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Book review

Control theory from the geometric viewpoint [Andrei A. Agrachev and Yuri L. Sachkov. Copyright 2003 World Scientific Publishing, ISBN: 3-540-21019-9]

Geometrical methods have had a profound impact in the development of modern nonlinear control theory. Fundamental results such as the orbit theorem, feedback linearization, disturbance decoupling or the various controllability tests for nonlinear systems are all deeply rooted on a geometric view of control theory. It is perhaps surprising, and possibly debatable, that in order to understand and appreciate the "essence" of linear control systems one has to delve into the intricacies of Lie brackets and Lie algebras. This is because only the geometric perspective offers the tools to study the properties of control systems that are invariant under (nonlinear) changes of coordinates and can therefore be considered intrinsic. Consider, for example, an inverted pendulum or the ball and beam system. It should be apparent that reachability or optimality properties for these systems do not depend on the particular reference frame chosen to write their equations of motion. These are intrinsic properties of these physical systems and thus require geometric techniques for its study.

"Control Theory from the Geometric Viewpoint" is a recent addition to the geometric control theory monograph/textbook literature having Jurdjevic (1997) as its closest neighbor and Nijmeijer and van der Schaft (1995), Isidori (1996) and Bloch (2003) as more distant ones. The book evolved from lecture notes for graduate courses taught by the first author at the International School for Advanced Studies in Trieste, Italy. The lecture notes style can be felt throughout the 24 chapters of the book treating a large number of topics ranging from controllability and reachability analysis to higher order conditions for optimality. This lecture notes style, patent on the relatively large number of treated topics in 400 pages, is the book's main handicap and merit. If, on the one hand, most chapters can be independently read thus allowing the reader to immediately dive into the topic of choice and quickly reach the zenith result, on the other hand, there is a certain lack of fluidity when one tries to read the book chapters consecutively. In the remaining lines I will try to articulate my own opinion, naturally conditioned by my taste and background, on the choice of topics and presentation as I go through some of the individual chapters.

The book starts with Chapter 1 where basic differential geometric concepts are reviewed. This chapter essentially serves to establish notation used throughout the book since it is too succinct for readers that are not familiar with differential geometry. The second chapter describes the chronological calculus, a very elegant tool developed by one of the authors and published originally in Russian. This chapter immediately distinguishes this book from its neighbors as chronological calculus is not discussed in any of them. I only regret that the authors did not invest more time in presenting chronological calculus in a more pedagogical fashion, perhaps following Nestruev's style (Nestruev, 2002). Chapter 2 also serves to separate the wheat from the chaff in the sense that it defines the level of mathematical sophistication expected from the reader. This is even more the case if one considers that chronological calculus is an essential tool in the proofs of many results presented in the book. Its first appearance occurs already in Chapter 3 to derive the closed formula expression for the solution of a linear control system in terms of the matrix exponential. Chapter 5 introduces the orbit theorem and some of its consequences such as Frobenius theorem and Nagano's Principle stating that, in general,[1] all the local information of analytic control systems is encoded on Lie brackets. If the reader has reached this point in the book, then he should be convinced that the geometric viewpoint is worth being pursued since according to Nagano's Principle, Lie brackets are pervasive in control theory. Chapters 6 and 7, detailing the orbits of the controlled rigid body and of simple point mass systems, illustrate the fact that orbits of control systems have, in general, dimension strictly greater than the number of inputs. This motivates the study of reachable sets presented in Chapter 8. This chapter exemplifies the presentation style of the book and the intended audience: a reader can have an idea of the available results on reachable sets by reading the 9 pages of Chapter 8 or gain a much deeper and detailed knowledge about the same topic by investing the necessary time and effort to study Chapters 3 and 4 in Jurdjevic (1997) totaling 60 pages. These are clearly different styles for different audiences. The first part of

[1] A technically more precise statement is obtained by eliminating "in general" and adding "bracket generating" immediately before "analytic".

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the book terminates with Chapter 9 where the problem of feedback linearization is treated. The remaining book is devoted to Optimal Control. The basic problem is introduced in Chapter 10 where its relationship with reachable sets is also described. In order to understand the geometric content of Pontryagin's maximum principle one needs the language of geometric mechanics, known as Symplectic geometry, which is reviewed in Chapter 11. After a brisk walk through tensors, symplectic forms and Hamiltonian vector fields we return to Pontryagin's Principle in Chapter 12. The following four chapters contain examples, a curious application, and the optimal control theory for linear control systems. In most of these chapters the influence of the excellent original source (Pontryagin, Boltyanskii, Gamkrelidze, & Mishchenko, 1962) can be felt quite vividly. Further examples are given in Chapter 19 after setting up the necessary geometric machinery in Chapter 18. These examples, on compact Lie groups, provide a different justification for a geometric version of Pontryagin's Principle applicable to control systems having smooth manifolds as state space. In my opinion, the real force of the book is only reached at the last five chapters. It could be argued that it is reached late but it would be difficult to talk about second order conditions for optimality without first introducing the first order conditions described by Pontryagin's Principle. These five chapters contain a very interesting exposition of second order optimality conditions, the Jacobi equation and the Curvature invariant drawing heavily on the authors own work. Furthermore, I am not aware of other textbook/monograph sources where this material can be found. The reader is also offered a reduction theorem and a final example, the rolling bodies, illustrating several results presented in the book.

"Control Theory from the Geometric Viewpoint" is a very welcome addition to the geometric control literature combining an exposition of standard topics with more recent ones based on the authors own research. It is a valuable resource deserving to sit on the bookshelf of every researcher using geometric control tools in his work. It also plays a role in the classroom where it can be used as a textbook for an advanced course in control theory targeted to a mathematically skilled audience. It can also be used for a challenging course taught to graduate engineering students provided that it is complemented with other sources covering the essential concepts of (symplectic) differential geometry. In this case only part of the book's topics can be discussed in class and the lecture notes style is definitely helpful in isolating the relevant chapters.

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About the reviewer

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Paulo Tabuada was the recipient of the Francisco de Holanda prize in 1998 for the best research project with an artistic or aesthetic component awarded by the Portuguese Science Foundation. He was a finalist for the Best Student Paper Award at both the 2001 American Control Conference and the 2001 IEEE Conference on Decision and Control and he was the recipient of a NSF CAREER award in 2005. He co-edited the volume *Networked Embedded Sensing and Control* published in Springer's Lecture Notes in Control and Information Sciences series.

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