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A competitive game formulation for large scale frictional contact problems

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ABSTRACT

Using a Total Lagrangian framework the discretized frictional contact conditions are presented in a unified manner. The equilibrium equations are used to eliminate the displacement quantities yielding a force method - like formulation. Then the frictional contact problem is viewed as a competitive two person game, leading to an integrable Quasivariational Inequality (QVI), solvable by the variational section algorithm. Beeing directly parallelizable, this algorithm is best suited for large scale problems.

INTRODUCTION

Unilateral frictional contact problems is a highly active research area with several applications (see e.g. Duvaut and Lions [1], Panagiotopoulos [2], Kikuchi and Oden [3], Kalker [4], [5], Curnier [6]). In a quite influential paper Panagiotopoulos [7] has suggested a two - stage iterative technique where a normal contact problem and a Coulomb friction problem is solved in each iteration. For linear structures this technique has been considered by Bisbos [8] as a two - person Nash game leading to the formulation of a discrete integrable QVI (cf. Aubin and Ekeland [9], Baiocchi and Capelo [10]). This way a one - stage iterative technique results, called the variational section algorithm, consisting in solving a Nonlinear Programming Problem (NLPP) in each iteration.

This paper extends the work of [8] in the case of nonlinear structures and presents the whole derivation in a unified manner. A parallel decomposition (cf. Bertsekas and Tsitsiklis [11]) of the variational section algorithm is discussed.

2. BASIC UNILATERAL FRICTIONAL CONTACT CONDITIONS

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Following a Total Lagrangian Formulation let us consider an assemblage of bodies (Fig. 1). The equilibrium positions are to be evaluated at discrete times $0, \Delta t, 2\Delta t, 3\Delta t, \ldots$ (load increments are applied at these discrete times too). Let ${}^0\boldsymbol{\xi}, {}^t\boldsymbol{\xi}$, ${}^{t+\Delta t}\boldsymbol{\xi}$ denote the coordinates at time $0,\ t,\ t+\Delta t$ and ${}^t\mathbf{u}, {}^{t+\Delta t}\mathbf{u}$ denote the displacements at times $t,\ t+\Delta t$ respectively :

$$ {}^t\boldsymbol{\xi} = {}^0\boldsymbol{\xi} + {}^t\mathbf{u}\,, \quad {}^{t+\Delta t}\boldsymbol{\xi} = {}^0\boldsymbol{\xi} + {}^{t+\Delta t}\mathbf{u} \tag{1} $$

The increments $\mathbf{u}$ in the displacements from time t to time $t+\Delta t$ are :

$$ \mathbf{u} = {}^{t+\Delta t}\mathbf{u} - {}^t\mathbf{u} \tag{2} $$

Let us now consider a pair of nodes A and B in contact at time $t+\Delta t$. We shall call it the j interfacial contact element with respective rotation matrix $\mathbf{R}^j = [\, \mathbf{R}_N^{(j)} \ \vdots\ \mathbf{R}_T^{(j)}\,]$. At time $t+\Delta t$ the final gap $w_N^{(j)}$ and the relative incremental slip $\mathbf{w}_T^{(j)}$ are given by :

$$ w_N^{(j)} = \mathbf{R}_N^{(j)T}\,({}^{t+\Delta t}\boldsymbol{\xi}_B - {}^{t+\Delta t}\boldsymbol{\xi}_A) \tag{3} $$
$$ \mathbf{w}_T^{(j)} = \mathbf{R}_T^{(j)T}\,(\mathbf{u}_B - \mathbf{u}_A) \tag{4} $$

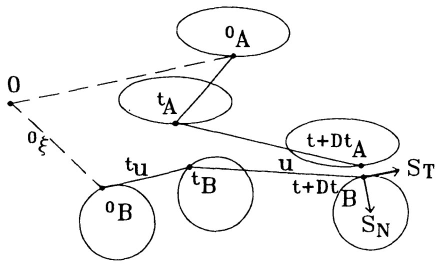

Figure 1. Assemblage of bodies in contact.

If $\mathbf{u}^{(j)} = (\mathbf{u}_A, \mathbf{u}_B)$ are the displacement increments of the j contact element we can define the resp. strain increments $\mathbf{e}^{(j)} = (e_N^{(j)}, \mathbf{e}_T^{(j)})$ as :

$$ \mathbf{e}^{(j)} = \mathbf{R}^{(j)T}\,(\mathbf{u}_A - \mathbf{u}_B) \tag{5} $$

obtaining through some matrix algebra the following representation :

$$ e_N^{(j)} = \mathbf{H}_N^{(j)T}\,\mathbf{u}^{(j)}\,, \qquad \mathbf{e}_T^{(j)} = \mathbf{H}_T^{(j)T}\,\mathbf{u}^{(j)} \tag{6} $$

where the matrices $\mathbf{H}_N^{(j)}$ and $\mathbf{H}_T^{(j)}$ are constructed from $\mathbf{R}_N^{(j)}$, $\mathbf{R}_T^{(j)}$ and eventual shape functions. Then Equations (3), (4) are transformed to :

$$ w_N^{(j)} = h_N^{(j)} - e_N^{(j)}\,, \qquad \mathbf{w}_T^{(j)} = -\,\mathbf{e}_T^{(j)} \tag{7} $$
$$ h_N^{(j)} = \mathbf{R}_N^{(j)T}\,({}^t\boldsymbol{\xi}_B - {}^t\boldsymbol{\xi}_A) \tag{8} $$

Let $s_N^{(j)}, \mathbf{s}_T^{(j)}$ be the stresses of the interfacial element . The inner nodal forces of the contact element in the global coordinate system are :

$$\mathbf{q}^{(j)} = \mathbf{H}_N^{(j)}\, s_N^{(j)} + \mathbf{H}_T^{(j)}\, \mathbf{s}_T^{(j)} \tag{9}$$

With the friction cone bound defined as $s_B^{(j)} = (\, c_f^{(j)} s_N^{(j)}\,)^2$ - where $c_f^{(j)}$ is the local friction coefficient - the Coulomb friction cone is given by :

$$\varphi_L^{(j)} = 1/2\,(s_B^{(j)} - \mathbf{s}_T^{(j)T}\, \mathbf{s}_T^{(j)}\,) \tag{10}$$

The unilateral contact conditions in the normal direction and the Coulomb stick / slip friction conditions in the tangential one are :

$$s_N^{(j)} w_N^{(j)} = 0\,,\ s_N^{(j)} \geq 0\,,\ w_N^{(j)} \geq 0 \tag{11}$$

$$\lambda^{(j)} \varphi_L^{(j)} = 0\,,\ \varphi_L^{(j)} \geq 0\,,\ \lambda^{(j)} \geq 0\,,\ \mathbf{w}_T^{(j)} = -\mathbf{s}_T^{(j)} \lambda^{(j)} \tag{12}$$

The respective structural relations for n DOFs and m interfacial elements are :

$$\mathbf{s}_N^T \mathbf{w}_N = 0\,,\ \mathbf{s}_N \geq \mathbf{0}\,,\ \mathbf{w}_N \geq \mathbf{0} \tag{13}$$

$$\boldsymbol{\varphi}_L = 1/2\,(\,\mathbf{s}_B - \mathbf{Q}^T(\mathbf{s}_T)\,\mathbf{s}_T) \tag{14}$$

$$\boldsymbol{\lambda}_L^T \boldsymbol{\varphi}_L = 0\,,\ \boldsymbol{\varphi}_L \geq \mathbf{0}\,,\ \boldsymbol{\lambda} \geq \mathbf{0}\,,\ \mathbf{w}_T = -\mathbf{Q}(\mathbf{s}_T)\,\boldsymbol{\lambda} \tag{15}$$

$$\mathbf{w}_N = -\mathbf{e}_N + \mathbf{h}_N = -\mathbf{H}_N^T\,\mathbf{u} + \mathbf{h}_N \tag{16}$$

$$\mathbf{w}_T = -\mathbf{e}_T = -\mathbf{H}_T^T\,\mathbf{u} \tag{17}$$

$$\mathbf{q} = \mathbf{H}_N\,\mathbf{s}_N + \mathbf{H}_T\,\mathbf{s}_T \tag{18}$$

where the $n \times m$ matrix $\mathbf{H}_N$, the $n \times 2m$ matrix $\mathbf{H}_T$, and the $2m \times m$ matrix function $\mathbf{Q}(\mathbf{s}_T)$ are constructed from their element counterpaprts through a Boolean assembly process. This relation set is to be completed with the structural equilibrium equations (internal forces equal to external forces) :

$$^{t+\Delta t}\mathbf{b}_{int} + \mathbf{q} = {}^{t+\Delta t}\mathbf{b}_{ext} \tag{19}$$

or through Equation (18) :

$$^{t+\Delta t}\mathbf{b}_{int} + \mathbf{H}_N\,\mathbf{s}_N + \mathbf{H}_T\,\mathbf{s}_T = {}^{t+\Delta t}\mathbf{b}_{ext} \tag{20}$$

In the next two sections we will use Equation (20) to eliminate $\mathbf{u}$ from Equations (16) and (17), to obtain $\mathbf{w}_N, \mathbf{w}_T$ as functions of $\mathbf{s}_N, \mathbf{s}_T$.

3. LINEAR ELASTIC STRUCTURE

In the case of linear elastic structure : a) ${}^t\boldsymbol{\xi} = {}^0\boldsymbol{\xi}$ holds and b) the displacement increments $\mathbf{u}$ are the displacements themselves. The matrices

$\mathbf{H}_N, \mathbf{H}_T$ are evaluated at the undeformed structure and Equation (7) becomes :

$$h_N^{(j)} = \mathbf{R}_N^{(j)T} \left({}^0\boldsymbol{\xi}_B - {}^0\boldsymbol{\xi}_A \right) \tag{21}$$

The equilibrium equations (20) obtain the form :

$$\mathbf{K}\ \mathbf{u} + \mathbf{H}_N\ \mathbf{s}_N + \mathbf{H}_T\ \mathbf{s}_T = \mathbf{p}\ , \qquad \mathbf{p} = \mathbf{b}_{ext} \tag{22}$$

yielding for invertible linear structural stiffness matrix $\mathbf{K}$:

$$\mathbf{u} = \mathbf{u}_0 - \mathbf{K}^{-1}\,\mathbf{H}_N\,\mathbf{s}_N - \mathbf{K}^{-1}\,\mathbf{H}_T\,\mathbf{s}_T\ , \qquad \mathbf{u}_0 = \mathbf{K}^{-1}\mathbf{p} \tag{23}$$

Setting :

$$\mathbf{w}_{N0} = \mathbf{h}_N - \mathbf{H}_N^T\,\mathbf{u}_0 \tag{24}$$
$$\mathbf{w}_{T0} = -\mathbf{H}_T^T\,\mathbf{u}_0 \tag{25}$$
$$\mathbf{B}_{NN} = \mathbf{H}_N^T\,\mathbf{K}^{-1}\,\mathbf{H}_N\ ,\ \mathbf{B}_{NT} = \mathbf{H}_N^T\,\mathbf{K}^{-1}\,\mathbf{H}_T\ , \tag{26}$$
$$\mathbf{B}_{TN} = \mathbf{H}_T^T\,\mathbf{K}^{-1}\,\mathbf{H}_N\ ,\ \mathbf{B}_{TT} = \mathbf{H}_T^T\,\mathbf{K}^{-1}\,\mathbf{H}_T \tag{27}$$

and combining Equations (16), (17) with (23) yields :

$$\mathbf{w}_N = \mathbf{B}_{NN}\,\mathbf{s}_N + \mathbf{B}_{NT}\,\mathbf{s}_T + \mathbf{w}_{N0} \tag{28}$$
$$\mathbf{w}_T = \mathbf{B}_{TN}\,\mathbf{s}_N + \mathbf{B}_{TT}\,\mathbf{s}_T + \mathbf{w}_{T0}\ . \tag{29}$$

4. NONLINEAR STRUCTURE

Let us turn our attention to the Total Lagrangian Formulation again. Within each increment a sequence of iterations is performed.The displacement increments $\mathbf{u}$ are approximated at the r iteration by :

$$\mathbf{u} = \sum_{k=1}^{r-1} \Delta\mathbf{u}_k + \Delta\mathbf{u}_r \tag{30}$$

Let us set :

$$\boldsymbol{v} = \sum_{k=1}^{r-1} \Delta\mathbf{u}_k\ , \quad \Delta\mathbf{u} = \Delta\mathbf{u}_r\ , \quad \mathbf{u} = \boldsymbol{v} + \Delta\mathbf{u} \tag{31}$$

and denote by $t+\Delta\tau$ the 'time' coresponding to ${}^t\mathbf{u}+\boldsymbol{v}$. Let us approximate the matrices $\mathbf{R}_N^{(j)}, \mathbf{R}_T^{(j)}, \mathbf{H}_N, \mathbf{H}_T$ by their value at 'time' ${}^t\mathbf{u}+\boldsymbol{v}$. As usual the internal forces ${}^{t+\Delta t}\mathbf{b}_{int}$ are approximated by :

$${}^{t+\Delta t}\mathbf{b}_{int} = {}^{t+\Delta\tau}\mathbf{b}_{int} + {}^{t+\Delta\tau}\mathbf{K}_t\ \Delta\mathbf{u} \tag{32}$$

Let us formally denote by $\mathbf{K}$ the tangent stiffness matrix ${}^{t+\Delta\tau}\mathbf{K}_t$ to obtain the equilibrium equations in the form :

$${}^{t+\Delta\tau}\mathbf{b}_{int} + \mathbf{K}\ \Delta\mathbf{u} + \mathbf{H}_N\ \mathbf{s}_N + \mathbf{H}_T\ \mathbf{s}_T = {}^{t+\Delta t}\mathbf{b}_{ext} \tag{33}$$

or similarly to (22) as :

$$\mathbf{K}\ \Delta\mathbf{u} + \mathbf{H}_N\ \mathbf{s}_N + \mathbf{H}_T\ \mathbf{s}_T = \Delta\mathbf{p}\,, \qquad \Delta\mathbf{p} = {}^{t+\Delta t}\mathbf{b}_{ext} - {}^{t+\Delta\tau}\mathbf{b}_{int} \tag{34}$$

Equation (34) is reformulated as :

$$\Delta\mathbf{u} = \Delta\mathbf{u}_0 - \mathbf{K}^{-1}\ \mathbf{H}_N\ \mathbf{s}_N - \mathbf{K}^{-1}\ \mathbf{H}_T\ \mathbf{s}_T\,, \qquad \Delta\mathbf{u}_0 = \mathbf{K}^{-1}\ \Delta\mathbf{p} \tag{35}$$

Equations (28) - (29) are directly recovered with data :

$$\mathbf{w}_{N0} = \mathbf{h}_N - \mathbf{H}_N^T\ (\boldsymbol{\upsilon} + \Delta\mathbf{u}_0) \tag{36}$$
$$\mathbf{w}_{T0} = -\ \mathbf{H}_T^T\ (\boldsymbol{\upsilon} + \Delta\mathbf{u}_0) \tag{37}$$

and by the Equations (26) and (27). The term $\mathbf{h}_N$ is naturally computed through the formula (8).

4. THE GAME PROBLEM

In the sequel we assume $\mathbf{K}$ symmetric yielding the relation $\mathbf{B}_{TN} = \mathbf{B}_{NT}^T$. Obviously $\mathbf{s}_N, \mathbf{w}_N \in R^m$ and $\mathbf{s}_T, \mathbf{w}_T \in R^{2m}$. Let us set $\mathbf{s} = (\mathbf{s}_N, \mathbf{s}_T)$ and consider the linear maps :

$$\mathbf{s}_T \mapsto \mathbf{d}_N(\mathbf{s}_T) \in R^m\ ,\ \mathbf{d}_N(\mathbf{s}_T) = \mathbf{B}_{NT}\ \mathbf{s}_T + \mathbf{w}_{N0} \tag{38}$$
$$\mathbf{s}_N \mapsto \mathbf{d}_T(\mathbf{s}_N) \in R^{2m}\ ,\ \mathbf{d}_T(\mathbf{s}_N) = \mathbf{B}_{TN}\ \mathbf{s}_N + \mathbf{w}_{T0} \tag{39}$$

and the following point - to - set valued maps :

$$\mathbf{s}_T \mapsto K_N(\mathbf{s}_T) \subset R^m\ :\ K_N(\mathbf{s}_T) = \{\ \mathbf{s}_N \in R^m \mid \mathbf{s}_N \geq \mathbf{0}\ \} \tag{40}$$
$$\mathbf{s}_N \mapsto K_T(\mathbf{s}_N) \subset R^{2m}\ :\ K_T(\mathbf{s}_N) = \{\ \mathbf{s}_T \in R^{2m} \mid \varphi_L \geq \mathbf{0}\ \} \tag{41}$$
$$\mathbf{s} = (\mathbf{s}_N, \mathbf{s}_T) \mapsto K_M(\mathbf{s}) \subset R^{3m}\ :\ K_M(\mathbf{s}) = K_N(\mathbf{s}_T)\ \times\ K_T(\mathbf{s}_N) \tag{42}$$

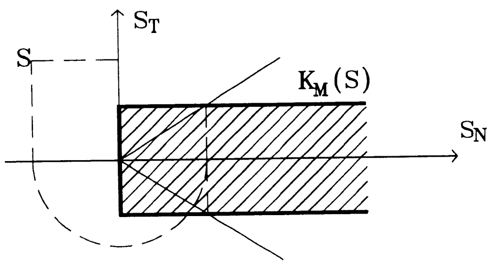

Figure 2. Point - to - set valued map of the QVI.

The simple Cartesian product map (42), depicted in Fig.2 , belongs to the QVI definition. Let us now consider the strain energy functionals :

$$J_M(\mathbf{s}) = 1/2\ \mathbf{s}_N^T\mathbf{B}_{NN}\mathbf{s}_N + 1/2\ \mathbf{s}_T^T\mathbf{B}_{TT}\mathbf{s}_T + \mathbf{s}_N^T\mathbf{B}_{NT}\mathbf{s}_T + \mathbf{s}_N^T\mathbf{w}_{N0} + \mathbf{s}_T^T\mathbf{w}_{T0} \tag{43}$$
$$J_N(\mathbf{s}_N) = 1/2\ \mathbf{s}_N^T\ \mathbf{B}_{NN}\ \mathbf{s}_N + \mathbf{s}_N^T\ \mathbf{d}_N \quad for\ \mathbf{s}_T\ fixed \tag{44}$$
$$J_T(\mathbf{s}_T) = 1/2\ \mathbf{s}_T^T\ \mathbf{B}_{TT}\ \mathbf{s}_T + \mathbf{s}_T^T\ \mathbf{d}_T \quad for\ \mathbf{s}_N\ fixed \tag{45}$$

In the sequel we will assume that these functionals are convex. Obviously the following relations hold for $\mathbf{w}_N, \mathbf{w}_T$:

$$\mathbf{w}_N = \mathbf{B}_{NN}\,\mathbf{s}_N + \mathbf{d}_N = \nabla_N\, J_M(\mathbf{s}_N, \mathbf{s}_T) = \nabla_N\, J_N(\mathbf{s}_N) \tag{46}$$

$$\mathbf{w}_T = \mathbf{B}_{TT}\,\mathbf{s}_T + \mathbf{d}_T = \nabla_T\, J_M(\mathbf{s}_N, \mathbf{s}_T) = \nabla_T\, J_T(\mathbf{s}_T) \tag{47}$$

By the Kuhn - Tucker theory, the relations (13), (38), (44) and (46) lead to the conclusion that $\mathbf{s}_N$ is the solution of the following Quadratic Programming Problem (QPP) :

$$\mathbf{s}_N = Argmin\,\{\, J_N(\mathbf{x}_N) \mid \mathbf{x}_N \in K_N \,\} \tag{48}$$

for $\mathbf{s}_T$ fixed. Similarly for $\mathbf{s}_N$ fixed, the relations (14), (15), (39), (41), (45) and (47) define $\mathbf{s}_T$ as the unique solution of the following convex NLPP :

$$\mathbf{s}_T = Argmin\,\{\, J_T(\mathbf{x}_T) \mid \mathbf{x}_T \in K_T \,\} \tag{49}$$

Obviously the Equation (48) describes the frictionless unilateral contact problem with given tangential loads, where the Equation (49) expresses the 3 - D Coulomb friction problem under normal loads fixed. The equivalent Variational Inequalities (VI) are :

$$\left.\begin{array}{l} Find\quad \mathbf{s}_N \in K_N \quad such \quad that\ : \\ \quad (\mathbf{x}_N - \mathbf{s}_N)^T \nabla_N\, J_N(\mathbf{s}_N)\ \geq \mathbf{0} \quad \forall\, \mathbf{x}_N \in K_N \end{array}\right\} \tag{50}$$

and

$$\left.\begin{array}{l} Find\quad \mathbf{s}_T \in K_T \quad such \quad that\ : \\ \quad (\mathbf{x}_T - \mathbf{s}_T)^T \nabla_T\, J_T(\mathbf{s}_T)\ \geq \mathbf{0} \quad \forall\, \mathbf{x}_T \in K_T \end{array}\right\} \tag{51}$$

Let us now imagine a game with two players playing consequtively, namely the N - player and the T - player. The respective costs, strategies and admissible strategy sets are $J_N, \mathbf{s}_N, K_N$ and $J_T, \mathbf{s}_T, K_T$. Every player aims to the minimization of his own cost by selecting his strategy within the respective admissible strategy set. By construction every player infuences the cost and the admissible strategy set of the other player. In the Game Theory language([9]) it is a constrained competitive (Nash) two - person nonzero game. Let us assume firstly that the T - player has selected its strategy $\mathbf{s}_T$ making $\mathbf{w}_{N0}, K_N$ fixed. Then the N - player selects his strategy according to Equation (48). Now $\mathbf{s}_N$ becomes fixed making $\mathbf{w}_{T0}, K_T$ constant. The strategy selection rule of the T - player is defined by Equation (49).

Let $(\mathbf{s}^*_N, \mathbf{s}^*_T)$ be the strategy point selected by the two players at some time instant of the game and $(\mathbf{s}_N, \mathbf{s}_T)$ the strategy selected immediately after. The mixed contact problem is solved if an equilibrium point (called a Nash point or a fixed point of the game) is found , i.e a point satisfying :

$$(\mathbf{s}^*_N, \mathbf{s}^*_T) = (\mathbf{s}_N, \mathbf{s}_T) \tag{52}$$

Adding the VIs (50) and (51) yieds the following QVI Problem :

$$\left.\begin{array}{ll} \textit{Find} \quad (\mathbf{s}_N, \mathbf{s}_T) \in R^{3m} \quad \textit{such} \quad \textit{that} : & \\ \text{i)} \quad (\mathbf{s}_N, \mathbf{s}_T) \in K_M(\mathbf{s}_N, \mathbf{s}_T) & \\ \text{ii)} \quad (\mathbf{x}_N - \mathbf{s}_N)^T \nabla_N J_M(\mathbf{s}_N, \mathbf{s}_T) + & \\ \quad\quad (\mathbf{x}_T - \mathbf{s}_T)^T \nabla_T J_M(\mathbf{s}_N, \mathbf{s}_T) \geq \mathbf{0} \quad \forall (\mathbf{x}_N, \mathbf{x}_T) \in K_M(\mathbf{s}_N, \mathbf{s}_T) & \end{array}\right\} \tag{53}$$

5. THE VARIATIONAL SECTION ALGORITHM

Fixing the set $K_M = K_M(\mathbf{z})$ for some $\mathbf{z} = (\mathbf{z}_N, \mathbf{z}_T)$ yields the respective variational section ([10]) of the QVI (53), which is a VI :

$$\left.\begin{array}{l} \textit{Find} \quad (\mathbf{s}_N, \mathbf{s}_T) \in K_M(\mathbf{z}) \quad \textit{such} \quad \textit{that} : \\ \quad (\mathbf{x}_N - \mathbf{s}_N)^T \nabla_N J_M(\mathbf{s}_N, \mathbf{s}_T) + \\ \quad (\mathbf{x}_T - \mathbf{s}_T)^T \nabla_T J_M(\mathbf{s}_N, \mathbf{s}_T) \geq \mathbf{0} \quad \forall (\mathbf{x}_N, \mathbf{x}_T) \in K_M(\mathbf{z}) \end{array}\right\} \tag{54}$$

The integrable VI (54) is equivalent to the following convex NLP :

$$(\mathbf{s}_N, \mathbf{s}_T) = \mathit{Argmin} \{ J_M(\mathbf{x}_N, \mathbf{x}_T) \mid (\mathbf{x}_N, \mathbf{x}_T) \in K_M(\mathbf{z}) \} \tag{55}$$

Concerning the mechanical meaning of Equation (55), we observe that this minimization problem corresponds to the complementary energy of a structure with frictionless unilateral contact conditions coupled with usual elastoplastic elements in the tangential direction. Problem (55) can be solved through an appropriate algorithm, as the SQP method [8]. Thus the following Variational Section algorithm occurs in a natural way :

0.Step : Set $k = 0$. Estimate some $\mathbf{s}^k$
1.Step : Set $k := k + 1$. Set $\mathbf{z} = \mathbf{s}^{k-1}$. Solve Problem (20) to obtain $\mathbf{s}^k$.
2.Step : If convergence is achieved, stop. Else goto Step 1.

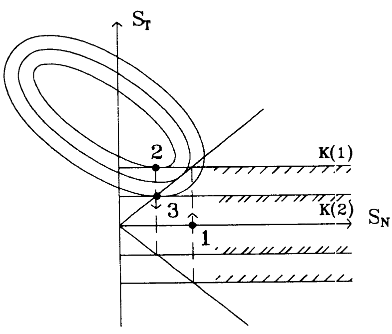

Figure 3. The Variational Section Algorithm.

The algorithm is depicted in Fig.3. The initial point 1 defines the cone $K(1) = K_M(1)$ and the solution of the resp. variational section is the point 2

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, yielding a new set K_M, the cone $K(2)$. The solution of the new variational section is the point 3, which belongs to $K(2)$, i.e. it is the desired solution of the QVI.

The two - stage technique of [7] can be interpreted now as a serial decomposition method to solve the QVI. Due to the Cartesian product nature (cf.[11]) of the point - to - set map (42), preserved in the variational sections, the method presented offers a series of parallelization possibilities, mostly suitable for a massively parallel computing environment. A direct implementation is to adopt a decomposition - coordination framework. During the decomposition phase the variational sections are solved in parallel for appropriate node groups, while the terms $\mathbf{w}_{N0}$, $\mathbf{w}_{T0}$ are computed in the coordination level. This way the parallelization degree approaches concurrent programming and quite large problems can be solved effectively.

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