Logic: A Brief Introduction

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Chapter 1 - Basic Training

1.1 Introduction

In this logic course, we are going to be relying on some "mental muscles" that may need some toning up. But don't worry, we recognize that it may take some time to get these muscles strengthened; so we are going to take it slow before we get up and start running. Are you ready? Well, let's begin our basic training with some preliminary conceptual stretching. Certainly all of us recognize that activities like riding a bicycle, balancing a checkbook, or playing chess are acquired skills. Further, we know that in order to acquire these skills, training and study are important, but practice is essential. It is also commonplace to acknowledge that these acquired skills can be exercised at various levels of mastery. Those who have studied the piano and who have practiced diligently certainly are able to play better than those of us who have not. The plain fact is that some people play the piano better than others. And the same is true of riding a bicycle, playing chess, and so forth.

Now let's stretch this agreement about such skills a little further. Consider the activity of thinking. Obviously, we all think, but it is seldom that we think about thinking. So let's try to stretch our thinking to include thinking about thinking. As we do, and if this does not cause too much of a cramp, we will see that thinking is indeed an activity that has much in common with other acquired skills. As such, it seems reasonable to think that thinking, like other acquired skills, can be improved with training, study and practice.

Thinking of thinking in this way may be a bit uncomfortable at first, especially if you are inclined to think that thinking is not an acquired skill. Perhaps you believe that thinking is more like a natural function such as eating or drinking, or perhaps like seeing or hearing, than it is like an acquired skill. This may seem plausible, since it is clear that even though not everyone knows how to ride a horse, every normal person eats and drinks, sees and hears, and does so as a matter of course. Accordingly, it would not occur to us to remark on how well one eats her food, as though some are expert eaters, while others are not. Of course, we may need lessons in manners, and perhaps we can all learn to chew our food more than we usually do, but ordinarily we do not think that some people eat better than others, or that some are masters of it, or that others need improvement, or that they need eating lessons, or need further study or more practice in eating. While we may have a good idea of what a chess master is, it seems unlikely that we would as readily know what an eating master would be.

It is therefore tempting to think of thinking as though it were more like eating than like the acquired skills of playing chess or riding a bicycle. Again, while not everyone plays chess or rides horses, everyone eats, and of course everyone thinks. Indeed, if you think of thinking as merely a natural function, as opposed to an acquired skill, you may conclude that you think just fine, or at least that you think as well as anybody else. You may even be offended by the idea that your thinking may be in need of improvement, or in need of further study and practice.

Suppose, however, that thinking is in important respects more like playing a piano or playing chess than it is like eating. Is there any reason to think so? Well, consider this: while it certainly seems to make sense to praise someone as being a good (or bad) thinker, the same does not hold in the case of eating. What would a good (or bad) eater be? This suggests that thinking has something in common with other acquired skills: we can be good or bad at it.

If we can be good at thinking, and not so obviously good at eating (if thinking is more of an acquired skill, like piano playing, than we may have thought at first) then it is reasonable to think that thinking is a skill that, with study and practice, we can improve and even master. Indeed, this is just what we want to suggest. Of course mastering the skill of good thinking will require--as most other such acquired skills do--the mastering of concepts, rules, techniques, and the like. Hard work, however, can be rewarding. We are convinced that if you work hard in this course, if you study hard and most importantly practice and then practice some more, you will improve your thinking skills and become a better and stronger thinker.

Well, what would it be like to become a master thinker? Good question. But before we try to answer it, we must first alert you to the fact that "thinking well" has various meanings. Such variations occur because there are many different skills available for human mastery. Some people are very good at thinking of new melodies--we call them composers; some people are very good at thinking of novel ways to express human emotions--we call them poets, or novelists; some claim to be better than others at thinking of God, or beauty, or of life in general--we call them mystics. Generally, however, we do not submit these kinds of thinking to appraisals of correctness or incorrectness, or of clarity or a lack of clarity. Moreover, we are not quite sure how we might go about improving these kinds of thinking skills. We take such skills to be natural gifts.

There is, however, another kind of thinking that is subject to being appraised as correct or incorrect. This is the thinking skill called reasoning. To be a good thinker in this sense is to be a person who reasons correctly--most of the time, at least. The master thinker rarely reasons incorrectly.

Even though we want you to think of reasoning well as an acquired skill, we must concede that it is also, in some respects, like a natural function insofar as it does not depend on having special gifts. Our assumption, in fact, is that most people reason and that most are capable of reasoning well. In this respect then, thinking well is not like those acquired skills that require special gifts. Although only a few can become accomplished pianists, most can be good thinkers, or at least significantly improve their thinking skills.

Clearly then, we cannot assume that everyone who thinks also reasons well. And unfortunately, as everyone knows, the consequences of bad (incorrect) reasoning are many and may be deep; such consequences range from lost time in solving practical and theoretical problems, to ruined lives caused by coming to unwarranted conclusions about what is true, important, and valuable.

We will begin this course in logic, then, assuming that it makes sense to say that we can improve our reasoning skills, and assuming further that you want to strengthen yours. The way to this improvement will be through study and practice. What you will study are some basic principles and techniques that will enable you, with practice, to distinguish good (correct) reasoning from bad (incorrect) reasoning. With this distinction firmly in mind, you will be able to submit your own reasoning, as well as the reasoning of others, to constructive criticism. This will enable you to listen more carefully to others and to evaluate more critically what they say. It should also make you a more careful and convincing speaker and writer.

So, let's pause and take a breath. I hope that you are beginning to see more clearly what we are going to be doing in this course in logic.

Got your breath? OK, stretching is over. Now it's time to begin our run. It's on to fundamentals.

1.2 Logic, Arguments, and Propositions

Well, where have we gotten so far? Far enough, we think, to get to a definition of the subject matter of this course in "logic."

Before we do this, however, let's make sure we do not confuse logic with psychology. While psychology does study thinking, it does not propose a system for distinguishing good reasoning from bad reasoning.

Keeping this in mind we can define "logic," the subject matter of this course, as follows:

Logic: **The basic principles and techniques that are used to distinguish correct (good) reasoning from incorrect (bad) reasoning**

If we are to use the term "logic" correctly, however, we must avoid a common confusion. The term is sometimes mistakenly thought to carry only the connotation of good or correct reasoning. For example, when someone has reasoned well (correctly), we say that he or she is being logical, and when someone has reasoned badly (incorrectly) we say that he or she is being illogical. The discipline of logic studies both correct and incorrect reasoning. Accordingly, when someone reasons illogically, we will say that he or she has made a logical mistake, that is, a mistake in logic.

Now, let's stretch this definition of "**logic**" a little further. As it turns out, **to reason correctly is to argue correctly**. In fact, an alternative definition of the subject matter of logic is as follows: the basic principles and techniques that are used to distinguish **good (correct) arguments** from **bad (incorrect) arguments**. But before we can say more about this distinction between good and bad arguments, we must define what logicians mean by the term "argument."

"Argument," like so many words, has various meanings. For many it is an entirely negative term connoting confrontation, conflict and disagreement. Clearly this is what is meant when a coed complains about her date, saying that all she and her boyfriend did was argue all night. The assumption here is that arguing does not make for a pleasant relationship. And most likely, the term "argument" conjures up for some the images of flying pots and pans and shouting, perhaps even violence. Along these lines, we think of arguments as something like school-yard fights in which the so-called “argument” consists of a heated disagreement in which one party simply denies what the other party affirms, perhaps coming to blows over their difference.

This image of an argument is precisely what is depicted in Monty Python’s famous gag about a person who buys a ticket that entitles him to an argument with a professional arguer. As the gag goes, two people are supposedly arguing, but in fact one person simply denies everything the other person asserts. Such an exchange might go as follows:

“There is life on other planets.”
“No, there's not!”
“Oh yes, there is!”
“Oh no, there's not”!
"Is!"
"Isn't!"
"Is!"
"Isn't!"

Back and forth it goes, getting nowhere. While this may be taken for an argument, in the logician’s sense of the term, it is not. We need a more serious definition. So let’s put this caricature of an argument aside, and get on to the logician’s definition.

In the first place an argument is made up of several statements. However, not every group of statements is an argument. What turns a group of statements into an argument is the **relation** between or among them.

Accordingly, we need to clarify this relation between or among statements, for it is this relation that transforms them into an argument.

But before we can do this, we need to define the term “statement” more precisely. As we will use it, the term is defined as **the content of an assertion that is either true or false.** To mark this special use of the term “statement” we will use the term “proposition” instead of “statement.” Keep in mind, however, that the terms are synonymous for all practical purposes.

Proposition:
The content of an assertion that is either true or false

As you may be realizing by now, when it comes to defining terms there is always room for confusion. To avoid one such confusion, you need to be careful to distinguish a proposition from a sentence. Propositions are usually expressed in sentences, but not every sentence necessarily expresses a proposition. Clearly the declarative sentence, “The cat is on the mat,” expresses a proposition, for what it asserts (its propositional content) is either true or false, depending of course on whether the cat is or is not on the mat. But consider the interrogative sentence “Is the cat on the mat?” This sentence does not assert anything, so it cannot be either true or false, and hence it cannot count as asserting a proposition. And the same can be said for other sentences in other grammatical forms, for example, the exclamation “Help!” Clearly a cry for help is neither true nor false; hence this sentence does not express a proposition.

But don’t get the wrong idea here: we don’t mean to be suggesting that all declarative sentences necessarily express propositions, or that only declarative sentences can express them. But again, this depends on how you define “declarative sentence.” Some define it as a sentence that makes a statement (asserts a proposition.) Well, of course, if we accept this definition, then by definition, all declarative sentences express propositions. If, however, we define “declarative sentence” as a sentence with a subject followed by a verb without intervening phrases or clauses, then some declarative sentences do not express propositions. For example, consider the sentence, “I do." By the second definition, this is a declarative sentence, but it would certainly be a mistake to think that it makes an assertion that is either true or false. In fact, philosophers usually call such utterances “performatives” for they do not assert some fact but are used to perform some action. In this case, "I do" may be a matter of taking someone to be your husband or wife.

By the same token, there are some sentences that are not declarative in grammatical form but nevertheless can be used to express propositions. Consider the interrogative sentence, “You certainly want to live, don’t you?” Such a sentence may reasonably be taken to contain an assertion: “You want to live” which is, of course, true or false, depending on whether you do or don’t want to live.

And there is another confusion that we must avoid. Don’t think that because we define “proposition” as the content of an assertion that is either true or false, that we must know whether the content is in fact true or false in order to know that a proposition is being expressed. Even though I do not know whether it is raining right now on the White House, the assertion “It is raining right now on the White House” does express a proposition. The reason for this is that even if we do not know whether it is or is not in fact raining there, we do know that it is either raining or not raining and not both at the same time. Even assertions such as “The universe is finite,” or “God exists” express propositions, despite the fact that we do not know whether what is asserted is true or false. What we do know is that what is asserted in these sentences cannot be both true and false. As we might put this, we know that any given proposition is either true or false, and cannot be both, and that this is so even if we do not know what its truth value is in fact.

There are a couple more confusions to avoid concerning propositions and sentences. Don’t think that just because two sentences are exactly the same that they therefore express the same proposition. They may not.

How can this be so? Consider the sentence “I have a headache” said by me (Ron Hall), and “I have a headache” said by Jon Smith. The sentences are exactly the same, but are used in different assertions and therefore entirely different propositions are expressed.

And don’t think that just because two sentences are not exactly the same that they cannot be used to express the same proposition. Consider the sentence “I have a headache” said by me, and “Ron has a headache” said by Jon. Now we have two different sentences used to express the very same proposition.

And one last thing: one sentence may express more than one proposition. For example, "The cat is on the mat; after all I saw her there just a minute ago." This sentence may be used to assert two propositions, "The cat is on the mat" and "I saw her there just a minute ago." As we shall see later, examples of this kind show that it is possible for an argument to be expressed in a single sentence.

The upshot here is that we need to pay attention to what speakers or writers are doing with sentences. We have to decide whether or not they are asserting something with the sentences that they say or write. If so, then these sentences express propositions, that is, make assertions that are either true or false. Next, we must try to discern what is being asserted, and we may even have to reformulate the sentence to make the propositional content clear.

When we have made these determinations, and when we have also determined that we are dealing with more than one proposition, we are ready to see whether or not the group of assertions is put forward as an argument. To make this determination, however, we must ascertain whether the propositions stand in that special relation to one another that we mentioned above. So we are brought back to the task of saying what this special relation is.

So let’s get on with it. In an argument propositions are related to each other by virtue of the roles they play. There are two roles: either giving or receiving support. In the image below, it is clear that the earth is giving support and the man is being supported.

Propositions give support when they are offered as evidence for the truth of another proposition. With this in mind, we can now define an argument as follows:

Argument:
A group of propositions is an argument if some of the propositions (the premises) are asserted as supporting the truth of another of the propositions (the conclusion)

Now, let's take this one step further and name these roles that propositions may play in an argument. Propositions used to give support in an argument are called **premises**, and the proposition that is receiving support is called the **conclusion** of the argument.

The following table summarizes these definitions:

Premise: **A proposition in an argument that provides support for the conclusion**
Conclusion: **The proposition in an argument that receives support from the premises**
Argument: **A group of propositions in which at least one of these propositions functions as premise and only one of which functions as the conclusion**

An argument can have several premises, but only one conclusion. We count arguments by counting conclusions, that is, every argument has only one conclusion. In addition, there is nothing intrinsic to a particular proposition that makes it either a premise or a conclusion. The sole determination of whether a particular proposition is a premise or a conclusion is its role in the argument. Indeed, the same proposition can be the conclusion of one argument and a premise in another. Again, if a proposition is playing a support role, it is a premise; if it is playing the role of being supported then it is a conclusion.

In order for an argument to be persuasive, it must begin with premises that are accepted as true or probably true. The reason for this is that the premises that are doing the supporting are not themselves supported. They must be accepted if the conclusion of the argument is to be accepted. It may help to think of premises as assumptions. Or as we might say, the premises of an argument constitute the foundation on which the conclusion is established.

The most useful arguments are the ones that arrive at conclusions that were previously not known. Correct useful arguments, in other words, provide us with conclusions that add to our knowledge. This is so because a correct and useful argument establishes or proves that a given proposition (the conclusion) is true or is probably true, depending on the truth of the premises and on the kind of argument it is. (In a moment we will say more about how conclusions depend on the truth of the premises, and about differences in kinds of arguments.)

It should be clear by now, that the way logicians think of arguments is quite different from the way that the Monty Python gag represents them. With our expanded concept of an argument, perhaps we are now in a position to understand why someone might rave about his or her date precisely because the two did nothing but argue all night. How exhilarating to think that it is possible for two people actually to have a meaningful and rational conversation. (Looking deeply into each other's eyes over candlelight can last only so long.) So perhaps it makes good sense to imagine the following comment about a date as follows: "What a great date! We argued all evening: she had her opinions, and I had mine, and we were willing to listen to and evaluate the reasons each had for these opinions. This was wonderful!"

Well, I can't promise you a lifetime of such rational conversations, but I hope you will begin to appreciate them and to seek them out. Or, as I might also say, I hope that in this course in logic you will begin to develop your appreciation of the importance and positive value that arguments can have in our lives.

Before we turn to the exercises I want to make sure that you have the important distinction between a premise and a conclusion firmly in hand. To do this I remind you to think of it in terms of the difference between **giving and receiving support**. Again, when a proposition is taken as receiving support, it is the conclusion of the argument. When a proposition is used to give support, it is a premise in the argument.

In the image below the man is receiving support from the woman. As we might say, the man is playing the role that a conclusion plays in an argument. As well, the woman is playing the role that a premise plays in an argument.

When you are doing the exercises, keep in mind the definitions that we have just offered. Be especially aware of the logician's special use of terms like "proposition" and "argument." And as I just illustrated, with regard to the logician's special use of the term "argument," be sure that you keep in mind the difference between the role of giving support and the role of receiving support.

Summary of Definitions

- ***Logic:*** **The basic principles and techniques that are used to distinguish correct (good) reasoning from incorrect (bad) reasoning**
- ***Proposition:*** **The content of an assertion that is either true or false**
- ***Argument:*** **A group of propositions is an argument if some of the propositions (the premises) are asserted as supporting the truth of another of the propositions (the conclusion)**
- ***Premise:*** **A proposition in an argument that is used to give support.**
- ***Conclusion:*** **The proposition in an argument that is taken to receive support**

1.3 Deduction and Induction

Arguments are usually divided into two primary kinds: inductive arguments (inductions) which are based upon inductive reasoning and deductive arguments (deductions) which are based upon deductive reasoning. Needless to say, then, we must distinguish these two kinds of arguments. But we must keep in mind that both are arguments. What is common to both kinds of reasoning is that conclusions are claimed to follow from premises. If the conclusions do follow, then the arguments are good ones, and if not, they are bad ones. The distinction between inductions and deductions, then, will turn on two different senses in which conclusions are said to follow from premises.

Broadly speaking, an inductive argument claims some support for its conclusion, whereas a deductive argument offers decisive support for its conclusion. More precisely, we define an inductive argument as follows:

Inductive argument: An argument in which it is claimed that the premises offer some evidence for the conclusion but do not guarantee the conclusion's truth

Of course the more likely it is that the conclusion follows from the premises, the better the induction. In appraising an inductive argument, we will accordingly say that it is either weak or strong—depending on how likely it is that the truth of the conclusion follows from the premises. It should be clear from this definition that the truth of the premises in an inductive argument is not taken to guarantee the truth of its conclusion. Rather, the premises are offered as evidence in support of the claim that the conclusion is probably true. With this definition in mind, it should be obvious that the following argument is an induction:

Every Full Professor in Harvard's Philosophy Department holds a Ph.D. degree
Every Full Professor in Yale's Philosophy Department holds a Ph.D. degree
In fact this is true of every Philosophy Department with which I am acquainted
Therefore it is likely that every Full Professor in every university Department of Philosophy holds a Ph.D. degree

This inductive argument is what is called a **generalization**. But inductive arguments take many forms. These forms include, among others, arguments from analogy, predictions, and sometimes both. Consider the following examples:

1. **Analogy:** *That nest looks like the one I watched a Robin make in my backyard. So it must also be a Robin's nest*
2**. Prediction:** *It usually rains in the late afternoon in the summer when the clouds gather on the horizon like they are doing now. So I expect it will rain this evening*
3. **Both:** *When I took ancient philosophy in college we read Plato's* Republic. *So if you take that course when you go to college you will read it too.*

Good and bad inductions are so in degree, not absolutely. Additional evidence, of course, can make an inductive argument stronger or weaker. In fact, in some cases one new piece of evidence can completely undermine the likelihood of its conclusion being true. For example, if I am trying to establish that "All swans are white," seeing another white one adds strength to my argument. However, it takes only one black swan to demonstrate that the inductive generalization, "All swans are white," regardless of how many white swans have been observed, is not warranted.

Perhaps this definition of induction is not one that you are familiar with. This is not surprising since there is a rather long tradition of defining induction as reasoning from particular cases to generalizations. In fact the first example of an inductive argument that we cited above fits this definition perfectly. The problem with defining induction this way, however, is that inductive arguments do not always move from particular cases to generalizations. For example, the following argument is an induction, yet it moves from the general to the particular.

Most university professors of philosophy hold the Ph.D. degree
Jon Smith is a university professor of philosophy
Therefore, Jon Smith probably holds the Ph.D. degree

So even though many inductive arguments do move from particular cases to generalizations, not all do. It is therefore a more precise definition of inductive reasoning to say that such arguments claim that the premises of the argument aim at making it likely (probable) that the conclusion is warranted. A strong inductive argument (a good one) succeeds in making the case (by offering premises) that the conclusion is very likely warranted. A weak inductive argument is one that is less successful and, without ceasing to be an induction, may be so weak that it would normally be considered bad reasoning. Consider this very weak induction: "All philosophers are arrogant. I recently had a conversation with two of them at a bar." probability of the conclusion being true is greater given the premises; and that they are weak or strong according to how high the likelihood of the conclusion's being true is, based on these premises.

Now, on to deductive arguments: unlike inductive arguments that offer some evidence for their conclusions, deductive arguments are ones whose premises are offered as a guarantee of the warrant of their conclusions. We can define a deductive argument precisely as follows:

Deductive Argument:
An argument in which it is claimed that the conclusion follows from the premises with necessity

If the conclusion of such an argument does in fact follow with necessity, we say the argument is a good (correct) one and if the conclusion does not follow from the premises with necessity, we say that the argument is a bad (incorrect) one. When we say that the conclusion follows from the premises with necessity in a correct deductive argument, we mean that this conclusion is not just probably true, but must be true, that it cannot fail to be true, if the premises were true. When the conclusion does follow necessarily from the premises, we say that this deductive argument is **valid**.

We will turn in the next section to an elaboration of this definition of validity. First, consider the following deductive arguments. Hopefully it is obvious to you that these are good arguments.

All rabbits are mammals
All mammals are warm-blooded
Therefore, all rabbits are warm-blooded
All men are mortal
Socrates is a man
Therefore Socrates is mortal

Given the premises of these arguments, the conclusions are not just likely to be true, but must be true, that is, they cannot be false.

Be cautious here: just because we know that an argument is a deduction, we do not know automatically that it is a good one. That is, what makes an argument a deduction is the claim that the truth of the conclusion follows from the truth of the premises with necessity. However, the claim that the conclusion follows from the premises with necessity does not mean that it actually does. In such cases, where the conclusion of a deductive argument may be false even if its premises are assumed to be true, the argument does not cease to be a deduction; it is simply a bad or incorrect deduction. Consider the following example of an incorrect deductive argument:

All rabbits are mammals
All rabbits have legs
Therefore, all mammals have legs

As in the case of inductions, our definition of deductions differs from the traditional view according to which deductive arguments move from general premises to particular conclusions. Many deductions do indeed follow this pattern. Indeed the argument above about Socrates is an example of a deductive argument that follows this pattern. However, while the first rabbit argument above is an example of a good deduction and the second rabbit argument that we just cited is an example of a bad deduction, both are cases of deductions that move from general premises to a general conclusion. As well, a deduction can also move from particular premises to a particular conclusion. Consider the following example of such a deduction (which by the way is valid):

Either Ron Hall is a professor of philosophy or he is an astronaut
He is not an astronaut
Therefore, Ron Hall is a professor of philosophy

Inductive Argument: Claims the conclusion is probably true based upon the premises
Strong: The sun has risen in the East for thousands upon thousands of mornings. So it will rise in the East tomorrow morning
Weak: John has a Ph.D. but is unemployed. So this degree does not help you get a job

So again, we will stay with our more precise definition of deductive arguments, according to which an argument is a deduction if and only if it claims that its conclusion follows from its premises with necessity.

While it is important to make the distinction between inductive and deductive arguments, this course will be centered solely on deductive reasoning. Do not take this to mean that we think that deduction is more important than induction. We do not. We recognize, however, that you will have opportunities elsewhere to study induction, since there are many courses offered in statistics and probability. This course in logic, no doubt, will be one of the few opportunities you will have to study deduction as such.

Perhaps the summary will help you to remember the differences between inductive and deductive arguments.

Deductive Argument: Claims that it would be impossible for the conclusion to be false if the premises were true
Correct (Valid): If John practices law in Florida, he must have passed the Florida Bar. John practices law in Florida, so he must have passed the Florida Bar.
Incorrect (Invalid): If John practices law in Florida, he must have passed the Florida Bar. John has passed the Florida Bar. Therefore, John practices law in Florida. (The conclusion here could be false even if the premises were true.)

1.4 Truth, Validity and Soundness

We must be careful not to think that arguments, inductive or deductive, can be evaluated as being either true or false. These terms are reserved solely for propositions. Remember also that the terms “true” and “false” do not apply to sentences, but only to the propositions that are expressed in them. This is so, because there are many sentences that are neither true nor false, because they do not express propositions. “Help!” was our earlier example of such a sentence that is neither true nor false.

Recall that deductive arguments make a claim. That claim is that the conclusion follows from the premises with necessity. Now of course not every claim is warranted. I may claim that I can beat you at tennis. You would be well advised, however, to invite me to the court to see if I can actually make good on this claim.

Similarly, if I claim that a conclusion follows from a set of premises with necessity, this does not mean that the conclusion actually does guarantee the conclusion. We might need to take this claim to a different sort of court, a court of appeals. Well, what are we to appeal to in trying to decide whether or not our claims are satisfied? Obviously we need a criterion. That is, we need a very precise test for judging whether or not the claim of any deductive argument is satisfied. If a deductive argument passes or fails this test (and the test is strictly pass/fail with nothing in between) we say that the argument is either valid or invalid. So what is this test?

Before I say what the criterion of validity is, and later what the criteria for soundness are, let's note some likely misuses of the terms. Notice that when we introduced the idea of validity above, we did not qualify it as "deductive validity." The reason for this is that the logical term "validity" applies only to deductive arguments. The logician reserves "validity/invalidity" for evaluating deductive arguments, just as the terms "weak" and "strong" are reserved for evaluating inductive arguments. The same is true in our use of the term "soundness": it is reserved for the evaluation of deductive arguments. That is, inductive arguments cannot be said to be either valid or invalid, nor can they be sound or unsound.

Of course the terms "valid" and "invalid" have uses beyond logic. We have valid or invalid driver's licenses for example. Sometimes people say things. Like: "that is a valid conclusion…" and we know what they mean. But in logic conclusions are either true or false since they are propositions. We can say that a conclusion has been reached validly, in which case we are not talking about its truth but its relation to its premises.

Well enough of these qualifications, we are now ready to make explicit our criterion for validity, and hence invalidity.

Validity:
An argument is valid if and only if it would be impossible for its conclusion to be false if its premises were true, i.e. the truth of the premises guarantees the truth of the conclusion

This criterion is deceptively simple. What is deceptive about it is that it is phrased in the subjunctive mood (which we seldom use anymore), not in the indicative mood, which we almost always use. But this is just a fancy way of saying that our criterion of validity (and hence invalidity) has nothing whatsoever to do with whether or not the premises or conclusion of a particular argument are true in fact (indicative mood), but has everything to do with whether or not the conclusion of a particular argument would be true, if its premises were true (subjunctive mood).

What is tricky here is to notice that even though the terms 'true" and "false" occur in the statement of the criterion of validity, that criterion has nothing to do with what is true or false in fact. Or another way to put this is to say that the judgment of validity (and hence invalidity) is a judgment about the ***relation*** between the premises and the conclusion of an argument, and not a judgment about truth.

Above we talked about conclusions following from premises. Now we are able to give a more precise meaning to this idea. To say that the conclusion of a deductive argument "follows from" the premises is just to say that that conclusion has a certain relation to the premises. Validity is more about this relation than it is about truth. That relation amounts to this**:**

In valid arguments the conclusion is so related to its premises that it would be impossible for the conclusion to be false if the premises were in fact true, regardless of whether or not they are

If you want to see just how little the notion of validity (and hence invalidity) has to do with truth, and how much it has to do with the relation between its premises and its conclusion, consider the following valid argument.

All roaches are millionaires
Ron Hall is a roach
Therefore Ron Hall is a millionaire

We don't think you will have much trouble agreeing that the first premise of this argument is in fact false. Indeed we hope that you will also concede that the second one is false. But take it from us, Ron Hall is no millionaire. Yet this is a perfectly valid argument. Why? It meets our criterion. **If** the premises were true, (and here they are not) then the conclusion would have to be true (even though in this case, it is not true in fact).

But think of the situation in which an argument has premises that are true in fact and even a conclusion that is true in fact. Would it have to be valid? No. Consider the following such argument:

All philosophers love wisdom
Jon Smith is a philosopher
Therefore, Jon Smith teaches logic

Now all of these propositions are true in fact, but the argument is not valid. The reason is that the conclusion, although true in fact, would not have to be true given the truth of the premises. By the same token, if we know that the premises of a given argument are in fact true, and the relation between these premises and its conclusion is valid, we would thereby know that the conclusion would have to be true, that is, could not be false.

As we pointed out above, in a deduction, to argue correctly is to argue validly. And validity is high praise for a deductive argument, even though it does not depend upon the truth of the propositions in the argument. Rather, validity depends solely on the relation between premises and conclusion. Having said this, we are ready to see that validity is not the highest praise we can give to deductive arguments. The reason for this is that truth is important when it comes making the final judgment as to whether or not we have reasoned well. The highest praise for an argument is to say that it is **sound**.

As in the case of validity, we have criteria for soundness (and hence for unsoundness). Those criteria are as follows:

Soundness:
An argument is sound if and only if
it is valid and has all true premises

Notice that there are two criteria for soundness (and hence for unsoundness). The first criterion is that of validity.

However, even though validity is necessary for soundness, and its absence is sufficient for unsoundness, it is not sufficient by itself. Sound arguments must not only be valid, they must also have all true premises. Likewise, having all true premises is not itself a sufficient condition for soundness, even though a single false premise is a sufficient condition for unsoundness. The two criteria, (1) validity and (2) all true premises, taken together are sufficient to determine that a particular argument is sound. Consider the following examples:

1. **Sound Argument:** *If Ron Hall is a philosopher, then Ron Hall loves wisdom. Ron Hall is a philosopher. Therefore Ron Hall loves wisdom.* (Valid, all true premises)
2. **Unsound Argument A**: *If Jon Smith is a philosopher, then Jon Smith loves wisdom. Jon Smith loves wisdom. Therefore Jon Smith is a philosopher.* (Invalid, all true premises and a true conclusion)
3. **Unsound Argument B**: *If Socrates is a philosopher then Socrates loves wisdom. Socrates does not love wisdom. Therefore Socrates is not a philosopher.* (Valid, one false premise and a false conclusion)

Be sure to notice this: even though the criteria of soundness do not mention the conclusion, the conclusion of a sound argument must be true. If an argument is valid, then if its premises were true, its conclusion would necessarily be true. But in sound arguments all the premises are true; hence the conclusion must be true as well.

The upshot of this is that there are two ways to evaluate an argument as being incorrect. We can say that it is invalid or we can say that it is unsound. Just because a proposition follows from a set of premises, we do not have to accept it as true. It is always open to us to call the truth of one or more of the premises into question.

This issue of truth, however, is not something that we can investigate in a logic course. We will have to leave this to others. In other words, in this course we will not be in the business of evaluating soundness. Rather, we will restrict our evaluations to questions of validity. But this is no small matter, for in order to reason soundly, we must first reason validly. And of course if we reason invalidly, there is no way for our arguments to be sound.

The following table represents all of the eight combinations and permutations of truth-values that are possible in arguments that have two premises.

Case	**Premises** 1	2	**Conclusion**
1st	**T**	**T**	**T**
2nd	**F**	**T**	**T**
3rd	**T**	**F**	**T**
4th	**F**	**F**	**T**
5th	**T**	**T**	**F**
6th	**F**	**T**	**F**
7th	**T**	**F**	**F**
8th	**F**	**F**	**F**

In the light of our discussion of validity and soundness ask yourself this question: "Which of these combinations of truth-values could represent a valid argument? Careful reflection ought to tell you that every one of these combinations **could** be valid, except one. Do you see which one? Clearly, the combination in case #5 could not ever be valid since it is the only one that directly violates our definition of validity. Remember, if the premises were true then the conclusion would have to be true. In case #5 the premises are true and the conclusion is false. Such a combination could never be valid.

Don't jump to the conclusion that all of the other combinations are valid. All that we can say is that arguments with any of the other combinations of truth-values **could be valid**. You can take any of the other combinations and construct a valid or an invalid argument. For example, we can take combination case 3 and construct both a valid and an invalid argument as follows:

1**. Valid:** *Either Ron Hall is a philosopher or he likes to walk. He does not like to walk. Therefore he is a philosopher*. (The first premise is true, the second is false, and the conclusion is true.)
2. **Invalid:** *If Jon Smith is a philosopher then he loves wisdom. Jon Smith is not a philosopher. Therefore, he loves wisdom.* (The first premise is true, the second is false, and the conclusion is true.)

We could make similar constructions on all of these combinations of truth-values except in combination case # 5. In this case, it is impossible to construct a valid argument with this combination of truth-values.

Now consider these same combinations in relation to soundness. Ask yourself this question: Which of these combinations could never be sound? Well obviously no sound argument can have any false premise. So we can eliminate cases #2, # 3, # 4, # 6, # 7, and #8, since in all of these at least one premise is false. We are left then with cases #1 and #5. But surely case #5 can be eliminated since it can never be valid, and every sound argument is valid. So we are left with case #1. Is such a combination always sound? If you reflect carefully you will see that the answer to this question is “no.” Case #1 is the only one of these 8 possibilities that **could** be sound. However, the fact remains that even this combination can be invalid. And hence we can construct an unsound argument with all true premises if the argument we construct is invalid. Consider the following example of an argument that is unsound because it is invalid.

All philosophers love wisdom
Jon Smith is a philosopher
Therefore Jon Smith is an avid sailor

All of the premises and the conclusion of this argument are true. Yet the argument is not sound precisely because it is not valid. It is not valid precisely because its conclusion could be false even though the premises are true, and even though the conclusion is true as a matter of fact.

Before we get to the Exercises, let’s summarize our remarks about truth, validity and soundness in the following table. In this table we see that deductive arguments can be valid, invalid, sound, or unsound:

VALID: Could not have a false conclusion if its premises were true and could be sound or unsound
INVALID: Could have a false conclusion if its premises were true but could not be sound
SOUND: Must be valid and have all true premises
UNSOUND: Has at least one false premise or is invalid