SOLVING SOLUTION AND MIXTURE VERBAL PROBLEMS

This type of problem involves mixing two different solutions of a certain ingredient to get a desired concentration of the ingredient. Before we can solve problems that involve concentrations, we must review certain concepts about percents. If you need to do this, go to the brush-up materials for solving percent problems on the Dolciani website.

1. Solution Problems

Basic Equation: amount of solution $\times$ concentration of substance = amount of substance

Example: 40 ounces (amount of solution) of a 25% solution of acid (concentration) contains 25(40) = 10 ounces of acid

Usual equation to solve for the variable:

Amount of substance in solution 1 + Amount of substance in solution 2 = Amount of substance in solution

2. Mixture Problems

Basic Equation: unit price $\times$ # units = cost (or value)

Example: 5 pounds of apples (# units) that sell for $1.20 per pound (unit price) costs 5(1.20) = $6

Using equation to solve for the variable:

Cost of ingredient 1 + Cost of ingredient 2 = Cost of mixture

Now let's look at an actual mixture problem. It is easiest when solving a mixture problem to make a table to get the information organized. No matter the story line of the problem, the table can be used and labelled as necessary. Let's look at a few examples.

Example 1:

Fatima's chemistry lab stocks an 8% acid solution and a 20% acid solution. How many ounces of each must she combine to produce 60 ounces of a mixture that is 10% acid?

Solution:

We want to find how much of each solution must be in the mixture. We assign variables to these two unknowns:

Let x = amount of 8% solution needed and let y = amount of 20% solution needed

Now we can make a table showing what we know about the two solutions and the mixture

	Amount of Solution	Amount of Acid
8% solution	x	$0.08x$
20% solution	y	$0.20y$
10% mixture	60	(0.10)(60)

The right hand column tells us how much acid is in the ingredients and the mixture. We obtain the right hand column by remembering that "and 8% acid solution" means that 8% of the solution is acid. Thus 8% of x ounces of solution is $0.08x$ ounces of acid. Similarly, 20% of y ounces is $0.20y$ ounces of acid; and 10% of 60 is (0.10)(60) ounces of acid.

Now we need to write two equations. We do so by noting that the amounts put into any mixture must add up to the total amount that is in the mixture. Therefore, for the solutions (middle column),

$$x + y = 60$$

and for the acid only (right column),

$$0.08x + 0.20y = (0.10)(60)$$

We thus have a systems of two equations in two unknowns that we can solve by substitution. We rewrite the first equation and substitute in the second.

$y = 60 - x$	First equation
$0.08x + 0.20(60 - x) = 6$	Substituting for y
$0.08 + 12 - 0.20x = 6$	Multiplying out
$-0.12x + 12 = 6$	Combining like terms
$-0.12x = -6$	Adding –12 to both sides
$x = 50$	Multiplying by $\frac{-1}{0.12}$

Now, by substituting 50 for x in the first equation, we have

$$50 + y = 60$$

$$y = 10$$

We then need to check the values for x and y to see that they total 60 ounces.

$$50 + 10 = 60$$

And we need to check that the concentration of the mixture is correct.

$$\frac{\text{Amount of acid}}{\text{Amount of solution}} \times 100\%$$

$$= \frac{0.08x + 0.20y}{60} \times 100\%$$

$$= \frac{0.08(50) + 0.20(10)}{60} \times 100\%$$

$$= \frac{4 + 2}{60} \times 100\%$$

$$= \frac{6}{60} \times 100\%$$

$$= \frac{1}{10} \times 100\% = 10\% \text{ Correct}$$

So the 10% mixture contains 50 ounces of 8% acid solution and 10 ounces of 20% acid solution.

Example 2:

A piggy bank has 40 nickels and dimes. The total amount of money in the bank $2.50. Find the amount of each coin.

Solution:

We need to find how many of each coin there are in the bank. We assign the variables that way.

Let x = number of nickels

Let y = number of dimes

Then we make a table showing what we know about the two coins, their monetary value, and the total amount in the bank. Because each nickel is worth 5 cents, the monetary value of all the nickels is $(0.05)x$, or $0.05x$. Similarly, because each dime is worth 10 cents, the monetary value of all the dimes in $(0.10)y$, or $0.10y$.

	Number Of Coins	Monetary Value
Nickels	x	$0.05x$
Dimes	y	$0.10y$
Bank	40	2.50

Next we need to write the two equations. We can write one for the number of coins and one for their value.

$$x + y = 40$$

$$0.05x + 0.10y = 2.50$$

We solve the first equation for y:

$$y = 40 - x$$

Solving the system by substituting into the second equation, we get

$0.05x + 0.10(40 - x) = 2.50$	Substituting for y
$0.05x + 4 - 0.10x = 2.50$	Multiplying out
$-0.05x + 4 = 2.50$	Combining like terms
$-0.05x = -1.50$	Adding –4 to both sides
$x = 30$	Multiplying by $\frac{-1}{0.05}$

Substituting 30 for x in the first equation yields

$$30 + y = 40$$

$$y = 10$$

Checking the value, we have

30 nickels:	$30(0.05) = \$1.50$	
10 dimes:	$10(0.10) = \$1.00$	
Total:	$\$2.50$	Correct

So there are 30 nickels and 10 dimes in the piggy bank.

Example 3: How many pounds of a 15% aluminum alloy must be added to 500 pounds of a 22% alloy to make a 20% alloy?

Solution:

Let x = amount of 15% alloy that must be added to the 500 pounds of the 22% alloy.

Then the total number of pounds of the new solution is x + 500.

A table or a diagram is very useful for these problems:

	amount of alloy	Concentration	amount of silver
15% alloy	x	.15	.15x
22% alloy	500	.22	.22(500)
solution	x + 500	.20	.20(x + 500)

The equation is $.15x+.22(500)=.20(x+500)$

Solving for x, you find that 200 pounds of a 15% alloy of aluminum must be added to a 500 pounds of a 22% alloy to obtain a 20% alloy.

Example 4: The owner of a fruit stand combined cranberry juice that cost $4.60 per gallon with 50 gallons of apple juice that cost $2.24 per gallon. How much cranberry juice was used to make the cranapple juice if the mixture costs $3.00 per gallon. Round to the nearest tenth.

Let x = # of gallons of cranberry juice that must be added. Then the total # of gallons of the mixture is x + 50.

	# gallons	price per gallon	total cost
cranberry juice	x	4.6	4.6x
apple juice	50	2.24	2.24(50)
cranapple juice	x + 500	3.0	3.0(x + 50)

The equation is: $4.6x+2.24(50)=3.0(x+500)$

Solving for x, you find that 23.8 gallons of cranberry juice costing $4.60 per pound is needed to combine with 50 gallons of apple juice costing $2.24 per gallon to make cranapple juice costing $3.00 per gallon.

Practice:

1. How many liters of a 25% solution of a drug must be mixed with a 55% solution of the drug to produce 50 liters of a 46% solution of this drug?

2. A pocketful of coins includes dimes and quarters. The total value of the 36 coins is $5.10. Find the number of each coin.

3. A hospital laboratory needs a 28% hydrogen chloride solution, but only 15% and 35% HCl solutions are in stock. How much of the 35% solution should be mixed with 100 fl oz of the 15% solution to get a 28% solution?

4. How many liters of a 10% dextrose solution should be mixed with 20 L of a 15% dextrose solution to obtain a 12% dextrose solution?

5. How many liters of a 30% NaCl solution should be mixed with 1 L of a solution that contains no NaCl to obtain a 24% NaCl solution?

6. 80 pounds of a dry powder drug with 30% concentration needs to be mixed with how many pounds of the dry powder drug with a 10% concentration to get a 20% dry powder drug?

7. An 8% solution of local anesthesia is needed. However, 40 fl oz of a 10% solution is the only type of this anesthesia in stock. How much neutral solution containing no anesthesia should be added to the 40 fl oz container of the 10% solution?

8. A pharmacist needs to fill an order for a 4% lidocaine solution topical. However, only 2% and 9% concentrations are in stock. How much of the 9% concentration should be mixed with 150g of the 2% concentration to get a 4% concentration?

9. How many ounces of a certain chemical that is 40% alcohol must be mixed with a second chemical that is 60% alcohol to get 40 ounces of a mixture that is 55% alcohol?

10. A sample of 200 ml of 15% HCl is combined with 200 ml of an unmarked concentration of HCl. What is the concentration if the resulting solution has a 14% concentration?

11. Mixed Nuts has 10% pecans. Nestor Nuts has 18% pecans. How many ounces of each should be mixed to get 20 ounces of a nut mixture that is 15% pecans?

12. Chocolate that sell for \$1.50 per pound is added to mints that sell for \$2.75 per pound to make a 25 pound mix that sells for \$2.50 per pound. How many pounds of each are there in the mix?

13. Sean has \$3.70 with quarters, nickels and dimes. The number of dimes is one less than the number of nickels, and the number of quarters is 3 less than the number of dimes. How much of each does he have?

Answers:

1. 15 liters of 25% and 35 liters of 55%
2. 26 dimes and 10 quarters
3. $\frac{1300}{7}$ or 185.71 oz of 35% solution
4. 30 liters of 10% solution
5. 4 liters of 30% solution
6. 80 pounds
7. 10oz is needed
8. 60g of 9% solution
9. 30 ox of 60%, 10 oz of 40%
10. 13%
11. Mixed: 7.5 ounces; Nestor Nuts: 12.5 ounces
12. 20 pounds of mints; 5 pounds of chocolate
13. 12 nickels, 11 dimes, 8 quarters