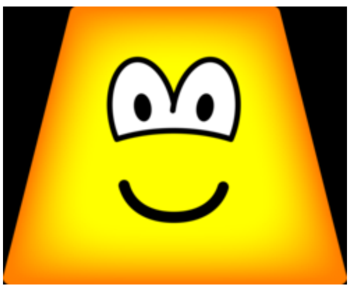
Unit 4
QUADRILATERALS
A polygon with 4 sides

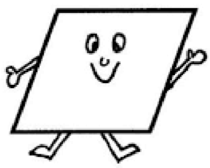
In this unit we will extensively study the properties of **quadrilaterals**, any figure with exactly four sides.

Lesson 1 Objective: We will begin unit 4 by first studying a very broad category known as a **trapezoid**.

U4 L1
Trapezoids and Parallelograms

There are many special types of quadrilaterals, including parallelograms, rectangles, squares, and kites (yes even that is a shape).





TRAPEZOID

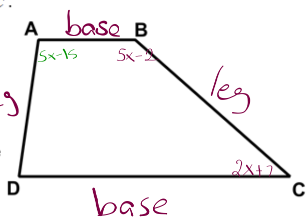
Any **quadrilateral** with **<u>at least</u> one pair of parallel sides**. This means it could have either one pair of parallel sides or two pairs of parallel sides.

***Exercise* #1:** In the diagram below, *ABCD* is a trapezoid with $\overline{AB} \parallel \overline{DC}$.

(a) What must be true of angle pair $\angle A$ and $\angle D$ and angle pair $\angle B$ and $\angle C$?

supplementary

(b) If $m\angle A = 5x - 15$, $m\angle B = 5x - 2$, and $m\angle C = 2x + 7$, then find the degree measure of $\angle D$. Show the work you used.



$5x - 2 + 2x + 7 = 180°$

$7x + 5 = 180$

$7x = 175$

$x = 25$

$m\angle A = 5(25) - 15$

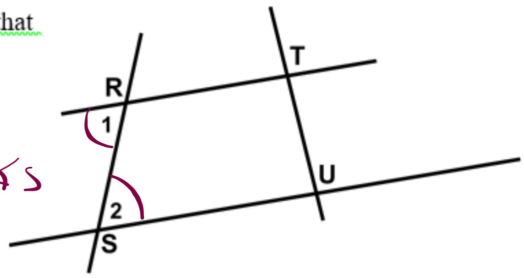
$= 125 - 15$

$m\angle A = 110$

$m\angle D = 70°$

Trapezoid is a very broad category of figure in geometry. Since they have only a single defining property (i.e. at least one pair of parallel sides), they do not give rise to many extra properties besides the one noted in *Exercise* #1. Still, it is important to know their basic definition should they arise in proof.

***Exercise* #2:** In the diagram shown below it is known that $\angle 1 \cong \angle 2$. Explain why quadrilateral *RTUS* must be a trapezoid.



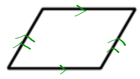
≅ alternate interior ∡s

Prove $\overline{RT} \parallel \overline{SU}$

A quad w/ @ least 1 pair of ∥ sides is trapezoid

PARALLELOGRAM

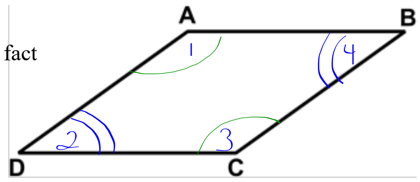
Any **quadrilateral** with **both** pairs of **opposite sides being parallel.**



***Exercise* #3:** Explain why all parallelograms are also trapezoids.

Because they have @ least 1 pair of || lines.

***Exercise* #4:** Given that *ABCD* is a parallelogram, answer the following questions:



(a) List all pairs of supplementary angles based on the fact that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

∠B & ∠C   ∠A & ∠B

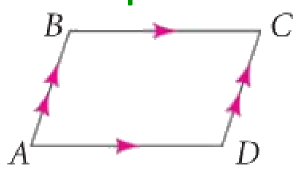
∠A & ∠D   ∠D & ∠C

(b) Prove below that $m\angle A = m\angle C$ (and thus opposite angles of a parallelogram are equal).

Supplements of the same are ≅.

- **Consecutive Angles of a Parallelogram Are Supplementary**

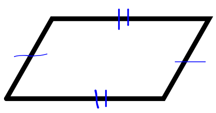
(based on what we know about parallel lines cut by a transversal!)



Angles of a polygon that share a side are **consecutive angles.** A parallelogram has opposite sides parallel. Its consecutive angles are same-side interior angles so they are supplementary. In ▱*ABCD*, consecutive angles *B* and *C* are supplementary, as are consecutive angles *C* and *D*.

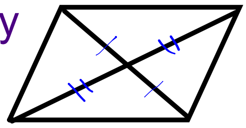
(Same-side interiors)

Definition of Parallelogram: A quadrilateral with both pairs of opposite sides parallel.



Properties of a Parallelogram

- Both pairs of opposite sides are
- ✓ Both pairs of opposite angles are
- ✓ Consecutive angles are supplementary
- Diagonals bisect each other →



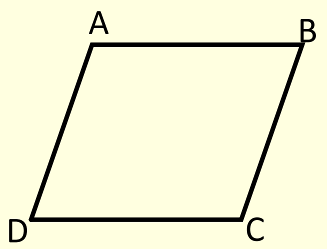
All of these properties can be proven using transformations & Congruent Triangles!

Here's another proof of the property

- Both pairs of opposite angles are

Given: ▱$ABCD$

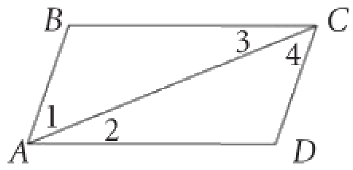
Prove: $\angle A \cong \angle C$, $\angle B \cong \angle D$



Statement	Reason
1.	1.
2.	2.
3.	3.

Let's prove this properties now!

- Both pairs of opposite sides are



Given: $\square ABCD$

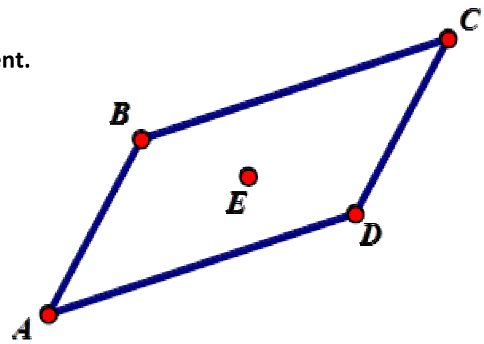
Prove: $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

<u>Proof by Symmetry (Informal – Transformational Approach)</u>

Given: Parallelogram ABCD

Prove that opposite sides of a parallelogram are congruent.

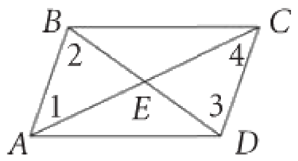


A parallelogram has 180 degree rotational symmetry. This means that if I rotate it 180 degrees about the center point E, then

Rotations preserve

Here's another proof of the property

- Diagonals bisect each other



Given: $\square ABCD$

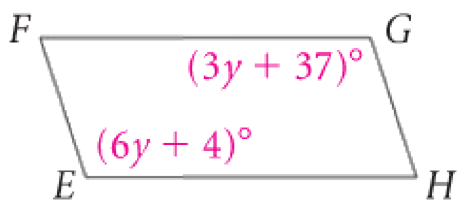
Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.

Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.** Given

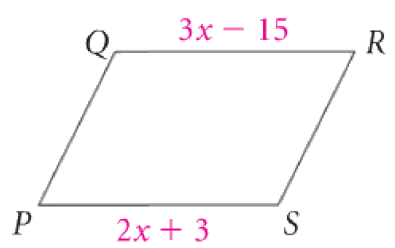
Now that this has been proven you may use it in a proof!!

Using these property algebraically:

1) Find the value of y in $\square EFGH$. Then find $m\angle E$, $m\angle G$, $m\angle F$, and $m\angle H$.



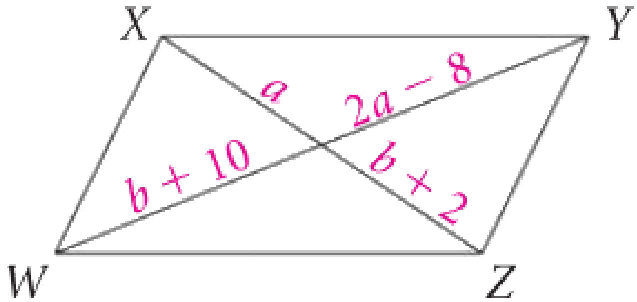
2) Find the value of x in $\square PQRS$. Then find QR and PS.



Using these property algebraically:

3) Find $m\angle S$ in $\square RSTW$.

4) Find the values of a and b.



Homework

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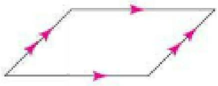
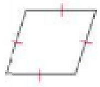
Definitions **Special Quadrilaterals**

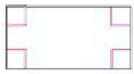
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.



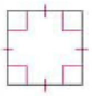
A **rhombus** is a parallelogram with four congruent sides.



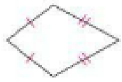
A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.



A **kite** is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The **isosceles trapezoid** at the right is a trapezoid whose nonparallel opposite sides are congruent.