5x + 1)\right)$

352.
$(3, 4)$

354.
$\left(\frac{196}{39}, -\frac{5}{13}\right)$

356.
$(31, -42, 87)$

358.
$\left(\frac{21}{40}, \frac{1}{20}, \frac{9}{8}\right)$

360.
$\left(\frac{18}{13}, \frac{15}{13}, -\frac{15}{13}\right)$

362.
$\left(x, y, \frac{1}{2}(1 - 2x - 3y)\right)$

364.
$\left(x, -\frac{x}{2}, -1\right)$

366.
$(125, -25, 0)$

368.
$(8, 1, -2)$

370.
$(1, 2, 3)$

372.
$\left(x, \frac{31}{28} - \frac{3x}{4}, \frac{1}{28}(-7x - 3)\right)$

374. No solutions exist.
376. 860 red velvet, 1,340 chocolate
378. 4% for account 1, 6% for account 2
380. $126
382. Banana was 3%, pumpkin was 7%, and rocky road was 2%
384. 100 almonds, 200 cashews, 600 pistachios

385. If A^{-1} is the inverse of $A,$ then $AA^{-1} = I,$ the identity matrix. Since A is also the inverse of $A^{-1}, A^{-1}A = I.$ You can also check by proving this for a 2×2 matrix.

387. No, because ad and bc are both 0, so $ad - bc = 0,$ which requires us to divide by 0 in the formula.

389. Yes. Consider the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$ The inverse is found with the following calculation:

$$A^{-1} = \frac{1}{0(0)-1(1)}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

391.

$$AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

393.

$$AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

395.

$$AB = BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

397.

$$\frac{1}{29}\begin{bmatrix} 9 & 2 \\ -1 & 3 \end{bmatrix}$$

399.

$$\frac{1}{69}\begin{bmatrix} -2 & 7 \\ 9 & 3 \end{bmatrix}$$

401. There is no inverse

403.

$$\frac{4}{7}\begin{bmatrix} 0.5 & 1.5 \\ 1 & -0.5 \end{bmatrix}$$

405.

$$\frac{1}{17}\begin{bmatrix} -5 & 5 & -3 \\ 20 & -3 & 12 \\ 1 & -1 & 4 \end{bmatrix}$$

407.

$$\frac{1}{209}\begin{bmatrix} 47 & -57 & 69 \\ 10 & 19 & -12 \\ -24 & 38 & -13 \end{bmatrix}$$

409.

$$\begin{bmatrix} 18 & 60 & -168 \\ -56 & -140 & 448 \\ 40 & 80 & -280 \end{bmatrix}$$

411.

$(-5, 6)$

413.

$(2, 0)$

415.
$\left(\frac{1}{3}, \ -\frac{5}{2}\right)$

417.
$\left(-\frac{2}{3}, \ -\frac{11}{6}\right)$

419.
$\left(7, \frac{1}{2}, \frac{1}{5}\right)$

421.
$(5, \ 0, \ -1)$

423.
$\frac{1}{34}(-35, \ -97, \ -154)$

425.
$\frac{1}{690}(65, \ -1136, \ -229)$

427.
$\left(-\frac{37}{30}, \frac{8}{15}\right)$

429.
$\left(\frac{10}{123}, -1, \frac{2}{5}\right)$

431.
$\frac{1}{2}\begin{bmatrix} 2 & 1 & -1 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$

433.
$\frac{1}{39}\begin{bmatrix} 3 & 2 & 1 & -7 \\ 18 & -53 & 32 & 10 \\ 24 & -36 & 21 & 9 \\ -9 & 46 & -16 & -5 \end{bmatrix}$

435.
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}$

437. Infinite solutions.
439. 50% oranges, 25% bananas, 20% apples
441. 10 straw hats, 50 beanies, 40 cowboy hats
443. Tom ate 6, Joe ate 3, and Albert ate 3.
445. 124 oranges, 10 lemons, 8 pomegranates
446. A determinant is the sum and products of the entries in the matrix, so you can always evaluate that product—even if it does end up being 0.
448. The inverse does not exist.

450.
-2

452.
7

454.
-4

456.
0

458.
$-7,\ 990.7$

460.
3

462.
-1

464.
224

466.
15

468.
-17.03

470.
$(1,\ 1)$

472.
$\left(\frac{1}{2},\ \frac{1}{3}\right)$

474.
$(2,\ 5)$

476.
$\left(-1,\ -\frac{1}{3}\right)$

478.
$(15,\ 12)$

480.
$(1,\ 3,\ 2)$

482.
$(-1,\ 0,\ 3)$

484.
$\left(\frac{1}{2},\ 1,\ 2\right)$

486.
$(2,\ 1,\ 4)$

488. Infinite solutions

490.
24

492.
1

494. Yes; 18, 38
496. Yes; 33, 36, 37
498. $7,000 in first account, $3,000 in second account.
500. 120 children, 1,080 adult
502. 4 gal yellow, 6 gal blue
504. 13 green tomatoes, 17 red tomatoes
506. Strawberries 18%, oranges 9%, kiwi 10%
508. 100 for movie 1, 230 for movie 2, 312 for movie 3
510. 20–29: 2,100, 30–39: 2,600, 40–49: 825
512. 300 almonds, 400 cranberries, 300 cashews

Review Exercises

514. No

516.
$(-2,\ 3)$

518.
$(4,\ -1)$

520. No solutions exist.

522.
$(300,\ 60,\ 000)$

523.
$(400,\ 30,\ 000)$

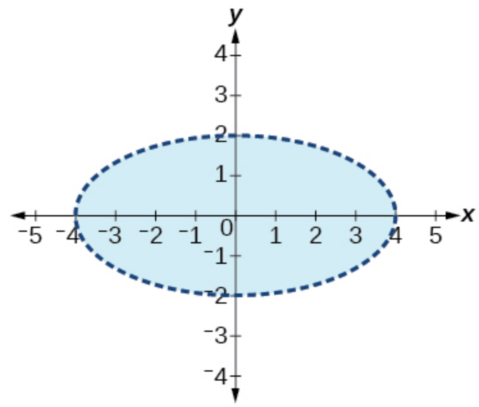
524.
$(10,\ -10,\ 10)$

526. No solutions exist.
528.
$(-1,\ -2,\ 3)$

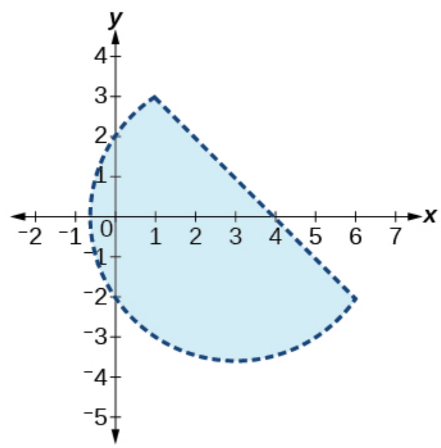
530.
$\left(x,\ \frac{8x}{5},\ \frac{14x}{5}\right)$

532. 11, 17, 33
534.
$(2,\ -3),\ (3,\ 2)$

536. No solution
538. No solution
540.



542.



544.
$\frac{2}{x+2},\ \frac{-4}{x+1}$

546.
$\frac{7}{x+5},\ \frac{-15}{(x+5)^2}$

548.
$\frac{3}{x-5},\ \frac{-4x+1}{x^2+5x+25}$

550.

$$\frac{x-4}{(x^2-2)}, \frac{5x+3}{(x^2-2)^2}$$

552.

$$\begin{bmatrix} -16 & 8 \\ -4 & -12 \end{bmatrix}$$

554. undefined; dimensions do not match

556. undefined; inner dimensions do not match

558.

$$\begin{bmatrix} 113 & 28 & 10 \\ 44 & 81 & -41 \\ 84 & 98 & -42 \end{bmatrix}$$

560.

$$\begin{bmatrix} -127 & -74 & 176 \\ -2 & 11 & 40 \\ 28 & 77 & 38 \end{bmatrix}$$

562. undefined; inner dimensions do not match

564.

$$\begin{aligned} x - 3z &= 7 \\ y + 2z &= -5 \end{aligned}$$ with infinite solutions

566.

$$\left[\begin{array}{ccc|c} -2 & 2 & 1 & 7 \\ 2 & -8 & 5 & 0 \\ 19 & -10 & 22 & 3 \end{array}\right]$$

568.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 12 \\ -1 & 4 & 0 & 0 \\ 0 & 1 & 2 & -7 \end{array}\right]$$

570. No solutions exist.

572. No solutions exist.

574.

$$\frac{1}{8}\begin{bmatrix} 2 & 7 \\ 6 & 1 \end{bmatrix}$$

576. No inverse exists.

578.

$$(-20, 40)$$

580.

$$(-1, 0.2, 0.3)$$

582. 17% oranges, 34% bananas, 39% apples

584. 0

586. 6

588.

$$\left(6, \frac{1}{2}\right)$$

590. $(x, 5x + 3)$

592.

$$\left(0, 0, -\frac{1}{2}\right)$$

Practice Test

594. Yes

596. No solutions exist.

598.

$$\frac{1}{20}(10, 5, 4)$$

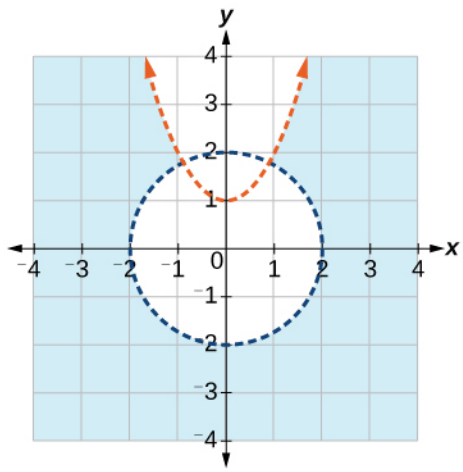
600.

$\left(x, \frac{16x}{5} - \frac{13x}{5}\right)$

602.

$(-2\sqrt{2}, -\sqrt{17}), \left(-2\sqrt{2}, \sqrt{17}\right), \left(2\sqrt{2}, -\sqrt{17}\right), \left(2\sqrt{2}, \sqrt{17}\right)$

604.



606.

$\frac{5}{3x+1} - \frac{2x+3}{(3x+1)^2}$

608.

$\begin{bmatrix} 17 & 51 \\ -8 & 11 \end{bmatrix}$

610.

$\begin{bmatrix} 12 & -20 \\ -15 & 30 \end{bmatrix}$

612.

$-\frac{1}{8}$

614.

$\left[\begin{array}{rrr|r} 14 & -2 & 13 & 140 \\ -2 & 3 & -6 & -1 \\ 1 & -5 & 12 & 11 \end{array}\right]$

616. No solutions exist.

618.

$(100, 90)$

620.

$\left(\frac{1}{100}, 0\right)$

622. 32 or more cell phones per day

Chapter 10

Try It

10.1.

$x^2 + \frac{y^2}{16} = 1$

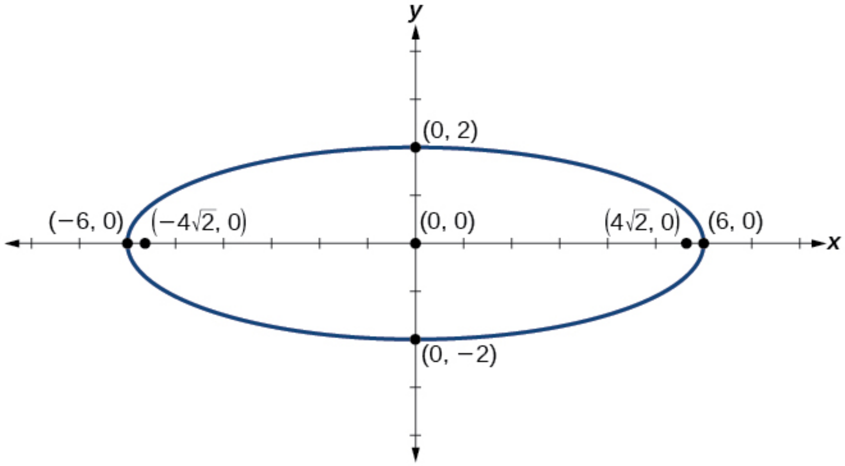
10.2.

$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{4} = 1$

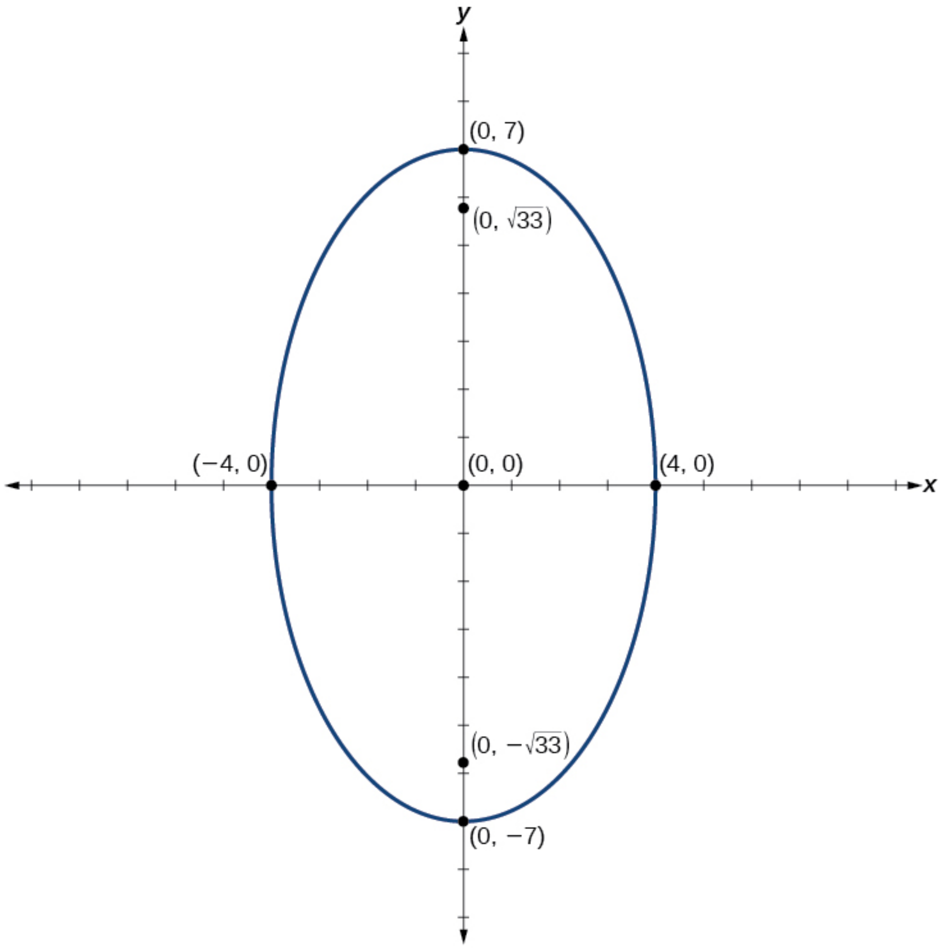
10.3. center:

$(0, 0)$; vertices:

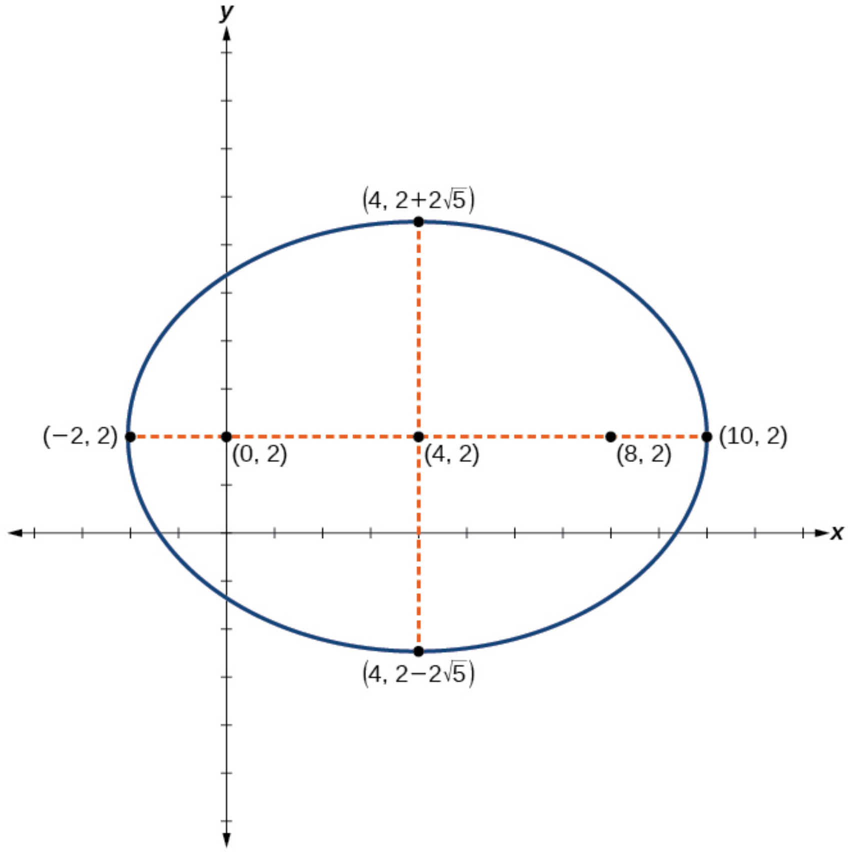
$(\pm 6,\ 0);$ co-vertices: $(0,\ \pm 2);$ foci: $\left(\pm 4\sqrt{2},\ 0\right)$



10.4. Standard form: $\frac{x^2}{16}+\frac{y^2}{49}=1;$ center: $(0,\ 0);$ vertices: $(0,\ \pm 7);$ co-vertices: $(\pm 4,\ 0);$ foci: $\left(0,\ \pm\sqrt{33}\right)$



10.5. Center: $(4,\ 2);$ vertices: $(-2,\ 2)$ and $(10,\ 2);$ co-vertices: $\left(4,\ 2-2\sqrt{5}\right)$ and $\left(4,\ 2+2\sqrt{5}\right);$ foci: $(0,\ 2)$ and $(8,\ 2)$



10.6.
$\frac{(x-3)^2}{4}+\frac{(y+1)^2}{16}=1;$ center: $(3,\ -1);$ vertices: $(3,\ -5)$ and $(3,\ 3);$ co-vertices: $(1,\ -1)$ and $(5,\ -1);$ foci: $\left(3,\ -1-2\sqrt{3}\right)$ and $\left(3,\ -1+2\sqrt{3}\right)$

10.7.

a.

$$\frac{x^2}{57,600}+\frac{y^2}{25,600}=1$$

b. The people are standing 358 feet apart.

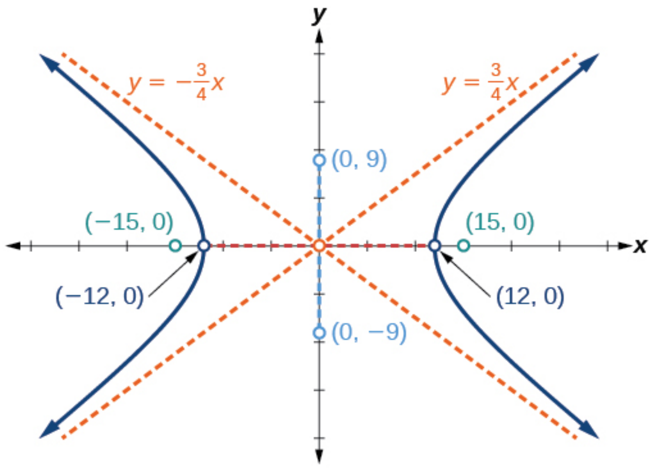
10.8. Vertices: $(\pm 3,\ 0);$ Foci: $\left(\pm\sqrt{34},\ 0\right)$

10.9.
$\frac{y^2}{4}-\frac{x^2}{16}=1$

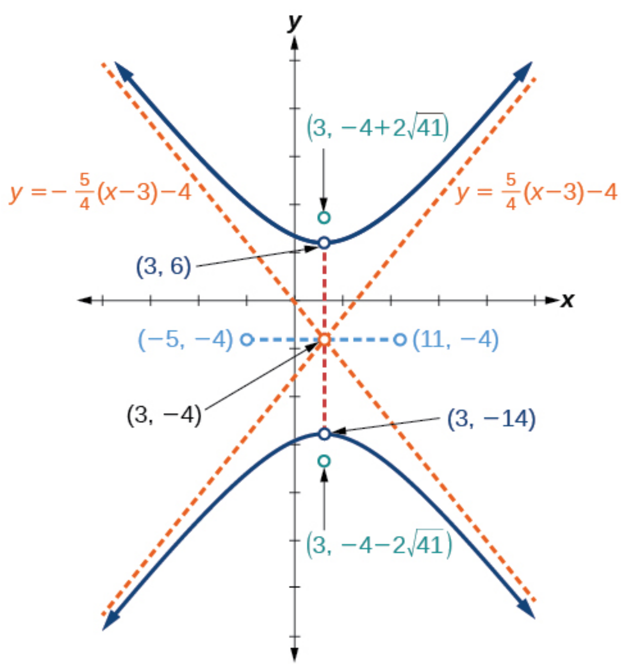
10.10.
$\frac{(y-3)^2}{25}+\frac{(x-1)^2}{144}=1$

10.11. vertices: $(\pm 12,\ 0);$ co-vertices:

$(0, \ \pm 9);$ foci:
$(\pm 15, \ 0);$ asymptotes:
$y = \ \pm \frac{3}{4}x;$

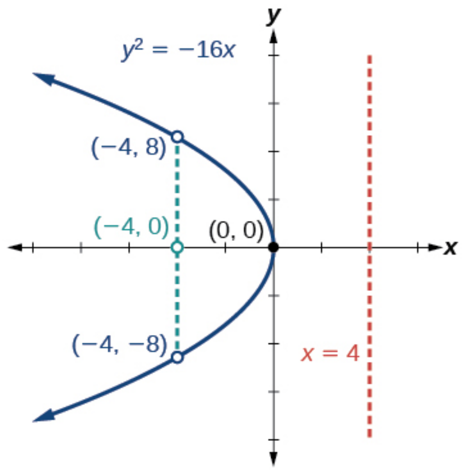


10.12. center:
$(3, \ -4);$ vertices:
$(3, \ -14)$ and
$(3, \ 6);$ co-vertices:
$(-5, \ -4);$ and
$(11, \ -4);$ foci:
$\left(3, \ -4 - 2\sqrt{41}\right)$ and
$\left(3, \ -4 + 2\sqrt{41}\right);$ asymptotes:
$y = \ \pm \frac{5}{4}(x - 3) - 4$

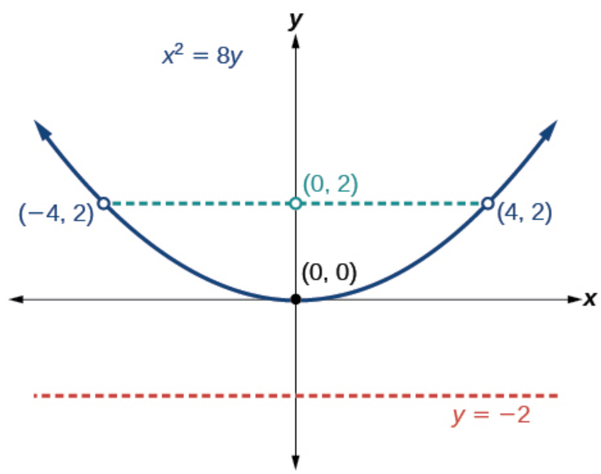


10.13. The sides of the tower can be modeled by the hyperbolic equation.
$\frac{x^2}{400} - \frac{y^2}{3600} = 1 \text{or } \frac{x^2}{20^2} - \frac{y^2}{60^2} = 1.$

10.14. Focus:
$(-4, 0);$ Directrix:
$x = 4;$ Endpoints of the latus rectum:
$(-4, \pm 8)$

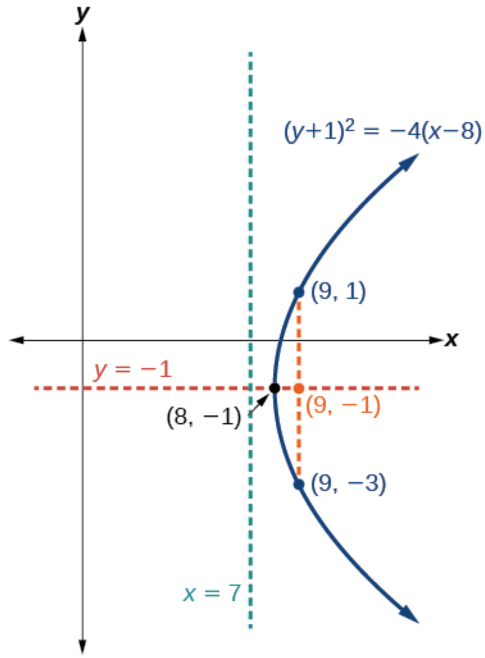


10.15. Focus:
$(0, 2);$ Directrix:
$y = -2;$ Endpoints of the latus rectum:
$(\pm 4, 2).$

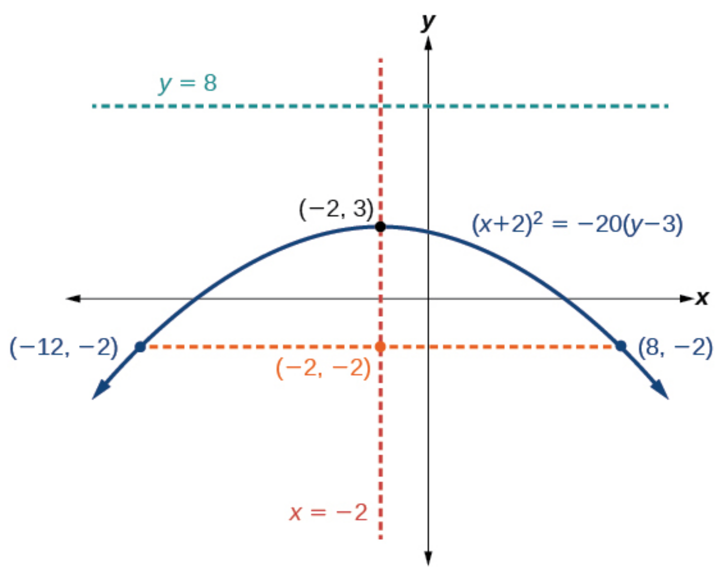


10.16.
$x^2 = 14y.$

10.17. Vertex:
$(8, -1);$ Axis of symmetry:
$y = -1;$ Focus:
$(9, -1);$ Directrix:
$x = 7;$ Endpoints of the latus rectum:
$(9, -3)$ and
$(9, 1).$



10.18. Vertex:
$(-2, 3);$ Axis of symmetry:
$x = -2;$ Focus:
$(-2, -2);$ Directrix:
$y = 8;$ Endpoints of the latus rectum:
$(-12, -2)$ and
$(8, -2).$



10.19.

a. $y^2 = 1280x$

b. The depth of the cooker is 500 mm

10.20.

a. hyperbola

b. ellipse

10.21.

$$\frac{x'^2}{4}+\frac{y'^2}{1}=1$$

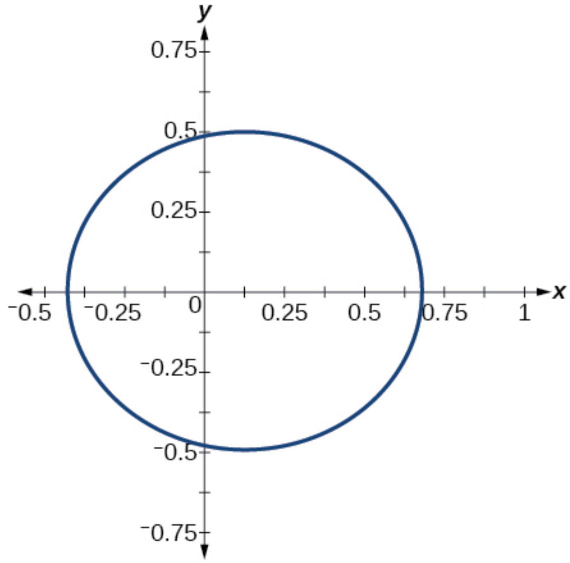
10.22.

a. hyperbola
b. ellipse

10.23. ellipse;

$$e=\frac{1}{3};\ x=-2$$

10.24.



10.25.

$$r=\frac{1}{1-\cos\theta}$$

10.26.

$$4-8x+3x^2-y^2=0$$

Section Exercises

1. An ellipse is the set of all points in the plane the sum of whose distances from two fixed points, called the foci, is a constant.
3. This special case would be a circle.
5. It is symmetric about the *x*-axis, *y*-axis, and the origin.
7. yes;

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$$

9. yes;

$$\frac{x^2}{\left(\frac{1}{2}\right)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$$

11.

$\frac{x^2}{2^2}+\frac{y^2}{7^2}=1;$ Endpoints of major axis $(0,\ 7)$ and $(0,\ -7).$ Endpoints of minor axis $(2,\ 0)$ and $(-2,\ 0).$ Foci at $\left(0,\ 3\sqrt{5}\right),\ \left(0,\ -3\sqrt{5}\right).$

13.

$\frac{x^2}{(1)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1;$ Endpoints of major axis $(1,\ 0)$ and $(-1,\ 0).$ Endpoints of minor axis $\left(0,\ \frac{1}{3}\right),\ \left(0,\ -\frac{1}{3}\right).$ Foci at $\left(\frac{2\sqrt{2}}{3},\ 0\right),\ \left(-\frac{2\sqrt{2}}{3},\ 0\right).$

15.

$\frac{(x-2)^2}{7^2}+\frac{(y-4)^2}{5^2}=1;$ Endpoints of major axis $(9,\ 4),\ (-5,\ 4).$ Endpoints of minor axis $(2,\ 9),\ (2,\ -1).$ Foci at $\left(2+2\sqrt{6},\ 4\right),\ \left(2-2\sqrt{6},\ 4\right).$

17.

$\frac{(x+5)^2}{2^2}+\frac{(y-7)^2}{3^2}=1;$ Endpoints of major axis $(-5,\ 10),\ (-5,\ 4).$ Endpoints of minor axis $(-3,\ 7),\ (-7,\ 7).$ Foci at $\left(-5,\ 7+\sqrt{5}\right),\ \left(-5,\ 7-\sqrt{5}\right).$

19.

$\frac{(x-1)^2}{3^2}+\frac{(y-4)^2}{2^2}=1;$ Endpoints of major axis $(4,\ 4),\ (-2,\ 4).$ Endpoints of minor axis $(1,\ 6),\ (1,\ 2).$ Foci at $\left(1+\sqrt{5},\ 4\right),\ \left(1-\sqrt{5},\ 4\right).$

21.

$\frac{(x-3)^2}{\left(3\sqrt{2}\right)^2}+\frac{(y-5)^2}{\left(\sqrt{2}\right)^2}=1;$ Endpoints of major axis $\left(3+3\sqrt{2},\ 5\right),\ \left(3-3\sqrt{2},\ 5\right).$ Endpoints of minor axis $\left(3,\ 5+\sqrt{2}\right),\ \left(3,\ 5-\sqrt{2}\right).$ Foci at $(7,\ 5),\ (-1,\ 5).$

23.

$\frac{(x+5)^2}{(5)^2}+\frac{(y-2)^2}{(2)^2}=1;$ Endpoints of major axis $(0,\ 2),\ (-10,\ 2).$ Endpoints of minor axis $(-5,\ 4),\ (-5,\ 0).$ Foci at $\left(-5+\sqrt{21},\ 2\right),\ \left(-5-\sqrt{21},\ 2\right).$

25.

$\frac{(x+3)^2}{(5)^2}+\frac{(y+4)^2}{(2)^2}=1;$ Endpoints of major axis $(2,\ -4),\ (-8,\ -4).$ Endpoints of minor axis $(-3,\ -2),\ (-3,\ -6).$ Foci at $\left(-3+\sqrt{21},\ -4\right),\ \left(-3-\sqrt{21},\ -4\right).$

27. Foci

$\left(-3, \ -1+\sqrt{11}\right), \left(-3, \ -1-\sqrt{11}\right)$

29. Focus

$(0, 0)$

31. Foci

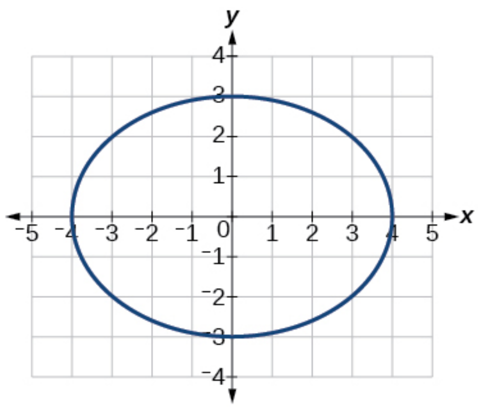
$(-10, 30), (-10, \ -30)$

33. Center

$(0, 0),$ Vertices

$(4, 0), (-4, 0), (0, 3), (0, \ -3),$ Foci

$\left(\sqrt{7}, 0\right), \left(-\sqrt{7}, 0\right)$

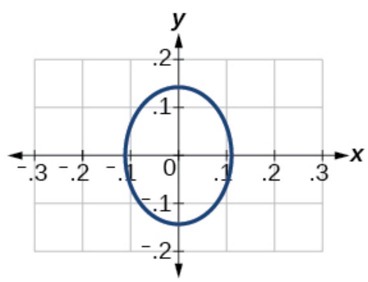


35. Center

$(0, 0),$ Vertices

$\left(\frac{1}{9}, 0\right), \left(-\frac{1}{9}, 0\right), \left(0, \frac{1}{7}\right), \left(0, \ -\frac{1}{7}\right),$ Foci

$\left(0, \frac{4\sqrt{2}}{63}\right), \left(0, \ -\frac{4\sqrt{2}}{63}\right)$

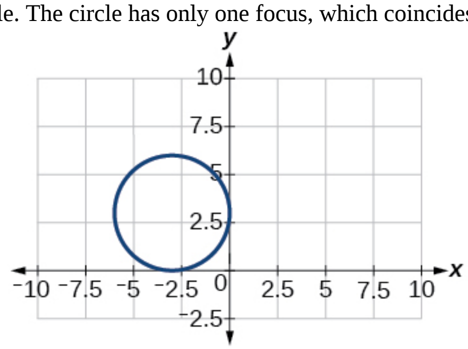


37. Center

$(-3, 3),$ Vertices

$(0, 3), (-6, 3), (-3, 0), (-3, 6),$ Focus

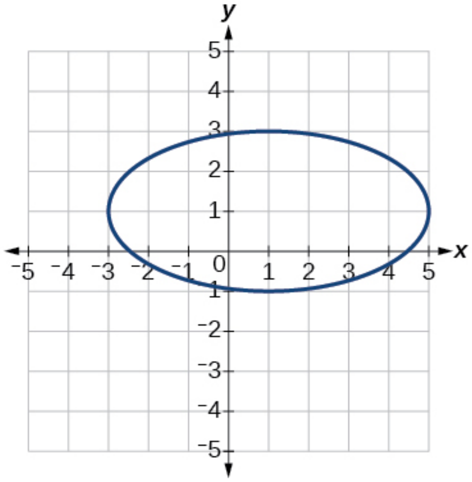
$(-3, 3)$ Note that this ellipse is a circle. The circle has only one focus, which coincides with the center.



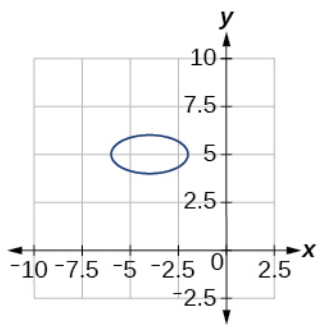
39. Center

$(1, 1),$ Vertices

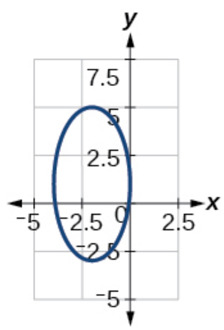
$(5,\ 1),\ (-3,\ 1),\ (1,\ 3),\ (1,\ -1),$ Foci
$\left(1,\ 1+4\sqrt{3}\right),\ \left(1,\ 1-4\sqrt{3}\right)$



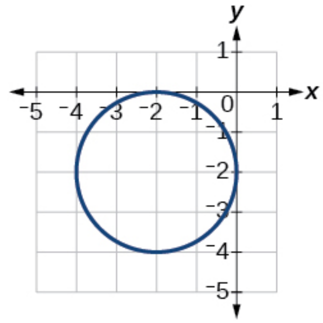
41. Center
$(-4,\ 5),$ Vertices
$(-2,\ 5),\ (-6,\ 4),\ (-4,\ 6),\ (-4,\ 4),$ Foci
$\left(-4+\sqrt{3},\ 5\right),\ \left(-4-\sqrt{3},\ 5\right)$



43. Center
$(-2,\ 1),$ Vertices
$(0,\ 1),\ (-4,\ 1),\ (-2,\ 5),\ (-2,\ -3),$ Foci
$\left(-2,\ 1+2\sqrt{3}\right),\ \left(-2,\ 1-2\sqrt{3}\right)$



45. Center
$(-2,\ -2),$ Vertices
$(0,\ -2),\ (-4,\ -2),\ (-2,\ 0),\ (-2,\ -4),$ Focus
$(-2,\ -2)$



47.
$$\frac{x^2}{25}+\frac{y^2}{29}=1$$
49.
$$\frac{(x-4)^2}{25}+\frac{(y-2)^2}{1}=1$$
51.
$$\frac{(x+3)^2}{16}+\frac{(y-4)^2}{4}=1$$
53.
$$\frac{x^2}{81}+\frac{y^2}{9}=1$$
55.
$$\frac{(x+2)^2}{4}+\frac{(y-2)^2}{9}=1$$
57.
$\text{Area}=12\pi$ square units

59.
$\text{Area}=2\sqrt{5}\pi$ square units

61.
$\text{Area}=9\pi$ square units

63.
$$\frac{x^2}{4h^2}+\frac{y^2}{\frac{1}{4}h^2}=1$$
65.
$\frac{x^2}{400}+\frac{y^2}{144}=1$. Distance = 17.32 feet

67. Approximately 51.96 feet

69. A hyperbola is the set of points in a plane the difference of whose distances from two fixed points (foci) is a positive constant.

71. The foci must lie on the transverse axis and be in the interior of the hyperbola.

73. The center must be the midpoint of the line segment joining the foci.

75. yes
$$\frac{x^2}{6^2}-\frac{y^2}{3^2}=1$$
77. yes
$$\frac{x^2}{4^2}-\frac{y^2}{5^2}=1$$
79.
$\frac{x^2}{5^2}-\frac{y^2}{6^2}=1;$ vertices: $(5,\ 0),\ (-5,\ 0);$ foci:

$\left(\sqrt{61},\ 0\right),\ \left(-\sqrt{61},\ 0\right);$ asymptotes:
$y = \frac{6}{5}x,\ y = -\frac{6}{5}x$

81.
$\frac{y^2}{2^2} - \frac{x^2}{9^2} = 1;$ vertices:
$(0,\ 2),\ (0,\ -2);$ foci:
$\left(0,\ \sqrt{85}\right),\ \left(0,\ -\sqrt{85}\right);$ asymptotes:
$y = \frac{2}{9}x,\ y = -\frac{2}{9}x$

83.
$\frac{(x-1)^2}{3^2} - \frac{(y-2)^2}{4^2} = 1;$ vertices:
$(4,\ 2),\ (-2,\ 2);$ foci:
$(6,\ 2),\ (-4,\ 2);$ asymptotes:
$y = \frac{4}{3}(x-1)+2,\ y = -\frac{4}{3}(x-1)+2$

85.
$\frac{(x-2)^2}{7^2} - \frac{(y+7)^2}{7^2} = 1;$ vertices:
$(9,\ -7),\ (-5,\ -7);$ foci:
$\left(2+7\sqrt{2},\ -7\right),\ \left(2-7\sqrt{2},\ -7\right);$ asymptotes:
$y = x-9,\ y = -x-5$

87.
$\frac{(x+3)^2}{3^2} - \frac{(y-3)^2}{3^2} = 1;$ vertices:
$(0,\ 3),\ (-6,\ 3);$ foci:
$\left(-3+3\sqrt{2},\ 1\right),\ \left(-3-3\sqrt{2},\ 1\right);$ asymptotes:
$y = x+6,\ y = -x$

89.
$\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{4^2} = 1;$ vertices:
$(3,\ 6),\ (3,\ 2);$ foci:
$\left(3,\ 4+2\sqrt{5}\right),\ \left(3,\ 4-2\sqrt{5}\right);$ asymptotes:
$y = \frac{1}{2}(x-3)+4,\ y = -\frac{1}{2}(x-3)+4$

91.
$\frac{(y+5)^2}{7^2} - \frac{(x+1)^2}{70^2} = 1;$ vertices:
$(-1,\ 2),\ (-1,\ -12);$ foci:
$\left(-1,\ -5+7\sqrt{101}\right),\ \left(-1,\ -5-7\sqrt{101}\right);$ asymptotes:
$y = \frac{1}{10}(x+1)-5,\ y = -\frac{1}{10}(x+1)-5$

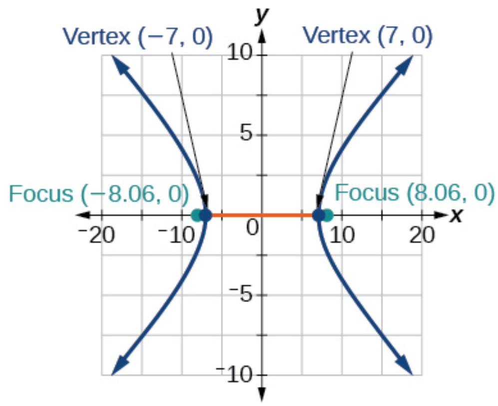
93.
$\frac{(x+3)^2}{5^2} - \frac{(y-4)^2}{2^2} = 1;$ vertices:
$(2,\ 4),\ (-8,\ 4);$ foci:

$\left(-3+\sqrt{29},\ 4\right), \left(-3-\sqrt{29},\ 4\right);$ asymptotes:
$y = \frac{2}{5}(x+3)+4,\ y = -\frac{2}{5}(x+3)+4$

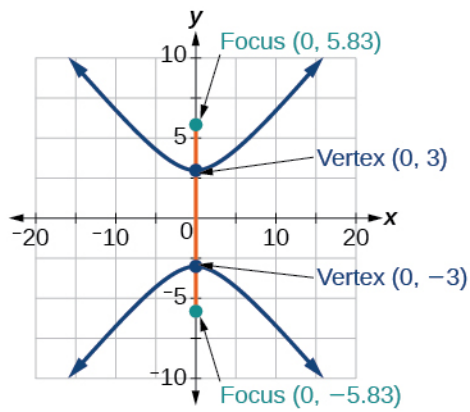
95.
$y = \frac{2}{5}(x-3)-4,\ y = -\frac{2}{5}(x-3)-4$

97.
$y = \frac{3}{4}(x-1)+1,\ y = -\frac{3}{4}(x-1)+1$

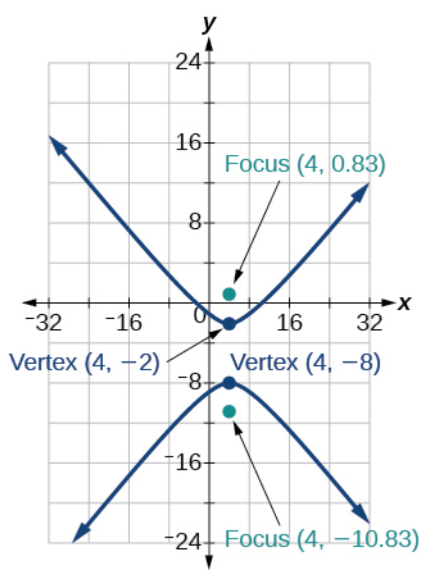
99.



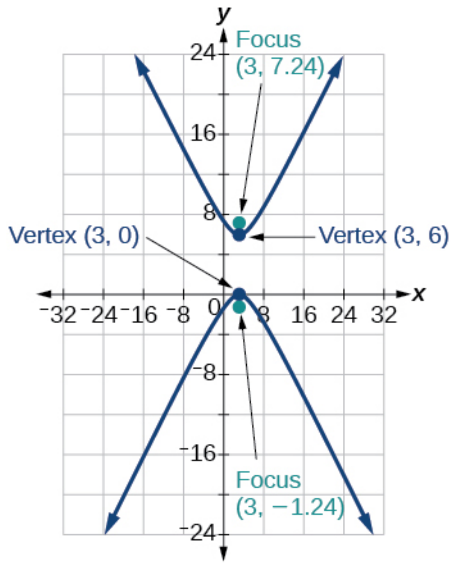
101.



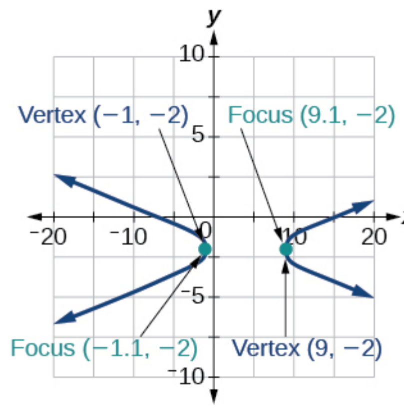
103.



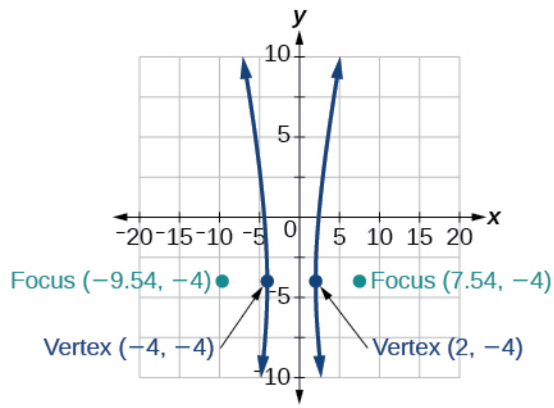
105.


y
Focus
(3, 7.24)
Vertex (3, 0)
Vertex (3, 6)
x
Focus
(3, −1.24)

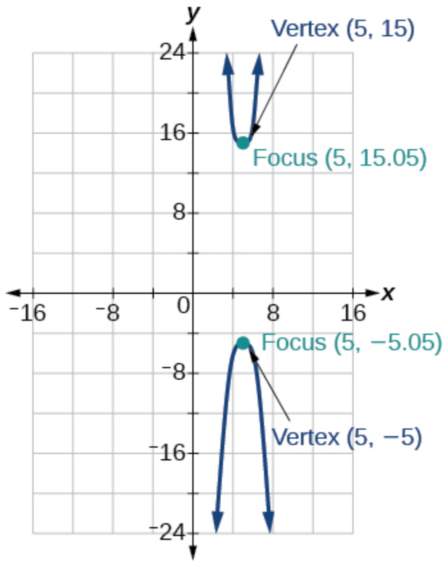
107.


y
Vertex (−1, −2)
Focus (9.1, −2)
x
Focus (−1.1, −2)
Vertex (9, −2)

109.


y
x
Focus (−9.54, −4)
Focus (7.54, −4)
Vertex (−4, −4)
Vertex (2, −4)

111.



113.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

115.

$$\frac{(x-6)^2}{25} - \frac{(y-1)^2}{11} = 1$$

117.

$$\frac{(x-4)^2}{25} - \frac{(y-2)^2}{1} = 1$$

119.

$$\frac{y^2}{16} - \frac{x^2}{25} = 1$$

121.

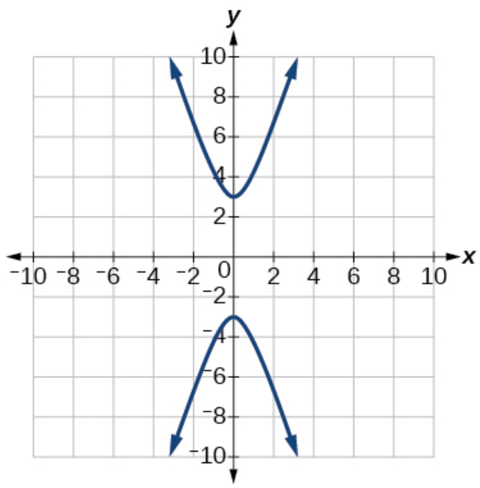
$$\frac{y^2}{9} - \frac{(x+1)^2}{9} = 1$$

123.

$$\frac{(x+3)^2}{25} - \frac{(y+3)^2}{25} = 1$$

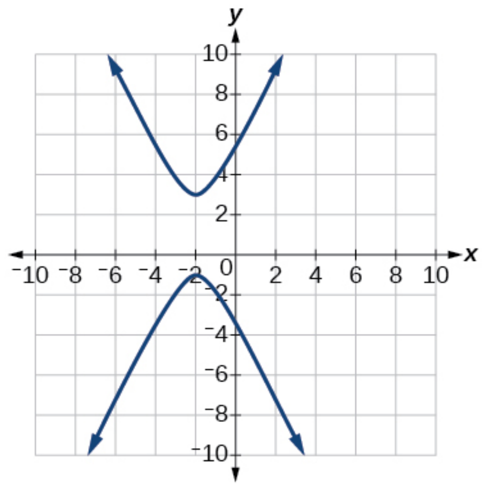
125.

$$y(x) = 3\sqrt{x^2+1},\ y(x) = -3\sqrt{x^2+1}$$



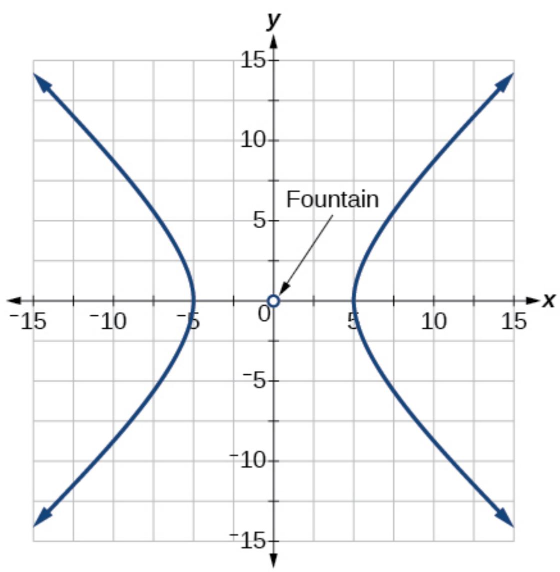
127.

$y(x) = 1 + 2\sqrt{x^2 + 4x + 5},\ y(x) = 1 - 2\sqrt{x^2 + 4x + 5}$



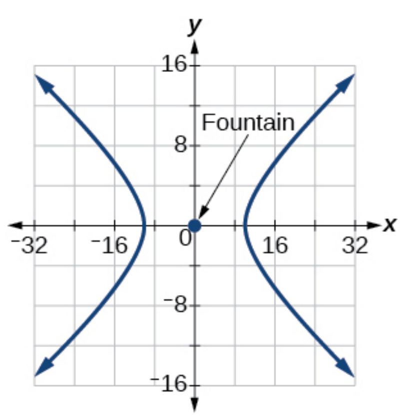
129.

$\frac{x^2}{25} - \frac{y^2}{25} = 1$



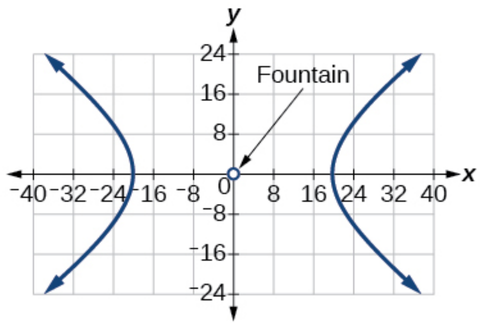
131.

$\frac{x^2}{100} - \frac{y^2}{25} = 1$



133.

$$\frac{x^2}{400} - \frac{y^2}{225} = 1$$



135.

$$\frac{(x-1)^2}{0.25} - \frac{y^2}{0.75} = 1$$

137.

$$\frac{(x-3)^2}{4} - \frac{y^2}{5} = 1$$

139. A parabola is the set of points in the plane that lie equidistant from a fixed point, the focus, and a fixed line, the directrix.
141. The graph will open down.
143. The distance between the focus and directrix will increase.
145. yes

$$y = 4(1)x^2$$

147. yes

$$(y-3)^2 = 4(2)(x-2)$$

149.

$$y^2 = \frac{1}{8}x,\ V:(0,\ 0);\ F:\left(\frac{1}{32},\ 0\right);\ d:x = -\frac{1}{32}$$

151.

$$x^2 = -\frac{1}{4}y,\ V:(0,\ 0);\ F:\left(0,\ -\frac{1}{16}\right);\ d:y = \frac{1}{16}$$

153.

$$y^2 = \frac{1}{36}x,\ V:(0,\ 0);\ F:\left(\frac{1}{144},\ 0\right);\ d:x = -\frac{1}{144}$$

155.

$$(x-1)^2 = 4(y-1),\ V:(1,\ 1);\ F:(1,\ 2);\ d:y = 0$$

157.

$$(y-4)^2 = 2(x+3),\ V:(-3,\ 4);\ F:\left(-\frac{5}{2},\ 4\right);\ d:x = -\frac{7}{2}$$

159.

$$(x+4)^2 = 24(y+1),\ V:(-4,\ -1);\ F:(-4,\ 5);\ d:y = -7$$

161.

$$(y-3)^2 = -12(x+1),\ V:(-1,\ 3);\ F:(-4,\ 3);\ d:x = 2$$

163.

$$(x-5)^2 = \frac{4}{5}(y+3),\ V:(5,\ -3);\ F:\left(5,\ -\frac{14}{5}\right);\ d:y = -\frac{16}{5}$$

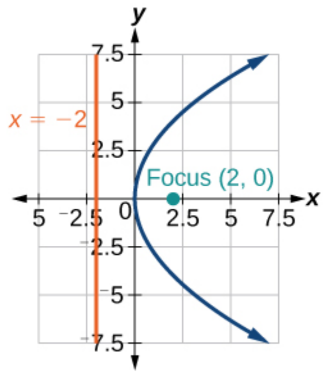
165.

$$(x-2)^2 = -2(y-5),\ V:(2,\ 5);\ F:\left(2,\ \frac{9}{2}\right);\ d:y = \frac{11}{2}$$

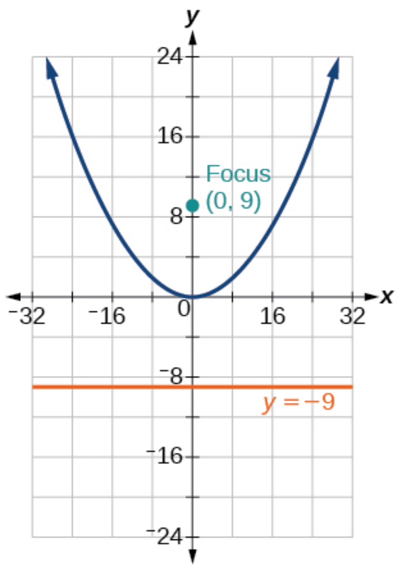
167.

$$(y-1)^2 = \frac{4}{3}(x-5),\ V:(5,\ 1);\ F:\left(\frac{16}{3},\ 1\right);\ d:x = \frac{14}{3}$$

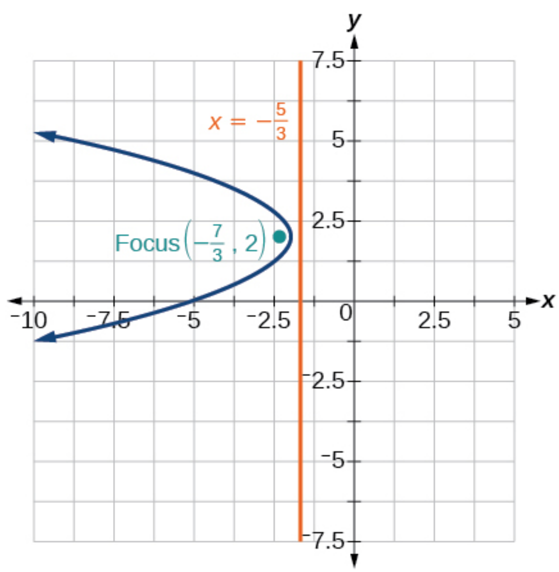
169.


y
x
x = −2
Focus (2, 0)
7.5
5
2.5
0
−2.5
−5
−7.5
2.5
5
7.5

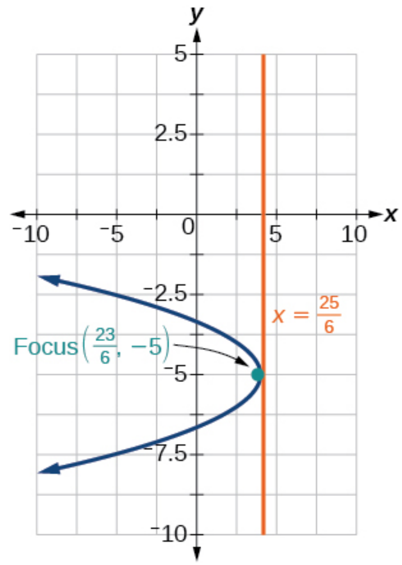
171.


y
x
Focus
(0, 9)
y = −9
24
16
8
0
−8
−16
−24
−32
−16
16
32

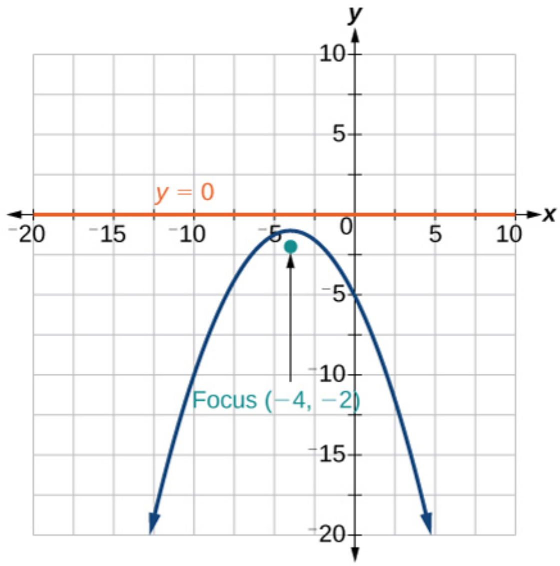
173.


y
x
x = −5/3
Focus (−7/3, 2)
7.5
5
2.5
0
−2.5
−5
−7.5
−10
−5
−2.5
2.5
5

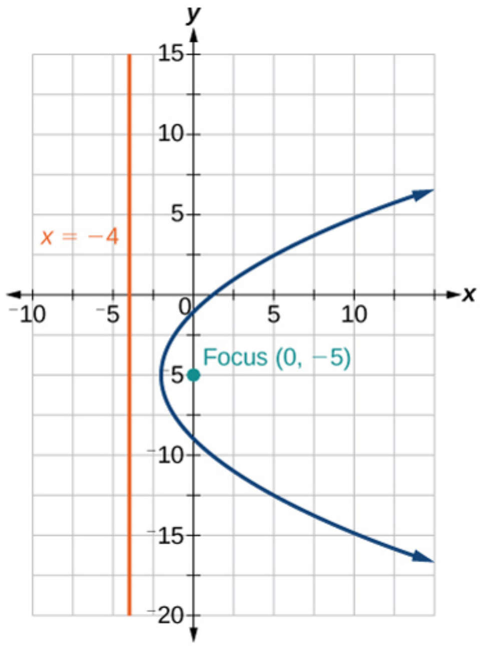
175.


y
5
2.5
−10
−5
0
5
10
x
−2.5
$x = \frac{25}{6}$
Focus$\left(\frac{23}{6}, -5\right)$
−5
−7.5
−10

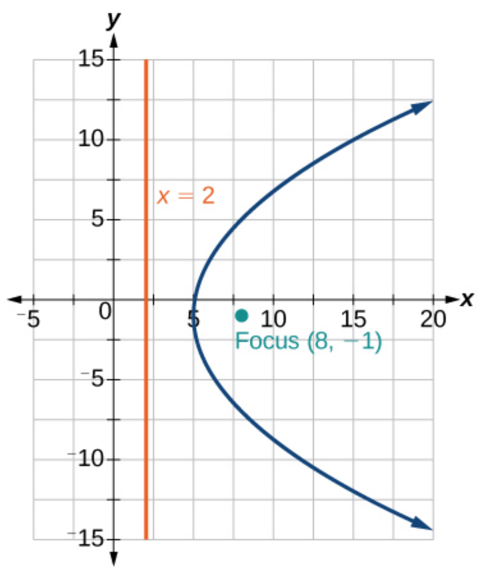
177.


y
10
5
y = 0
−20
−15
−10
−5
0
5
10
x
−5
−10
Focus (−4, −2)
−15
−20

179.



181.



183.

$x^2 = -16y$

185.

$(y-2)^2 = 4\sqrt{2}(x-2)$

187.

$\left(y+\sqrt{3}\right)^2 = -4\sqrt{2}\left(x-\sqrt{2}\right)$

189.

$x^2 = y$

191.

$(y-2)^2 = \frac{1}{4}(x+2)$

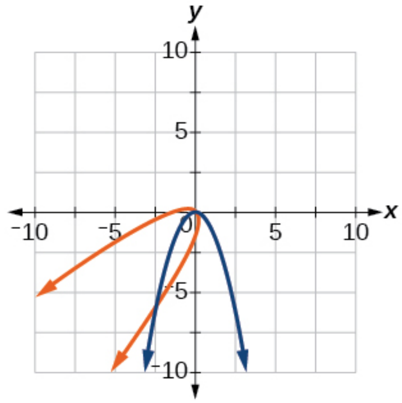
193.

$\left(y-\sqrt{3}\right)^2 = 4\sqrt{5}\left(x+\sqrt{2}\right)$

195.

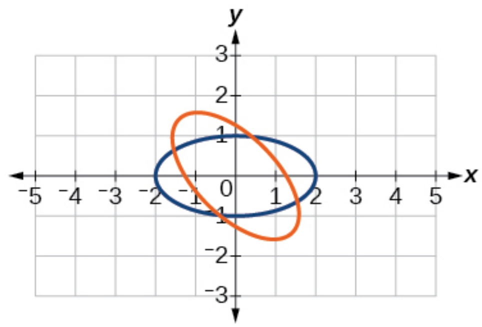
$y^2 = -8x$

197.
$(y+1)^2 = 12(x+3)$
199.
$(0,\ 1)$
201. At the point 2.25 feet above the vertex.
203. 0.5625 feet
205.
$x^2 = -125(y-20),$ height is 7.2 feet
207. 2304 feet
209. The xy term causes a rotation of the graph to occur.
211. The conic section is a hyperbola.
213. It gives the angle of rotation of the axes in order to eliminate the xy term.
215.
$AB = 0,$ parabola
217.
$AB = -4 < 0,$ hyperbola
219.
$AB = 6 > 0,$ ellipse
221.
$B^2 - 4AC = 0,$ parabola
223.
$B^2 - 4AC = 0,$ parabola
225.
$B^2 - 4AC = -96 < 0,$ ellipse
227.
$7x'^2 + 9y'^2 - 4 = 0$
229.
$3x'^2 + 2x'y' - 5y'^2 + 1 = 0$
231.
$\theta = 60°,\ 11x'^2 - y'^2 + \sqrt{3}x' + y' - 4 = 0$
233.
$\theta = 150°,\ 21x'^2 + 9y'^2 + 4x' - 4\sqrt{3}y' - 6 = 0$
235.
$\theta \approx 36.9°,\ 125x'^2 + 6x' - 42y' + 10 = 0$
237.
$\theta = 45°,\ 3x'^2 - y'^2 - \sqrt{2}x' + \sqrt{2}y' + 1 = 0$
239.
$\frac{\sqrt{2}}{2}(x' + y') = \frac{1}{2}(x' - y')^2$



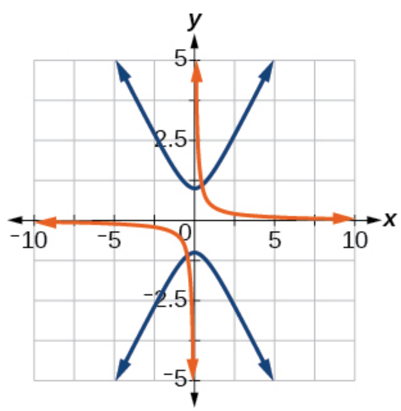
241.

$$\frac{(x' - y')^2}{8} + \frac{(x' + y')^2}{2} = 1$$



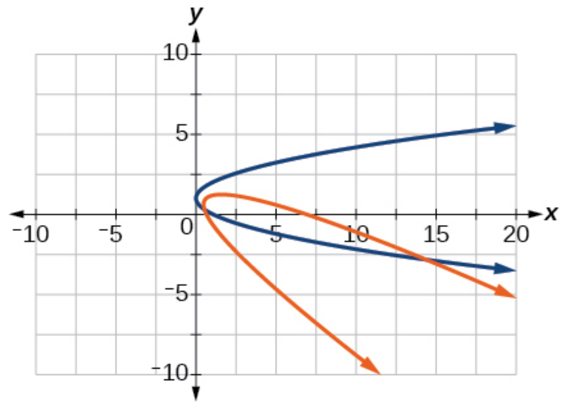
243.

$$\frac{(x' + y')^2}{2} - \frac{(x' - y')^2}{2} = 1$$

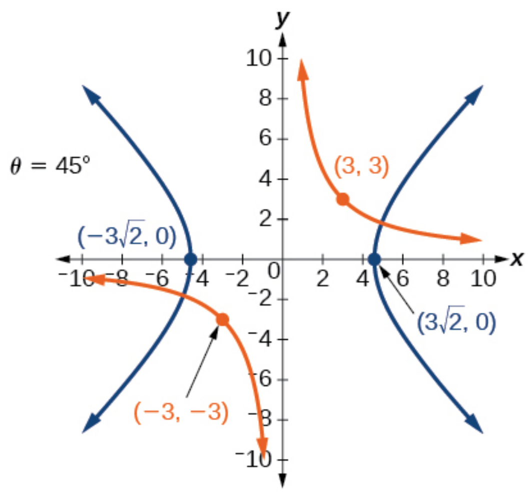


245.

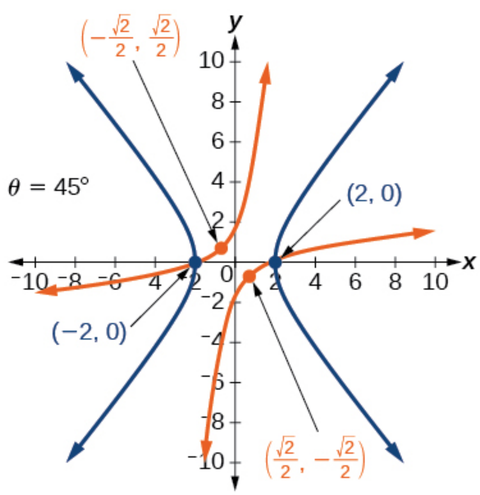
$$\frac{\sqrt{3}}{2}x' - \frac{1}{2}y' = \left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' - 1\right)^2$$


y
10
5
−5
−10
x
−10
−5
0
5
10
15
20

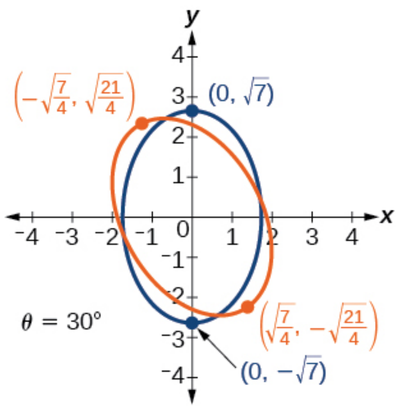
247.


y
x
θ = 45°
(3, 3)
(−3√2, 0)
(3√2, 0)
(−3, −3)

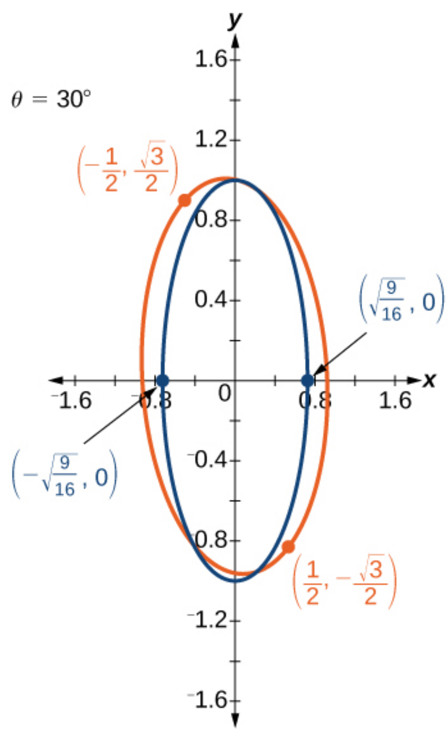
249.


y
x
θ = 45°
(−√2/2, √2/2)
(2, 0)
(−2, 0)
(√2/2, −√2/2)

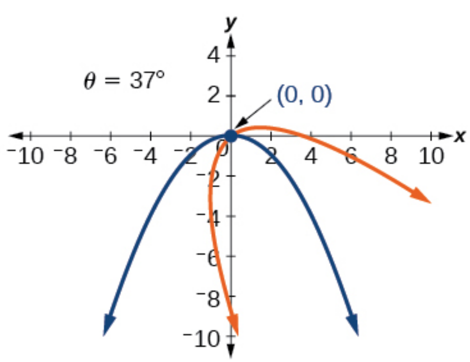
251.


$\left(-\sqrt{\frac{7}{4}}, \sqrt{\frac{21}{4}}\right)$
$(0, \sqrt{7})$
$\theta = 30°$
$\left(\sqrt{\frac{7}{4}}, -\sqrt{\frac{21}{4}}\right)$
$(0, -\sqrt{7})$

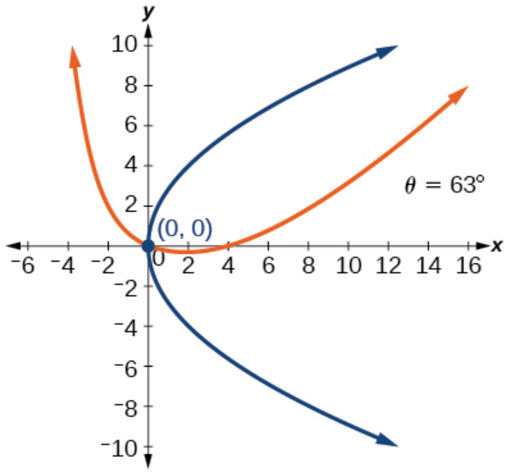
253.


$\theta = 30°$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\sqrt{\frac{9}{16}}, 0\right)$
$\left(-\sqrt{\frac{9}{16}}, 0\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

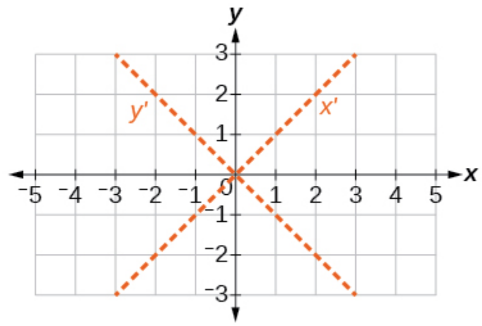
255.


$\theta = 37°$
(0, 0)

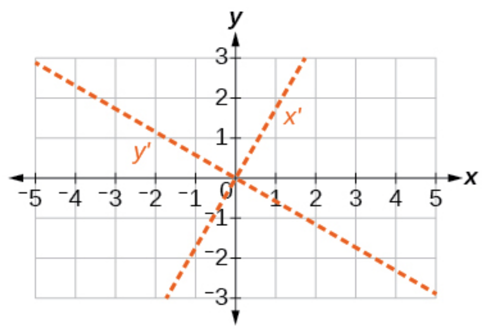
257.



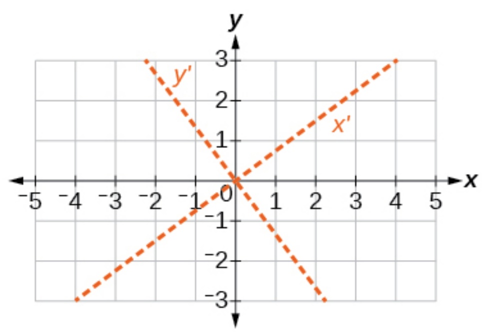
259.
$\theta = 45^\circ$



261.
$\theta = 60^\circ$



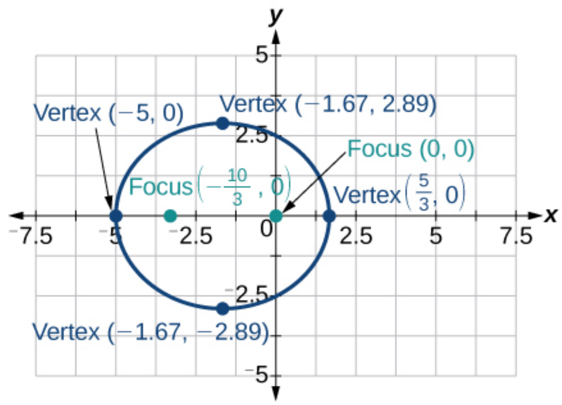
263.
$\theta \approx 36.9^\circ$



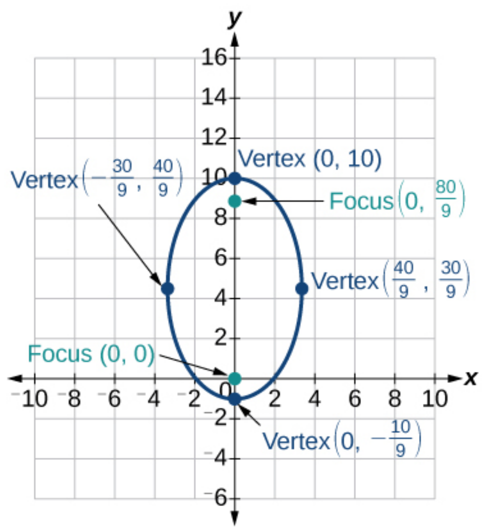
265.
$-4\sqrt{6} < k < 4\sqrt{6}$

267.
$k = 2$

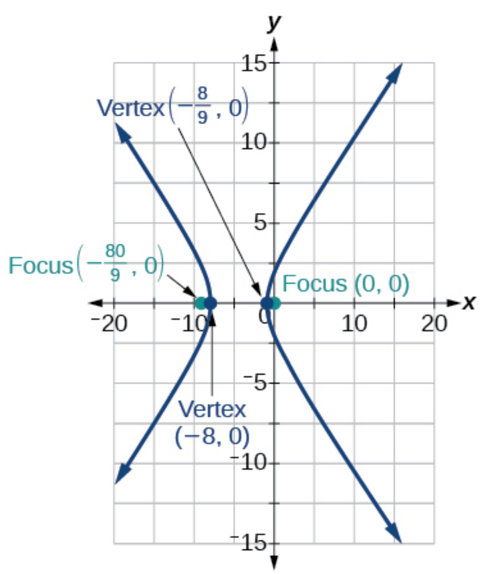
269. If eccentricity is less than 1, it is an ellipse. If eccentricity is equal to 1, it is a parabola. If eccentricity is greater than 1, it is a hyperbola.
271. The directrix will be parallel to the polar axis.
273. One of the foci will be located at the origin.
275. Parabola with
$e = 1$ and directrix
$\frac{3}{4}$ units below the pole.
277. Hyperbola with
$e = 2$ and directrix
$\frac{5}{2}$ units above the pole.
279. Parabola with
$e = 1$ and directrix
$\frac{3}{10}$ units to the right of the pole.
281. Ellipse with
$e = \frac{2}{7}$ and directrix
2 units to the right of the pole.
283. Hyperbola with
$e = \frac{5}{3}$ and directrix
$\frac{11}{5}$ units above the pole.
285. Hyperbola with
$e = \frac{8}{7}$ and directrix
$\frac{7}{8}$ units to the right of the pole.
287.
$25x^2 + 16y^2 - 12y - 4 = 0$
289.
$21x^2 - 4y^2 - 30x + 9 = 0$
291.
$64y^2 = 48x + 9$
293.
$96y^2 - 25x^2 + 110y + 25 = 0$
295.
$3x^2 + 4y^2 - 2x - 1 = 0$
297.
$5x^2 + 9y^2 - 24x - 36 = 0$
299.


Vertex (−5, 0)
Vertex (−1.67, 2.89)
Focus (0, 0)
Focus (−10/3, 0)
Vertex (5/3, 0)
Vertex (−1.67, −2.89)

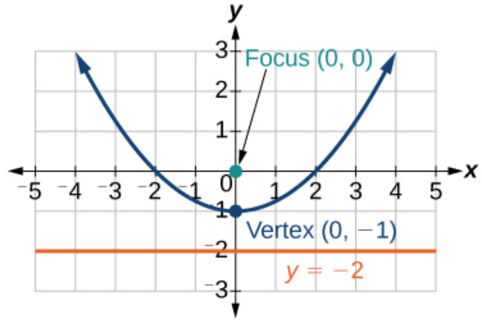
301.


Vertex (0, 10)
Vertex (−30/9, 40/9)
Focus (0, 80/9)
Vertex (40/9, 30/9)
Focus (0, 0)
Vertex (0, −10/9)

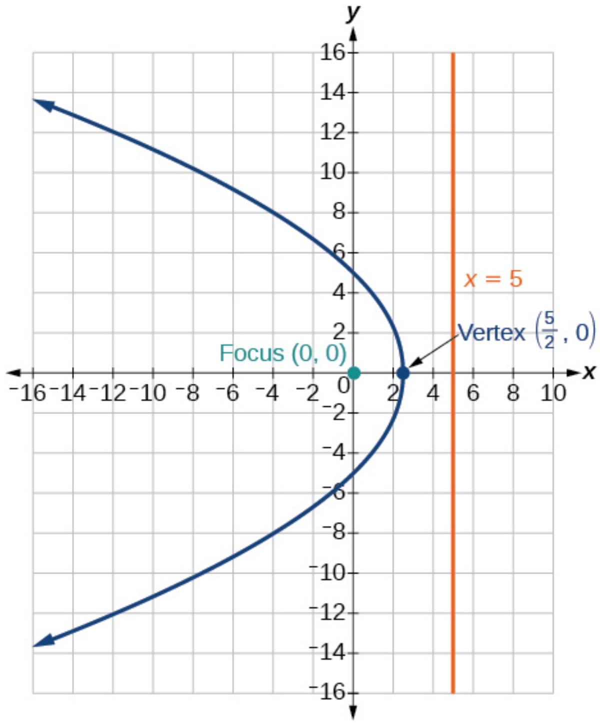
303.


Vertex (−8/9, 0)
Focus (−80/9, 0)
Focus (0, 0)
Vertex (−8, 0)

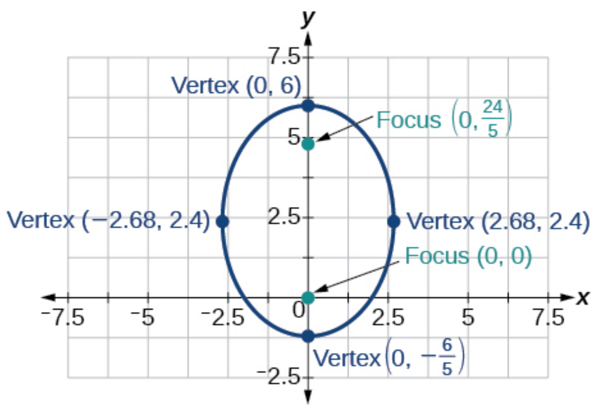
305.



307.



309.



311.

$$r = \frac{4}{5 + \cos\theta}$$

313.

$$r = \frac{4}{1 + 2\sin\theta}$$

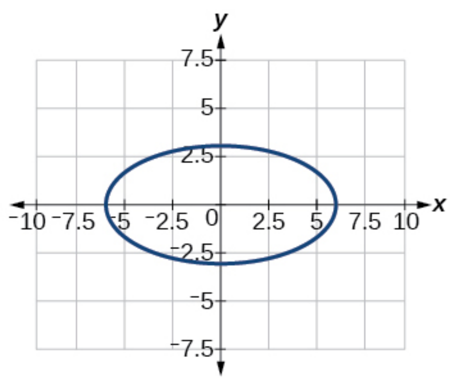
315.

$$r = \frac{1}{1 + \cos\theta}$$

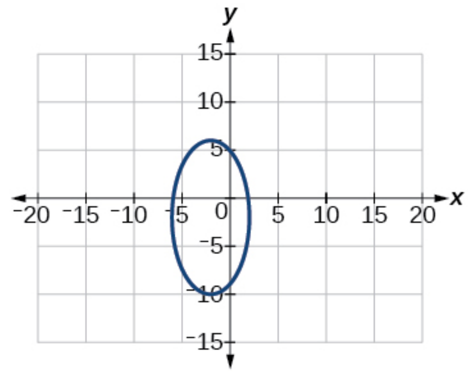
317.
$r=\frac{7}{8-28\cos\theta}$
319.
$r=\frac{12}{2+3\sin\theta}$
321.
$r=\frac{15}{4-3\cos\theta}$
323.
$r=\frac{3}{3-3\cos\theta}$
325.
$r=\pm\frac{2}{\sqrt{1+\sin\theta\cos\theta}}$
327.
$r=\pm\frac{2}{4\cos\theta+3\sin\theta}$

Review Exercises

329.
$\frac{x^2}{5^2}+\frac{y^2}{8^2}=1;$ center: $(0, 0);$ vertices: $(5, 0), (-5, 0), (0, 8), (0, -8);$ foci: $\left(0, \sqrt{39}\right), \left(0, -\sqrt{39}\right)$
331.
$\frac{(x+3)^2}{1^2}+\frac{(y-2)^2}{3^2}=1\ (-3, 2);\ (-2, 2), (-4, 2), (-3, 5), (-3, -1);\ \left(-3, 2+2\sqrt{2}\right), \left(-3, 2-2\sqrt{2}\right)$
333. center: $(0, 0);$ vertices: $(6, 0), (-6, 0), (0, 3), (0, -3);$ foci: $\left(3\sqrt{3}, 0\right), \left(-3\sqrt{3}, 0\right)$



335. center: $(-2, -2);$ vertices: $(2, -2), (-6, -2), (-2, 6), (-2, -10);$ foci: $\left(-2, -2+4\sqrt{3}, \right), \left(-2, -2-4\sqrt{3}\right)$



337.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

339. Approximately 35.71 feet

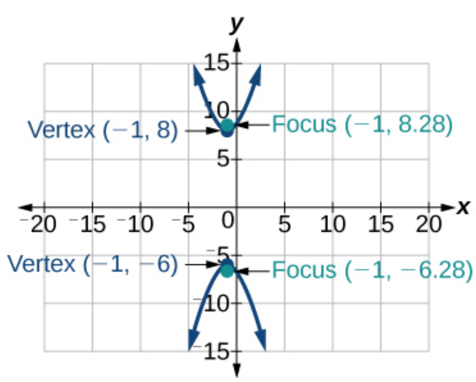
341.

$$\frac{(y+1)^2}{4^2} - \frac{(x-4)^2}{6^2} = 1;$$ center: $(4, -1);$ vertices: $(4, 3), (4, -5);$ foci: $\left(4, -1 + 2\sqrt{13}\right), \left(4, -1 - 2\sqrt{13}\right)$

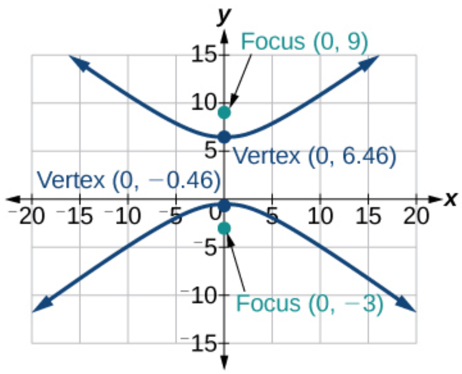
343.

$$\frac{(x-2)^2}{2^2} - \frac{(y+3)^2}{\left(2\sqrt{3}\right)^2} = 1;$$ center: $(2, -3);$ vertices: $(4, -3), (0, -3);$ foci: $(6, -3), (-2, -3)$

345.



347.



349.

$$\frac{(x-5)^2}{1} - \frac{(y-7)^2}{3} = 1$$

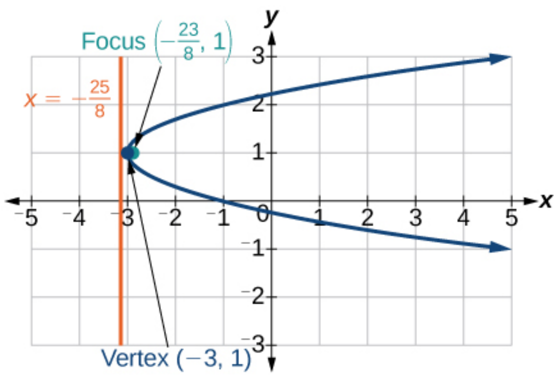
351.

$(x+2)^2 = \frac{1}{2}(y-1);$ vertex: $(-2,\ 1);$ focus: $\left(-2,\ \frac{9}{8}\right);$ directrix: $y = \frac{7}{8}$

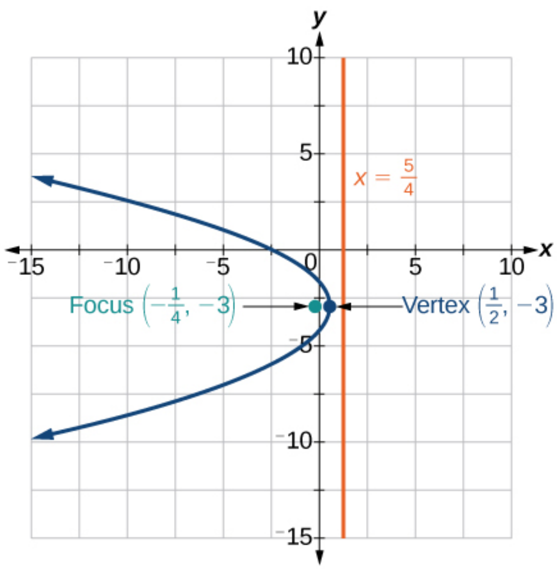
353.

$(x+5)^2 = (y+2);$ vertex: $(-5,\ -2);$ focus: $\left(-5,\ -\frac{7}{4}\right);$ directrix: $y = -\frac{9}{4}$

355.



357.



359.

$(x-2)^2 = \left(\frac{1}{2}\right)(y-1)$

361.

$B^2 - 4AC = 0,$ parabola

363.

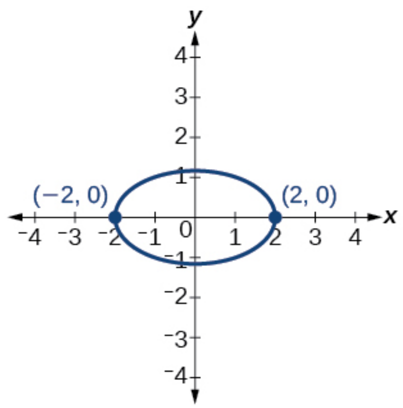
$B^2 - 4AC = -31 < 0,$ ellipse

365.

$\theta = 45°, \; x'^2 + 3y'^2 - 12 = 0$

367.

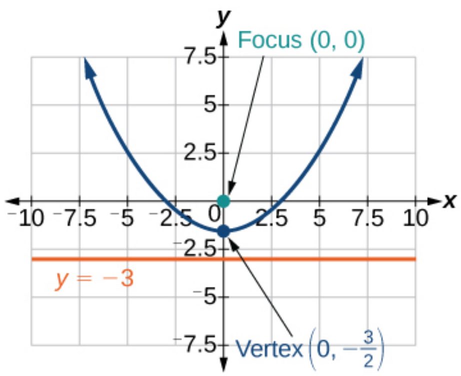
$\theta = 45°$



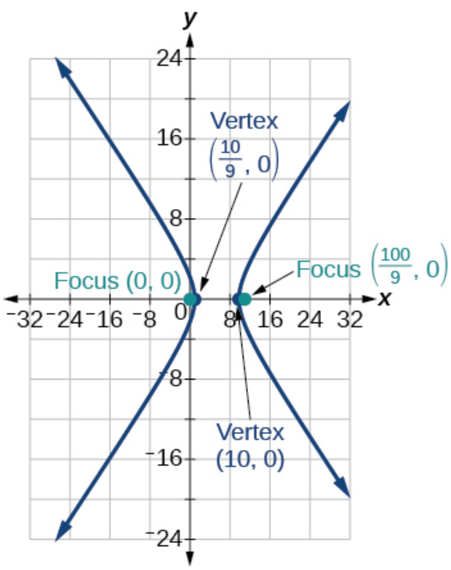
369. Hyperbola with $e = 5$ and directrix 2 units to the left of the pole.

371. Ellipse with $e = \frac{3}{4}$ and directrix $\frac{1}{3}$ unit above the pole.

373.



375.



377.

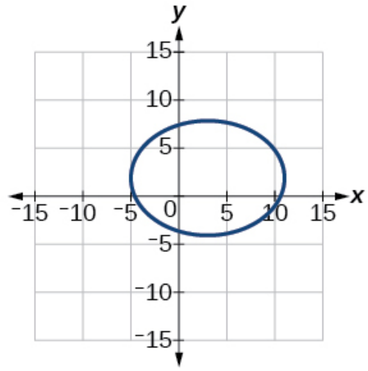
$r = \frac{3}{1 + \cos \quad \theta}$

Practice Test

379.

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1;$ center: $(0, 0);$ vertices: $(3, 0), (–3, 0), (0, 2), (0, -2);$ foci: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$

381. center: $(3, 2);$ vertices: $(11, 2), (-5, 2), (3, 8), (3, -4);$ foci: $(3 + 2\sqrt{7}, 2), (3 - 2\sqrt{7}, 2)$



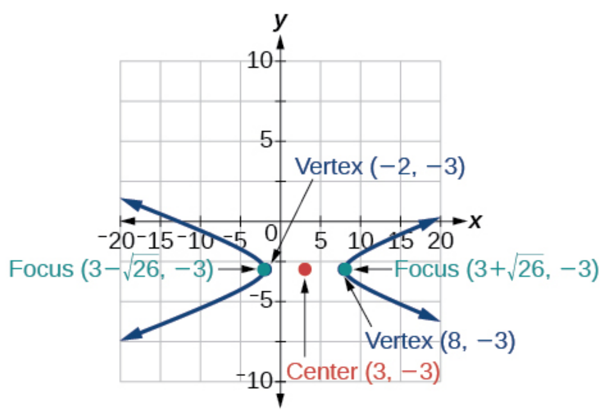
383.

$$\frac{(x-1)^2}{36}+\frac{(y-2)^2}{27}=1$$

385.

$\frac{x^2}{7^2}-\frac{y^2}{9^2}=1;$ center: $(0,\ 0);$ vertices $(7,\ 0),\ (-7,\ 0);$ foci: $(\sqrt{130},\ 0),\ (-\sqrt{130},\ 0);$ asymptotes: $y=\ \pm\frac{9}{7}x$

387. center: $(3,\ -3);$ vertices: $(8,\ -3),\ (-2,\ -3);$ foci: $(3+\sqrt{26},\ -3),\ (3-\sqrt{26},\ -3);$ asymptotes: $y=\ \pm\frac{1}{5}(x-3)-3$



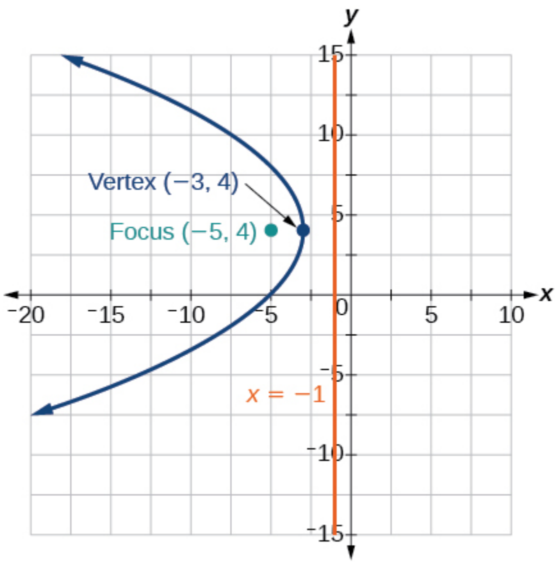
389.

$$\frac{(y-3)^2}{1}-\frac{(x-1)^2}{8}=1$$

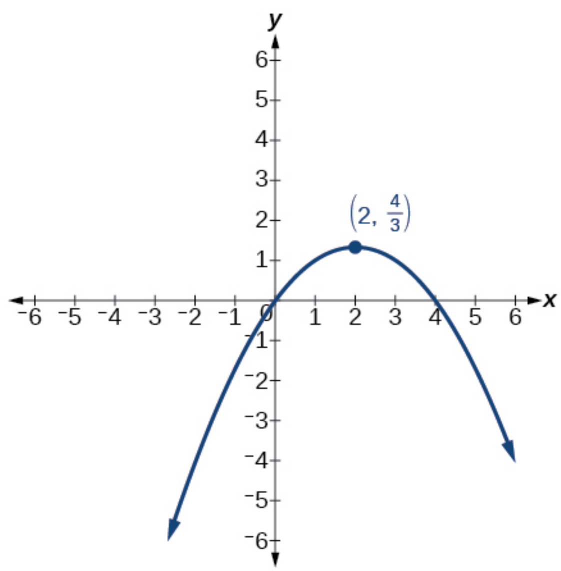
391.

$(x-2)^2=\frac{1}{3}(y+1);$ vertex: $(2,\ -1);$ focus: $\left(2,\ -\frac{11}{12}\right);$ directrix: $y=\ -\frac{13}{12}$

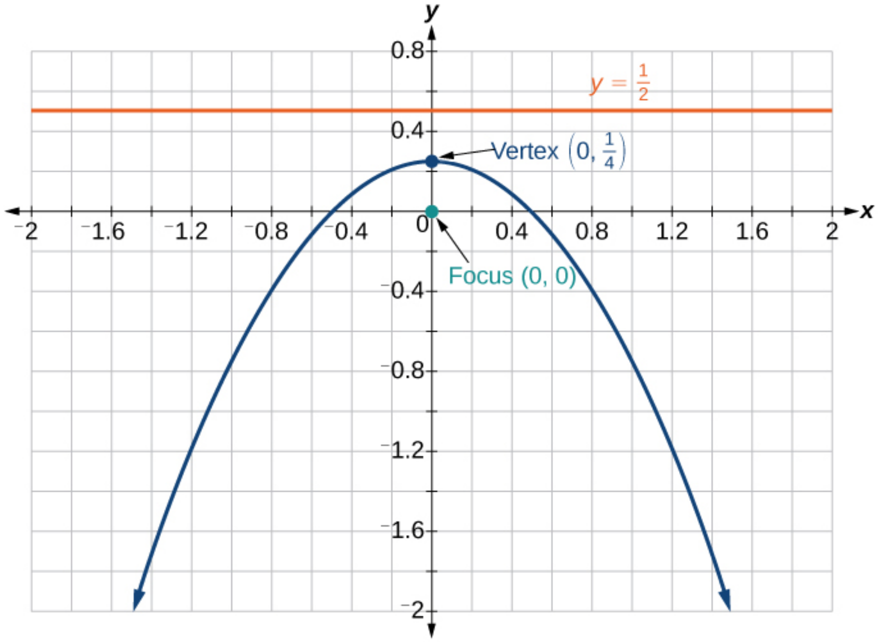
393.



395. Approximately
8.49 feet
397. parabola;
$\theta \approx 63.4°$
399.
$x'^2 - 4x' + 3y' = 0$



401. Hyperbola with
$e = \frac{3}{2},$ and directrix
$\frac{5}{6}$ units to the right of the pole.
403.



Chapter 11

Try It

11.1. The first five terms are
$\{1, 6, \quad 11, \quad 16, \quad 21\}.$

11.2. The first five terms are
$\left\{-2, \quad 2, \quad -\frac{3}{2}, \quad 1, \quad -\frac{5}{8}\right\}.$

11.3. The first six terms are
$\{2, \ 5, \ 54, \ 10, \ 250, \ 15\}.$

11.4.
$a_n = (-1)^{n+1} 9^n$

11.5.
$a_n = -\frac{3^n}{4n}$

11.6.
$a_n = e^{n-3}$

11.7.
$\{2, \quad 5, \quad 11, \quad 23, \quad 47\}$

11.8.
$\left\{0, \quad 1, \quad 1, \quad 1, \quad 2, \quad 3, \quad \frac{5}{2}, \ \frac{17}{6}\right\}.$

11.9. The first five terms are
$\left\{1, \quad \frac{3}{2}, \quad 4, \ 15, \ 72\right\}.$

11.10. The sequence is arithmetic. The common difference is
$-2.$

11.11. The sequence is not arithmetic because
$3 - 1 \neq 6 - 3.$

11.12.
$\{1, \quad 6, \quad 11, \quad 16, \quad 21\}$

11.13.
$a_2 = 2$

11.14.
$a_1 = 25$
$a_n = a_{n-1} + 12, \text{ for } n \geq 2$
11.15.
$a_n = 53 - 3n$
11.16. There are 11 terms in the sequence.
11.17. The formula is
$T_n = 10 + 4n,$ and it will take her 42 minutes.
11.18. The sequence is not geometric because
$\frac{10}{5} \neq \frac{15}{10}$.
11.19. The sequence is geometric. The common ratio is
$\frac{1}{5}$.
11.20.
$\left\{18,\ 6,\ 2,\ \frac{2}{3},\ \frac{2}{9}\right\}$
11.21.
$a_1 = 2$
$a_n = \frac{2}{3}a_{n-1} \text{ for } n \geq 2$

11.22.
$a_6 = 16,\ 384$
11.23.
$a_n = -(-3)^{n-1}$
11.24.
a.
$P_n \quad = \quad 293 \cdot 1.026a^n$
b. The number of hits will be about 333.
11.25. 38
11.26.
26.4
11.27.
328
11.28.
-280
11.29. $2,025
11.30.
$\approx 2,\ 000.00$
11.31. 9,840
11.32. $275,513.31
11.33. The sum is defined. It is geometric.
11.34. The sum of the infinite series is defined.
11.35. The sum of the infinite series is defined.
11.36. 3
11.37. The series is not geometric.
11.38.
$-\frac{3}{11}$
11.39. $92,408.18
11.40. 7
11.41. There are 60 possible breakfast specials.
11.42. 120
11.43. 60
11.44. 12

11.45.
$P(7, 7) = 5, 040$

11.46.
$P(7, 5) = 2, 520$

11.47.
$C(10, 3) = 120$

11.48. 64 sundaes
11.49. 840
11.50.
a. 35
b. 330

11.51.
a.
$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
b.
$8x^3 + 60x^2y + 150xy^2 + 125y^3$

11.52.
$-10, 206x^4y^5$

11.53.

Outcome	Probability
Roll of 1	
Roll of 2	
Roll of 3	
Roll of 4	
Roll of 5	
Roll of 6	

11.54.
$\frac{2}{3}$

11.55.
$\frac{7}{13}$

11.56.
$\frac{2}{13}$

11.57.
$\frac{5}{6}$

11.58.
a. $\frac{1}{91}$; b. $\frac{5}{91}$; c. $\frac{86}{91}$

Section Exercises

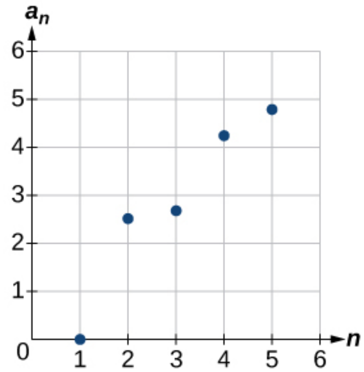
1. A sequence is an ordered list of numbers that can be either finite or infinite in number. When a finite sequence is defined by a formula, its domain is a subset of the non-negative integers. When an infinite sequence is defined by a formula, its domain is all positive or all non-negative integers.

3. Yes, both sets go on indefinitely, so they are both infinite sequences.
5. A factorial is the product of a positive integer and all the positive integers below it. An exclamation point is used to indicate the operation. Answers may vary. An example of the benefit of using factorial notation is when indicating the product It is much easier to write than it is to write out
$13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1.$
7. First four terms:
$-8, \quad -\frac{16}{3}, \quad -4, \quad -\frac{16}{5}$
9. First four terms:
$2, \ \frac{1}{2}, \ \frac{8}{27}, \ \frac{1}{4}.$
11. First four terms:
$1.25, \quad -5, \ 20, \quad -80.$
13. First four terms:
$\frac{1}{3}, \ \frac{4}{5}, \ \frac{9}{7}, \ \frac{16}{9}.$
15. First four terms:
$-\frac{4}{5}, \ 4, \quad -20, \ 100$
17.
$\frac{1}{3}, \ \frac{4}{5}, \ \frac{9}{7}, \ \frac{16}{9}, \ \frac{25}{11}, \ 31, \ 44, \ 59$
19.
$-0.6, \ -3, \ -15, \ -20, \ -375, \ -80, \ -9375, \ -320$
21.
$a_n = n^2 + 3$
23.
$a_n = \frac{2^n}{2n} \text{ or } \frac{2^{n-1}}{n}$
25.
$a_n = \left(-\frac{1}{2}\right)^{n-1}$
27. First five terms:
$3, \quad -9, \ 27, \quad -81, \ 243$
29. First five terms:
$-1, \ 1, \quad -9, \ \frac{27}{11}, \ \frac{891}{5}$
31.
$\frac{1}{24}, \ 1, \frac{1}{4}, \ \frac{3}{2}, \ \frac{9}{4}, \ \frac{81}{4}, \ \frac{2187}{8}, \ \frac{531,441}{16}$
33.
$2, \ 10, \ 12, \ \frac{14}{5}, \ \frac{4}{5}, \ 2, \ 10, \ 12$
35.
$a_1 = -8, \ a_n = a_{n-1} + n$
37.
$a_1 = 35, \ a_n = a_{n-1} + 3$
39.
720
41.
$665,280$
43. First four terms:
$1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}$

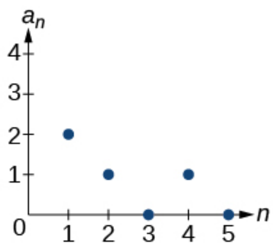
45. First four terms:

$-1, 2, \frac{6}{5}, \frac{24}{11}$

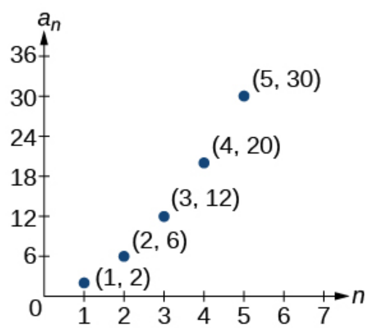
47.



49.



51.



53.

$a_n = 2^{n-2}$

55.

$a_1 = 6, \ a_n = 2a_{n-1} - 5$

57. First five terms:

$\frac{29}{37}, \frac{152}{111}, \frac{716}{333}, \frac{3188}{999}, \frac{13724}{2997}$

59. First five terms:

$2, 3, 5, 17, 65537$

61.

$a_{10} = 7, 257, 600$

63. First six terms:

$0.042, 0.146, 0.875, 2.385, 4.708$

65. First four terms:

$5.975, 32.765, 185.743, 1057.25, 6023.521$

67. If

$a_n = -421$ is a term in the sequence, then solving the equation

$-421 = -6 - 8n$ for

n will yield a non-negative integer. However, if

$-421 = -6 - 8n,$ then
$n = 51.875$ so
$a_n = -421$ is not a term in the sequence.

69.
$a_1 = 1, a_2 = 0, a_n = a_{n-1} - a_{n-2}$

71.
$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)\cdot(n+1)\cdot(n)\cdot(n-1)\cdot...\cdot3\cdot2\cdot1}{(n-1)\cdot...\cdot3\cdot2\cdot1} = n(n+1)(n+2) = n^3 + 3n^2 + 2n$

72. A sequence where each successive term of the sequence increases (or decreases) by a constant value.

74. We find whether the difference between all consecutive terms is the same. This is the same as saying that the sequence has a common difference.

76. Both arithmetic sequences and linear functions have a constant rate of change. They are different because their domains are not the same; linear functions are defined for all real numbers, and arithmetic sequences are defined for natural numbers or a subset of the natural numbers.

78. The common difference is
$\frac{1}{2}$

80. The sequence is not arithmetic because
$16 - 4 \neq 64 - 16.$

82.
$0, \frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}$

84.
$0, -5, -10, -15, -20$

86.
$a_4 = 19$

88.
$a_6 = 41$

90.
$a_1 = 2$

92.
$a_1 = 5$

94.
$a_1 = 6$

96.
$a_{21} = -13.5$

98.
$-19, -20.4, -21.8, -23.2, -24.6$

100.
$a_1 = 17; \quad a_n = a_{n-1} + 9 \;\; n \geq 2$

102.
$a_1 = 12; \quad a_n = a_{n-1} + 5 \;\; n \geq 2$

104.
$a_1 = 8.9; \quad a_n = a_{n-1} + 1.4 \;\; n \geq 2$

106.
$a_1 = \frac{1}{5}; \quad a_n = a_{n-1} + \frac{1}{4} \;\; n \geq 2$

108.
$_1 = \frac{1}{6}; \quad a_n = a_{n-1} - \frac{13}{12} \;\; n \geq 2$

110.
$a_1 = 4; \; a_n = a_{n-1} + 7; \; a_{14} = 95$

112. First five terms:
$20, 16, 12, 8, 4.$

114.
$a_n = 1 + 2n$

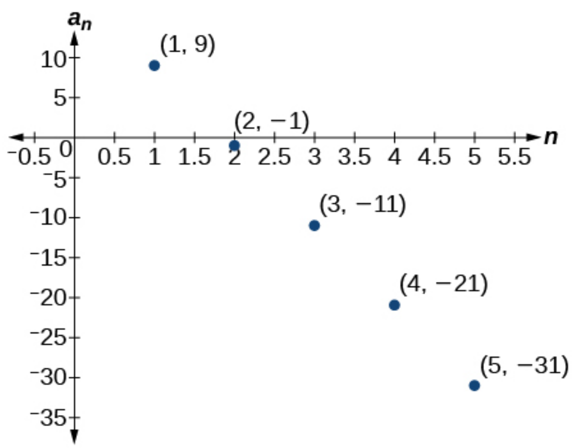
116.
$a_n = -105 + 100n$

118.
$a_n = 1.8n$

120.
$a_n = 13.1 + 2.7n$

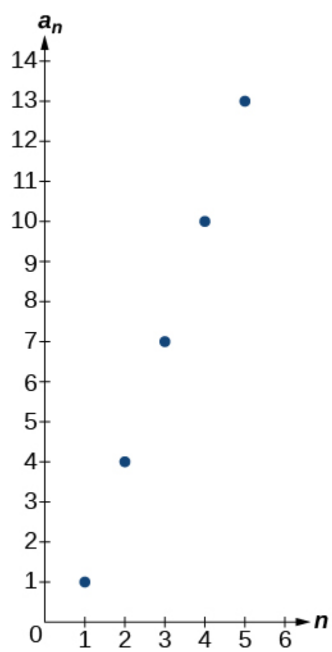
122.
$a_n = \frac{1}{3}n - \frac{1}{3}$

124. There are 10 terms in the sequence.
126. There are 6 terms in the sequence.
128. The graph does not represent an arithmetic sequence.
130.

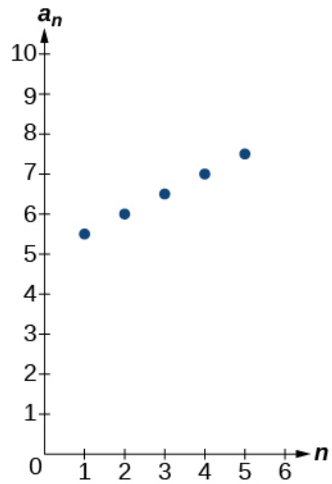


132.
1, 4, 7, 10, 13, 16, 19

134.



136.



138. Answers will vary. Examples:
$a_n = 20.6n$ and
$a_n = 2 + 20.4\text{n}.$

140.
$a_{11} = -17a + 38b$

142. The sequence begins to have negative values at the 13th term,
$a_{13} = -\frac{1}{3}$

144. Answers will vary. Check to see that the sequence is arithmetic. Example: Recursive formula:
$a_1 = 3,\ a_n = a_{n-1} - 3.$ First 4 terms:
$3,\ 0,\ -3,\ -6\ \ a_{31} = -87$

146. A sequence in which the ratio between any two consecutive terms is constant.
148. Divide each term in a sequence by the preceding term. If the resulting quotients are equal, then the sequence is geometric.
150. Both geometric sequences and exponential functions have a constant ratio. However, their domains are not the same. Exponential functions are defined for all real numbers, and geometric sequences are defined only for positive integers. Another difference is that the base of a geometric sequence (the common ratio) can be negative, but the base of an exponential function must be positive.
152. The common ratio is
-2

154. The sequence is geometric. The common ratio is 2.
156. The sequence is geometric. The common ratio is
$-\frac{1}{2}.$

158. The sequence is geometric. The common ratio is
$5.$

160.
$5,\ 1,\ \frac{1}{5},\ \frac{1}{25},\ \frac{1}{125}$

162.
$800,\ 400,\ 200,\ 100,\ 50$

164.
$a_4 = -\frac{16}{27}$

166.
$a_7 = -\frac{2}{729}$

168.
$7,\ 1.4,\ 0.28,\ 0.056,\ 0.0112$

170.
$a = -32,\quad a_n = \frac{1}{2}a_{n-1}$

172.
$a_1 = 10, \quad a_n = -0.3a_{n-1}$

174.
$a_1 = \frac{3}{5}, \quad a_n = \frac{1}{6}a_{n-1}$

176.
$a_1 = \frac{1}{512}, \ a_n = -4a_{n-1}$

178.
$12, \ -6, \ 3, \ -\frac{3}{2}, \ \frac{3}{4}$

180.
$a_n = 3^{n-1}$

182.
$a_n = 0.8 \cdot (-5)^{n-1}$

184.
$a_n = -\left(\frac{4}{5}\right)^{n-1}$

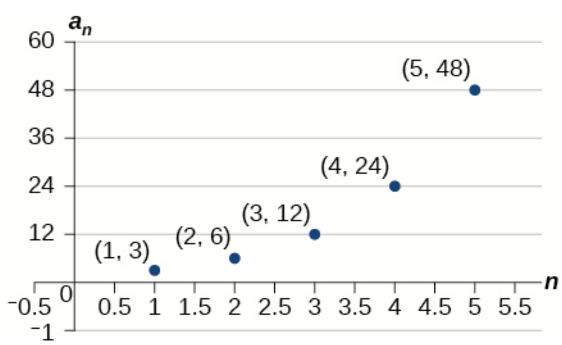
186.
$a_n = 3 \cdot \left(-\frac{1}{3}\right)^{n-1}$

188.
$a_{12} = \frac{1}{177,\ 147}$

190. There are
12 terms in the sequence.

192. The graph does not represent a geometric sequence.

194.



196. Answers will vary. Examples:
$a_1 = 800, \quad a_n = 0.5a_{n-1}$ and
$a_1 = 12.5, \quad a_n = 4a_{n-1}$

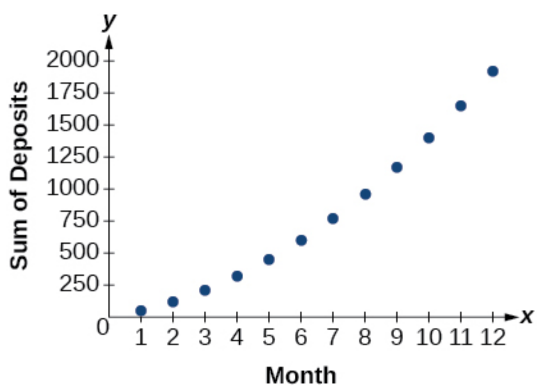
198.
$a_5 = 256b$

200. The sequence exceeds
100 at the 14^{th} term,
$a_{14} \approx 107.$

202.
$a_4 = -\frac{32}{3}$ is the first non-integer value

204. Answers will vary. Example: Explicit formula with a decimal common ratio:
$a_n = 400 \cdot 0.5^{n-1};$ First 4 terms:
$400, \ 200, \ 100, \ 50; \ a_8 = 3.125$

206. An nth partial sum is the sum of the first n terms of a sequence.
208. A geometric series is the sum of the terms in a geometric sequence.
210. An annuity is a series of regular equal payments that earn a constant compounded interest.
212.
$$\sum_{n=0}^{4} 5n$$
214.
$$\sum_{k=1}^{5} 4$$
216.
$$\sum_{k=1}^{20} 8k+2$$
218.
$$S_5 = \frac{5\left(\frac{3}{2}+\frac{7}{2}\right)}{2}$$
220.
$$S_{13} = \frac{13(3.2+5.6)}{2}$$
222.
$$\sum_{k=1}^{7} 8 \cdot 0.5^{k-1}$$
224.
$$S_5 = \frac{9\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}} = \frac{121}{9} \approx 13.44$$
226.
$$S_{11} = \frac{64\left(1-0.2^{11}\right)}{1-0.2} = \frac{781,249,984}{9,765,625} \approx 80$$
228. The series is defined.
$$S = \frac{2}{1-0.8}$$
230. The series is defined.
$$S = \frac{-1}{1-\left(-\frac{1}{2}\right)}$$
232.



234. Sample answer: The graph of S_n seems to be approaching 1. This makes sense because

$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$ is a defined infinite geometric series with

$$S = \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)} = 1.$$

236. 49
238. 254
240.

$$S_7 = \frac{147}{2}$$

242.

$$S_{11} = \frac{55}{2}$$

244.

$$S_7 = 5208.4$$

246.

$$S_{10} = -\frac{1023}{256}$$

248.

$$S = -\frac{4}{3}$$

250.

$$S = 9.2$$

252. \$3,705.42
254. \$695,823.97
256.

$$a_k = 30 - k$$

258. 9 terms
260.

$$r = \frac{4}{5}$$

262. \$400 per month
264. 420 feet
266. 12 feet
268. There are $m + n$ ways for either event A or event B to occur.

270. The addition principle is applied when determining the total possible of outcomes of either event occurring. The multiplication principle is applied when determining the total possible outcomes of both events occurring. The word “or” usually implies an addition problem. The word “and” usually implies a multiplication problem.

272. A combination;

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

274.

$$4 + 2 = 6$$

276.

$$5 + 4 + 7 = 16$$

278.

$$2 \times 6 = 12$$

280.

$$10^3 = 1000$$

282.

$$P(5, 2) = 20$$

284.

$$P(3, 3) = 6$$

286.
$P(11,\ 5) = 55,\ 440$

288.
$C(12,\ 4) = 495$

290.
$C(7,\ 6) = 7$

292.
$2^{10} = 1024$

294.
$2^{12} = 4096$

296.
$2^{9} = 512$

298.
$\frac{8\,!}{3\,!} = 6720$

300.
$\frac{12\,!}{3\,!2\,!3\,!4\,!}$

302. 9

304. Yes, for the trivial cases
$r = 0$ and
$r = 1.$ If
$r = 0,$ then
$C(n,\ r) = P(n,\ r) = 1.$ If
$r = 1,$ then
$r = 1,$
$C(n,\ r) = P(n,\ r) = n.$

306.
$\frac{6\,!}{2\,!} \times 4\,! = 8640$

308.
$6 - 3 + 8 - 3 = 8$

310.
$4 \times 2 \times 5 = 40$

312.
$4 \times 12 \times 3 = 144$

314.
$P(15,\ 9) = 1,\ 816,\ 214,\ 400$

316.
$C(10,\ 3) \times C(6,\ 5) \times C(5,\ 2) = 7,\ 200$

318.
$2^{11} = 2048$

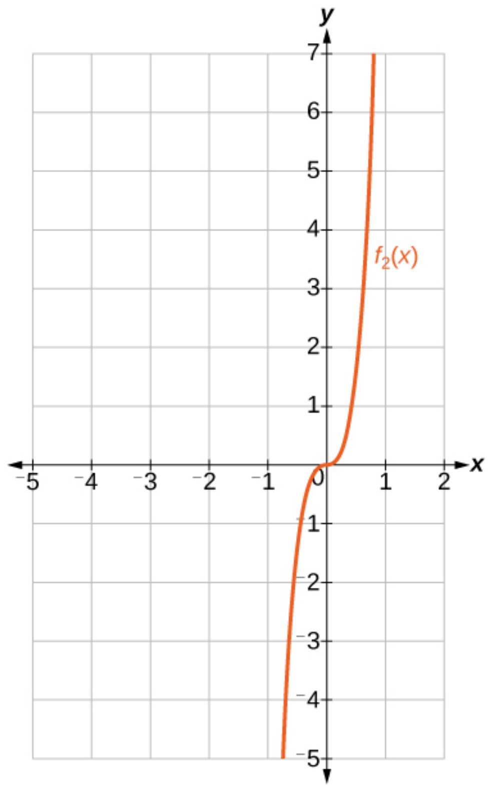
320.
$\frac{20\,!}{6\,!6\,!8\,!} = 116,\ 396,\ 280$

322. A binomial coefficient is an alternative way of denoting the combination
$C(n,\ r).$ It is defined as
$\binom{n}{r} = C(n,\ r) = \frac{n\,!}{r\,!(n - r)\,!}.$

324. The Binomial Theorem is defined as
$(x + y)^n = \sum_{k = 0}^{n} \binom{n}{k} x^{n - k} y^k$ and can be used to expand any binomial.

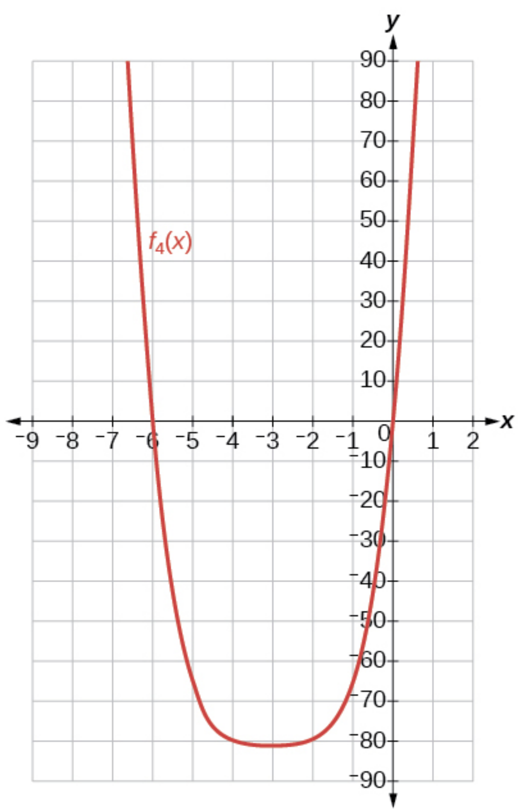
326. 15

328. 35
330. 10
332. 12,376
334.
$64a^3 - 48a^2 b + 12ab^2 - b^3$
336.
$27a^3 + 54a^2 b + 36ab^2 + 8b^3$
338.
$1024x^5 + 2560x^4 y + 2560x^3 y^2 + 1280x^2 y^3 + 320xy^4 + 32y^5$
340.
$1024x^5 - 3840x^4 y + 5760x^3 y^2 - 4320x^2 y^3 + 1620xy^4 - 243y^5$
342.
$\frac{1}{x^4} + \frac{8}{x^3 y} + \frac{24}{x^2 y^2} + \frac{32}{xy^3} + \frac{16}{y^4}$
344.
$a^{17} + 17a^{16} b + 136a^{15} b^2$
346.
$a^{15} - 30a^{14} b + 420a^{13} b^2$
348.
$3,\ 486,\ 784,\ 401a^{20} + 23,\ 245,\ 229,\ 340a^{19} b + 73,\ 609,\ 892,\ 910a^{18} b^2$
350.
$x^{24} - 8x^{21} \sqrt{y} + 28x^{18} y$
352.
$-720x^2 y^3$
354.
$220,\ 812,\ 466,\ 875,\ 000y^7$
356.
$35x^3 y^4$
358.
$1,\ 082,\ 565a^3 b^{16}$
360.
$\frac{1152y^2}{x^7}$
362.
$f_2(x) = x^4 + 12x^3$



364.

$f_4(x) = x^4 + 12x^3 + 54x^2 + 108x$



366.

$590,\ 625x^5y^2$

368.
$k-1$
369.
$\binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k};$ Proof:

$$\begin{aligned}
&\binom{n}{k-1}+\binom{n}{k} \\
&=\frac{n!}{k!(n-k)!}+\frac{n!}{(k-1)!(n-(k-1))!} \\
&=\frac{n!}{k!(n-k)!}+\frac{n!}{(k-1)!(n-k+1)!} \\
&=\frac{(n-k+1)n!}{(n-k+1)k!(n-k)!}+\frac{kn!}{k(k-1)!(n-k+1)!} \\
&=\frac{(n-k+1)n!+kn!}{k!(n-k+1)!} \\
&=\frac{(n+1)n!}{k!((n+1)-k)!} \\
&=\frac{(n+1)!}{k!((n+1)-k)!} \\
&=\binom{n+1}{k}
\end{aligned}$$

370. The expression
$(x^3+2y^2-z)^5$ cannot be expanded using the Binomial Theorem because it cannot be rewritten as a binomial.
371. probability; The probability of an event is restricted to values between
0 and
$1,$ inclusive of
0 and
$1.$
373. An experiment is an activity with an observable result.
375. The probability of the *union of two events* occurring is a number that describes the likelihood that at least one of the events from a probability model occurs. In both a union of sets
A and B and a union of events
A and $B,$ the union includes either
A or B or both. The difference is that a union of sets results in another set, while the union of events is a probability, so it is always a numerical value between
0 and
$1.$
377.
$\frac{1}{2}.$
379.
$\frac{5}{8}.$
381.
$\frac{1}{2}.$
383.
$\frac{3}{8}.$
385.
$\frac{1}{4}.$
387.
$\frac{3}{4}.$

389.
$\frac{3}{8}$.

391.
$\frac{1}{8}$.

393.
$\frac{15}{16}$.

395.
$\frac{5}{8}$.

397.
$\frac{1}{13}$.

399.
$\frac{1}{26}$.

401.
$\frac{12}{13}$.

403.

	1	**2**	**3**	**4**	**5**	**6**
1	(1, 1) 2	(1, 2) 3	(1, 3) 4	(1, 4) 5	(1, 5) 6	(1, 6) 7
2	(2, 1) 3	(2, 2) 4	(2, 3) 5	(2, 4) 6	(2, 5) 7	(2, 6) 8
3	(3, 1) 4	(3, 2) 5	(3, 3) 6	(3, 4) 7	(3, 5) 8	(3, 6) 9
4	(4, 1) 5	(4, 2) 6	(4, 3) 7	(4, 4) 8	(4, 5) 9	(4, 6) 10
5	(5, 1) 6	(5, 2) 7	(5, 3) 8	(5, 4) 9	(5, 5) 10	(5, 6) 11
6	(6, 1) 7	(6, 2) 8	(6, 3) 9	(6, 4) 10	(6, 5) 11	(6, 6) 12

405.
$\frac{5}{12}$.

407.
0.

409.
$\frac{4}{9}$.

411.
$\frac{1}{4}$.

413.
$\frac{3}{4}$

415.
$$\frac{21}{26}$$

417.
$$\frac{C(12,\ 5)}{C(48,\ 5)} = \frac{1}{2162}$$

419.
$$\frac{C(12,\ 3)C(36,\ 2)}{C(48,\ 5)} = \frac{175}{2162}$$

421.
$$\frac{C(20,\ 3)C(60,\ 17)}{C(80,\ 20)} \approx 12.49\%$$

423.
$$\frac{C(20,\ 5)C(60,\ 15)}{C(80,\ 20)} \approx 23.33\%$$

425.
$$20.50 + 23.33 - 12.49 = 31.34\%$$

427.
$$\frac{C(40000000,\ 1)C(277000000,\ 4)}{C(317000000,\ 5)} = 36.78\%$$

429.
$$\frac{C(40000000,\ 4)C(277000000,\ 1)}{C(317000000,\ 5)} = 0.11\%$$

Review Exercises

431.
$$2,\ 4,\ 7,\ 11$$

433.
$$13,\ 103,\ 1003,\ 10003$$

435. The sequence is arithmetic. The common difference is
$$d = \frac{5}{3}.$$

437.
$$18,\ 10,\ 2,\ -6,\ -14$$

439.
$$a_1 = -20,\ \ a_n = a_{n-1} + 10$$

441.
$$a_n = \frac{1}{3}n + \frac{13}{24}$$

443.
$$r = 2$$

445.
$$4,\ 16,\ 64,\ 256,\ 1024$$

447.
$$3,\ 12,\ 48,\ 192,\ 768$$

449.
$$a_n = -\frac{1}{5} \cdot \left(\frac{1}{3}\right)^{n-1}$$

451.
$$\sum_{m=0}^{5} \left(\frac{1}{2}m + 5\right).$$

453.
$$S_{11} = 110$$

455.
$S_9 \approx 23.95$

457.
$S = \frac{135}{4}$

459. \$5,617.61
461. 6
463.
$10^4 = 10{,}000$

465.
$P(18,\ 4) = 73{,}440$

467.
$C(15,\ 6) = 5005$

469.
$2^{50} = 1.13 \times 10^{15}$

471.
$\frac{8\,!}{3\,!2\,!} = 3360$

473.
490,314

475.
$131{,}072a^{17} + 1{,}114{,}112a^{16}b + 4{,}456{,}448a^{15}b^2$

477.

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

479.
$\frac{1}{6}$

481.
$\frac{5}{9}$

483.
$\frac{4}{9}$

485.
$1 - \frac{C(350,\ 8)}{C(500,\ 8)} \approx 94.4\%$

487.
$\frac{C(150,\ 3)C(350,\ 5)}{C(500,\ 8)} \approx 25.6\%$

Practice Test

488.
$-14,\ -6,\ -2,\ 0$

490. The sequence is arithmetic. The common difference is $d = 0.9.$

492.
$a_1 = -2,\ a_n = a_{n-1} - \frac{3}{2};\ a_{22} = -\frac{67}{2}$

494. The sequence is geometric. The common ratio is $r = \frac{1}{2}.$

496.
$a_1 = 1,\ a_n = -\frac{1}{2} \cdot a_{n-1}$

498.
$\sum_{k=-3}^{15}\left(3k^2 - \frac{5}{6}k\right)$

500.
$S_7 = -2604.2$

502. Total in account: $\$140,\ 355.75;$ Interest earned: $\$14,\ 355.75$

504.
$5 \times 3 \times 2 \times 3 \times 2 = 180$

506.
$C(15,\ 3) = 455$

508.
$\frac{10!}{2!3!2!} = 151{,}200$

510.
$\frac{429x^{14}}{16}$

512.
$\frac{4}{7}$

514.
$\frac{5}{7}$

516.
$\frac{C(14,\ 3)C(26,\ 4)}{C(40,\ 7)} \approx 29.2\%$

Chapter 12

Try It

12.1.
$a = 5,$ $f(x) = 2x^2 - 4,$ and $L = 46.$

12.2. a. 0; b. 2; c. does not exist; d. $-2;$ e. 0; f. does not exist; g. 4; h. 4; i. 4

12.3.

$$\lim_{x \to 0}\left(\frac{20\sin(x)}{4x}\right) = 5$$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	4.9916708	4.9999167	4.9999992	Error	4.9999992	4.9999167	4.9916708

$$\lim_{x \to 0^-}\left(\frac{20\sin(x)}{4x}\right) \longrightarrow 5 \qquad\qquad 5 \longleftarrow \lim_{x \to 0^+}\left(\frac{20\sin(x)}{4x}\right)$$

12.4. does not exist
12.5. 26
12.6. 59
12.7. 10
12.8.
-64

12.9.
-3

12.10.
$-\frac{1}{50}$

12.11.
$-\frac{1}{8}$

12.12.
$2\sqrt{3}$

12.13.
-1

12.14.

a. removable discontinuity at $x = 6;$
b. jump discontinuity at $x = 4$

12.15. yes
12.16. No, the function is not continuous at $x = 3.$ There exists a removable discontinuity at $x = 3.$

12.17.
$x = 6$

12.18. 3
12.19.
$f'(a) = 6a + 7$

12.20.
$$f'(a) = \frac{-15}{(5a + 4)^2}$$

12.21.
$\frac{3}{2}$

12.22. 0
12.23.
$-2,$ 0, 0, -3

12.24.

a. After zero seconds, she has traveled 0 feet.
b. After 10 seconds, she has traveled 150 feet east.
c. After 10 seconds, she is moving eastward at a rate of 15 ft/sec.
d. After 20 seconds, she is moving westward at a rate of 10 ft/sec.
e. After 40 seconds, she is 100 feet westward of her starting point.

12.25. The graph of
f is continuous on
$(-\infty,\ 1) \cup (1,\ 3) \cup (3,\ \infty).$ The graph of
f is discontinuous at
$x = 1$ and
$x = 3.$ The graph of
f is differentiable on
$(-\infty,\ 1) \cup (1,\ 3) \cup (3,\ \infty).$ The graph of
f is not differentiable at
$x = 1$ and
$x = 3.$

12.26.
$y = 19x - 16$

12.27. –68 ft/sec, it is dropping back to Earth at a rate of 68 ft/s.

Section Exercises

1. The value of the function, the output, at
$x = a$ is
$f(a).$ When the
$\lim_{x \to a} f(x)$ is taken, the values of
x get infinitely close to
a but never equal
$a.$ As the values of
x approach
a from the left and right, the limit is the value that the function is approaching.

3. –4
5. –4
7. 2
9. does not exist
11. 4
13. does not exist
15. Answers will vary.
16. Answers will vary.
17. Answers will vary.
18. Answers will vary.
19. Answers will vary.
20. Answers will vary.
21. Answers will vary.
23. 7.38906
25. 54.59815
27.
$e^6 \approx 403.428794,$
$e^7 \approx 1096.633158,$
e^n

29.
$\lim_{x \to -2} f(x) = 1$

31.
$\lim_{x \to 3} \left(\frac{x^2 - x - 6}{x^2 - 9} \right) = \frac{5}{6} \approx 0.83$

33.
$\lim_{x \to 1} \left(\frac{x^2 - 1}{x^2 - 3x + 2} \right) = -2.00$

35.

$$\lim_{x \to 1}\left(\frac{10 - 10x^2}{x^2 - 3x + 2}\right) = 20.00$$

37.

$$\lim_{x \to \frac{-1}{2}}\left(\frac{x}{4x^2 + 4x + 1}\right)$$ does not exist. Function values decrease without bound as x approaches –0.5 from either left or right.

39.

$$\lim_{x \to 0}\frac{7\tan x}{3x} = \frac{7}{3}$$

x	$f(x)$	
−0.1	2.34114234	$\lim_{x \to 0^-}\frac{7\tan x}{3x}$
−0.01	2.33341114	↓
−0.001	2.33333411	$\frac{7}{3}$
0	Error	
0.001	2.33333411	$\frac{7}{3}$
0.01	2.33341114	↑
0.1	2.34114234	$\lim_{x \to 0^+}\frac{7\tan x}{3x}$

40.

x	$f(x)$	
3.9	−152.1	$\lim_{x \to 4^-}\frac{x^2}{x-4}$
3.99	−1592.01	↓
3.999	−15992.001	$-\infty$
4	Error	
4.001	16008.001	$+\infty$
4.01	1608.1	↑
4.1	168.1	$\lim_{x \to 4^+}\frac{x^2}{x-4}$

41.

$$\lim_{x \to 0}\frac{2\sin x}{4\tan x} = \frac{1}{2}$$

x	$f(x)$	
−0.1	0.49750208	$\lim_{x\to 0^-} \frac{2\sin x}{4\tan x}$
−0.01	0.49997500	↓
−0.001	0.49999975	$\frac{1}{2}$
0	Error	
0.001	0.49999975	$\frac{1}{2}$
0.01	0.49997500	↑
0.1	0.49750208	$\lim_{x\to 0^+} \frac{2\sin x}{4\tan x}$

43.

$\lim_{x\to 0} e^{e^{-\frac{1}{x^2}}} = 1.0$

45.

$\lim_{x\to -1^-} \frac{|x+1|}{x+1} = \frac{-(x+1)}{(x+1)} = -1$ and

$\lim_{x\to -1^+} \frac{|x+1|}{x+1} = \frac{(x+1)}{(x+1)} = 1;$ since the right-hand limit does not equal the left-hand limit,

$\lim_{x\to -1} \frac{|x+1|}{x+1}$ does not exist.

47.

$\lim_{x\to -1} \frac{1}{(x+1)^2}$ does not exist. The function increases without bound as x approaches -1 from either side.

49.

$\lim_{x\to 0} \frac{5}{1-e^{\frac{2}{x}}}$ does not exist. Function values approach 5 from the left and approach 0 from the right.

51. Through examination of the postulates and an understanding of relativistic physics, as $v \to c,$ $m \to \infty.$ Take this one step further to the solution, (12.33)

$$\lim_{v\to c^-} m = \lim_{v\to c^-} \frac{m_o}{\sqrt{1-(v^2/c^2)}} = \infty$$

53. If f is a polynomial function, the limit of a polynomial function as x approaches a will always be $f(a).$

55. It could mean either (1) the values of the function increase or decrease without bound as x approaches $c,$ or (2) the left and right-hand limits are not equal.

57.

$\frac{-10}{3}$

59. 6

61.
$\frac{1}{2}$
63. 6
65. does not exist
67.
-12
69.
$-\frac{\sqrt{5}}{10}$
71.
-108
73. 1
75. 6
77. 1
79. 1
81. does not exist
83.
$6+\sqrt{5}$
85.
$\frac{3}{5}$
87. 0
89.
-3
91. does not exist; right-hand limit is not the same as the left-hand limit.
93. 2
95. Limit does not exist; limit approaches infinity.
97.
$4x+2h$
99.
$2x+h+4$
101.
$\frac{\cos(x+h)-\cos(x)}{h}$
103.
$\frac{-1}{x(x+h)}$
105.
$\frac{-1}{\sqrt{x+h}+\sqrt{x}}$
107.
$f(x)=\frac{x^2+5x+6}{x+3}$
109. does not exist
111. 52
113. Informally, if a function is continuous at
$x=c,$ then there is no break in the graph of the function at
$f(c),$ and
$f(c)$ is defined.
115. discontinuous at
$a=-3$;
$f(-3)$ does not exist
117. removable discontinuity at
$a=-4$;
$f(-4)$ is not defined

119. Discontinuous at $a = 3$; $\lim_{x \to 3} f(x) = 3,$ but $f(3) = 6,$ which is not equal to the limit.

121. $\lim_{x \to 2} f(x)$ does not exist.

123. $\lim_{x \to 1^-} f(x) = 4; \lim_{x \to 1^+} f(x) = 1$. Therefore, $\lim_{x \to 1} f(x)$ does not exist.

125. $\lim_{x \to 1^-} f(x) = 5 \neq \lim_{x \to 1^+} f(x) = -1$. Thus $\lim_{x \to 1} f(x)$ does not exist.

127. $\lim_{x \to -3^-} f(x) = -6$, $\lim_{x \to -3^+} f(x) = -\frac{1}{3}$ Therefore, $\lim_{x \to -3} f(x)$ does not exist.

129. $f(2)$ is not defined.

131. $f(-3)$ is not defined.

133. $f(0)$ is not defined.

135. Continuous on $(-\infty, \infty)$

137. Continuous on $(-\infty, \infty)$

139. Discontinuous at $x = 0$ and $x = 2$

141. Discontinuous at $x = 0$

143. Continuous on $(0, \infty)$

145. Continuous on $[4, \infty)$

147. Continuous on $(-\infty, \infty)$.

149. 1, but not 2 or 3

151. 1 and 2, but not 3

153. $f(0)$ is undefined.

155. $(-\infty, 0) \cup (0, \infty)$

157. At $x = -1,$ the limit does not exist. At $x = 1,$

$f(1)$ does not exist. At
$x = 2,$ there appears to be a vertical asymptote, and the limit does not exist.

159.
$\frac{x^3 + 6x^2 - 7x}{(x + 7)(x - 1)}$

161. The function is discontinuous at
$x = 1$ because the limit as
x approaches 1 is 5 and
$f(1) = 2.$

163. The slope of a linear function stays the same. The derivative of a general function varies according to
$x.$ Both the slope of a line and the derivative at a point measure the rate of change of the function.

165. Average velocity is 55 miles per hour. The instantaneous velocity at 2:30 p.m. is 62 miles per hour. The instantaneous velocity measures the velocity of the car at an instant of time whereas the average velocity gives the velocity of the car over an interval.

167. The average rate of change of the amount of water in the tank is 45 gallons per minute. If
$f(x)$ is the function giving the amount of water in the tank at any time
$t,$ then the average rate of change of
$f(x)$ between
$t = a$ and
$t = b$ is
$f(a) + 45(b - a).$

169.
$f'(x) = -2$

171.
$f'(x) = 4x + 1$

173.
$f'(x) = \frac{1}{(x - 2)^2}$

175.
$\frac{-16}{(3 + 2x)^2}$

177.
$f'(x) = 9x^2 - 2x + 2$

179.
$f'(x) = 0$

181.
$-\frac{1}{3}$

183. undefined

185.
$f'(x) = -6x - 7$

187.
$f'(x) = 9x^2 + 4x + 1$

189.
$y = 12x - 15$

191.
$k = -10$ or
$k = 2$

193. Discontinuous at
$x = -2$ and
$x = 0.$ Not differentiable at –2, 0, 2.

195. Discontinuous at
$x = 5.$ Not differentiable at -4, –2, 0, 1, 3, 4, 5.

197.
$f(0) = -2$

199.
$f(2) = -6$

201.
$f'(-1) = 9$

203.
$f'(1) = -3$

205.
$f'(3) = 9$

207. Answers vary. The slope of the tangent line near $x = 1$ is 2.

209. At 12:30 p.m., the rate of change of the number of gallons in the tank is –20 gallons per minute. That is, the tank is losing 20 gallons per minute.

211. At 200 minutes after noon, the volume of gallons in the tank is changing at the rate of 30 gallons per minute.

213. The height of the projectile after 2 seconds is 96 feet.

215. The height of the projectile at $t = 3$ seconds is 96 feet.

217. The height of the projectile is zero at $t = 0$ and again at $t = 5.$ In other words, the projectile starts on the ground and falls to earth again after 5 seconds.

219.
36π

221. $50.00 per unit, which is the instantaneous rate of change of revenue when exactly 10 units are sold.

223. $21 per unit

225. $36

227.
$f'(x) = 10a - 1$

229.
$\frac{4}{(3-x)^2}$

Review Exercises

230. 2

232. does not exist

234.
Discontinuous at $x = -1 (\lim_{x \to a} f(x)$ does not exist), $x = 3$ (jump discontinuity), and $x = 7$ ($\lim_{x \to a} f(x)$ does not exist).

235. 3

237.
$\lim_{x \to -2} f(x) = 1$

239. 2

241.
-15

243. 3

245. 12

247.
-10

249.
$-\frac{1}{9}$

251. At $x = 4,$ the function has a vertical asymptote.

253. removable discontinuity at
$a = -\frac{5}{2}$

255. continuous on
$(-\infty, \infty)$

257. removable discontinuity at
$x = 2.$
$f(2)$ is not defined, but limits exist.

259. discontinuity at
$x = 0$ and
$x = 2.$ Both
$f(0)$ and
$f(2)$ are not defined.

261. removable discontinuity at
$x = -2. f(-2)$ is not defined.

263. 0

265.
$\frac{\ln(x+h) - \ln(x)}{h}$

267.
$= 4$

269.
$y = -8x + 16$

271.
12π

Practice Test

272. 3

274. 0

276.
-1

278.
$\lim_{x \to 2^-} f(x) = -\frac{5}{2}a$ and
$\lim_{x \to 2^+} f(x) = 9$ Thus, the limit of the function as
x approaches 2 does not exist.

279.
$-\frac{1}{50}$

281. 1

283. removable discontinuity at
$x = 3$

285.
$f'(x) = -\frac{3}{2a^{\frac{3}{2}}}$

287. discontinuous at –2,0, not differentiable at –2,0, 2.

289. not differentiable at
$x = 0$ (no limit)

291. the height of the projectile at
$t = 2$ seconds

293. the average velocity from
$t = 1$ to $t = 2$

295.
$\frac{1}{3}$

297. 0
299. 2
300.
$x=1$
302.
$y=-14x-18$
304. The graph is not differentiable at
$x=1$ (cusp).
306.
$f'(x)=8x$
308.
$f'(x)=-\frac{1}{(2+x)^2}$
310.
$f'(x)=-3x^2$
312.
$f'(x)=\frac{1}{2\sqrt{x-1}}$