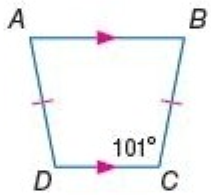
6-6 Trapezoids and Kites

Find each measure.

1. $m\angle D$



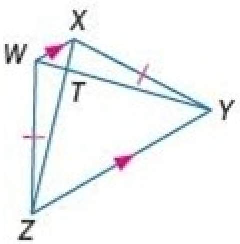
SOLUTION:

The trapezoid *ABCD* is an isosceles trapezoid. So, each pair of base angles is congruent. Therefore, $m\angle D = m\angle C = 101$.

ANSWER:

101

2. *WT*, if *ZX* = 20 and *TY* = 15



SOLUTION:

The trapezoid *WXYZ* is an isosceles trapezoid. So, the diagonals are congruent. Therefore, *WY* = *ZX*.

WT + *TY* = *ZX*

WT + 15 = 20

WT = 5

ANSWER:

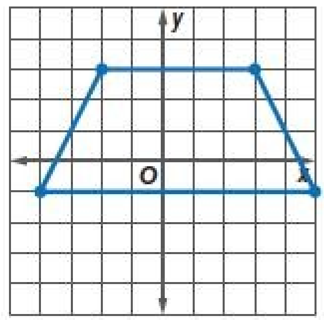
5

COORDINATE GEOMETRY Quadrilateral *ABCD* has vertices *A* (–4, –1), *B*(–2, 3), *C*(3, 3), and *D*(5, –1).

3. Verify that *ABCD* is a trapezoid.

SOLUTION:

First graph the points on a coordinate grid and draw the trapezoid.



Use the slope formula to find the slope of the sides of the trapezoid.

$$m_{AB} = \frac{3-(-1)}{-2-(-4)} = 2$$

$$m_{BC} = \frac{3-3}{3-(-2)} = 0$$

$$m_{CD} = \frac{-1-3}{5-3} = -2$$

$$m_{DA} = \frac{-1-(-1)}{-4-5} = 0$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *ABCD* is a trapezoid.

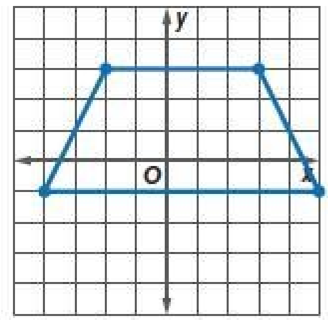
ANSWER:

$\overline{BC} \parallel \overline{AD}, \overline{AB} \nparallel \overline{CD}$; *ABCD* is a trapezoid.

4. Determine whether *ABCD* is an isosceles trapezoid. Explain.

SOLUTION:

Refer to the graph of the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{AB} = \frac{3-(-1)}{-2-(-4)} = 2$$

$$m_{BC} = \frac{3-3}{3-(-2)} = 0$$

$$m_{CD} = \frac{-1-3}{5-3} = -2$$

$$m_{DA} = \frac{-1-(-1)}{-4-5} = 0$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *ABCD* is a trapezoid.

Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$AB = \sqrt{(-2-(-4))^2 + (3-(-1))^2} = \sqrt{20}$$

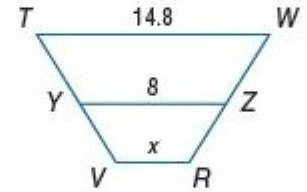
$$CD = \sqrt{(5-3)^2 + (-1-3)^2} = \sqrt{20}$$

The lengths of the legs are equal. Therefore, *ABCD* is an isosceles trapezoid.

ANSWER:

isosceles; $AB = \sqrt{20} = CD$

5. **GRIDDED REPSONSE** In the figure, $\overline{YZ}$ is the midsegment of trapezoid *TWRV*. Determine the value of *x*.



SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{TW}$ and $\overline{VR}$ are the bases and $\overline{YZ}$ is the midsegment. So,

$$YZ = \frac{TW + VR}{2}.$$

$$8 = \frac{14.8 + x}{2}$$

Solve for *x*.

$16 = 14.8 + x$

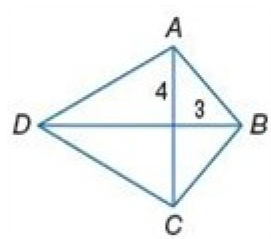
$1.2 = x$

ANSWER:

1.2

CCSS SENSE-MAKING If *ABCD* is a kite, find each measure.

6. *AB*



SOLUTION:

By the Pythagorean Theorem,

$AB^2 = 4^2 + 3^2 = 25$

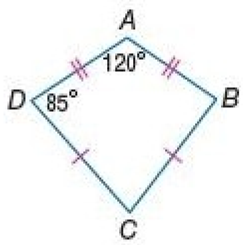
$AB = \sqrt{25} = 5$

ANSWER:

5

6-6 Trapezoids and Kites

7. $m\angle C$



SOLUTION:

$\angle A$ is an obtuse angle and $\angle C$ is an acute angle. Since a kite can only have one pair of opposite congruent
angles and $\angle A \ncong \angle C$.

$\angle B \cong \angle D = 85$.

The sum of the measures of the angles of a quadrilateral is 360.

$m\angle A + m\angle B + m\angle C + m\angle D = 360$

$120 + 85 + m\angle C + 85 = 360$

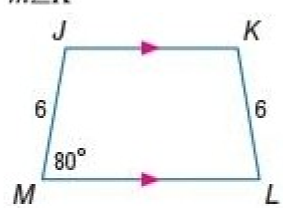
$m\angle C = 70$

ANSWER:

70

Find each measure.

8. $m\angle K$



SOLUTION:

The trapezoid *JKLM* is an isosceles trapezoid so each pair of base angles is congruent. So, $\angle M \cong \angle L$ and $\angle J \cong \angle K$.

The sum of the measures of the angles of a quadrilateral is 360.

Let $m\angle J = m\angle K = x$.

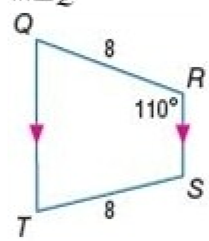
$m\angle M + m\angle L + m\angle K + m\angle J = 360$	Sum of angles is 360.
$80 + 80 + x + x = 360$	Substitute.
$2x = 200$	Combine like terms.
$x = 100$	Divide each side by 2.

So, $m\angle K = 100$.

ANSWER:

100

9. $m\angle Q$



SOLUTION:

The trapezoid *QRST* is an isosceles trapezoid so each pair of base angles is congruent. So, $\angle R \cong \angle S$ and $\angle Q \cong \angle T$.

The sum of the measures of the angles of a quadrilateral is 360.

Let $m\angle Q = m\angle T = x$.

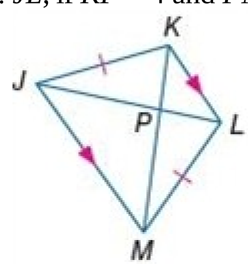
$m\angle Q + m\angle R + m\angle S + m\angle T = 360$	Sum of angles is 360.
$x + 110 + 110 + x = 360$	Substitute.
$2x = 140$	Combine like terms.
$x = 70$	Divide each side by 2.

So, $m\angle Q = 70$.

ANSWER:

70

10. *JL*, if *KP* = 4 and *PM* = 7



SOLUTION:

The trapezoid *JKLM* is an isosceles trapezoid. So, the diagonals are congruent. Therefore, *KM* = *JL*.

KM = *KP* + *PM*

= 4 + 7

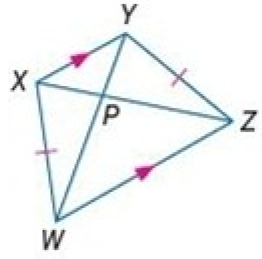
= 11

JL = *KM* = 11

ANSWER:

11

11. *PW*, if *XZ* = 18 and *PY* = 3



SOLUTION:

The trapezoid *WXYZ* is an isosceles trapezoid. So, the diagonals are congruent. Therefore, *YW* = *XZ*.

YP + *PW* = *XZ*.

3 + *PW* = 18

PW = 15

ANSWER:

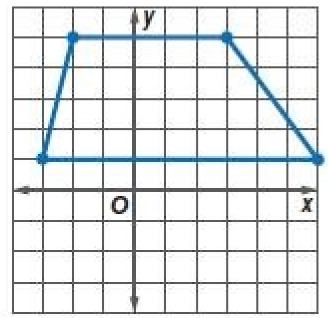
15

COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

12. *A*(–2, 5), *B*(–3, 1), *C*(6, 1), *D*(3, 5)

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{AB} = \frac{1-5}{-3-(-2)} = 4$$

$$m_{BC} = \frac{1-1}{6-(-3)} = 0$$

$$m_{CD} = \frac{5-1}{3-6} = -\frac{4}{3}$$

$$m_{DA} = \frac{5-5}{-2-3} = 0$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *ABCD* is a trapezoid.

Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$AB = \sqrt{(-3-(-2))^2 + (1-5)^2} = \sqrt{(-1)^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17}$$

$$CD = \sqrt{(3-6)^2 + (5-1)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

The lengths of the legs are not equal. Therefore, *ABCD* is not an isosceles trapezoid.

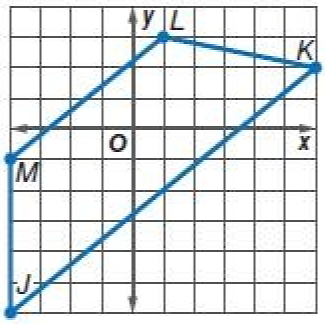
ANSWER:

$\overline{BC} \parallel \overline{AD}, \overline{AB} \nparallel \overline{CD}$; *ABCD* is a trapezoid, but not isosceles since $AB = \sqrt{17}$ and *CD* = 5.

13. $J(-4, -6)$, $K(6, 2)$, $L(1, 3)$, $M(-4, -1)$

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{JK} = \frac{2-(-6)}{6-(-4)} = \frac{4}{5}$$

$$m_{KL} = \frac{3-2}{1-6} = -\frac{1}{5}$$

$$m_{LM} = \frac{-1-3}{-4-1} = \frac{4}{5}$$

$$m_{MJ} = \frac{-6-(-1)}{-4-(-4)} = \text{ undefined}$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *JKLM* is a trapezoid.
Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$KL = \sqrt{(1-6)^2 + (3-2)^2} = \sqrt{(-5)^2 + 1^2} = \sqrt{25+1} = \sqrt{26}$$

$$JM = \sqrt{(-4-(-4))^2 + (-1-(-6))^2} = \sqrt{0^2 + (-5)^2} = \sqrt{25} = 5$$

The lengths of the legs are not equal. Therefore, *JKLM* is not an isosceles trapezoid.

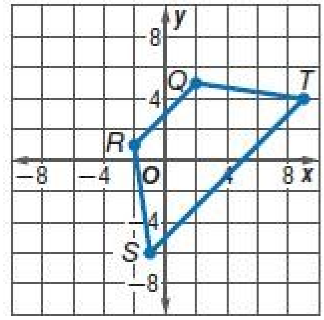
ANSWER:

$\overline{JK} \parallel \overline{LM}, \overline{KL} \nparallel \overline{JM}$; *JKLM* is a trapezoid, but not isosceles since $KL = \sqrt{26}$ and $JM = 5$.

14. $Q(2, 5)$, $R(-2, 1)$, $S(-1, -6)$, $T(9, 4)$

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{QR} = \frac{1-5}{-2-2} = 1$$

$$m_{RS} = \frac{-6-1}{-1+2} = -7$$

$$m_{ST} = \frac{4-(-6)}{9-(-1)} = 1$$

$$m_{TQ} = \frac{5-4}{2-9} = -\frac{1}{7}$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *QRST* is a trapezoid.
Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$RS = \sqrt{(-1-(-2))^2 + (-6-1)^2} = \sqrt{1^2 + (-7)^2} = \sqrt{1+49} = \sqrt{50}$$

$$QT = \sqrt{(9-2)^2 + (4-5)^2} = \sqrt{7^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50}$$

The lengths of the legs are equal. Therefore, *QRST* is an isosceles trapezoid.

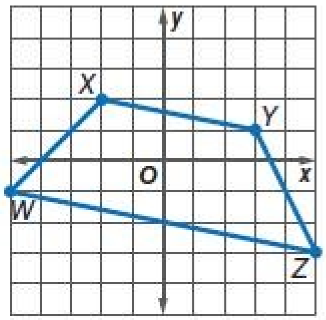
ANSWER:

$\overline{QR} \parallel \overline{ST}, \overline{RS} \nparallel \overline{QT}$; *QRST* is a trapezoid, Isosceles since $RS = \sqrt{50} = QT$.

15. $W(-5, -1)$, $X(-2, 2)$, $Y(3, 1)$, $Z(5, -3)$

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{WX} = \frac{2-(-1)}{-2-(-5)} = 1$$

$$m_{XY} = \frac{1-2}{3-(-2)} = -\frac{1}{5}$$

$$m_{YZ} = \frac{-3-1}{5-3} = -2$$

$$m_{ZW} = \frac{-1-(-3)}{-5-5} = -\frac{1}{5}$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral *WXYZ* is a trapezoid.
Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$XZ = \sqrt{(5-(-2))^2 + (-3-2)^2} = \sqrt{7^2 + (-5)^2} = \sqrt{49+25} = \sqrt{74}$$

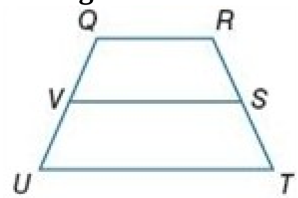
$$WY = \sqrt{(3-(-5))^2 + (1-(-1))^2} = \sqrt{8^2 + 2^2} = \sqrt{64+4} = \sqrt{68}$$

The lengths of the legs are not equal. Therefore, *WXYZ* is not an isosceles trapezoid.

ANSWER:

$\overline{XY} \parallel \overline{WZ}, \overline{WX} \nparallel \overline{YZ}$; *WXYZ* is a trapezoid, but not isosceles since $XZ = \sqrt{74}$ *and* $WY = \sqrt{68}$.

For trapezoid *QRTU, V* and *S* are midpoints of the legs.



16. If $QR = 12$ and $UT = 22$, find VS.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$

$$VS = \frac{12+22}{2}$$
$$= 17$$

ANSWER:

17

17. If $QR = 4$ and $UT = 16$, find VS.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$

$$VS = \frac{4+16}{2}$$
$$= 10$$

ANSWER:

10

18. If *VS* = 9 and *UT* = 12, find *QR*.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$
$$9 = \frac{QR + 12}{2}$$
$$18 = QR + 12$$
$$6 = QR$$

ANSWER:

6

19. If *TU* = 26 and *SV* = 17, find *QR*.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$
$$17 = \frac{QR + 26}{2}$$
$$34 = QR + 26$$
$$8 = QR$$

ANSWER:

8

20. If *QR* = 2 and *VS* = 7, find *UT*.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$
$$7 = \frac{2 + UT}{2}$$
$$14 = 2 + UT$$
$$12 = UT$$

ANSWER:

12

21. If *RQ* = 5 and *VS* = 11, find *UT*.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

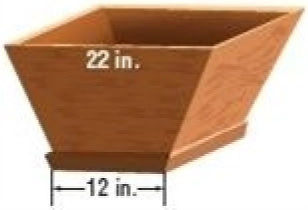
$\overline{QR}$ and $\overline{UT}$ are the bases and $\overline{VS}$ is the midsegment. So,

$$VS = \frac{QR + UT}{2}.$$
$$11 = \frac{5 + UT}{2}$$
$$22 = 5 + UT$$
$$17 = UT$$

ANSWER:

17

22. **DESIGN** Juana is designing a window box. She wants the end of the box to be a trapezoid with the dimensions shown. If she wants to put a shelf in the middle for the plants to rest on, about how wide should she make the shelf?



SOLUTION:

The length of the midsegment of the trapezoid is about the width of the shelf.
By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. The length of the bases are 22 inches and 12 inches. So, the length of the midsegment is:

$$\frac{22+12}{2}=17 \text{ in.}$$

Therefore, without knowing the thickness of the sides of the box, the width of the shelf is about 17 inches.

ANSWER:

17 in

23. **MUSIC** The keys of the xylophone shown form a trapezoid. If the length of the lower pitched *C* is 6 inches long, and the higher pitched *D* is 1.8 inches long, how long is the *G* key?

SOLUTION:

The *G* key is the midsegment of the trapezoid.
By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. So, the length of the *G* key is

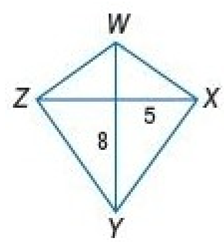
$$\frac{6+1.8}{2}=3.9 \text{ in.}$$

ANSWER:

3.9 in

CCSS SENSE-MAKING If *WXYZ* is a kite, find each measure.

24. *YZ*



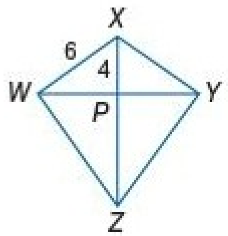
SOLUTION:

By the Pythagorean Theorem,
$XY^2 = 8^2 + 5^2 = 89$
$XY = \sqrt{89}$
A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. So, $\overline{XY} \cong \overline{ZY}$.
Therefore, $YZ = XY = \sqrt{89}$.

ANSWER:

$\sqrt{89}$

25. *WP*



SOLUTION:

By the Pythagorean Theorem,

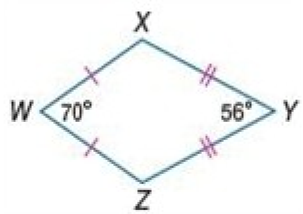
$WP^2 = WX^2 - XP^2 = 6^2 - 4^2 = 20$

$WP = \sqrt{20}$

ANSWER:

$\sqrt{20}$

26. $m\angle X$



SOLUTION:

A kite can only have one pair of opposite congruent angles and $\angle W \not\cong \angle Y$, so $\angle X \cong \angle Z$.

Let $m\angle X = m\angle Z = x$.

The sum of the measures of the angles of a quadrilateral is 360.

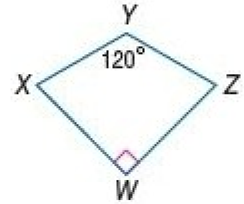
$m\angle W + m\angle X + m\angle Y + m\angle Z = 360$	Sum of angles is 360
$70 + x + 56 + x = 360$	Substitute.
$2x = 234$	Combine like terms.
$x = 117$	Divide each side by 2.

So, $m\angle X = 117$.

ANSWER:

117

27. $m\angle Z$



SOLUTION:

A kite can only have one pair of opposite congruent angles and $\angle W \not\cong \angle Y$, so $\angle X \cong \angle Z$.

Let $m\angle X = m\angle Z = x$.

The sum of the measures of the angles of a quadrilateral is 360.

$m\angle W + m\angle X + m\angle Y + m\angle Z = 360$	Sum of angles is 360
$90 + x + 120 + x = 360$	Substitute.
$2x = 150$	Combine like terms.
$x = 75$	Divide each side by 2.

So, $m\angle Z = 75$.

ANSWER:

75

PROOF Write a paragraph proof for each theorem.

28. Theorem 6.21

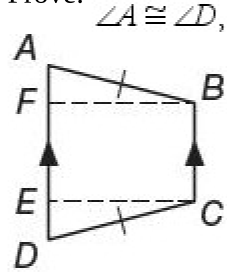
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is an isosceles trapezoid. $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \cong \overline{CD}$. You need to prove that $\angle A \cong \angle D$, $\angle ABC \cong \angle DCB$. Use the properties that you have learned about trapezoids to walk through the proof.

Given: *ABCD* is an isosceles trapezoid.

$\overline{BC} \parallel \overline{AD}$, $\overline{AB} \cong \overline{CD}$

Prove: $\angle A \cong \angle D$, $\angle ABC \cong \angle DCB$



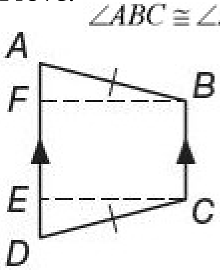
Proof: Draw auxiliary segments so that $\overline{BF} \perp \overline{AD}$ and $\overline{CE} \perp \overline{AD}$.

Since $\overline{BC} \parallel \overline{AD}$ and parallel lines are everywhere equidistant, $\overline{BF} \cong \overline{CE}$. Perpendicular lines form right angles, so $\angle BFA$ and $\angle CED$ are right angles. So, ΔBFA and ΔCED are right triangles by definition. Therefore, $\Delta BFA \cong \Delta CED$ by HL. $\angle A \cong \angle D$ by CPCTC. Since $\angle CBF$ and $\angle BCE$ are right angles and all right angles are congruent, $\angle CBF \cong \angle BCE$. $\angle ABF \cong \angle DCE$ by CPCTC. So, $\angle ABC \cong \angle DCB$ by angle addition.

ANSWER:

Given: *ABCD* is an isosceles trapezoid.
$\overline{BC} \parallel \overline{AD}, \overline{AB} \cong \overline{CD}$

Prove: $\angle A \cong \angle D,$ $\angle ABC \cong \angle DCB$



Proof: Draw auxiliary segments so that $\overline{BF} \perp \overline{AD}$ and $\overline{CE} \perp \overline{AD}$.

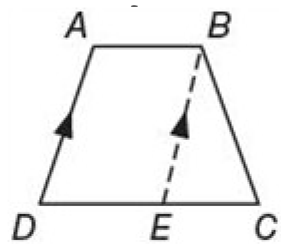
Since $\overline{BC} \parallel \overline{AD}$ and parallel lines are everywhere equidistant, $\overline{BF} \cong \overline{CE}$. Perpendicular lines form right angles, so $\angle BFA$ and $\angle CED$ are right angles. So, ΔBFA and ΔCED are right triangles by definition. Therefore, $\Delta BFA \cong \Delta CED$ by HL. $\angle A \cong \angle D$ by CPCTC. Since $\angle CBF$ and $\angle BCE$ are right angles and all right angles are congruent, $\angle CBF \cong \angle BCE$. $\angle ABF \cong \angle DCE$ by CPCTC. So, $\angle ABC \cong \angle DCB$ by angle addition.

29. Theorem 6.22

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a trapezoid; $\angle D \cong \angle C$. You need to prove that *ABCD* is an isosceles trapezoid. Use the properties that you have learned about trapezoids to walk through the proof.

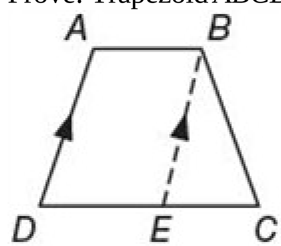
Given: *ABCD* is a trapezoid;
$\angle D \cong \angle C$
Prove: Trapezoid *ABCD* is isosceles.



Proof: By the Parallel Postulate, we can draw the auxillary line $\overline{EB} \parallel \overline{AD}$. $\angle D \cong \angle BEC$, by the Corr. $\angle s$ Thm. We are given that $\angle D \cong \angle C$, so by the Trans. Prop, $\angle BEC \cong \angle C$. So, ΔEBC is isosceles and $\overline{EB} \cong \overline{BC}$. From the def. of a trapezoid, $\overline{AB} \parallel \overline{DE}$. Since both pairs of opposite sides are parallel, *ABED* is a parallelogram. So, $\overline{AD} \cong \overline{EB}$. By the Transitive Property, $\overline{BC} \cong \overline{AD}$. Thus, *ABCD* is an isosceles trapezoid.

ANSWER:

Given: *ABCD* is a trapezoid;
$\angle D \cong \angle C$
Prove: Trapezoid *ABCD* is isosceles.



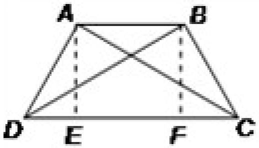
Proof: By the Parallel Postulate, we can draw the auxillary line $\overline{EB} \parallel \overline{AD}$. $\angle D \cong \angle BEC$, by the Corr. $\angle s$ Thm. We are given that $\angle D \cong \angle C$, so by the Trans. Prop, $\angle BEC \cong \angle C$. So, ΔEBC is isosceles and $\overline{EB} \cong \overline{BC}$. From the def. of a trapezoid, $\overline{AB} \parallel \overline{DE}$. Since both pairs of opposite sides are parallel, *ABED* is a parallelogram. So, $\overline{AD} \cong \overline{EB}$. By the Transitive Property, $\overline{BC} \cong \overline{AD}$. Thus, *ABCD* is an isosceles trapezoid.

30. Theorem 6.23

SOLUTION:

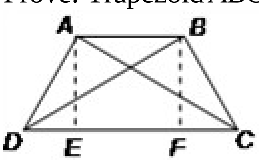
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a trapezoid; $\overline{AC} \cong \overline{BD}$. You need to prove that trapezoid *ABCD* is isosceles. Use the properties that you have learned about trapezoids to walk through the proof.

Given: *ABCD* is a trapezoid;
$\overline{AC} \cong \overline{BD}$
Prove: Trapezoid *ABCD* is isosceles.



Proof: It is given that *ABCD* is a trapezoid with $\overline{AC} \cong \overline{BD}$. Draw auxiliary segments so that $\overline{AE} \perp \overline{DC}$ and $\overline{BF} \perp \overline{DC}$. Since perpendicular lines form right angles, $\angle AEF$ and $\angle BFE$ are right angles. Therefore, ΔAEC and ΔBFD are right triangles by definition. $\overline{AE} \parallel \overline{BF}$ because two lines in a plane perpendicular to the same line are parallel $\overline{AE} \cong \overline{BF}$ since opposite sides of a trapezoid are congruent. $\Delta AEC \cong \Delta BFD$ by HL and $\angle ACD \cong \angle BDC$ by CPCTC. Since $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence, $\Delta AEC \cong \Delta BFD$ (SAS). $\overline{AD} \cong \overline{BC}$ by CPCTC, so trapezoid *ABCD* is isosceles.

ANSWER:
Given: *ABCD* is a trapezoid;
$\overline{AC} \cong \overline{BD}$
Prove: Trapezoid *ABCD* is isosceles.

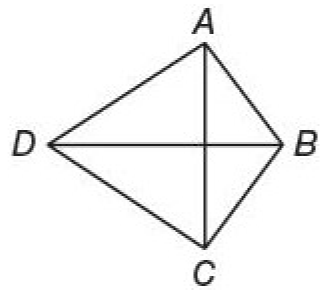


Proof: It is given that *ABCD* is a trapezoid with $\overline{AC} \cong \overline{BD}$. Draw auxiliary segments so that $\overline{AE} \perp \overline{DC}$ and $\overline{BF} \perp \overline{DC}$. Since perpendicular lines form right angles, $\angle AEF$ and $\angle BFE$ are right angles. Therefore, ΔAEC and ΔBFD are right triangles by definition. $\overline{AE} \parallel \overline{BF}$ because two lines in a plane perpendicular to the same line are parallel $\overline{AE} \cong \overline{BF}$ since opposite sides of a trapezoid are congruent. $\Delta AEC \cong \Delta BFD$ by HL and $\angle ACD \cong \angle BDC$ by CPCTC. Since $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence, $\Delta AEC \cong \Delta BFD$ (SAS). $\overline{AD} \cong \overline{BC}$ by CPCTC, so trapezoid *ABCD* is isosceles.

31. Theorem 6.25

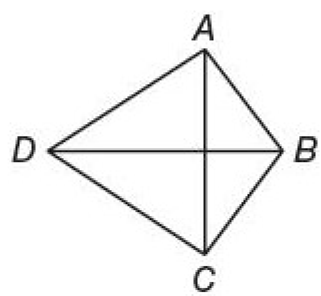
SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a kite with $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$. You need to prove $\overline{BD} \perp \overline{AC}$. Use the properties that you have learned about kites to walk through the proof.

Given: *ABCD* is a kite with $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$



Prove: $\overline{BD} \perp \overline{AC}$
Proof: We know that $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$. So, *B* and *D* are both equidistant from *A* and *C*.
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. The line that contains *B* and *D* is the perpendicular bisector of $\overline{AC}$, since only one line exists through two points. Thus, $\overline{BD} \perp \overline{AC}$.

ANSWER:
Given: *ABCD* is a kite with $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$



Prove: $\overline{BD} \perp \overline{AC}$
Proof: We know that $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$. So, *B* and *D* are both equidistant from *A* and *C*.
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. The line that contains *B* and *D* is the perpendicular bisector of $\overline{AC}$, since only one line exists through two points. Thus, $\overline{BD} \perp \overline{AC}$.

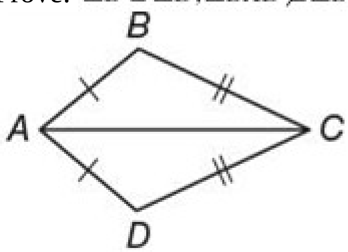
32. Theorem 6.26

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a kite. You need to prove $\angle B \cong \angle D, \angle BAD \not\cong \angle BCD$. Use the properties that you have learned about kites to walk through the proof.

Given: *ABCD* is a kite

Prove: $\angle B \cong \angle D, \angle BAD \not\cong \angle BCD$

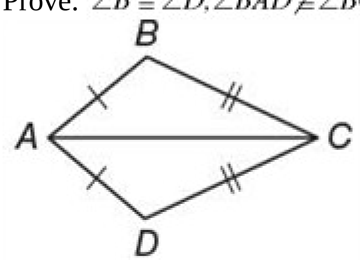


Proof: We know that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{CD}$ by the definition of a kite. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property. Therefore, $\Delta ABC \cong \Delta ADC$ by SSS.
$\angle B \cong \angle D$ by CPCTC.
If $\angle BAD \cong \angle BCD$, then *ABCD* is a parallelogram by definition, which cannot be true because we are given that *ABCD* is a kite. Therefore, $\angle BAD \not\cong \angle BCD$.

ANSWER:

Given: *ABCD* is a kite

Prove: $\angle B \cong \angle D, \angle BAD \not\cong \angle BCD$



Proof: We know that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{CD}$ by the definition of a kite. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property. Therefore, $\Delta ABC \cong \Delta ADC$ by SSS.
$\angle B \cong \angle D$ by CPCTC.
If $\angle BAD \cong \angle BCD$, then *ABCD* is a parallelogram by definition, which cannot be true because we are given that *ABCD* is a kite. Therefore, $\angle BAD \not\cong \angle BCD$.

33. **PROOF** Write a coordinate proof for Theorem 6.24.

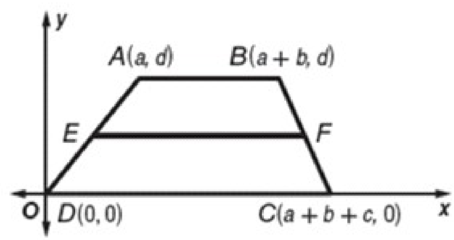
SOLUTION:

Begin by positioning trapezoid *ABCD* on a coordinate plane. Place vertex *D* at the origin with the longer base along the *x*-axis. Let the distance from *D* to *A* be *a* units, the distance from *A* to *B* *b* units, and the distance from *B* to *C* *c* units. Let the length of the bases be *a* units and the height be *c* units. Then the rest of the vertices are *A*(*a*, *d*), *B*(*a* + *b*, d), and *C*(*a* + *b* + *c*, 0). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a trapezoid with median $\overline{EF}$ and you need to prove $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$ and $EF = \frac{1}{2}(AB + DC)$. Use the properties that you have learned about trapezoids to walk through the proof.

Given: *ABCD* is a trapezoid with median $\overline{EF}$.

Prove: $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$ and $EF = \frac{1}{2}(AB + DC)$

Proof:



By the definition of the median of a trapezoid, *E* is the midpoint of $\overline{AD}$ and *F* is the midpoint of $\overline{BC}$.

Midpoint *E* is $\left(\frac{a+0}{2}, \frac{d+0}{2}\right)$ or $\left(\frac{a}{2}, \frac{d}{2}\right)$.

Midpoint *F* is

$\left(\frac{a+b+a+b+c}{2}, \frac{d+0}{2}\right)$ or $\left(\frac{2a+2b+c}{2}, \frac{d}{2}\right)$.

The slope of $\overline{AB} = 0$, the slope of $\overline{EF} = 0$, and the slope of $\overline{DC} = 0$. Thus, $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$.

$$AB = \sqrt{\left[(a+b)-a\right]^2 + (d-d)^2} = \sqrt{b^2} \text{ or } b$$

$$DC = \sqrt{\left[(a+b+c)-0\right]^2 + (0-0)^2}$$
$$= \sqrt{(a+b+c)^2} \text{ or } a+b+c$$

$$EF = \sqrt{\left(\frac{2a+2b+c-a}{2}\right) + \left(\frac{d}{2} - \frac{d}{2}\right)^2}$$
$$= \sqrt{\left(\frac{a+2b+c}{2}\right)^2} \text{ or } \frac{a+2b+c}{2}$$

$$\frac{1}{2}(AB+DC)=\frac{1}{2}[b+(a+b+c)]$$
$$=\frac{1}{2}(a+2b+c)$$
$$=\frac{a+2b+c}{2}$$
$$=EF$$

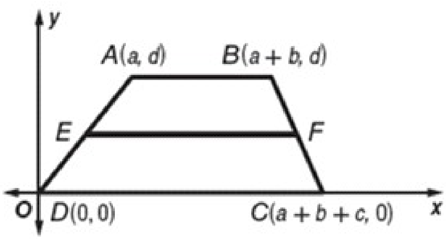
Thus, $EF=\frac{1}{2}(AB+DC)$.

ANSWER:

Given: $ABCD$ is a trapezoid with median $\overline{EF}$.

Prove: $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$ and $EF=\frac{1}{2}(AB+DC)$

Proof:



By the definition of the median of a trapezoid, E is the midpoint of $\overline{AD}$ and F is the midpoint of $\overline{BC}$.

Midpoint E is $\left(\frac{a+0}{2},\frac{d+0}{2}\right)$ or $\left(\frac{a}{2},\frac{d}{2}\right)$.

Midpoint F is $\left(\frac{a+b+a+b+c}{2},\frac{d+0}{2}\right)$ or $\left(\frac{2a+2b+c}{2},\frac{d}{2}\right)$.

The slope of $\overline{AB}=0$, the slope of $\overline{EF}=0$, and the slope of $\overline{DC}=0$. Thus, $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$.

$$AB=\sqrt{[(a+b)-a]^2+(d-d)^2}=\sqrt{b^2} \text{ or } b$$

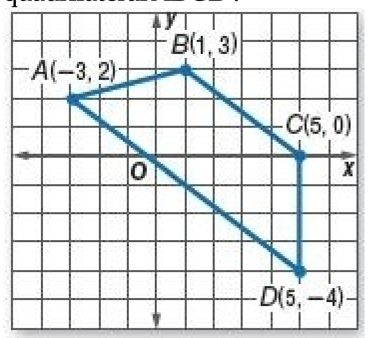
$$DC=\sqrt{[(a+b+c)-0]^2+(0-0)^2}$$
$$=\sqrt{(a+b+c)^2} \text{ or } a+b+c$$

$$EF=\sqrt{\left(\frac{2a+2b+c-a}{2}\right)+\left(\frac{d}{2}-\frac{d}{2}\right)^2}$$
$$=\sqrt{\left(\frac{a+2b+c}{2}\right)^2} \text{ or } \frac{a+2b+c}{2}$$

$$\frac{1}{2}(AB+DC)=\frac{1}{2}[b+(a+b+c)]$$
$$=\frac{1}{2}(a+2b+c)$$
$$=\frac{a+2b+c}{2}$$
$$=EF$$

Thus, $\frac{1}{2}(AB+DC)=EF$.

34. **COORDINATE GEOMETRY** Refer to quadrilateral $ABCD$.



a. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.
b. Is the midsegment contained in the line with equation $y = -x + 1$? Justify your answer.
c. Find the length of the midsegment.

SOLUTION:

a. Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{AB}=\frac{3-2}{1-(-3)}=\frac{1}{4}$$
$$m_{BC}=\frac{0-3}{5-1}=-\frac{3}{4}$$
$$m_{CD}=\frac{-4-0}{5-5}=\text{undefined}$$
$$m_{AD}=\frac{-4-2}{5-(-3)}=-\frac{3}{4}$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral $ABCD$ is a trapezoid.
Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$AB = \sqrt{(1-(-3))^2 + (3-2)^2} = \sqrt{17}$$
$$CD = \sqrt{(5-5)^2 + (-4-0)^2} = \sqrt{16} = 4$$

The lengths of the legs are not equal. Therefore, *ABCD* is not an isosceles trapezoid.

b. By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. Here, the slope of the bases of the trapezoid is $-\frac{3}{4}$. But the slope of the line with the equation $y = -x + 1$ is –1. So, they are not parallel.

c. Use the Distance formula to find the lengths of the bases.

$$AD = \sqrt{(5-(-3))^2 + (-4-2)^2} = \sqrt{8^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$$
$$BC = \sqrt{(5-1)^2 + (0-3)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

The length of the midpoint is one half the sum of the lengths of the bases. So, the length is
$\frac{10+5}{2} = 7.5$ units.

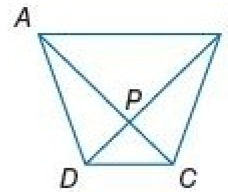
ANSWER:

a. $\overline{BC} \parallel \overline{AD}, \overline{AB} \nparallel \overline{CD}$; *ABCD* is a trapezoid, but not isosceles, because $AB = \sqrt{17}$ and *CD* = 4.

b. No; it is not $\parallel$ to the bases which have slopes of $-\frac{3}{4}$, while $y = -x + 1$ has a slope of –1.

c. 7.5 units

ALGEBRA ***ABCD*** **is a trapezoid.**



35. If *AC* = 3*x* – 7 and *BD* = 2*x* + 8, find the value of *x* so that *ABCD* is isosceles.

SOLUTION:

The trapezoid *ABCD* will be an isosceles trapezoid if the diagonals are congruent.

$AC = BD$
$3x - 7 = 2x + 8$
$x = 15$

When *x* = 15 *ABCD* is an isosceles trapezoid.

ANSWER:

15

36. If $m\angle ABC = 4x + 11$ and $m\angle DAB = 2x + 33$, find the value of *x* so that *ABCD* is isosceles.

SOLUTION:

The trapezoid *ABCD* will be an isosceles trapezoid if each pair of base angles is congruent.

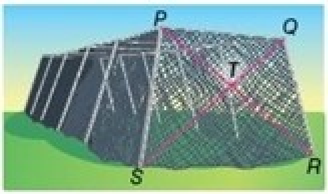
$m\angle ABC = m\angle ABC$
$4x + 11 = 2x + 33$
$2x = 22$
$x = 11$

When *x* = 11, *ABCD* is an isosceles trapezoid.

ANSWER:

11

SPORTS The end of the batting cage shown is an isosceles trapezoid. If *PT* = 12 feet, *ST* = 28 feet, and $m\angle PQR = 110$, find each measure.



37. *TR*

SOLUTION:

Since the trapezoid *PQRS* is an isosceles trapezoid the diagonals are congruent. By SSS Postulate, $\Delta PRS \cong \Delta QSR$. By CPCTC, $\angle QSR \cong \angle PRS$. So, ΔTSR is an isosceles triangle, then *TR* = *ST* = 28 ft.

ANSWER:

28 ft

38. *SQ*

SOLUTION:

Since the trapezoid *PQRS* is an isosceles trapezoid, the diagonals are congruent. By SSS Postulate, $\Delta RPQ \cong \Delta SQP$. So, $\angle RPQ \cong \angle SQP$ by CPCTC. Therefore, ΔTPQ is an isosceles triangle and then *TQ* = *TP* = 12 ft.

$SQ = ST + TQ$
$= 28 + 12$
$= 40$

Therefore, *SQ* = 40 ft.

ANSWER:

40 ft

39. $m\angle QRS$

SOLUTION:

Since the trapezoid *ABCD* is an isosceles trapezoid, both pairs of base angles are congruent. So, $m\angle QPS = m\angle PQR = 110$ and $m\angle QRS = m\angle PSR$. Let $m\angle QRS = m\angle PSR = x$.
The sum of the measures of the angles of a quadrilateral is 360.

$m\angle P + m\angle Q + m\angle R + m\angle S = 360$	Sum of angles is 360
$110 + 110 + x + x = 360$	Substitute.
$2x = 140$	Combine like terms.
$x = 70$	Divide each side by 2.

So, $m\angle QRS = 70$.

ANSWER:

70

40. $m\angle QPS$

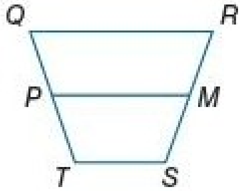
SOLUTION:

Since the trapezoid *ABCD* is an isosceles trapezoid, each pair of base angles is congruent. So, $m\angle QPS = m\angle PQR = 110$.

ANSWER:

110

ALGEBRA For trapezoid *QRST*, *M* and *P* are midpoints of the legs.



41. If $QR = 16$, $PM = 12$, and $TS = 4x$, find x.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{TS}$ are the bases and $\overline{PM}$ is the midsegment. So,

$$PM = \frac{QR + TS}{2}.$$

$$12 = \frac{16 + 4x}{2}$$

Solve for x.

$$24 = 16 + 4x$$

$$8 = 4x$$

$$2 = x$$

ANSWER:

2

42. If $TS = 2x$, $PM = 20$, and $QR = 6x$, find x.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{TS}$ are the bases and $\overline{PM}$ is the midsegment. So,

$$PM = \frac{QR + TS}{2}.$$

$$20 = \frac{6x + 2x}{2}$$

Solve for x.

$$40 = 8x$$

$$5 = x$$

ANSWER:

5

43. If $PM = 2x$, $QR = 3x$, and $TS = 10$, find PM.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{TS}$ are the bases and $\overline{PM}$ is the midsegment. So,

$$PM = \frac{QR + TS}{2}.$$

$$2x = \frac{3x + 10}{2}$$

Solve for x.

$$4x = 10 + 3x$$

$$x = 10$$

ANSWER:

20

44. If $TS = 2x + 2$, $QR = 5x + 3$, and $PM = 13$, find TS.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{QR}$ and $\overline{TS}$ are the bases and $\overline{PM}$ is the midsegment. So,

$$PM = \frac{QR + TS}{2}.$$

$$13 = \frac{5x + 3 + 2x + 2}{2}$$

Solve for x.

$$26 = 7x + 5$$

$$21 = 7x$$

$$3 = x$$

ANSWER:

8

SHOPPING The side of the shopping bag shown is an isosceles trapezoid. If $EC = 9$ inches, $DB = 19$ inches, $m\angle ABE = 40$, and $m\angle EBC = 35$, find each measure.

45. AE

SOLUTION:

The trapezoid *PQRS* is an isosceles trapezoid. So, the diagonals are congruent.

$AC = BD$

$AE + EC = BD$

$AE = 19 - 9 = 10$ inches.

ANSWER:

10 in.

46. AC

SOLUTION:

The trapezoid *PQRS* is an isosceles trapezoid. So, the diagonals are congruent.

$AC = BD = 19$ in.

ANSWER:

19 in.

47. $m\angle BCD$

SOLUTION:

The trapezoid *ABCD* is an isosceles trapezoid. So, each pair of base angles is congruent.

$$\begin{aligned} m\angle A &= m\angle B \\ &= m\angle ABE + m\angle EBC \\ &= 40 + 35 \\ &= 75 \end{aligned}$$

Let $m\angle BCD = m\angle ADC = x$.

The sum of the measures of the angles of a quadrilateral is 360.

$$\begin{aligned} m\angle A + m\angle B + m\angle C + m\angle D &= 360 && \text{Sum of angles is 360} \\ 75 + 75 + x + x &= 360 && \text{Substitute.} \\ 2x &= 210 && \text{Combine like terms.} \\ x &= 105 && \text{Divide each side by 2.} \end{aligned}$$

So, $m\angle BCD = 105$.

ANSWER:

105

48. $m\angle EDC$

SOLUTION:

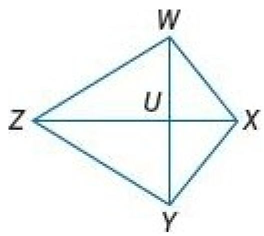
By Alternate Interior Angle Theorem,

$m\angle EDC = m\angle ABE = 40$

ANSWER:

40

ALGEBRA *WXYZ* is a kite.



49. If $m\angle WXY = 120, m\angle WZY = 4x,$ and $m\angle ZWX = 10x,$ find $m\angle ZYX$.

SOLUTION:

$\angle WZY$ is an acute angle and $\angle WXY$ is an obtuse angle. A kite can only have one pair of opposite congruent angles and $\angle WXY \ncong \angle WZY$.
So, $m\angle ZYX = m\angle ZWX = 10x$.

The sum of the measures of the angles of a quadrilateral is 360.

$$\begin{aligned} m\angle ZWX + m\angle WXY + m\angle ZYX + m\angle WZY &= 360 \\ 10x + 120 + 10x + 4x &= 360 \\ 24x &= 240 \\ x &= 10 \end{aligned}$$

Therefore, $m\angle ZYX = 10(10) = 100$.

ANSWER:
100

50. If $m\angle WXY = 13x + 24, m\angle WZY = 35,$ and $m\angle ZWX = 13x + 14,$ find $m\angle ZYX$.

SOLUTION:

$\angle WZY$ is an acute angle and $\angle WXY$ is an obtuse angle. A kite can only have one pair of opposite congruent angles and $\angle WXY \ncong \angle WZY$.

So, $m\angle ZYX = m\angle WXY = 13x + 14$
The sum of the measures of the angles of a quadrilateral is 360.

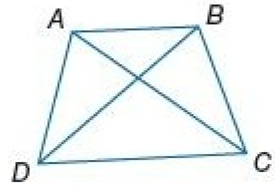
$$\begin{aligned} m\angle ZWX + m\angle WXY + m\angle ZYX + m\angle WZY &= 360 \\ 13x + 14 + 13x + 24 + 13x + 14 + 35 &= 360 \\ 39x &= 273 \\ x &= 7 \end{aligned}$$

Therefore, $m\angle ZYX = 13(7) + 14 = 105$.

ANSWER:
105

CCSS ARGUMENTS Write a two-column proof.

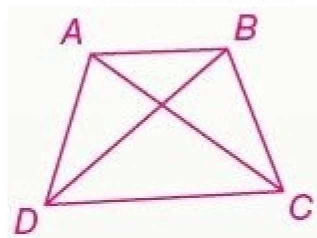
51. **Given:** *ABCD* is an isosceles trapezoid.
Prove: $\angle DAC \cong \angle CBD$



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is an isosceles trapezoid.. You need to prove $\angle DAC \cong \angle CBD$. Use the properties that you have learned about trapezoids to walk through the proof.

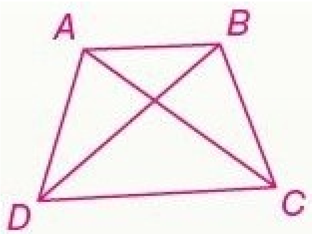
Given: *ABCD* is an isosceles trapezoid.
Prove: $\angle DAC \cong \angle CBD$



Statements(Reasons)
1. *ABCD* is an isosceles trapezoid. (Given)
2. $\overline{AD} \cong \overline{BC}$ (Def. of isos. trap.)
3. $\overline{DC} \cong \overline{DC}$ (Refl. Prop.)
4. $\overline{AC} \cong \overline{BD}$ (Diags. of isos. trap. are $\cong$.)

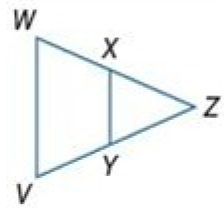
5.
$\triangle ADC \cong \triangle BCD$ (SSS)
6. $\angle DAC \cong \angle CBD$ (CPCTC)

ANSWER:
Given: $ABCD$ is an isosceles trapezoid.
Prove: $\angle DAC \cong \angle CBD$



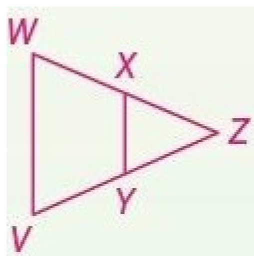
Statements(Reasons)
1. $ABCD$ is an isosceles trapezoid. (Given)
2. $\overline{AD} \cong \overline{BC}$ (Def. of isos. trap.)
3. $\overline{DC} \cong \overline{DC}$ (Refl. Prop.)
4. $\overline{AC} \cong \overline{BD}$ (Diags. of isos. trap. are $\cong$.)
5.
$\triangle ADC \cong \triangle BCD$ (SSS)
6. $\angle DAC \cong \angle CBD$ (CPCTC)

52. **Given:** $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$.
Prove: $WXYV$ is an isosceles trapezoid.



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$. You need to prove that $WXYV$ is an isosceles trapezoid.. Use the properties that you have learned about trapezoids to walk through the proof.

Given: $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$.
Prove: $WXYV$ is an isosceles trapezoid.

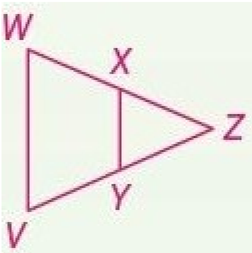


Statements(Reasons)
1. $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$. (Given)
2. $\frac{1}{2}\overline{WZ} = \frac{1}{2}\overline{ZV}$ (Mult. Prop.)
3. $WX = VY$ (Def. of midpt.)
4. $\overline{WX} \cong \overline{VY}$ (Def. of $\cong$ segs.)
5. $\angle W \cong \angle ZXY$ (Given)
6. $\overline{XY} \parallel \overline{WV}$ (If corr. $\angle$s are $\cong$, lines are $\parallel$.)
7. $WXYV$ is an isosceles trapezoid. (Def. of isos. trap.)

ANSWER:

Given: $\overline{WZ} \cong \overline{ZV}, \overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$.

Prove: *WXYV* is an isosceles trapezoid.



Statements(Reasons)

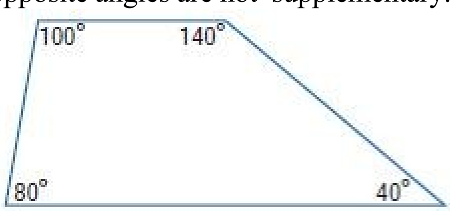
1. $\overline{WZ} \cong \overline{ZV}, \overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$. (Given)
2. $\frac{1}{2}\overline{WZ} = \frac{1}{2}\overline{ZV}$ (Mult. Prop.)
3. $WX = VY$ (Def. of midpt.)
4. $\overline{WX} \cong \overline{VY}$ (Def. of $\cong$ segs.)
5. $\angle W \cong \angle ZXY$ (Given)
6. $\overline{XY} \parallel \overline{WV}$ (If corr. $\angle$s are $\cong$, lines are $\parallel$.)
7. *WXYV* is an isosceles trapezoid. (Def. of isos. trap.)

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

53. The opposite angles of a trapezoid are supplementary.

SOLUTION:

The opposite angles of an isosceles trapezoid are supplementary but if a trapezoid is not isosceles, the opposite angles are not supplementary.



So, the statement is *sometimes* true.

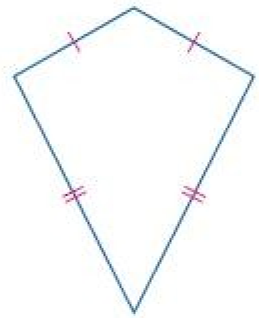
ANSWER:

Sometimes; opp $\angle s$ are supplementary in an isosceles trapezoid.

54. One pair of opposite sides are parallel in a kite.

SOLUTION:

In a kite, exactly two pairs of adjacent sides are congruent.



So, the statement is *never* true.

ANSWER:

Never; exactly two pairs of adjacent sides are congruent.

55. A square is a rhombus.

SOLUTION:

By definition, a square is a quadrilateral with 4 right angles and 4 congruent sides. Since by definition, a rhombus is a quadrilateral with 4 congruent sides a square is always a rhombus but a rhombus is not a square. So, the statement is *always* true.

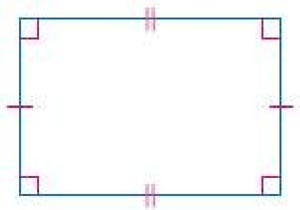
ANSWER:

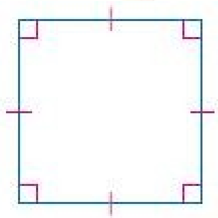
Always; by def., a square is a quadrilateral with 4 rt $\angle s$ and 4 $\cong$ sides. Since by def., a rhombus is a quadrilateral with 4 $\cong$ sides, a square is always a rhombus.

56. A rectangle is a square.

SOLUTION:

If the rectangle has 4 congruent sides, then it is a square. Otherwise, it is not a square.





So, the statement is *sometimes* true.

ANSWER:

Sometimes; if the rectangle has 4 $\cong$ sides, then it is a square. Otherwise, it is not a square.

57. A parallelogram is a rectangle.

SOLUTION:

A rectangle is a parallelogram but a parallelogram is a rectangle only if the parallelogram has 4 right angles or congruent diagonals. So, the statement is *sometimes* true.

ANSWER:

Sometimes; only if the parallelogram has 4 rt $\angle s$ and/or congruent diagonals, is it a rectangle.

58. **KITES** Refer to the kite. Using the properties of kites, write a two-column proof to show that ΔMNR is congruent to ΔPNR.

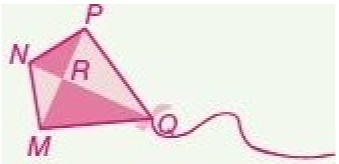
SOLUTION:

Sample answer:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *MNPQ* is a kite. You need to prove $\Delta MNR \cong \Delta PNR$. Use the properties that you have learned about kites to walk through the proof.

Given: Kite *MNPQ*

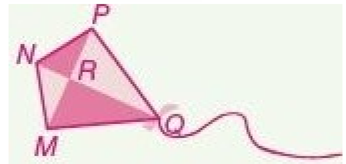
Prove:
$\Delta MNR \cong \Delta PNR$



Proof:
Statements (Reasons)
1. $MNPQ$ is a kite. (Given)
2. $\overline{NM} \cong \overline{NP}, \overline{MQ} \cong \overline{PQ}$ (Def. of a kite)
3. $\overline{QN} \cong \overline{QN}$ (Refl. Prop.)
4. $\Delta NMQ \cong \Delta NPQ$ (SSS)
5. $\angle MNR \cong \angle PNR$ (CPCTC)
6. $\overline{NR} \cong \overline{NR}$ (Refl. Prop.)
7. $\Delta MNR \cong \Delta PNR$ (SAS)

ANSWER:
Sample answer:
Given: Kite $MNPQ$

Prove:
$\Delta MNR \cong \Delta PNR$



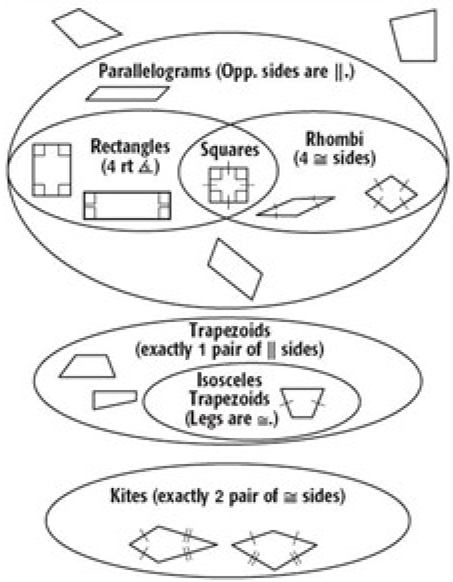
Proof:
Statements (Reasons)
1. $MNPQ$ is a kite. (Given)
2. $\overline{NM} \cong \overline{NP}, \overline{MQ} \cong \overline{PQ}$ (Def. of a kite)
3. $\overline{QN} \cong \overline{QN}$ (Refl. Prop.)
4. $\Delta NMQ \cong \Delta NPQ$ (SSS)
5. $\angle MNR \cong \angle PNR$ (CPCTC)
6. $\overline{NR} \cong \overline{NR}$ (Refl. Prop.)
7. $\Delta MNR \cong \Delta PNR$ (SAS)

59. **VENN DIAGRAM** Create a Venn diagram that incorporates all quadrilaterals, including trapezoids, isosceles trapezoids, kites, and quadrilaterals that cannot be classified as anything other than quadrilaterals.

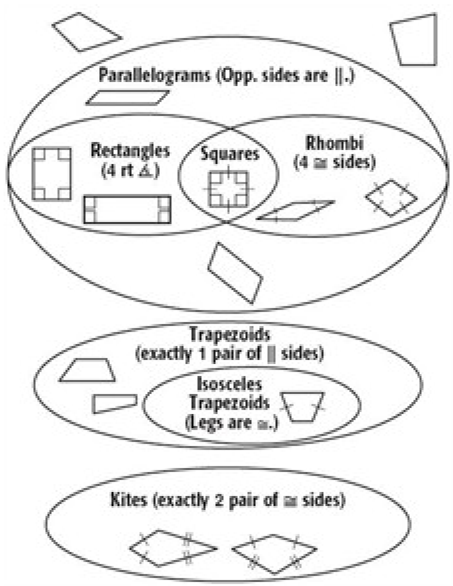
SOLUTION:
To create a Venn diagram that organizes all of the quadrilaterals first review the properties of each: parallelogram, rectangle, square, rhombus, trapezoid, isosceles trapezoid, and kites. In a Venn diagram, each type of quadrilateral is represented by an oval. Ovals that overlap show a quadrilateral that can be classified as different quadrilaterals.

Parallelograms have opposite sides that are parallel and congruent. Rectangles, squares, and rhombi also have these characteristics. So, draw a large circle to represent parallelograms. Then look at the relationship between rectangles, squares, and rhombi. A square is a rectangle but a rectangle is not necessarily a square. A square is a rhombus but rhombi are not necessarily squares. Two overlapping ovals represent rectangles and rhombi with squares in the overlapping section.

Neither trapezoids nor kites are parallelograms. A kite cannot be a trapezoid and a trapezoid cannot be a kite. So draw separate ovals to represent these quadrilaterals. Lastly, an isosceles trapezoid is a trapezoid so draw a smaller circle inside the trapezoid oval to represent these quadrilaterals.



ANSWER:

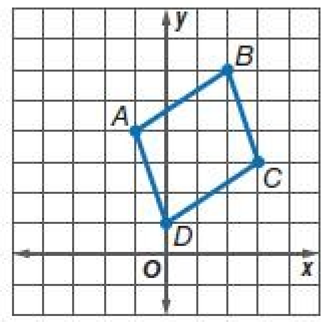


COORDINATE GEOMETRY Determine whether each figure is a *trapezoid,* a *parallelogram,* a *square,* a *rhombus,* or a *quadrilateral* given the coordinates of the vertices. Choose the most specific term. Explain.

60. *A*(–1, 4), *B*(2, 6), *C*(3, 3), *D*(0, 1)

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{AB} = \frac{6-4}{2-(-1)} = \frac{2}{3}$$

$$m_{BC} = \frac{3-6}{3-2} = -3$$

$$m_{CD} = \frac{1-3}{0-3} = \frac{2}{3}$$

$$m_{AD} = \frac{1-4}{0-(-1)} = -3$$

The slopes of each pair of opposite sides are equal. So, the two pairs of opposite sides are parallel. Therefore, the quadrilateral *ABCD* is a parallelogram.
None of the adjacent sides have slopes whose product is –1. So, the angles are not right angles.
Use the Distance Formula to find the lengths of the sides of the parallelogram.

$$AB = \sqrt{(2-(-1))^2 + (6-4)^2} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$
$$BC = \sqrt{(3-2)^2 + (3-6)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$
$$CD = \sqrt{(0-3)^2 + (1-3)^2} = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$
$$AD = \sqrt{(0-(-1))^2 + (1-4)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$

No consecutive sides are congruent. Therefore, the quadrilateral *ABCD* is a parallelogram.

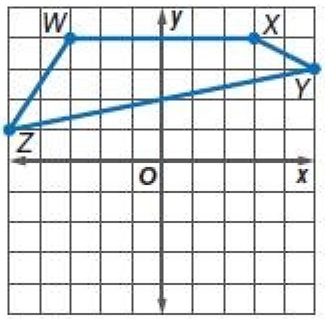
ANSWER:

Parallelogram; opp. sides ∥, no rt. ∠s, no consecutive sides ≅.

61. $W(-3, 4), X(3, 4), Y(5, 3), Z(-5, 1)$

SOLUTION:

First graph the trapezoid.



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{WX} = \frac{4-4}{3-(-3)} = 0$$

$$m_{XY} = \frac{3-4}{5-3} = -\frac{1}{2}$$

$$m_{YZ} = \frac{1-3}{-5-5} = \frac{1}{5}$$

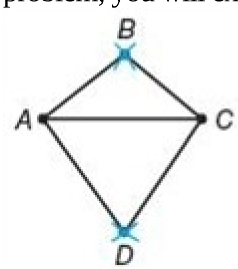
$$m_{WZ} = \frac{1-4}{-5-(-3)} = \frac{3}{2}$$

There are no parallel sides. Therefore, *WXYZ* can be classified only as a quadrilateral.

ANSWER:

quadrilateral; no parallel sides.

62. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportions in kites.



a. GEOMETRIC Draw a segment. Construct a noncongruent segment that perpendicularly bisects the first segment.
Connect the endpoints of the segments to form a quadrilateral *ABCD*. Repeat the process two times. Name the additional quadrilaterals *PQRS* and *WXYZ*.

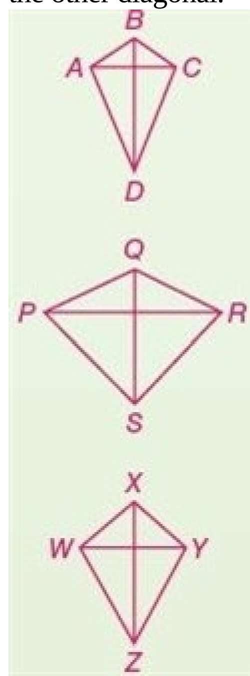
b. TABULAR Copy and complete the table below.

Figure	Side	Length	Side	Length	Side	Length	Side	Length
ABCD	AB		BC		CD		DA	
PQRS	PQ		QR		RS		SP	
WXYZ	WX		XY		YZ		ZW	

c. VERBAL Make a conjecture about a quadrilateral in which the diagonals are perpendicular, exactly one diagonal is bisected, and the diagonals are not congruent.

SOLUTION:

a. Sample answer: using a compass and straightedge to construct the kites ensures accuracy in that the diagonals are perpendicular and one diagonal bisects the other diagonal.



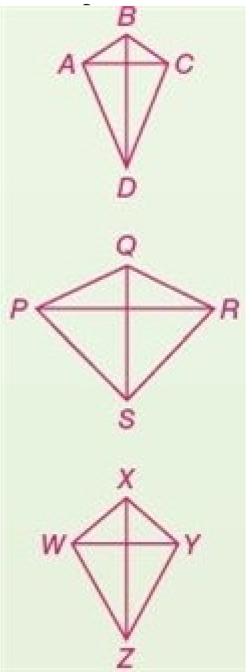
b. Use a ruler to measure each segment listed in the table. Use a centimeter ruler and measure to the nearest tenth.

Figure	Side	Length	Side	Length	Side	Length	Side	Length
ABCD	AB	0.8 cm	BC	0.8 cm	CD	1.6 cm	DA	1.6 cm
PQRS	PQ	1.4 cm	QR	1.4 cm	RS	1.8 cm	SP	1.8 cm
WXYZ	WX	0.4 cm	XY	0.4 cm	YZ	1.5 cm	ZW	1.5 cm

c. Look for the pattern in the measurements taken. Each kite has 2 pairs of congruent consecutive sides. If the diagonals of a quadrilateral are perpendicular, exactly one is bisected, and the diagonals are not congruent, then the quadrilateral is a kite.

ANSWER:

a. Sample answer:



b.

Figure	Side	Length	Side	Length	Side	Length	Side	Length
ABCD	*AB*	0.8 cm	*BC*	0.8 cm	*CD*	1.6 cm	*DA*	1.6 cm
PQRS	*PQ*	1.4 cm	*QR*	1.4 cm	*RS*	1.8 cm	*SP*	1.8 cm
WXYZ	*WX*	0.4 cm	*XY*	0.4 cm	*YZ*	1.5 cm	*ZW*	1.5 cm

c. If the diagonals of a quadrilateral are perpendicular, exactly one is bisected, and the diagonals are not congruent, then the quadrilateral is a kite.

PROOF Write a coordinate proof of each statement.

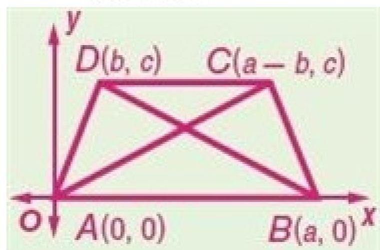
63. The diagonals of an isosceles trapezoid are congruent.

SOLUTION:

Begin by positioning trapezoid *ABCD* on a coordinate plane. Place vertex *A* at the origin. Let the length of the longer base be *a* units, the length of the shorter base be *b* units, and the height be *c* units. Then the rest of the vertices are *B*(*a*, 0), *C*(*a* - *b*, *c*), and *D*(*b*, *c*). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given isosceles trapezoid *ABCD* with $\overline{AD} \cong \overline{BC}$ and you need to prove that $\overline{BD} \cong \overline{AC}$. Use the properties that you have learned about trapezoids to walk through the proof.

Given: isosceles trapezoid *ABCD* with $\overline{AD} \cong \overline{BC}$

Prove: $\overline{BD} \cong \overline{AC}$



Proof:

$DB = \sqrt{(a-b)^2 + (0-c)^2}$ or $\sqrt{(a-b)^2 + c^2}$

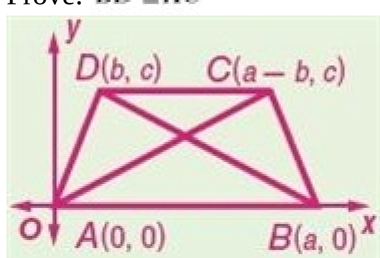
$AC = \sqrt{((a-b)-0)^2 + (c-0)^2}$ or $\sqrt{(a-b)^2 + c^2}$

$BD = AC$ therefore $\overline{BD} \cong \overline{AC}$.

ANSWER:

Given: isosceles trapezoid *ABCD* with $\overline{AD} \cong \overline{BC}$

Prove: $\overline{BD} \cong \overline{AC}$



Proof:

$DB = \sqrt{(a-b)^2 + (0-c)^2}$ or $\sqrt{(a-b)^2 + c^2}$

$AC = \sqrt{((a-b)-0)^2 + (c-0)^2}$ or $\sqrt{(a-b)^2 + c^2}$

$BD = AC$ therefore $\overline{BD} \cong \overline{AC}$.

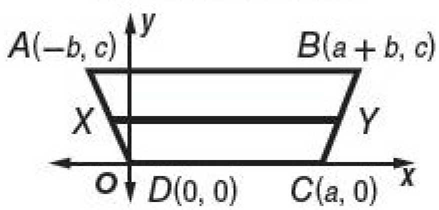
64. The median of an isosceles trapezoid is parallel to the bases.

SOLUTION:

Begin by positioning isosceles trapezoid *ABCD* on a coordinate plane. Place vertex *D* at the origin. Let the length of the shorter base be *a* units, the longer length be a + 2b and the height be *c* units. Then the rest of the vertices are $C(a, 0)$, $B(a + b, c)$, and $A(-b, c)$. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is an isosceles trapezoid with median $\overline{XY}$ and you need to prove that $\overline{XY} \parallel \overline{AB}$ and $\overline{XY} \parallel \overline{DC}$. Use the properties that you have learned about trapezoids to walk through the proof.

Given: *ABCD* is an isosceles trapezoid with median $\overline{XY}$.

Prove: $\overline{XY} \parallel \overline{AB}$ and $\overline{XY} \parallel \overline{DC}$



The midpoint of $\overline{AD}$ is X. The coordinates are $\left(\frac{-b}{2}, \frac{c}{2}\right)$.

The midpoint of $\overline{BC}$ is $Y\left(\frac{2a+b}{2}, \frac{c}{2}\right)$. Use the slope formula to find the slope of the bases and the median.

$$m_{AB} = \frac{c-c}{a+b-(-b)} = 0$$

$$m_{DC} = \frac{0-0}{a-0} = 0$$

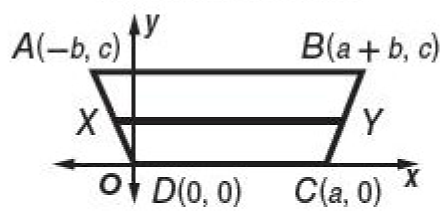
$$m_{XY} = \frac{\left(\frac{c}{2} - \frac{c}{2}\right)}{\left(\frac{2a+b}{2} - \left(-\frac{b}{2}\right)\right)} = 0$$

Thus, $\overline{XY} \parallel \overline{AB}$ and $\overline{XY} \parallel \overline{DC}$.

ANSWER:

Given: *ABCD* is an isosceles trapezoid with median $\overline{XY}$.

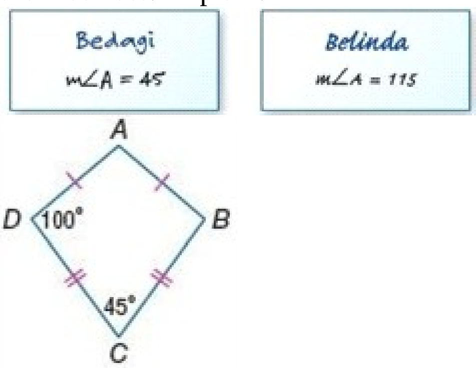
Prove: $\overline{XY} \parallel \overline{AB}$ and $\overline{XY} \parallel \overline{DC}$



The midpoint of $\overline{AD}$ is X. The coordinates are $\left(\frac{-b}{2}, \frac{c}{2}\right)$.

The midpoint of $\overline{BC}$ is $Y\left(\frac{2a+b}{2}, \frac{c}{2}\right)$. The slope of $\overline{AB} = 0$, the slope of $\overline{XY} = 0$, and the slope of $\overline{DC} = 0$. Thus, $\overline{XY} \parallel \overline{AB}$ *and* $\overline{XY} \parallel \overline{DC}$.

65. **ERROR ANALYSIS** Bedagi and Belinda are trying to determine $m\angle A$ in kite *ABCD* shown. Is either of them correct? Explain.



SOLUTION:

$\angle C$ is an acute angle and $\angle A$ is an obtuse angle. Since a kite can only have one pair of opposite congruent angles and

$\angle A \ncong \angle C$, so $\angle B \cong \angle D = 100$.

The sum of the measures of the angles of a quadrilateral is 360.

$$m\angle A + m\angle B + m\angle C + m\angle D = 360$$
$$m\angle A + 100 + 45 + 100 = 360$$
$$m\angle A = 115$$

Therefore, Belinda is correct.

ANSWER:

Belinda; $m\angle D = m\angle B$.

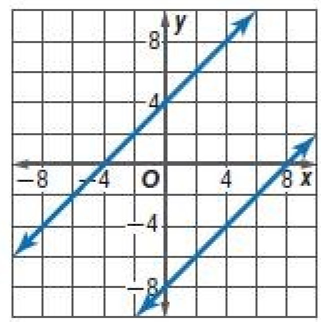
$$m\angle A + m\angle B + m\angle C + m\angle D = 360 \text{ or}$$
$$m\angle A + 100 + 45 + 100 = 360.$$

$$m\angle A = 115$$

66. **CHALLENGE** If the parallel sides of a trapezoid are contained by the lines $y = x + 4$ and $y = x - 8$, what equation represents the line contained by the midsegment?

SOLUTION:

First graph both lines.



By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. So, the slope of the line containing the midsegment is 1. Since the midsegment is equidistant from both the bases, the y-intercept of the line containing the midsegment will be the average of the y-intercepts of the bases, $\frac{4+(-8)}{2} = -2$.

Therefore, the equation is $y = x - 2$.

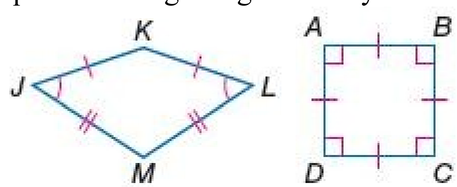
ANSWER:

$y = x - 2$

67. **CCSS ARGUMENTS** Is it *sometimes, always,* or *never* true that a square is also a kite? Explain.

SOLUTION:

A square has all 4 sides congruent, while a kite does not have any opposite sides congruent. A kite has exactly one pair of opposite angles congruent. A square has 4 right angles so they are all congruent.



Therefore, the statement is *never* true.

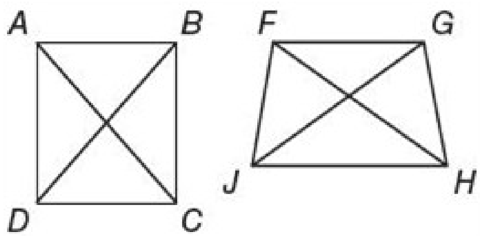
ANSWER:

Never; a square has all 4 sides $\cong$, while a kite does not have any opposite sides congruent.

68. **OPEN ENDED** Sketch two noncongruent trapezoids *ABCD* and *FGHJ* in which $\overline{AC} \cong \overline{FH}$ and $\overline{BD} \cong \overline{GJ}$.

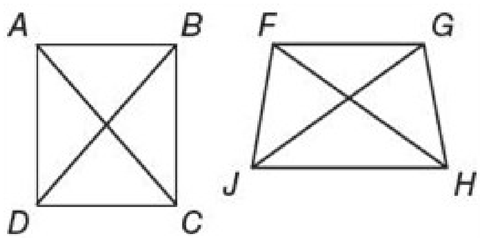
SOLUTION:

Sample answer: congruent trapezoids have corresponding sides and angles congruent. Draw the diagonals *AC* and *BD* first and then connect the edges such that $\overline{AB} \parallel \overline{DC}$. Draw diagonals *FH* and *GJ* that are the same length as *AC* and *BD*, respectively. If the diagonals are drawn such that the vertical angles are of different measure than those in *ABCD*, then *FG* and *JH* will be different than *AB* and *DC*.



ANSWER:

Sample answer:



69. **WRITING IN MATH** Describe the properties a quadrilateral must possess in order for the quadrilateral to be classified as a trapezoid, an isosceles trapezoid, or a kite. Compare the properties of all three quadrilaterals.

SOLUTION:

A quadrilateral is a trapezoid if:

- there is exactly one pair of sides are parallel.

A trapezoid is an isosceles trapezoid if:

- the legs are congruent.

A quadrilateral is a kite if:

- there is exactly 2 pairs of congruent consecutive sides, and
- opposite sides are not congruent.

A trapezoid and a kite both have four sides. In a trapezoid and isosceles trapezoid, both have exactly one pair of parallel sides.

ANSWER:

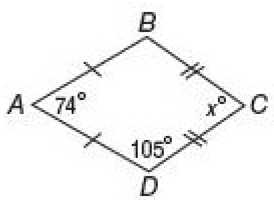
A quadrilateral must have exactly one pair of sides parallel to be a trapezoid. If the legs are congruent, then the trapezoid is an isosceles trapezoid. If a quadrilateral has exactly two pairs of consecutive congruent sides with the opposite sides not congruent, the quadrilateral is a kite. A trapezoid and a kite both have four sides. In a trapezoid and isosceles trapezoid, both have exactly one pair of parallel sides.

70. **ALGEBRA** All of the items on a breakfast menu cost the same whether ordered with something else or alone. Two pancakes and one order of bacon costs $4.92. If two orders of bacon cost $3.96, what does one pancake cost?
A $0.96
B $1.47
C $1.98
D $2.94

SOLUTION:
If two orders of bacon cost $3.96, one order costs
$\frac{3.96}{2} = \$1.98.$
Let x be the cost of one pancake. Then,
$2x + 1.98 = 4.92.$
Solve for x.
$2x = 2.94$
$x = 1.47$
So, each pancake costs $1.47. Therefore, the correct choice is B.

ANSWER:
B

71. **SHORT RESPONSE** If quadrilateral *ABCD* is a kite, what is $m\angle C$?



SOLUTION:
If *ABCD* is a kite with
$\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$, then,
$\angle B \cong \angle D$ and $\angle A \not\cong \angle C$.
So, $m\angle B = 105$.
The sum of the measures of the angles of a quadrilateral is 360.
$m\angle A + m\angle B + m\angle C + m\angle D = 360$
$74 + 105 + x + 105 = 360$
$x = 76$

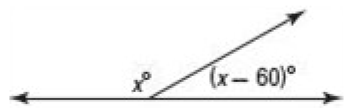
ANSWER:
76

72. Which figure can serve as a counterexample to the conjecture below?
If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.
F square
G rhombus
H parallelogram
J isosceles trapezoid

SOLUTION:
The diagonals of an isosceles trapezoid are congruent, but it is not a rectangle. Therefore, the correct choice is J.

ANSWER:
J

73. **SAT/ACT** In the figure below, what is the value of x?

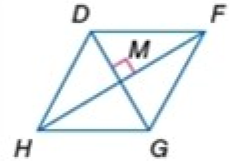


A 60
B 120
C 180
D 240
E 300

SOLUTION:
The two angles form a linear pair. So, sum of their measures is 180.
$x + x - 60 = 180$
$2x = 240$
$x = 120$
Therefore, the correct choice is B.

ANSWER:
B

ALGEBRA Quadrilateral *DFGH* is a rhombus. Find each value or measure.



74. If $m\angle FGH = 118$, find $m\angle MHG$.

SOLUTION:

Each pair of opposite angles of a rhombus is congruent. So,
$\angle FGH \cong \angle FDH$ and $\angle DHG \cong \angle DFG$.
The sum of the measures of the angles of a rhombus is 360.
Let $m\angle DHG = m\angle DFG = x$.

$118 + 118 + x + x = 360$ Angle sum is 360.
$2x = 124$ Combine like terms.
$x = 62$ Divide each side by 2.

A diagonal bisects the angle in a rhombus.

$$m\angle MHG = \frac{1}{2}m\angle DHG$$
$$= \frac{1}{2}(62)$$
$$= 31$$

ANSWER:

31

75. If $DM = 4x - 3$ and $MG = x + 6$, find DG.

SOLUTION:

The diagonals of a rhombus bisect each other.

$DM = MG$
$4x - 3 = x + 6$
$3x = 9$
$x = 3$

Next, find DG.

$DG = DM + MG$
$= 4x - 3 + x + 6$
$= 4(3) - 3 + 3 + 6$
$= 18$

ANSWER:

18

76. If $DF = 10$, find FG.

SOLUTION:

All the four sides of a rhombus are congruent. So, $FG = DF = 10$.

ANSWER:

10

77. If $HM = 12$ and $HD = 15$, find MG.

SOLUTION:

By the Pythagorean Theorem,
$DM^2 = 15^2 - 12^2 = 81$
$DM = \sqrt{81} = 9$
The diagonals of a rhombus bisect each other. So, $MG = DM = 9$.

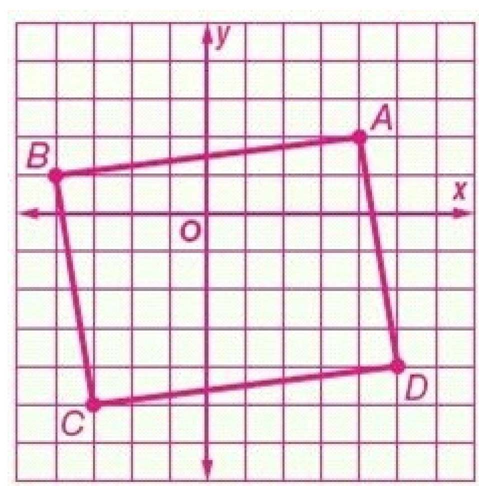
ANSWER:

9

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

78. $A(4, 2)$, $B(-4, 1)$, $C(-3, -5)$, $D(5, -4)$; Distance Formula

SOLUTION:



First, Use the Distance formula to find the lengths of the sides of the quadrilateral.

$$AB = \sqrt{(1-2)^2 + (-4-4)^2} = \sqrt{(-1)^2 + (-8)^2} = \sqrt{1+64} = \sqrt{65}$$
$$BC = \sqrt{(-5-1)^2 + (-3-(-4))^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{36+1} = \sqrt{37}$$
$$CD = \sqrt{(-4-(-5))^2 + (5-(-3))^2} = \sqrt{1^2 + 8^2} = \sqrt{1+64} = \sqrt{65}$$
$$AD = \sqrt{(-4-2)^2 + (5-4)^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{36+1} = \sqrt{37}$$

$AB = \sqrt{65} = CD, BC = \sqrt{37} = DA$, so $ABCD$ is a parallelogram.
A parallelogram is a rectangle if the diagonals are

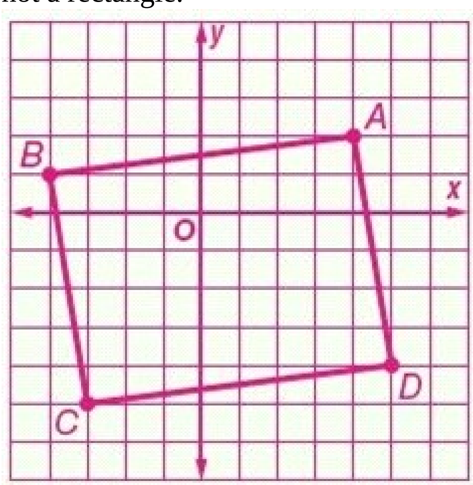
congruent. Use the Distance formula to find the lengths of the diagonals.

$$AC = \sqrt{(-5-2)^2 + (-3-4)^2} = \sqrt{(-7)^2 + (-7)^2} = \sqrt{49+49} = \sqrt{98}$$
$$BD = \sqrt{(-4-1)^2 + (5-(-4))^2} = \sqrt{(-5)^2 + 9^2} = \sqrt{25+81} = \sqrt{106}$$

So the diagonals are not congruent. Thus, *ABCD* is not a rectangle.

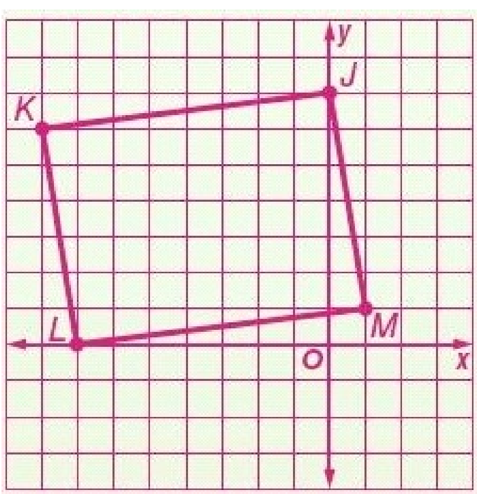
ANSWER:

No; $AB = \sqrt{65} = CD, BC = \sqrt{37} = DA,$ so *ABCD* is a parallelogram. $BD = \sqrt{106}; AC = \sqrt{98}. BD \neq AC,$ so the diagonals are not congruent. Thus, *ABCD* is not a rectangle.



79. *J*(0, 7), *K*(–8, 6), *L*(–7, 0), *M*(1, 1); Slope Formula

SOLUTION:



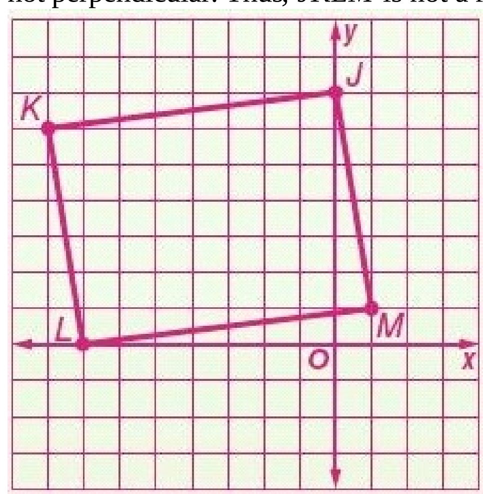
Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{JK} = \frac{6-7}{-8-0} = \frac{1}{8}$$
$$m_{KL} = \frac{0-6}{-7-(-8)} = -6$$
$$m_{LM} = \frac{1-0}{0-(-7)} = \frac{1}{8}$$
$$m_{JM} = \frac{1-7}{1-0} = -6$$

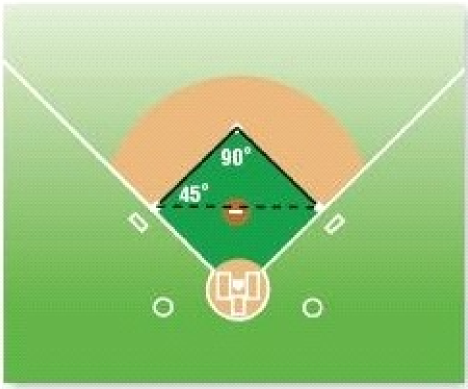
The slopes of each pair of opposite sides are equal. So, the two pairs of opposite sides are parallel. Therefore, the quadrilateral *JKLM* is a parallelogram. None of the adjacent sides have slopes whose product is –1. So, the angles are not right angles. Therefore, *JKLM* is not a rectangle.

ANSWER:

No; slope of $\overline{JK} = \frac{1}{8} =$ slope of $\overline{LM}$ and slope of $\overline{KL} = -6 =$ slope of $\overline{MJ}$. So, *JKLM* is a parallelogram. The product of the slopes of consecutive sides $\neq -1$, so the consecutive sides are not perpendicular. Thus, *JKLM* is not a rectangle.



80. **BASEBALL** A batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain.



SOLUTION:

By Theorem 5.10, if one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle. The angle opposite the side from third base to second base is 45 degrees which is smaller than the angle opposite the side from third to first, 90 degrees. Therefore, the distance from third to second is shorter than the distance from third to first. So, he should throw it to the second base.

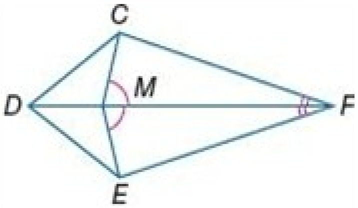
ANSWER:

Second base; the angle opposite the side from third base to second base is smaller than the angle opposite the side from third to first. Therefore, the distance from third to second is shorter than the distance from third to first.

81. **PROOF** Write a two-column proof.

Given: $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$

Prove: $\Delta DMC \cong \Delta DME$

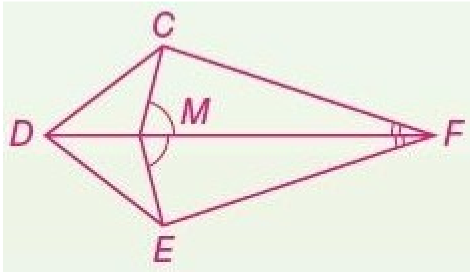


SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\angle CMF \cong \angle EMF, \angle CFM \cong \angle EFM$. You need to prove $\Delta DMC \cong \Delta DME$. Use the properties that you have learned about triangle congruence to walk through the proof.

Given: $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$

Prove: $\Delta DMC \cong \Delta DME$



Proof:

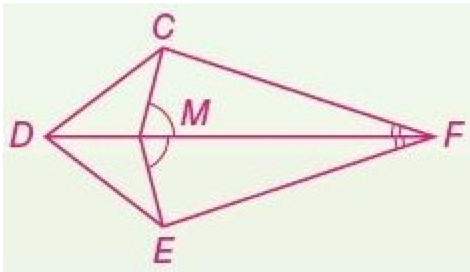
Statements (Reasons)

1. $\angle CMF \cong \angle EMF, \angle CFM \cong \angle EFM$ (Given)
2. $\overline{MF} \cong \overline{MF}, \overline{DM} \cong \overline{DM}$ (Reflexive Property)
3. $\Delta CMF \cong \Delta EMF$ (ASA)
4. $\overline{CM} \cong \overline{EM}$ (CPCTC)
5. $\angle DMC$ and $\angle CMF$ are supplementary and $\angle DME$ and $\angle EMF$ are supplementary. (Supplement Th.)
6. $\angle DMC \cong \angle DME$ ($\angle$s suppl. To $\cong \angle$s are $\cong$.)
7. $\Delta DMC \cong \Delta DME$ (SAS)

ANSWER:

Given: $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$

Prove: $\Delta DMC \cong \Delta DME$



Proof:

Statements (Reasons)

1. $\angle CMF \cong \angle EMF, \angle CFM \cong \angle EFM$ (Given)
2. $\overline{MF} \cong \overline{MF}, \overline{DM} \cong \overline{DM}$ (Reflexive Property)
3. $\Delta CMF \cong \Delta EMF$ (ASA)
4. $\overline{CM} \cong \overline{EM}$ (CPCTC)
5. $\angle DMC$ and $\angle CMF$ are supplementary and

$\angle DME$ and $\angle EMF$ are supplementary. (Supplement Th.)
6. $\angle DMC \cong \angle DME$ ($\angle$s suppl. To $\cong \angle$s are $\cong$.)
7. $\Delta DMC \cong \Delta DME$ (SAS)

Write an expression for the slope of each segment given the coordinates and endpoints.

82. $(x, 4y)$, $(–x, 4y)$

SOLUTION:

Use the slope formula to find the slope of the line joining the given points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4y - 4y}{-x - x} \\ &= \frac{0}{-2x} \\ &= 0 \end{aligned}$$

ANSWER:

0

83. $(–x, 5x)$, $(0, 6x)$

SOLUTION:

Use the slope formula to find the slope of the line joining the given points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6x - 5x}{0 - (-x)} \\ &= \frac{x}{x} \\ &= 1 \end{aligned}$$

ANSWER:

1

84. (y, x), (y, y)

SOLUTION:

Use the slope formula to find the slope of the line joining the given points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{y - x}{y - y} \\ &= \frac{y - x}{0} \\ &= \text{undefined} \end{aligned}$$

ANSWER:

undefined