Leture 4

Chapter 2

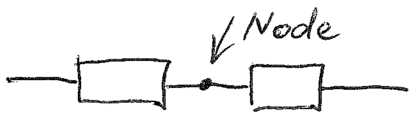
- Kirchhoff's ~~Circuit~~ current Law (KCL)
- Kirchhoff's voltage Law (KVL)

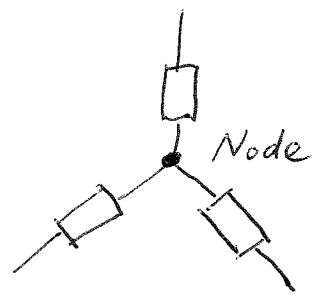
§2.1–2.3 PP. 53–89

In last Chapter, we introduced basic electrical quantities (current and voltage) and their relationship with electrical power and energy; we also learned basic circuit elements: sources and resistance. To analyze circuits, we still have to know fundamental laws that govern the behavior of voltages and currents in a circuit. This is what we are going to learn today. There are two fundamental laws, one governs the current; the other dictates the voltge. Before introducing the two laws, we first get to know some new terminology.

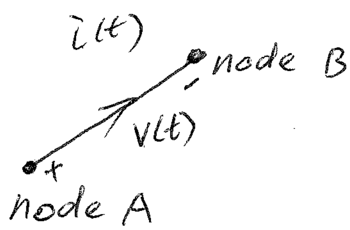
Some New Terminology

- Node: the connection point of one or more circuit elements





- Branch: Symbolically, a line segment connecting two nodes.

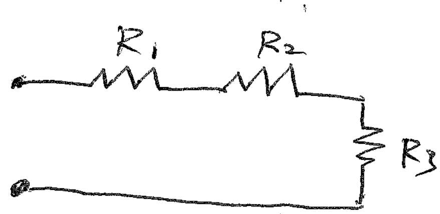


It represents a generic two-terminal circuit element.

- Series circuit:

A sequential connection of 2-terminal circuit elements end to end.

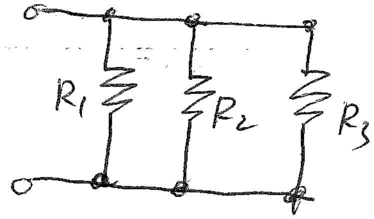
Example:



- Parallel circuit:

Two-terminal circuit elements share both terminals in common. In other words, one terminal is wired together; the other is also wired together.

Example:



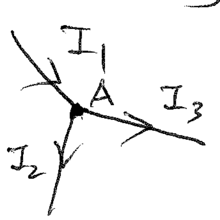
Kirchhoff's current Law (KCL)

- The algebraic sum of the current entering (or leaving) a node is zero for every instant of time.

  Mathematically written as

$$\boxed{\begin{array}{l} \sum I_{in} = 0 \\ \text{or} \quad \sum I_{out} = 0 \end{array}}$$

- The above law can be understood in the following way.



Consider current I_1 entering node A. Imagine, this means a group of ~~electrons~~ charges ~~enter nod~~ flow into node A. Due to charge conservation, the same amount of charges will leave node A via path 2 and path 3. As a result,

$$I_1 = I_2 + I_3$$

Hence, $I_1 - I_2 - I_3 = 0 \implies \sum I_{in} = 0$

current entering node A from path 1

current entering node A from path 2

current entering node A from path 3

Similarly, $-I_1 + I_2 + I_3 = 0$

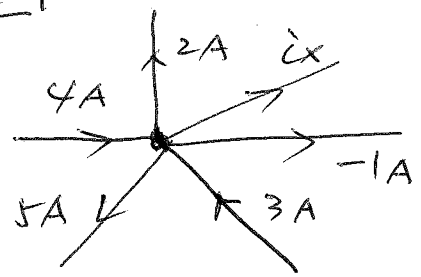
Current leaving node A from path 1

Current leaving node A from path 2

Current leaving node A from path 3

i.e. the algebraic sum of the current leaving a node is also zero for every instant of time.

Example 1



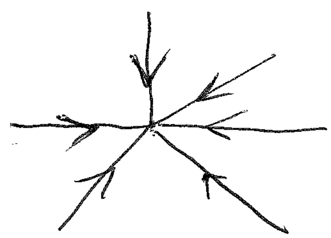
Find i_x.

Solution

$$\Sigma I_{in} = 0$$

$$4 - 2 + 1 - 5 + 3 - i_x = 0 \qquad (1)$$

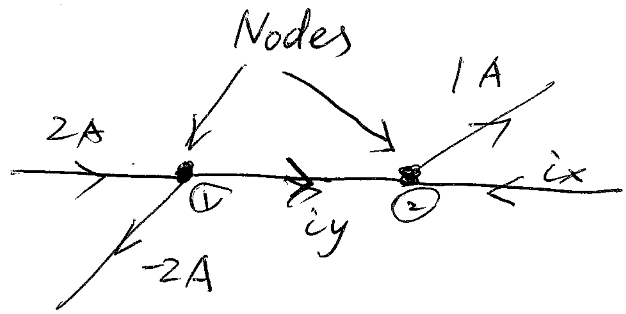
The above is written by setting the reference direction of each branch as the direction entering the circuit node, i.e.



If the ~~and~~ current given in the original problem agrees with the reference direction in that specific branch, then use that current as it is; otherwise, put '−' sign in front of the current.

From (1) $\quad i_x = 1 \quad (A)$

Example 2



Find i_x

Solution At node 1, using KCL

$$2-(-2)-i_y=0$$

$$\Rightarrow \quad i_y = 4\ (A)$$

At node 2, using KCL

$$i_x + i_y - 1 = 0$$

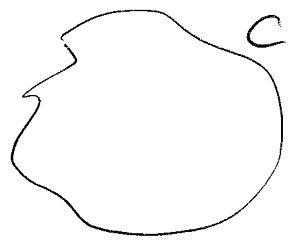
$$i_x = 1 - i_y = \boxed{-3}\ (A)$$

- KCL is also valid for Gaussian curves (surfaces)

What are Gaussian curves (surfaces)?

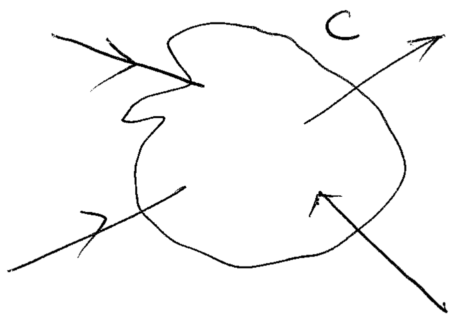
They are "closed" curves (surfaces).

Example



For the above closed contour C.

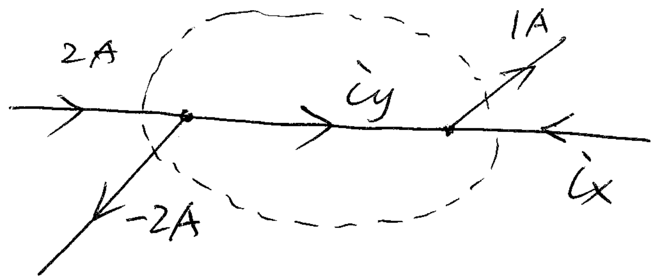
KCL also holds true. So



$\sum I_{in} = 0$

or $\sum I_{out} = 0$

We can resolve Example 2 by using the concept of Gaussian curves (surfaces)



Dashed line denotes a closed curve.
We apply KCL on this closed curve

$$\sum I_{in} = 0$$

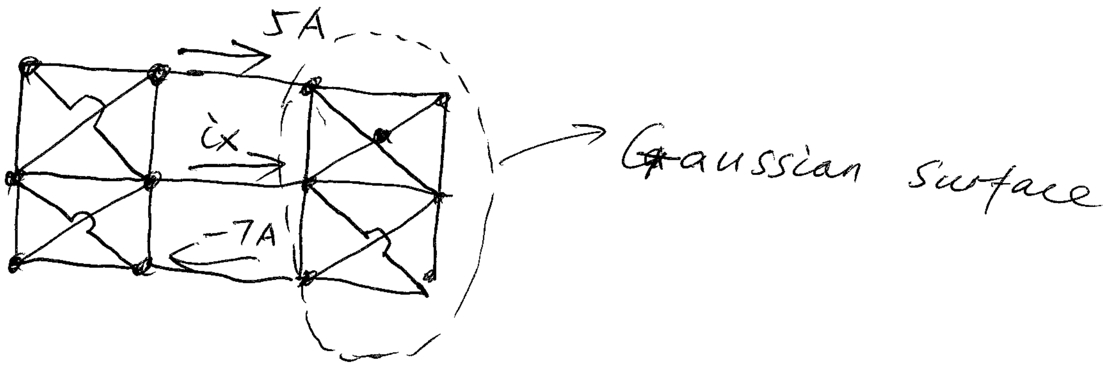
$$\Rightarrow \quad 2 - (-2) - 1 + i_x = 0$$

$$\Rightarrow \quad 4 - 1 + i_x = 0$$

$$\Rightarrow \quad \boxed{i_x = -3} \quad (A)$$

Same answer as we got by using node-based KCL.

Example 4 Find i_x



$$\sum I_{in} = 0$$

$$\Rightarrow i_x + 5 - (-7) = 0$$

$$\Rightarrow i_x = -12 \quad (A)$$

Kirchhoff's Voltage Law (KVL)

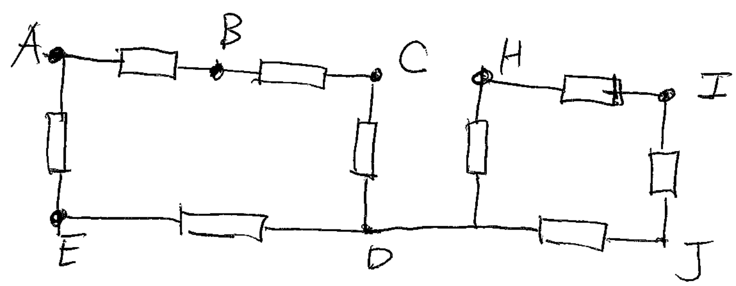
- The algebraic sum of the voltage drops around any closed path (or closed node sequence) is zero at every instant of time.

$$\boxed{\sum V_{drop} = 0}$$

Definition:

- Closed path: a connection of circuit elements that ends and begins ~~at~~ at the same node, and traverse each node only once.
- Closed node sequence: A finite sequence of nodes that begins and ends at the same node.

Example:



Closed path: $A \to B \to C \to D \to E$, $H \to I \to J \to D$

Closed node sequence: ABCHIJDEA, ABCHDEA

- For any pair of nodes j and k, the voltage drop from node j to node k is

$$V_{jk} = V_j - V_k$$

V_j : electric potential at node j
also knowns as the voltage at node j
With respect to reference, since

$$V_j = V_j - 0$$
$$= V_j - V_{ref}$$

$\rightarrow$ reference (node) point where potential is zero

The reference node is usually called 'ground'

Similarly,

V_K : node voltage at node k, i.e. the Voltage drop from node k to a reference node.

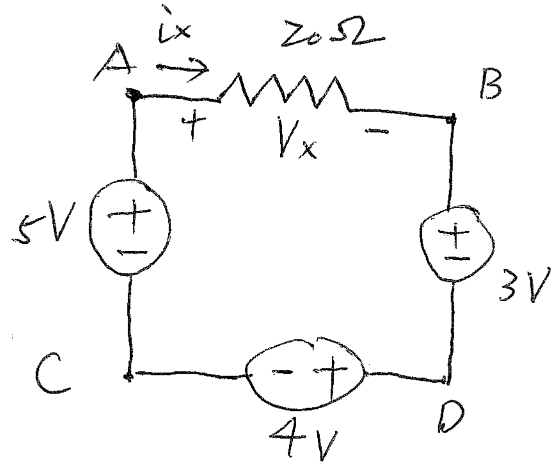
- Now, it is not difficult to understand KVL. Take any closed node sequence as an example, say $\underline{A}BCHDE\underline{A}$

$$\Sigma V_{drop} = V_{AB} + V_{BC} + V_{CH} + V_{HD} + V_{DE} + V_{EA}$$
$$= V_A - \cancel{V_B} + \cancel{V_B} - \cancel{V_C} + \cancel{V_C} - V_H + \cdots - \cancel{V_E} + \cancel{V_E} - V_A$$
$$= 0$$

- You can also make an analogy between KVL and the gravitational potential drops around any closed path.

For example, you climb a mountain starting from a point at the foot of the hill, get to the top of the hill, then return to your starting point. The gravitational potential difference is zero although you went through a long and difficult path, because the path is closed.

<u>Example</u> Find V_x and i_x



<u>Solution</u> Here, ~~ABDC~~ $A \to B \to D \to C$ is clearly a closed path.

We can apply KVL onto this path.

$$V_{AB} + V_{BD} + V_{DC} + V_{CA} = 0$$

$$V_x + 3 + 4 - 5 = 0$$

$$\Rightarrow \quad V_x = 5 - 4 - 3 = -2 \quad (V)$$

$$i_x = \frac{V_x}{R} = \frac{-2}{20} = -0.1 \quad (A)$$

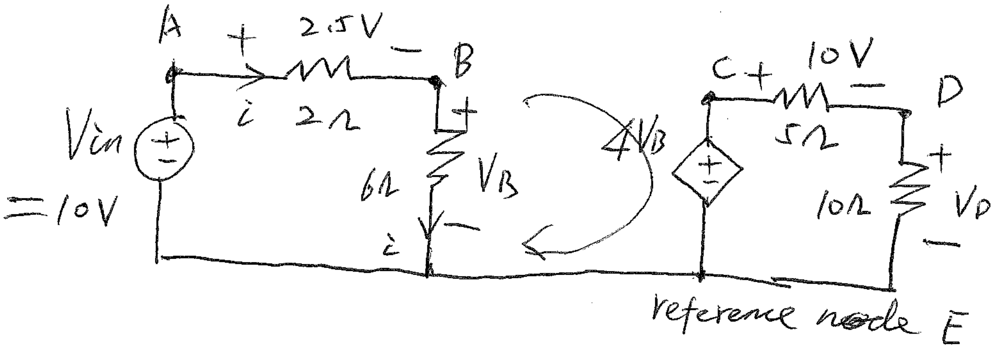
You can also use a reversed direction to calculate voltage drop, i.e. $A \to C \to D \to B$ (Couter-clockwise)

then $V_{AC} + V_{CD} + V_{DB} + V_{BA} = 0$

$$\Rightarrow \quad 5 + (-4) + (-3) + (-V_x) = 0$$

$$\Rightarrow \quad V_x = -2 \ (V) \quad \text{the same answer}$$

Example What is V_{BC} ?



Solution BCEB is certainly a closed-node sequence. There are also many others such as ABCEA, ABCDEA. You can choose any of them to figure out V_{BC}. Let's use BCEB as an example.

$$V_{BC} + V_{CE} + V_{EB} = 0$$

$$V_{BC} + 4V_B - V_B = 0$$

$$\Rightarrow V_{BC} = -3V_B$$

As long as we find V_B, we know V_{BC}.

$$V_B = 6i = 6 \cdot \frac{2.5}{2} = 7.5\ V$$

$$\therefore V_{BC} = -22.5\ (V)$$