

MATH 103 Word Problems Worksheet

Study Technique: As you work through the problems try to find which problems are similar so instead of answering many different types of questions you are answering only a few.

Some types of word problems to get you started: rates, density, arc length, volumes of revolution and probability distribution functions.

Practice:

1. Find the area between the curves $x = 3y - y^2$ and $x + y = 3$.
2. The volume of the solid generated by rotating the region bounded by $y = e^x$, $y = 0$, between $x = 0$ and $x = 4$ around the x -axis. Find the volume of this region rotated about the line $x = -1$.
3. What is the length of the $y = x^{3/2}$ over the interval $[0, 1]$?
4. An express mail truck delivers mail to various companies situated along a central avenue and often goes back and forth as new mail arrives. Over some period of time, $0 \leq t \leq 10$, its velocity (in km per hour) can be described by the function:

$$v(t) = (t - 5)e^t.$$

- (a) Find the displacement over this period of time.
 - (b) How much gasoline was consumed during this period of time if the vehicle uses 1/2 liters per km.
5. The acceleration of a particle is

$$a(t) = \frac{1}{t^2 + 1}.$$

If we say its initial position is zero meters and after one second the particle stops for a moment what is the position function at time t ?

6. A hiker is climbing over a hill which has height $f(t) = \frac{1}{2} + \sin(t)$ km above sea level where t is measured in hours. What is the average altitude of the hiker during its walk if they walked for π hours.
7. A beam 4 meters long bar has density

$$\rho(x) = \frac{10}{25 - x^2}$$

kg / m distance x from the left end of the bar.

- (a) How much does the bar weigh?
- (b) Where is the center of mass?
- (c) Where will you cut the bar to have two pieces of equal mass?

8. Consider a particle that is initially traveling at 1 meters per second. The particle constantly decelerates for 4 seconds until it is traveling backwards at 4 meters per second. Then the particle linearly accelerates for 4 more seconds until it is at rest. Draw the position, velocity and acceleration functions. If the particle initially is 5 meters forward where is the particle at the end of the 8 seconds? What was the total distance traveled?

9. The percentage of customers who call IT that have their phone call answered by t hours is

$$f(t) = -\frac{1}{(t+1)^3} + 1.$$

What is the average weight time of a customer? What percentage of customers have to wait at least two hours? What is the median wait time? What is the standard deviation of the wait time?

10. The probability that the height of a basketball player is x meters is given by the probability density distribution

$$p(x) = Cxe^{-x/2}$$

where $0 < x < 3$.

- (a) What is the constant C ?
- (b) What is the most probable height?
- (c) What is the mean height?

11. An open cylindrical barrel of diameter 7ft and height 21ft is filled to a depth of 10ft by muddy water. The density of the water varies linearly with the depth of the water. If the density at the bottom is 80 pounds per cubic foot and the density at the top is 30 pounds per cubic foot find the total weight of the muddy water in the cylindrical barrel.

12. Two cities get school funding depending on their population. City A's population is growing at a rate $A(t) = 4e^{-t}$ thousand people per year and city B's population is growing at $B(t) = 6e^{-2t}$.

- (a) Say each city initially has the same population. Which city is larger after $\ln(2) \approx .7$ years?
- (b) Consider the far future. Which city has the larger population? By how much?
- (c) When is does city B have the largest lead over city A? By how much?

13. A cup of tea is 100°C is in a room 20°C . By Newton's law of cooling the temperature of the tea will change at a rate proportional to the difference between the object's temperature and the surrounding temperature. How long does it take for the tea to reach the drinkable temperature of 70°C ? Let the constant of proportionality be $k \geq 0$.