UIL Number Sense Contest

Shortcuts for the More Experienced

Larry White

UIL State Number Sense Contest Director texasmath@centex.net http://www.uiltexas.org/academics/number-sense

UIL High School Number Sense Test Problem Sequencing

Problems 41 - 50***

- 1) Laws of Exponents
- 2) Right Triangle Problems
- 3) Coordinate Geometry Problems
- 4) Regular Polygon Problems
- 5) Inequalities

Problems 51 – 60 ***

- 1) Applications of Theorems from Geometry
- 2) Direct and Inverse Variation
- 3) Sequences & Series (Finite & Infinite)

Problems 51 — 70 ***

- 1) Complex Numbers
- 2) Logarithms & Logarithmic Equations
- 3) Permutations & Combinations
- 4) Probability
- 5) Conics
- 6) Binomial Theorem (Expansion)

Problems 61 — 70 ***

- 1) Volume & Surface Area
- 2) Greatest Integer
- **3)** Application of Remainder Theorem
- 4) Trigonometry
- 5) Determinants
- 6) Matrices
- 7) Vectors
- 8) Composite Functions

Problems~71-80

- 1) Value of Domain of a Given Function
- 2) Bases Involving Decimal Fractions
- 3) Polar/Rectangular Coordinates
- 4) Modular Arithmetic
- 5) Limits
- 6) Derivative
- 7) Slopes of Tangent Lines
- 8) Horizontal & Vertical Asymptotes
- 9) Determining Critical Values
- **10) Maximum & Minimum Problems**
- **11) Definite Integration**

*** A type of problem from a particular section could appear later in the test. Example: A base problem could appear as problem #55, but should not appear earlier than problem #21.

Special Numbers -- What Pops into Your Mind?

1728	1024
1331	289
1.732	2.828
3.141	2.718
1.618	720
0.08333	0.0625
1,1,2,3,5,8,	2,1,3,4,7,11,
1,3,6,10,15,21,	7,24,25

Notes:

Mental Math -- How fast can you work these?

- 1. 6.25% of 48 =
- 2. 144 ÷ 0.08333... =
- 3. 6! × 5! =
- 4. Truncate $\sqrt{7}$ to a whole number
- 5. The 10th term of 1,1,2,3,5,8, ... is
- 6. The 9th triangular number is

Notes:

Math Magic (Number Sense Tricks)

- A. Memorize the first 35 squares, the first 15 cubes, and the square roots of 2, 3, 5, 6, 7, 8, & 10.
- B. Know the "One-sies" equivalents. (Fractions-Decimals-Percents)
- C. $\frac{12}{17} + \frac{17}{12} = ?$ (Is it a trick? Is it magic? See proof)

$$\frac{12}{17} + \frac{17}{12} = 2 \frac{25}{204}$$
 (Is it magic ?)

$$\frac{a}{b} + \frac{b}{a}$$
 Proof
Let $x = \frac{a}{b} + \frac{b}{a}$
 $x = \frac{(a^2 + b^2)}{ab}$ (common denominator)
 $x - 2 = \frac{(a^2 + b^2)}{ab} - 2$ (subtract 2 from both sides)
 $x - 2 = \frac{(a^2 + b^2 - 2ab)}{ab}$ (common denominator)
 $x - 2 = \frac{(a - b)^2}{ab}$ (binomial square)
 $x = 2 + \frac{(a - b)^2}{ab}$ (solve for x)

Any questions on any of these?

(41)	$16_7 + 25_7 + 34_7 = $ 7
(42)	$\sqrt{16 \times 18 + 1}$ =
(43)	96 × 0.3125 =
(44)	The leg opposite the 45° angle in a right triangle is $\sqrt{18}$. The hypotenuse is
(45)	The sum of the product of the roots taken two at a time of $3x^3 + 4x^2 - 17x - 6 = 0$ is
(46)	If $xy = -1$ and $x + y = 5$ then $x^3 + y^3 = $
(47)	If $9^{(x)} = 2187$ then $9^{(x-1)} = $
(48)	Find k, so that 917k55 is the smallest 6-digit number divisible by 11
(49)	The slope of a line containing the points (3, 2) and (-4, 5) is
*(50)	$41\frac{2}{3}\%$ of 3690 — 58.7 =
(51)	If $\log_x 108 - \log_x 4 = 3$ then $x = $
(52)	Let (10 + 5i) (8 - 4i) = a + bi. Find a + b
(53)	If A is 24% more than B and B is 25% more than C, then A is% more than C.
(54)	$48^2 + 76^2 = $
(55)	$11^4 \div 14$ has a remainder of
(56)	If $\frac{x}{8}$ has a remainder of 7 and $\frac{y}{8}$ has a remainder of 5 then $\frac{xy}{8}$ has a remainder of
(57)	422 × 311 =
(58)	How many different sets of 5 books can be made from 8 different books?
(59)	The Cartesian product of the sets {f,i,v,e} and {f,o,u,r} contain how many ordered pairs?

Any questions on any of these?

- (61) 405 × 111 = _____
- (62) (234₇ + 432₇) ÷ 6 has a remainder of _____
- (63) $\sin\left(\arccos(\frac{24}{25})\right) =$ _____
- (64) A bag contains 12 white and k yellow golf balls. Find k if the probability of randomly drawing a yellow ball is 25%.
- (65) If $g(x) = 3x^2 4x + 2$, then g(g(1)) = _____

(66) 1 + 3 + 8 + 21 + ... + 144 =_____

- (67) $A = \begin{bmatrix} -1 & 3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ 5 & -1 \end{bmatrix}$. $|AB| = _$
- (68) The Greatest Integer Function is written as f(x) = [x]. Find $\begin{bmatrix} \tan \frac{2\pi}{3} \end{bmatrix}$.

(69)
$$\sin(\frac{5\pi}{6}) - \cos(\frac{4\pi}{3}) + \tan(3\pi) =$$

- *(70) 14 × 24 × 34 × 44 = _____
- (71) $6! \div 5! + 4! \div 3! 2! \div 1! =$
- (72) $\sqrt{103041} =$ _____
- (73) $\frac{1}{6} + \frac{1}{15} + \frac{1}{20} + \frac{1}{24} =$ _____
- (74) The next term of 2, 3, 4, 6, 9, 14, ... is _____
- (75) The horizontal asymptote for $f(x) = \frac{3-4x}{x-5}$ is $y = _$
- (76) If $f(x) = 3x^2 4x + 2$, then f'(-1) =_____

(77) If the rectangular coordinates of the polar coordinates $(2, \frac{\pi}{4})$ are (x, y), then $x \times y =$ _____

- (78) $\int_{2}^{4} (x+3) dx =$ _____
- (79) Change $\frac{11}{36}$ to a base 6 decimal.
- *(80) 428.571 x 349 = _____