

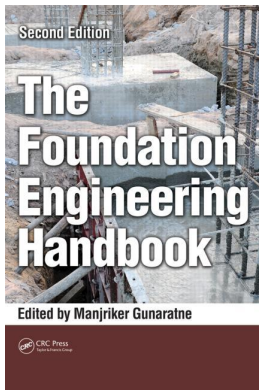
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3

Spread Footings: Analysis and Design

Manjriker Gunaratne

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A shallow spread footing is designed for a building column in order to safely transmit the structural load to the ground without exceeding the bearing capacity of the ground and causing excessive settlements. The system that encompasses the footing and the ground influenced by the footing is generally referred to as the foundation.

3.1 Design Criteria

3.1.1 Bearing Capacity Criterion

The maximum contact stress that can be borne by the foundation is termed the ultimate bearing capacity of the foundation. If the contact ground stress imposed by the structural load exceeds the ultimate bearing capacity, the shear stresses induced in the ground would cause plastic shear deformation within the foundation’s influence zone (Figure 3.1). This overloading condition can lead to either a global or a punching shear failure, which would

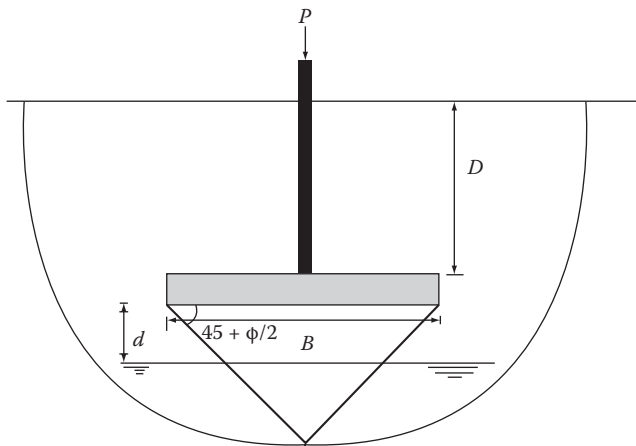


FIGURE 3.1
Influence zone of a shallow footing.

result in immediate sinking of the footing without prior warning. Therefore, the following condition must be satisfied for safety from bearing capacity failure, the following

$$\frac{P}{A} \leq \frac{q_{ult}}{F}, \tag{3.1a}$$

where

q_{ult} = ultimate bearing capacity of the foundation (kN/m², kPa, or ksf)

P = total load at the footing level (structural + refill soil load) (kN or kips)

A = footing area (m² or ft²)

F = appropriate safety factor that accounts for the uncertainties involved in the determination of the structural loads (P) and the ultimate bearing capacity (q_{ult})

3.1.2 Settlement Criterion

The designer must also ensure that the footing does not undergo either excessive total settlement as a unit or differential settlement within the footing. Excessive settlement of the foundation generally occurs as a result of irreversible compressive deformation taking place immediately or in the long term. Excessive time-dependent settlement occurs in saturated compressible clays with prior warning through cracking, tilting, and other signs of building distress. On the other hand, significant immediate settlement can occur in loose sands or compressible clays and silts. Therefore, the footing must be proportioned to limit its estimated settlements (δ_{est}) within tolerable settlements (δ_{tol}).

$$\delta_{est} \leq \delta_{tol} \tag{3.1b}$$

3.2 Evaluation of Bearing Capacity

Based on the discussion in Section 3.1.1, a foundation derives its bearing capacity from the shear strength of the subsoil within the influence area (Figure 3.1) and the embedment of the footing (D). Over the years, many eminent geotechnical engineers have suggested expressions for the ultimate bearing capacity of foundations that have also been verified on various occasions by load tests (e.g., Plate load test in Section 3.2.7.3). Several common expressions for the ultimate bearing capacity are provided next.

3.2.1 Bearing Capacity Evaluation in Homogeneous Soil

Terzaghi's bearing capacity expression

$$q_{ult} = cN_c s_c + qN_q + 0.5B\gamma N_\gamma s_\gamma \tag{3.2}$$

Meyerhoff's bearing capacity expression

For vertical loads

$$q_{ult} = cN_c s_c d_c + qN_q s_q d_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma \tag{3.3}$$

For inclined loads

$$q_{ult} = cN_c d_c i_c + qN_q d_q i_q + 0.5B\gamma N_\gamma d_\gamma i_\gamma \tag{3.4}$$

Hansen’s bearing capacity expression

$$q_{ult} = cN_c s_c d_c i_c g_c b_c + qN_q s_q d_q i_q g_q b_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \tag{3.5}$$

For undrained conditions

$$q_{ult} = 5.14s_u(1 + s'_c + d'_c - i'_c - g'_c - b'_c) + q \tag{3.6}$$

Vesic’s bearing capacity expression

$$q_{ult} = cN_c s_c d_c i_c g_c b_c + qN_q s_q d_q i_q g_q b_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \tag{3.7}$$

where

- c = cohesive strength
- ϕ = friction angle
- N_i = bearing capacity factors (Table 3.1)
- q = effective vertical stress at the footing base level
- γ = unit weight of surcharge soil
- s = shape factors (Tables 3.2a, 3.2b, and 3.2c)
- d = depth factors (Tables 3.2a, 3.2b, and 3.2c)
- i = inclination factors (Tables 3.2a, 3.2b, 3.2c, 3.3a, and 3.3b; see also Figure 3.2)
- g = ground slope factors (Tables 3.3a and 3.3b)
- b = base tilt factors (Tables 3.3a and 3.3b)

TABLE 3.1
Bearing Capacity Factors

ϕ	Terzaghi’s (1943) Expression			Hansen, Meyerhoff, and Vesic’s Expressions		Hansen (1970)	Meyerhoff (1951, 1963)	Vesic (1973, 1975)
	N_c	N_q	N_γ	N_c	N_q	N_γ	N_γ	N_γ
0	5.7	1.0	0.0	5.14	1.0	0.0	0.0	0.0
5	7.3	1.6	0.5	6.49	1.6	0.1	0.1	0.4
10	9.6	2.7	1.2	8.34	2.5	0.4	0.4	1.2
15	12.9	4.4	2.5	11.0	3.9	1.2	1.1	2.6
20	17.7	7.4	5.0	14.8	6.4	2.9	2.9	5.4
25	25.1	12.7	9.7	20.1	10.7	6.8	6.8	12.5
30	37.2	22.5	19.7	30.1	18.4	15.1	15.7	22.4
35	57.8	41.4	42.4	46.4	33.5	34.4	37.6	48.1
40	95.7	81.3	100	75.3	64.1	79.4	93.6	109.3
45	172	173	298	134	135	201	262.3	271.3

TABLE 3.2a

Shape and Depth Factors for Hansen’s Expression

Shape Factors	Depth Factors
$s'_c = 0.2 \frac{B}{L}$ for $\phi = 0^\circ$	$d'_c = 0.4k$ for $\phi = 0^\circ$
$s_c = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L}$	$d_c = 1.0 + 0.4k$
$s_q = 1.0 + \frac{B}{L} \sin \phi$	$k = D/B$ for $D/B \leq 1$
$s_\gamma = 1.0 - 0.4 \frac{B}{L}$	$k \text{ (rad)} = \tan^{-1} (D/B)$ for $D/B > 1$
	$d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 k$
	$d_\gamma = 1.00$

Source: Hansen, J.B., A revised and extended formula for bearing capacity, Danish Geotechnical Institute, Copenhagen, Bulletin No. 28, 1970. Reproduced with permission from Bowles, J.E., *Foundation Analysis and Design*, McGraw-Hill, New York, 2002.

TABLE 3.2b

Shape, Depth, and Inclination Factors for Meyerhoff’s Expression

Shape factor	$s_c = 1 + 0.2K_p \frac{B}{L}$	
	$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$	$\phi > 10^\circ$
	$s_q = s_\gamma = 1$	$\phi = 0^\circ$
Depth factor	$d_c = 1 + 0.2\sqrt{K_p} \frac{D}{B}$	
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p} \frac{D}{B}$	$\phi > 10^\circ$
	$d_q = d_\gamma = 1$	$\phi = 0^\circ$
Inclination factor	$i_c = i_q = \left(1 - \frac{\theta^\circ}{90^\circ}\right)^2$	
	$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2$	$\phi > 0^\circ$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0^\circ$

Note: θ is the load inclination to the vertical and $K_p = \tan^2(45 + \phi/2)$.

Finally, appropriate safety factors recommended for various construction situations are given in Tables 3.4a, 3.4b, and 3.4c.

TABLE 3.2c
Shape and Depth Factors for Vesic's Expression

Shape Factors	Depth Factors
$s_c = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L}$	$d'_c = 0.4k$ for $\phi = 0^\circ$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \leq 1$ k (rad) = $\tan^{-1}(D/B)$ for $D/B > 1$
$s_q = 1.0 + \frac{B}{L} \tan \phi$	$d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 k$
$s_\gamma = 1.0 - 0.4 \frac{B}{L}$	$d_\gamma = 1.00$

Source: Vesic, A.S., *J. Soil Mech. Found. Eng. Div., ASCE*, 99, SM 1, 45-763, 1973. *Foundation Engineering Handbook*, 1st ed., Winterkorn, H.F., Fang, H.Y. (eds.), Van Nostrand-Reinhold, New York, 1975. Reproduced with permission from Bowles, J.E., *Foundation Analysis and Design*, McGraw-Hill, New York, 2002.

TABLE 3.3a
Inclination, Ground Slope, and Base Tilt Factors for Hansen's Expression (Figure 3.2)

Load Inclination Factors	Factors for Base on Slope (β)	Factors for Tilted Base (η)
$i'_c = 0.5 - 0.5 \sqrt{1 - \frac{H_i}{A_f C_a}}$	$g'_c = \frac{\beta^\circ}{147^\circ}$	$b'_c = \frac{\eta^\circ}{147^\circ}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta^\circ}{147^\circ}$	$b_c = 1 - \frac{\eta^\circ}{147^\circ}$
$i_q = \left[1 - \frac{0.5 H_i}{V + A_f C_a \cot \phi} \right]^{\alpha_1}$	$g_q = g_\gamma = (1 - 0.5 \tan \beta)^5$	$b_q = \exp(-0.0349 \eta^\circ \tan \phi)$
$2 \leq \alpha_1 \leq 5$	(β° measured clockwise from horizontal)	$b_\gamma = \exp(-0.0471 \eta^\circ \tan \phi)$ (η is measured counter-clockwise from horizontal)
$i_\gamma = \left[1 - \frac{(0.7 - \theta^\circ / 450^\circ) H_i}{V + A_f C_a \cot \phi} \right]^{\alpha_2}$		
$2 \leq \alpha_2 \leq 5$		

Source: Hansen, J.B., A revised and extended formula for bearing capacity, Danish Geotechnical Institute, Copenhagen, Bulletin No. 28, 1970. Reproduced with permission from Bowles, J.E., *Foundation Analysis and Design*, McGraw-Hill, New York, 2002.

Note: Primed factors are for $\phi = 0$. C (cohesion) = attraction between the same material; C_a (adhesion) = attraction between two different materials (e.g., concrete and soil). Hence, $C_a < C$; Bowles (2002) suggests $C_a = (0.6 - 1.0)C$. The actual value depends on the concrete finish. If concrete foundation base is smooth then, C_a would be higher than that of a rough base.

TABLE 3.3b

Inclination, Ground Slope, and Base Tilt Factors for Vesic's Expression (Figure 3.2)

Load Inclination Factors	Factors for Base on Slope (β)	Factors for Tilted Base (η)
$i'_c = 1 - \frac{mH_i}{A_f C_a N_c}$	$g'_c = \frac{\beta}{5.14}$	$b'_c = g'_c$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = i_q - \frac{1 - i_q}{5.14 \tan \phi}$	$b_c = 1 - \frac{2\eta}{5.14 \tan \phi}$
$i_q = \left[1.0 - \frac{H_i}{V + A_f C_a \cot \phi} \right]^m$	$g_q = g_\gamma = (1.0 - \tan \beta)^2$ (β° measured clockwise from horizontal)	$b_q = b_\gamma = (1.0 - \eta \tan \phi)^2$ (η measured counterclockwise from horizontal)
$i_\gamma = \left[1.0 - \frac{H_i}{V + A_f C_a \cot \phi} \right]^{m+1}$		
When H is parallel to B		
$m = m_B = \frac{2 + B/L}{1 + B/L}$		
When H is parallel to L		
$m = m_L = \frac{2 + L/B}{1 + L/B}$		
When H has components parallel to both B and L		
$m^2 = m_B^2 + m_L^2$		

Source: Vesic, A.S., *J. Soil Mech. Found. Eng. Div., ASCE*, 99, SM 1, 45–763, 1973. *Foundation Engineering Handbook*, 1st ed., edited by Winterkorn, H.F. and Fang, H.Y., Van Nostrand-Reinhold, New York, 1975. Reproduced with permission from Bowles, J.E., *Foundation Analysis and Design*, McGraw-Hill, New York, 2002.

Note: Primed factors are for $\phi = 0$. C (cohesion) = attraction between the same material; C_a (adhesion) = attraction between two different materials (e.g., concrete and soil). Hence, $C_a < C$; Bowles (2002) suggests $C_a = (0.6 - 1.0)C$. The actual value depends on the concrete finish. If concrete foundation base is smooth then, C_a would be higher than that of a rough base.

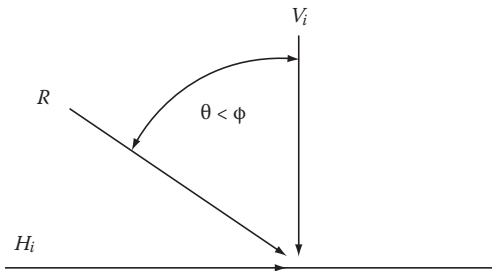


FIGURE 3.2
Guide for obtaining inclination factors.

TABLE 3.4a

Factors of Safety on Ultimate Geotechnical Capacity of Spread Footings for Bearing Capacity and Sliding Failure

Failure Condition	Required Minimum Factor of Safety (FS)
Bearing capacity of footing on soil or rock	3.0
Sliding resistance of footing on soil or rock	1.5

Source: AASHTO, Standard Specifications for Highway Bridges, American Association for State Highway and Transportation Officials, Washington, DC, 1996.

TABLE 3.4b

Factors of Safety on Ultimate Bearing Capacity of Spread Footings on Soils

Basis for Soil Strength Estimate	Suggested Minimum Factor of Safety (FS)
Standard penetration tests	3.0
Laboratory/field strength tests	2.5

Source: AASHTO, Standard Specifications for Highway Bridges, American Association for State Highway and Transportation Officials, Washington, DC, 1996.

TABLE 3.4c

Variable Factors of Safety on Ultimate Bearing Capacity of Spread Footings

Category	Typical Structures	Category Characteristics	Required Minimum Factor of Safety (FS)			
			Permanent Structures		Temporary Structures	
			Complete Soil Exploration	Limited Soil Exploration	Complete Soil Exploration	Limited Soil Exploration
A	Railway bridges Warehouses Blast furnaces Hydraulic Retaining Walls Silos	Maximum design load likely to occur often; consequences of failure disastrous	3.0	4.0	2.3	3.0
B	Highway bridges Light industrial and public buildings	Maximum design load may occur occasionally; consequences of failure serious	2.5	3.5	2.0	2.6
C	Apartment and office buildings	Maximum design load unlikely to occur	2.0	3.0	2.0	2.3

Source: AASHTO, Standard Specifications for Highway Bridges, American Association for State Highway and Transportation Officials, Washington, DC, 1996.

3.2.1.1 Mathematical Expressions for Meyerhof's and Hansen's Bearing Capacity Factors

For spreadsheet applications, the following mathematical expressions of the bearing capacity factors would be quite useful.

$$N_q = e^{\pi \tan \phi} \tan^2 (45 + \phi/2) \quad (3.8a)$$

$$N_c = (N_q - 1)\cot \phi \tag{3.8b}$$

For Meyerhoff's expression

$$N_\gamma = 1.5 (N_q - 1)\tan (1.4 \phi) \tag{3.9a}$$

For Hanson's expression

$$N_\gamma = 1.5 (N_q - 1)\tan \phi \tag{3.9b}$$

Example 3.1

For the column shown in Figure 3.3a, design a suitable footing to carry a column load of 400 kN, in a subsoil that can be considered homogenous silty clay with the following properties: unit weight (γ) = 17 kN/m³, internal friction (ϕ) = 15°, cohesion (C) = 20 kPa

- Case (1). Assume that the groundwater table is not in the vicinity.
- Case (2). Assume that the groundwater table is 0.5 m above the footing.

Solution

First, one must decide on a suitable footing shape and depth. In the case of the footing shape, unless there are limitations in spacing such as the close proximity to the property line, there is generally no reason for one not to use a square or a circular footing. Hence, in this design one can assume a circular footing.

As for the foundation depth, typically one would seek some significant embedment that does not reach the groundwater table or a weak layer known to be underlying the foundation soil. In the current case, obviously none of these can be used as a criterion to select the footing depth. Therefore, one could assume a depth approximately equal to the minimum footing dimension (diameter) of the footing. However, once the design parameters are obtained, one can reevaluate this criterion to verify that the depth is realistic from a construction point of view.

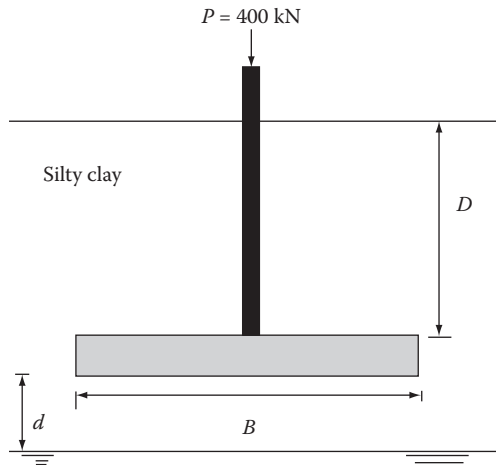


FIGURE 3.3a
Illustration for Example 3.1.

The tables indicate the following bearing capacity parameters:

- Terzaghi's factors (Table 3.1)

$$\begin{array}{lll} N_c = 12.9 & N_q = 4.4 & N_\gamma = 2.5 \\ s_c = 1.3 & s_q = 0.6 & \end{array}$$

- Hansen's factors (Table 3.1)

$$\begin{array}{lll} N_c = 10.97 & N_q = 3.9 & N_\gamma = 1.2 \\ s_c = 1.359 & s_q = 1.26 & s_\gamma = 0.6 \\ d_c = 1.4 & d_q = 1.294 & d_\gamma = 1.0 \end{array}$$

The vertical effective stress at the footing base level (q) = (17)(depth) = $17B$.

Then, the following expressions can be written for the ultimate bearing capacity:

- Terzaghi method (Equation 3.2)

$$\begin{aligned} q_{\text{ult}} &= 20(12.9)(1.3) + (17B)(4.4) + 0.5(17)(B)(2.5)(0.6) \\ &= 335.4 + 87.55B \end{aligned}$$

- Hansen method (Equation 3.5)

$$\begin{aligned} q_{\text{ult}} &= 20(10.97)(1.359)(1.4) + (17B)(3.9)(1.26)(1.294) + 0.5(17)(B)(1.2)(0.6)(1.0) \\ &= 417.4 + 114.22B \end{aligned}$$

Contact stress at the foundation level = $4 \times 400/(\pi B^2) + 17B$ = stresses imposed by the column and the recompacted soil (Figure 3.3a).

The following criterion can be applied to compare the contact stress and the ultimate bearing capacity with a safety factor of 2.5.

$$4 \times 400/(\pi B^2) + 17B = q_{\text{ult}}/(2.5)$$

From Terzaghi's expression,

$$\begin{aligned} 509.3/B^2 + 17B &= (335.4 + 87.55B)/2.5 \\ B &= 1.75 \text{ m} \end{aligned}$$

From Hansen's expression,

$$\begin{aligned} 509.3/B^2 + 17B &= (417.4 + 114.22B)/2.5 \\ B &= 1.55 \text{ m} \end{aligned}$$

Although the two solutions are different, one realizes that the disparity is insignificant from a construction point of view. Furthermore, in both cases, the footing depth obtained is within practical limits.

Case (2). Assume that the water table is 0.5 m above the footing.

Using Hansen's expression (Equation 3.5),

$$\begin{aligned} q_{\text{ult}} &= 20(10.97)(1.359)(1.4) + [17B - (9.8)(0.5)](3.9)(1.26)(1.294) + 0.5(17 - 9.8)(B)(1.2)(0.6)(1.0) \\ &= 386.27 + 110.69B \end{aligned}$$

$$509.3/B^2 + 17B = (386.27+110.69B)/2.5$$

$$B = 1.62 \text{ m}$$

It is noted that a slightly larger area is needed to counteract the loss of foundation strength due to the groundwater table.

Example 3.2

Design a flooring for the same soil conditions, when a horizontal load of 50 kN also acts along the footing base as shown in Figure 3.3b.

Case (1). Groundwater table is not in the vicinity.

- Hansen’s factors (Table 3.1)

$N_c = 10.97$	$N_q = 3.9$	$N_\gamma = 1.2$
$s_c = 1.359$	$s_q = 1.26$	$s_\gamma = 0.6$
$d_c = 1.4$	$d_q = 1.294$	$d_\gamma = 1.0$

(Table 3.3a)

$$H_i = 50 \text{ kN}$$

$$V = 400 \text{ kN}$$

$$A_f = (\pi B^2)/4$$

C_a = Adhesion between soil and concrete = 2/3 cohesion (see the footnote in Table 3.3a)
 = 2/3(20) = 13.33 kPa

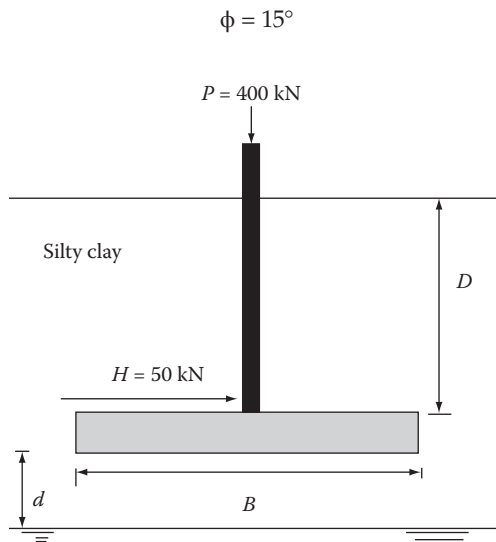


FIGURE 3.3b
 Illustration for Example 3.2

$$i_q = \left[1 - \frac{0.5H_i}{V + A_f C_a \cot \phi} \right]^{\alpha_1} \quad (\text{use 2 as the exponent without other information})$$

$$i_\gamma = \left[1 - \frac{(0.7 - \theta^\circ/450^\circ)H_i}{V + A_f C_a \cot \phi} \right]^{\alpha_2} \quad (\text{use 2 as the exponent without other information})$$

$$\tan \theta = 50/400, \text{ then } \theta = 7.1^\circ$$

Since B is to be determined, we do not want to complicate the equations by having a B^2 term in the i factors. Therefore, it is convenient to set up a spreadsheet (assume B values) and perform the design. In this case, one can assume that B is approximately 1.0 (only in the i factors expressions) to obtain an explicit solution.

$$i_c = i_q - \frac{1 - i_q}{N_q - 1}$$

$$i_c = 0.85 \quad i_q = 0.89, \quad i_\gamma = 0.85$$

- Hansen method (Equation 3.5)

$$\begin{aligned} q_{\text{ult}} &= 20(10.97)(1.359)(1.4)(0.85) + (17B)(3.9)(1.26)(1.294)(0.89) + 0.5(17)(B)(1.2)(0.6)(1.0)(0.85) \\ &= 354.8 + 101.65B \end{aligned}$$

Contact stress at the foundation level = $4 \times 400/(\pi B^2) + 17B$ = stresses imposed by the column and the recompacted soil (Figure 3.3b).

The following criterion can be applied to compare the contact stress and the ultimate bearing capacity with a safety factor of 2.5.

$$4 \times 400/(\pi B^2) + 17B = q_{\text{ult}}/(2.5)$$

From Hansen's expression,

$$509.3/B^2 + 17B = (354.8 + 101.65B)/2.5$$

$$B = 1.75 \text{ m}$$

It is seen that a larger footing is needed to support an inclined load.

(Case 2). When the groundwater table is in 0.5 m below the footing, one can solve the problem like in Example 3.1 with the above determined i factors.

3.2.2 Net Ultimate Bearing Capacity

If the structural (column) load is to be used in the bearing capacity criterion (Equation 3.1) to design the footing, then one has to strictly use the corresponding bearing capacity

that excludes the effects of the soil overburden. This is known as the net ultimate bearing capacity of the ground and it is expressed as

$$q_{n,ult} = q_{ult} - q_f \tag{3.10}$$

where q denotes the total overburden stress.

On the other hand, the net load increase on the ground would be the structural load only, if it is assumed that concrete counteracts the soil removed to lay the footing.

Then, Equation 3.1 can be modified as

$$P_{\text{structural}} / A \leq \frac{q_{n,ult}}{F} \tag{3.11}$$

3.2.3 Foundations on Stiff Soil Overlying a Soft Clay Stratum

One can expect a punching type of bearing capacity failure if the surface layer is relatively thin and stiffer than the underlying softer layer. In this case, if one assumes that the stiff stratum (i.e., stiff clay, medium dense or dense sand) where the footing is founded satisfies the bearing capacity criterion with respect to the surface layers, then the next most critical criterion is that the stress induced by the footing (Figure 3.4) at the interface of the stiff soil/soft clay must meet the relatively low bearing capacity of the soft layer. The distributed stress can be computed by the following equations:

For rectangular spread footings

$$\Delta p = q \left[\frac{BL}{(B + d_c)(L + d_c)} \right] \tag{3.12a}$$

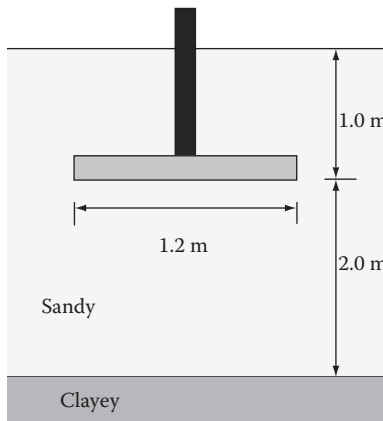


FIGURE 3.4
Illustration for Example 3.3.

For square or circular spread footings

$$\Delta p = q \left[\frac{B}{(B + d_c)} \right]^2 \quad (3.12b)$$

For strip footings

$$\Delta p = q \left[\frac{B}{(B + d_c)} \right] \quad (3.12c)$$

Example 3.3

Assume that the square footing shown in Figure 3.4 has been well designed to be founded in the sand layer overlying the soft clay layer. Check the bearing capacity criterion in the clay layer (undrained cohesion = 20 kPa).

If Hansen's bearing capacity equation (Equation 3.5) is used to estimate the *net* ultimate bearing capacity of the clay layer,

$$q_{n,ult} = cN_c s_c d_c i_c g_c b_c + q(N_q - 1) s_q d_q i_q g_q b_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \quad (3.5)$$

Under undrained conditions, since $O_u = 0$,

$$N_c = 5.14 \quad N_q = 1.0 \quad N_\gamma = 0 \text{ (Table 3.1)}$$

$$q_{n,ult} = cN_c s_c d_c i_c g_c b_c \text{ (Equation 3.5)}$$

$$s_c = 1.195 \text{ (square footing)}$$

$$d_c = 1.0 + 0.4 \left(\frac{3.0}{1.2} \right)$$

$$\begin{aligned} q_{n,ult} &= (20)(5.14)(1.195)(2.0) \\ &= 245.69 \text{ kPa} \end{aligned}$$

Alternatively, from Equation 3.6

$$q_{n,ult} = 5.14s_u(1 + s'_c + d'_c - i'_c - g'_c - b'_c) + q - q$$

From Table 3.2a,

$$s'_c = 0.2$$

$d'_c = 0.4k = 0.4 \left(\frac{3.0}{1.2} \right)$ (since $d/b = 3.0/1.2$ when one considers that the bearing capacity of the clay layer with respect to the distributed load from the footing).

Also, $i'_c = 0$, $g'_c = 0$ and $b'_c = 0$

Hence,

$$q_{n,ult} = 5.14s_u(1 + s'_c + d'_c - i'_c - g'_c - b'_c) + q - q = 5.14(20)(1 + 0.2 + 1) = 226.16 \text{ kPa}$$

The *net* stress applied on the soft clay can be estimated as

$$\begin{aligned} \Delta p &= q \left[\frac{B}{(B + d_c)} \right]^2 \\ &= [500 / (1.2)^2] [(1.2 / 3.2)^2] \\ &= 48.8 \text{ kPa} \end{aligned} \tag{3.12b}$$

Factor of safety (FOS) = 226.16/48.8 = 4.63 (satisfactory)

3.2.4 Foundations on Soft Soil Overlying a Hard Stratum

When foundations are constructed on thin clayey surface layers overlying relatively hard strata (Figure 3.5), the mechanism of bearing capacity failure transforms into one in which the footing tends to squeeze the soft layer away while sinking in. In such cases, the net ultimate bearing capacity of the surface layer can be obtained from the following expressions (Tomlinson and Boorman, 1995):

Circular/square footings

$$q_{n,ult} = \left(\frac{B}{2d} + \pi + 1 \right) S_u \quad \text{for } \frac{B}{d} \geq 2 \tag{3.13}$$

Strip footings

$$q_{n,ult} = \left(\frac{B}{3d} + \pi + 1 \right) S_u \quad \text{for } \frac{B}{d} \geq 6 \tag{3.14}$$

where

B = footing dimension

d = thickness of the surface layer

S_u = undrained strength of the surface layer

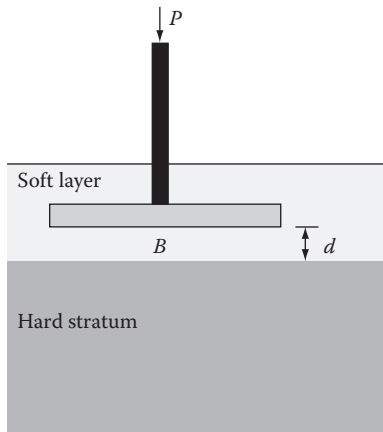


FIGURE 3.5
Soft surface layer overlying a harder layer.

It must be noted that if the criteria $\frac{B}{d} \geq 2$ and $\frac{B}{d} \geq 6$ are not satisfied for circular and strip footings, respectively, then the foundation can be treated as one placed in a homogeneous clay layer. For homogeneous cases, the bearing capacity estimation can be performed based on the methods discussed in the Section 3.2.1.

3.2.5 Bearing Capacity in Soils Mixed in Layers

When the subsurface constitutes an alternating (sandwiched) mixture of two distinct soil types as shown in Figure 3.6, one can use engineering judgment to estimate the bearing capacity. As an example, Figure 3.6 has the following layers as identified by the cone penetration test (CPT) results (Section 2.5):

1. SM (silty sand), which is sand contaminated with a significant portion of silt. As expected, the cone resistance q_c profile peaks out for sand.
2. CL or ML (clay and silt). As one would expect, the q_c profile drops for clay or silt (if the shaft friction, f_s , profile was provided, it would be relatively high for these layers).

In order to estimate the bearing capacity, the q_c values have to be averaged within the influence zone (Section 3.2.7.1). Because the soil types are not physically separated into two distinct layers, and because SM and CL (or ML) have very different engineering properties, it is conceptually incorrect to average the q_c values across the entire influence zone. Hence, the only way to address this is to assume one soil type at a time and obtain two bearing capacity estimates, an upper bound and a lower bound for the actual bearing capacity.

Step 1. Assume SM type only with a continuous linear q_c profile (with depth) defined by the peaks in Figure 3.6, thus ignoring the presence of clay and silt (CL/ML). Then, one deals with a silty sand only and the corresponding bearing capacity estimate would be Q_{ult2} (the upper bound).

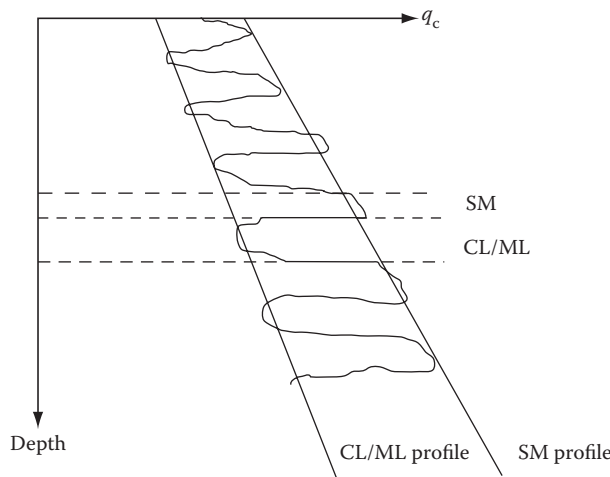


FIGURE 3.6
Bearing capacity of soils mixed in layers.

Step 2. Assume CL/ML type only with a continuous linear q_c profile (with depth) defined by the troughs (indentations), thus ignoring the presence of sand (SM) and assuming undrained conditions. Then, one deals with clay/silt only and the corresponding bearing capacity estimate would be Q_{ult2} (the lower bound).

Then, the effective bearing capacity could be estimated from the following inequality:

$$Q_{ult2} < Q_{ult} < Q_{ult1} \tag{3.15}$$

3.2.6 Bearing Capacity of Eccentric Footings

The pressure distribution on the bottom of an eccentric footing can be determined from combined axial and bending stresses, as shown in Figure 3.7. One also realizes that, in order to prevent tensile forces at the bottom that tends to uplift the footing, the following conditions must be satisfied.

$$e_x \leq \frac{B}{6} \quad e_y \leq \frac{L}{6} \tag{3.16a}$$

The above conditions are modified for rock as follows:

$$e_x \leq \frac{B}{4} \quad e_y \leq \frac{L}{4} \tag{3.16b}$$

For the load and resistance factor design (LRFD) method (Section 3.4), the following modifications are made in the maximum eccentricity criteria (for no tension at the footing/soil interface) in view of load factoring:

$$e_x \leq \frac{B}{4} \tag{3.17a}$$

and

$$e_y \leq \frac{L}{4} \tag{3.17b}$$

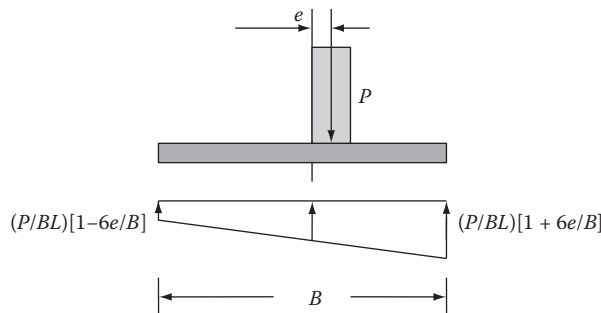


FIGURE 3.7
Bottom pressure distribution on rigid eccentric footings.

The above conditions are modified for rock as follows:

$$e_x \leq \frac{3B}{8} \tag{3.17c}$$

$$e_y \leq \frac{3L}{8} \tag{3.17d}$$

Because the contact pressure is nonuniform at the bottom of the footing (Figure 3.7), Meyerhoff (1963) and Hansen (1970) suggested the following effective footing dimensions to be used in order to compute the bearing capacity of an eccentrically loaded rectangular footing. For eccentricities in both X and Y directions (Figure 3.8),

$$B' = B - 2e_x \tag{3.18a}$$

$$L' = L - 2e_y \tag{3.18b}$$

At times, a horizontal load that has two components, that is, H_B parallel to B and H_L parallel to L , can act on the column producing two eccentricities e_x and e_y on the footing. In such cases, shape factors (Tables 3.2a, 3.2b, and 3.2c) are computed twice by interchanging B' and L' . Also, i factors (Tables 3.3a and 3.3b) are computed twice by replacing H_i once with H_L and then with H_B . Finally, the B' term in the q_{ult} expression also gets replaced by L' . Thus, in such cases, one would obtain two distinct q_{ult} values. The lesser of them is compared to P/A for the footing design.

In the case of circular footings having load eccentricity e and radius R , one must first locate the diameter corresponding to the eccentricity (point E in Figure 3.8b) and then construct a circular arc centered at F (EF = CE) with a radius equal to that of the footing. Then, the shaded area represents the effective footing area. Because the effective footing area is not of a geometrically regular shape, typically this is transformed into an equivalent rectangular footing of dimensions B' and L' . The effective dimensions can be found from the following expression:

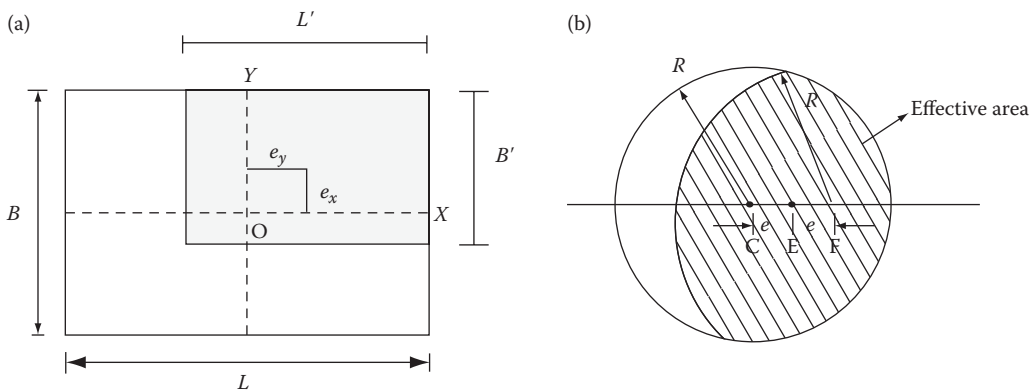


FIGURE 3.8
 (a) Rectangular footings with eccentricity. (b) Circular footings with eccentricity.

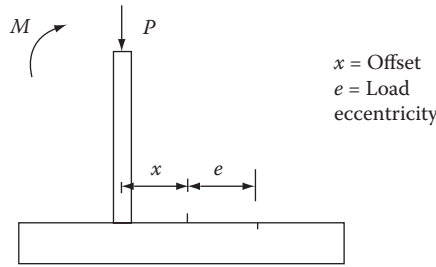


FIGURE 3.9
Designing footings to avoid eccentricity.

$$B'L' = 2R \left[R \cos^{-1} \left(\frac{e}{R} \right) - e \right] \tag{3.19}$$

However, it must be noted that the unmodified B and L must be used when determining the depth factors (d) in the bearing capacity equations.

When footings are to be designed for a column that carries an unbalanced moment, M , and an axial force, P , that are fixed in magnitude, the resulting eccentricity ($e = M/P$) induced on the footings can be avoided by offsetting the column by a distance of $x = -e$, as shown in Figure 3.9. It is seen how the axial force in the column creates an equal and opposite moment to counteract the moment in the column. However, this technique cannot be used to prevent footing eccentricities when eccentricities are introduced by variable moments due to wind and wave loading.

3.2.7 Bearing Capacity Using *In Situ* Test Data

3.2.7.1 CPT Data

Cone penetration data can be used to obtain the undrained strength of saturated fine-grained soils using the following expression (Equation 2.6d):

$$S_u = \frac{q_c - P_o}{N_k}, \tag{3.20}$$

where N_k is the cone factor that ranges between 15 and 20 for normally consolidated clay and 27 and 30 for overconsolidated clays. Bowles (2002) suggests the following expression for N_k

$$N_k = 13 + \frac{5.5}{50} PI, \tag{3.21}$$

where PI is the plasticity index.

To determine an average q_c for a footing design, one would consider a footing influence zone that extends $2B$ below the footing and $1/2D$ above the footing.

3.2.7.2 Standard Penetration Test Data

Parry (1977) provided the following expression for the allowable bearing capacity of spread footings on cohesionless soils.

For $D_f < B$,

$$q_{n,\text{all}} = 30N_{55} \left(\frac{s}{25.4} \right) \quad (3.22)$$

where N_{55} is the corrected SPT blow count corresponding to a 55% hammer efficiency and s is the settlement (in mm). A modified and more versatile form of this expression is provided in Section 4.4 under mat footing design.

Typically, when SPT data are provided, one can use the following correlation to estimate an equivalent angle of friction ϕ for the soil and determine the bearing capacity using the methods presented in Section 3.2.

$$\phi = 25 + 28 \left(\frac{N_{55}}{q} \right)^{1/2} \quad (3.23)$$

The footing influence zone suggested in Section 3.2.7.1 can also be used for computations involving Equations 3.22 and 3.23.

3.2.7.3 Plate Load Test Data

Figure 3.10 shows a typical plot of plate-load test results on a sand deposit. When one scrutinizes Figure 3.10, it is seen that the ultimate bearing capacity of the plate can be estimated from the eventual flattening of the load–deflection curve. Knowing the ultimate bearing capacity of the plate, one can predict the expected bearing capacity of a footing to be placed on the same location using the following expressions:

Clayey soils

$$q_{u(f)} = q_{u(p)} \quad (3.24)$$

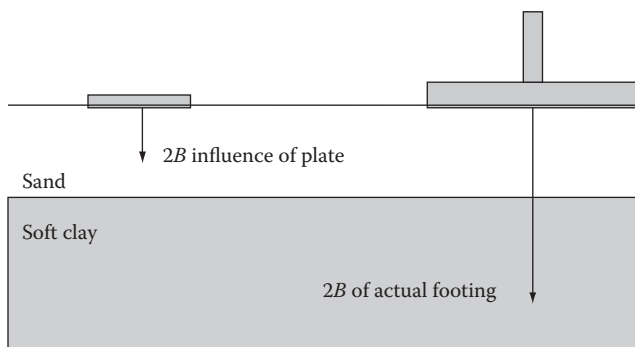


FIGURE 3.10
Illustration of influence zones.

Sandy soils

$$q_{u(f)} = q_{u(p)} \left(\frac{B_f}{B_p} \right) \tag{3.25}$$

where B_p is the plate diameter and B_f is the equivalent foundation diameter, which can be determined as the diameter of a circle having an area equal to that of the footing.

It must be noted that the above expressions can be applied if it is known that the influence zones (Figure 3.10) of both the plate and the footing are confined to the same type of soil and the effects of the groundwater table would be similar in both cases.

3.2.8 Presumptive Load-Bearing Capacity

The building codes of some cities suggest bearing capacities for a certain building sites based on the classification of the predominant soil type at that site. Tables 3.4a, 3.4b, 3.4c, 3.5, 3.6a,

TABLE 3.5
Presumptive Bearing Capacities from Indicated Building Codes (kPa)

Soil Description	Chicago, 1995	Natl. Board of Fire Underwriters, 1976	BOCA, 1993 ^a	Uniform Building Code, 1991 ^b
Clay, very soft	25			
Clay, soft	75	100	100	100
Clay, ordinary	125			
Clay, medium stiff	175	100		100
Clay, stiff	210		140	
Clay, hard	300			
Sand, compact and clean	240	—	140	200
Sand, compact and silty	100	}		
Inorganic silt, compact	125	}		
Sand, loose and fine		}	140	210
Sand, loose and coarse, or sand-gravel mixture, or compact and fine		140–400	240	300
Gravel, loose and compact coarse sand	300	}	240	300
		}		
Sand-gravel, compact		—	240	300
Hardpan, cemented sand, cemented gravel	600	950	340	
Soft rock				
Sedimentary layered rock (hard shale, sandstone, siltstone)			6000	1400
Bedrock	9600	9600	6000	9600

Source: Bowles, J.E., *Foundation Analysis and Design*, McGraw-Hill, New York, 2002. With permission.

Note: Values converted from pounds per square foot to kilopascals and rounded. Soil descriptions vary widely between codes. The following represents author’s interpretations.

^a Building Officials and Code Administrators International, Inc.

^b Bowles (2002) interpretation.

TABLE 3.6a

Presumptive Bearing Capacities for Foundations in Granular Soils Based on SPT Data (at a Minimum Depth of 0.75 m below Ground Level)

Description of Soil	N value in SPT	Presumed Bearing Value (kN/m ²) for Foundation of Width		
		1 m	2 m	4 m
Very dense sands and gravels	>50	800	600	500
Dense sands and gravels	30–50	500–800	400–600	300–500
Medium-dense sands and gravels	10–30	150–500	100–400	100–300
Loose sands and gravels	5–10	50–150	50–100	30–100

Source: Tomlinson, M.J., Boorman, R., *Foundation Design and Construction*, Longman Scientific and Technical, Brunthill, Harlow, England, 1995. With permission.

Note: The water table is assumed not to be above the base of foundation. Presumed bearing values for pad foundations up to 3 m wide are approximately twice the above values.

TABLE 3.6b

Presumptive Bearing Capacities for Foundations in Clayey Soils Based on Undrained Shear Strength (at a Minimum Depth of 1 m below Ground Level)

Description	Undrained Shear Strength (kN/m ²)	Presumed Bearing Value (kN/m ²) for Foundation of Width		
		1 m	2 m	4 m
Hard boulder clays, hard-fissured clays (e.g., deeper London and Gault Clays)	>300	800	600	400
Very stiff boulder clay, very stiff “blue” London Clay	150–300	400–800	300–500	150–250
Stiff-fissured clays (e.g., stiff “blue” and brown London Clay), stiff weathered boulder clay	75–150	200–400	150–250	75–125
Firm normally consolidated clays (at depth), fluvio-glacial and lake clays, upper weathered “brown” London Clay	40–75	100–200	75–100	50–75
Soft normally consolidated alluvial clays (e.g., marine, river and estuarine clays)	20–40	50–100	25–50	Negligible

Source: Tomlinson, M.J., Boorman, R., *Foundation Design and Construction*, Longman Scientific and Technical, Brunthill, Harlow, England, 1995. With permission.

3.6b, and 3.7 present a comprehensive list of presumptive bearing capacities for various soil types. However, it must be noted that these values do not reflect the foundation shape, depth, load inclination, location of the water table, and the settlements associated with the sites. Hence, the use of these bearing capacity factors are advocated primarily in situations where a preliminary idea of the potential foundation size is needed for the subsequent site investigation followed by detailed design.

TABLE 3.7

Presumptive Bearing Capacities for Foundations on Rock Surface (Settlement Not Exceeding 50 mm)

Rock Group	Strength Grade	Discontinuity Spacing (mm)	Presumed Allowable Bearing Value (kN/m ²)	
Pure limestones and dolomites, carbonate sandstones of low porosity	Strong	60 to >1000	>12,500 ^a	
		>600	>10,000 ^b	
	Moderately strong	200–600	7500–10,000	
		60–200	3000–7500	
		Moderately weak	600 to >1000	>5000 ^a
			200–600	3000–5000
	Weak	60–200	1000–3000	
		>600	>1000 ^a	
		200–600	750–1000	
	Igneous, oolitic, and marly limestones; well-cemented sandstones; indurated carbonate mudstones; metamorphic rocks (including slates and schists with flat cleavage/foliation)	Strong	200 to >1000	10,000 to >12,500 ^a
			60–200	5000–10,000
		Moderately strong	600 to >1000	8000 to >100,000 ^a
200–600			4000–8000	
60–200			1500–4000	
Moderately weak			600 to >1000	3000 to >5000 ^a
		200–600	1500–3000	
		60–200	500–1500	
Weak		600 to >1000	750 to >1000 ^a	
		>200	See note ^b	
		All	See note ^b	
Very marly limestones: poorly cemented sandstones; cemented mudstones and shales; slates and schists with steep cleavage/foliation		Strong	600 to >1000	10,000 to >12,500 ^b
	200–600		5000–10,000	
	Moderately strong	60–200	2500–5000	
		600 to >1000	4000 to >6000 ^b	
		200–600	2000 to >4000	
		60–200	750–2000	
	Moderately weak	600 to >1000	2000 to >3000 ^b	
		200–600	750–2000	
		60–200	250–750	
	Weak	600 to >1000	500–750	
		200–600	250–500	
		<200	See note ^b	
Very weak	All	See note ^b		

(continued)

TABLE 3.7 (Continued)

Presumptive Bearing Capacities for Foundations on Rock Surface (Settlement Not Exceeding 50 mm)

Rock Group	Strength Grade	Discontinuity Spacing (mm)	Presumed Allowable Bearing Value (kN/m ²)
Uncemented mudstones and shales	Strong	200–600	250–5000
		60–200	1250–2500
	Moderately strong	200–600	1000–2000
		60–200	1300–1000
	Moderately weak	200–600	400–1000
		60–200	125–400
	Weak	200–600	150–250
		60–200	See note ^b
Very weak	All	See note ^b	

Source: Tomlinson, M.J., Boorman, R., *Foundation Design and Construction*, Longman Scientific and Technical, Brunthill, Harlow, England, 1995. With permission.

Note: Presumed bearing values for square foundations up to 3 m wide are approximately twice the above values, or equal to the above values if settlements are to be limited to 25 mm.

^a Bearing pressures must not exceed the unconfined compression strength of the rock if the joints are tight. Where the joints open the bearing pressure must not exceed half the unconfined compression strength of the rock.

^b Bearing pressures for these weak or closely jointed rocks should be assessed after visual inspection, supplemented as necessary by field or laboratory tests to determine their strength and compressibility.

3.3 Settlement Analysis

Methodologies used for computation of ground settlement under building foundations have been discussed in detail in Section 1.5. Therefore in this section, a number of techniques commonly used to evaluate the ground stress increase due to footings will be reviewed. Then, a number of examples will be provided to illustrate the application of these techniques.

3.3.1 Stress Distribution in Subsurface Soils due to Foundation Loading

3.3.1.1 Analytical Methods

The vertical stress induced in the subsurface by a concentrated vertical load, such as the load on a relatively small footing founded on an extensive soil mass, can be approximately estimated by Boussinesq's elastic theory as follows:

$$\Delta\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}} \quad (3.26)$$

where r and z are indicated in Figure 3.11.

Equation 3.26 can be used to derive the magnitude of vertical stress imposed at any depth z vertically below the center of a circular foundation (of radius R) carrying a distributed load of q as (Figure 3.12)

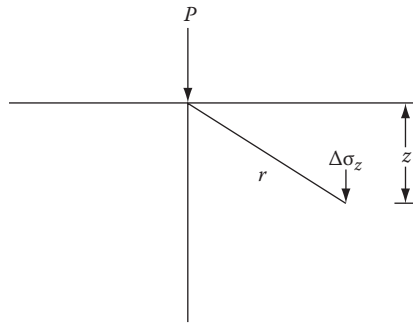


FIGURE 3.11
Stress increase due to a concentrated load.

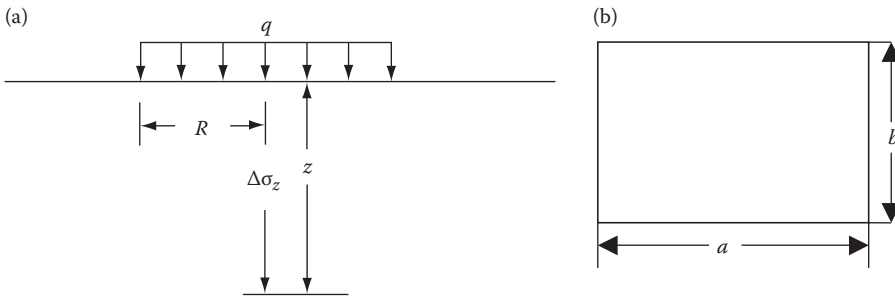


FIGURE 3.12
(a) Stress increase due to a distributed circular footing. (b) Stress increase due to a distributed rectangular footing.

$$\Delta\sigma_z = q \left[1 - \frac{1}{[1 + (R/z)^2]^{3/2}} \right] \tag{3.27}$$

Stress increments in the horizontal (x and y) and vertical (z) directions due to other shapes of uniformly loaded footings (e.g., rectangular, strip) can be estimated based on analytical expressions presented by Harr (1966).

3.3.1.2 Approximate Stress Distribution Method

At times, it is more convenient to estimate the subsurface stress increments due to footings using approximate distributions. A commonly used distribution is the 2:1 distribution shown in Figure 3.13. Based on Figure 3.13, it can be seen that the stress increment caused by a uniformly loaded rectangular footing ($B \times L$) at a depth of z is

$$\Delta\sigma_z = q \left[\frac{BL}{(B+z)(L+z)} \right] \tag{3.28}$$

Example 3.4

Assume that it is necessary to compute the ultimate consolidation settlement and the 10-year settlement of the 1.5×1.5 m footing carrying a 200-kN load as shown in Figure

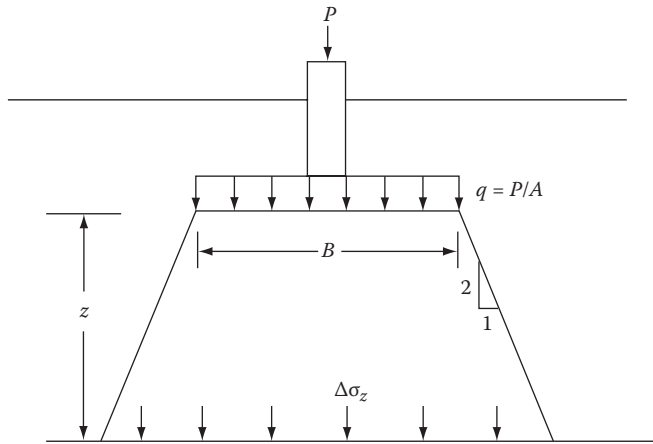


FIGURE 3.13
Approximate estimation of subsurface vertical stress increment.

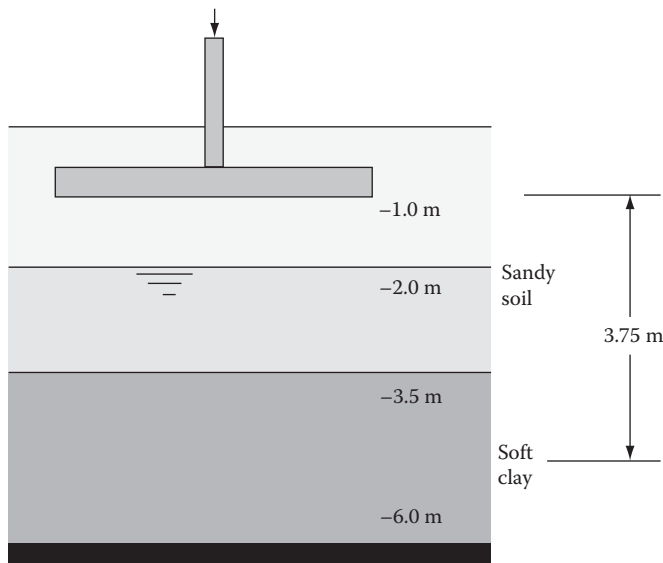


FIGURE 3.14
Illustration for Example 3.4.

3.14. Soil properties are provided in Table 3.8. Also assume that the laboratory consolidation characteristics of a representative sample (from the mid-plane area of the clay layer) are represented by Figure 3.15, and the coefficient of consolidation (C_v) of the clay was determined to be $1.0 \times 10^{-8} \text{ m}^2/\text{s}$ based on the methodology presented in Section 1.5.

From Figure 3.15,

$$\text{Preconsolidation pressure } (p_c) = 60 \text{ kPa}$$

$$\text{Contact pressure } (q) = 200/(1.5)^2 = 88.89 \text{ kPa}$$

$$\text{Overburden pressure at the footing depth} = 16.5 \times 1.0 = 16.5 \text{ kPa}$$

TABLE 3.8
Soil Properties for Example 3.3

Layer	Unit Weight (kN/m ³)	SPT(<i>N'</i>)	<i>E</i> (MPa)
Dry sand	16.5	15	11.5
Wet sand	17.5	14	10.7
Soft clay	18.0	10	2.56

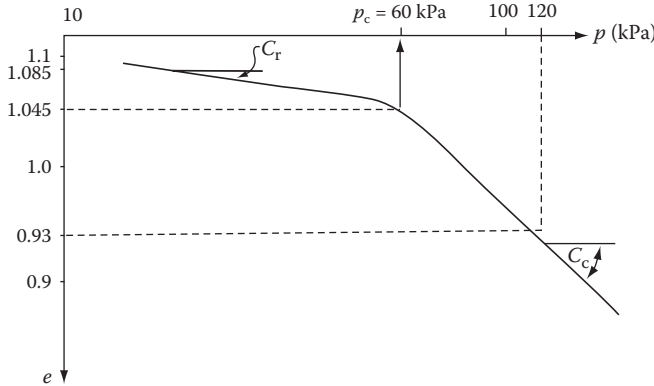


FIGURE 3.15
Laboratory consolidation curve.

The average stress increase in the clay layer can be obtained using the Newmark’s influence chart (reproduced in Figure 3.16) by considering the mid-plane depth of clay. This can be obtained from Figure 3.16 by mapping the footing to the scale indicated at the bottom of the figure, that is, d_c (the depth from the footing to the location where the stress increase is needed) = the distance indicated as OQ. In this example, one can see that $d_c = 3.75$ m.

The stress increase at a depth d_c can be found using Equation 1.19

$$\Delta p = NqI, \tag{1.19}$$

where N and I are the number of elements of the Newmark’s chart covered by the scaled footing and I is the influence factor of the diagram. For the chart shown in Figure 3.16, $I = 0.001$. If the footing were to behave as a flexible footing, the center settlement would be the maximum, whereas the corner settlement would be the minimum within the footing. Thus,

$$\Delta p_{\text{center}} = (4 \times 19) \times 88.89 \times 0.001 = 6.75 \text{ kPa}$$

$$\Delta p_{\text{conner}} = (58) \times 88.89 \times 0.001 = 5.2 \text{ kPa}$$

On the other hand, if the footing were to behave as a rigid footing, then the average stress increase at the mid-plane level of the clay layer within the footing can be determined by using an appropriate stress attenuation (Figure 3.13). Using the commonplace 2:1 stress attenuation (Equation 3.28), one can estimate the stress increase as:

$$\Delta p = q \left[\frac{BL}{(B + d_c)(L + d_c)} \right]$$

where B and L are the footing dimensions.

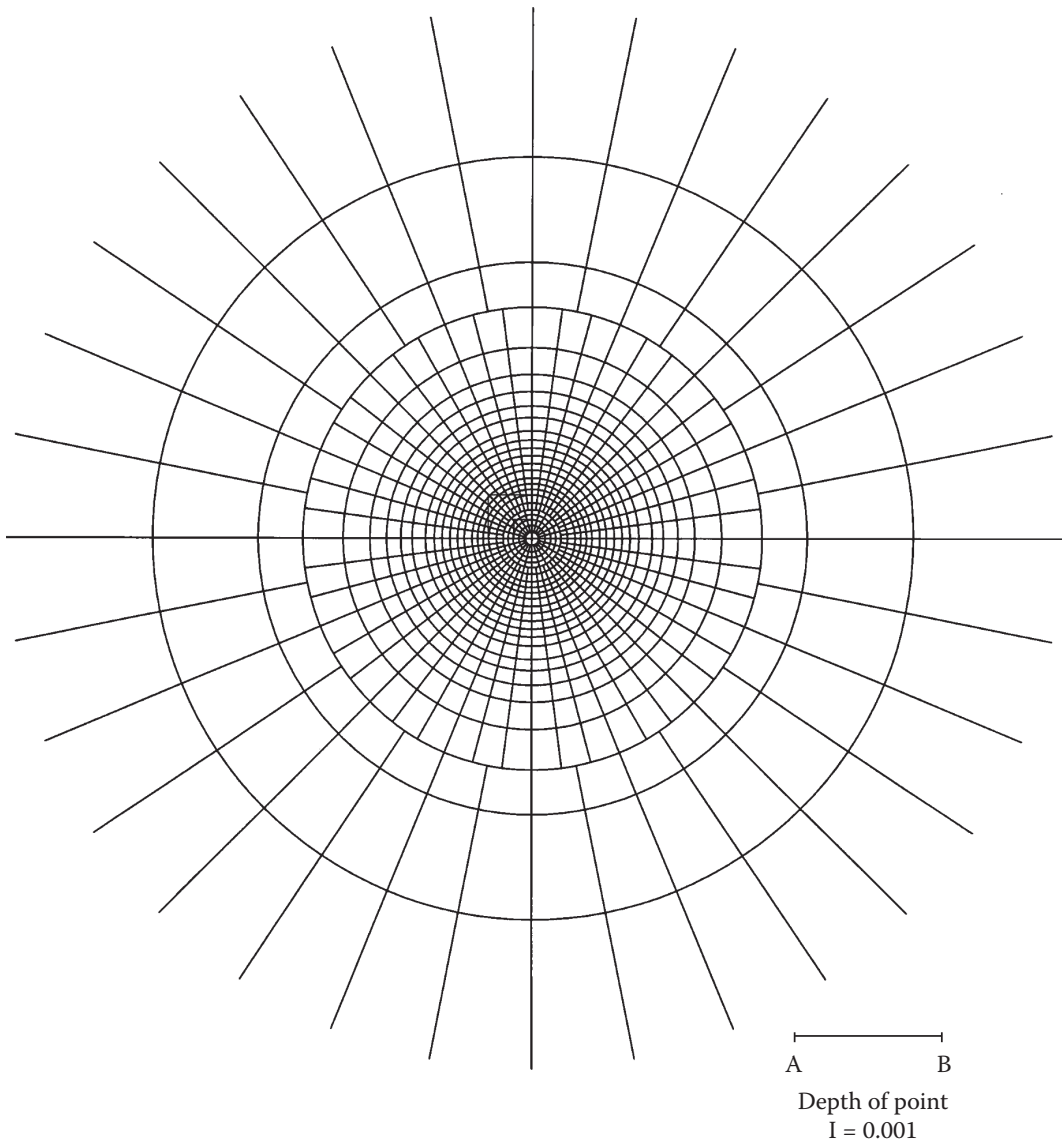


FIGURE 3.16

Use of Newmark's chart in Example 3.3. (From Holtz, R.D., Kovacz, W.D., *An Introduction to Geotechnical Engineering*, Prentice Hall, Englewood Cliffs, NJ, 1981.)

Thus,

$$\Delta p_{\text{average}} = 88.89[1.5/(1.5 + 3.75)]^2 = 7.256 \text{ kPa}$$

It must be noted that if one were to have averaged the above stress estimates for the center and corner of the footing, one would have obtained,

$$\Delta p_{\text{average}} = \frac{1}{2}(6.75 + 5.2) = 5.975 \text{ kPa}$$

Because the estimates are significantly different, the author suggests using the averages of the estimates in Figure 3.15 as opposed to the approximate estimate obtained from Figure 3.13. The average effective overburden pressure at the mid-plane of the clay layer is found from Equation 1.4b as

$$\sigma'_{vo} = 16.5(2) + 17.5(1.5) + 18.0(1.25) - 9.8(2.75) = 54.8 \text{ kPa}$$

Since $\sigma'_{vo} < p_c$, one can assume that the overall clay layer is in an overconsolidated state.

Ultimate Settlement beneath the Center of (Flexible) Footing

The following expression can be used to estimate the ultimate consolidation settlement since $\sigma'_{vo} + \Delta p_{center} > p_c$ (Figures 1.20c and 3.15)

$$s_{center} = \left[\frac{H}{1 + e_0} \right] \left[C_r \log \frac{p_c}{\sigma'_{vo}} + C_c \log \frac{\sigma'_{vo} + \Delta p}{p_c} \right]$$

$$s_{center} = \left[\frac{2.5}{1 + 1.06} \right] \left[0.064 \log \frac{60}{54.8} + 0.382 \log \frac{54.8 + 6.75}{60} \right] = 8.19 \text{ mm}$$

Ultimate Settlement beneath the Corner of (Flexible) Footing

The following expression can be used to estimate the ultimate consolidation settlement since $\sigma'_{vo} + \Delta p_{corner} < p_c$ (Figures 1.20b and 3.15):

$$s_{center} = \left[\frac{H}{1 + e_0} \right] \left[C_r \log \frac{\sigma'_{vo} + \Delta p}{\sigma'_{vo}} \right]$$

$$s_{center} = \left[\frac{2.5}{1 + 1.06} \right] \left[0.064 \log \frac{54.8 + 5.2}{54.8} \right] = 3.06 \text{ mm}$$

Average Ultimate Settlement of Footing (Rigid)

The following expression can be used to estimate the average ultimate consolidation settlement since $\sigma'_{vo} + \Delta p_{average} > p_c$ (Figures 1.20c and 3.15)

$$s_{average} = \left[\frac{H}{1 + e_0} \right] \left[C_r \log \frac{p_c}{\sigma'_{vo}} + C_c \log \frac{\sigma'_{vo} + \Delta p}{p_c} \right]$$

$$s_{average} = \left[\frac{2.5}{1 + 1.06} \right] \left[0.064 \log \frac{60}{54.8} + 0.382 \log \frac{54.8 + 5.975}{60} \right] = 5.64 \text{ mm}$$

Estimation of 10-Year Settlement

The settlement of the footing at any intermediate time (t) can be estimated by using the average degree of consolidation, U_{ave} , of the clay layer corresponding to the particular time t in combination with any one of the above ultimate settlement estimates.

$$s_t = U_{ave} s_{ult} \tag{3.29}$$

Using Terzaghi's theory of one dimensional (1-D) consolidation (Terzaghi, 1943), the average degree of consolidation at time t , U_{ave} can be determined from Table 1.8 knowing the

Time Factor (T) corresponding to the time t . T can be determined using the following expression:

$$T = \frac{C_v t}{H_{dr}^2}, \tag{1.16}$$

where H_{dr} is the longest path accessible to draining pore water in the clay layer. In Figure 3.14, one can see that, for this example, $H_{dr} = 2.5$ m.

Then,

$$T = \frac{10^{-8}(10 \times 365 \times 24 \times 60 \times 60)}{2.5^2} = 0.504$$

From Table 1.8,

$$U_{ave} = 0.77$$

Example 3.5

Assume that it is necessary to compute the ultimate total differential settlement of the foundation shown in Figure 3.14, for which the strain influence factor plot is shown in Figure 3.17. The average CPT values for the three layers are given in Table 3.9.

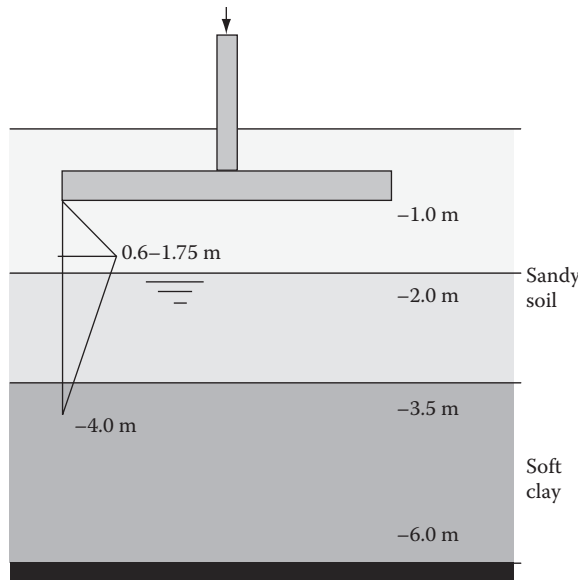


FIGURE 3.17
Immediate settlement computation.

TABLE 3.9

Soil Properties Used in Example 3.5

Soil Type	q_c (MPa)	E_s MPa
Dry sand	2.875	11.5 ($E_s = 4q_c$ from Table 1.7)
Wet sand	2.675	10.7
Clay	5	10 ($E_s = 2q_c$ from Table 1.7)

Solution

For the above data,

$$\text{Contact pressure } (\Delta\sigma) = 200/(1.5)^2 \text{ kPa} = 88.89 \text{ kPa}$$

$$\text{Overburden pressure at footing depth } (q) = 16.5 \times 1.0 \text{ kPa} = 16.5 \text{ kPa}$$

Immediate Settlement. Areas of the strain-influence diagram covered by different elastic moduli are

$$A_1 = 0.5(0.1 \times 0.75) + 0.5(0.75 \times 0.6) + 0.5(0.25)(0.533 + 0.6) = 0.41 \text{ m}$$

$$A_2 = 0.5(1.5)(0.533 + 0.133) = 0.5 \text{ m}$$

$$A_3 = 0.5(0.5)(0.133) = 0.033 \text{ m}$$

Then, by applying Equation 1.13, one obtains the immediate settlement as

$$s_{\text{center}} = \{1 - 0.5[16.5/(88.89 - 16.5)]\}[1.0][88.89 - 16.5][0.41(1.0)/(11.5 \times 10^3) + 0.5/(10.7 \times 10^3) + 0.033/(2.57 \times 10^3)] = 5.9 \text{ mm}$$

From Equation 1.12, s_{corner} can be deduced as $0.5(5.87) = 2.95 \text{ mm}$.

Therefore, the total settlement at the center of the footing will be 14.06 (= 8.19 + 5.87) mm or 0.55 in, whereas that at the corner will be 6.0 (3.06 + 2.94) mm or 0.24 in.

Total Settlement Check. Most building codes stipulate the maximum allowable total settlement to be 1.0 in. Hence, the above value is acceptable.

Differential Settlement Check. The differential settlement is equal to

$$(s_{\text{center}} - s_{\text{corner}})/\text{distance from center to corner}$$

$$\text{or } (14.00 - 6.00)/(1.06)/1000 = 0.007.$$

According to most building codes, the maximum allowable differential settlement to prevent structural cracks in concrete is 0.013. Hence, the differential settlement criterion is also satisfied.

3.3.2 Settlement Computation Based on Plate Load Test Data

The immediate settlement of a shallow footing can be determined from a plate load test performed at the same location and the depth that the footing would be constructed. For the same magnitude in the contact stress level, settlement of the foundation can be estimated based on the settlement of the plate and the following expressions:

Clayey soils

$$s_f = s_p \frac{B_f}{B_p} \tag{3.30}$$

Sandy soils

$$s_f = s_p \left(\frac{2B_f}{B_f + B_p} \right)^2, \tag{3.31}$$

where B_p is the plate diameter and B_f is the equivalent foundation diameter, which can be determined as the diameter of a circle having an area equal to that of the footing.

3.3.3 Computation of Settlement in Organic Soils

Foundations constructed in organic soils exhibit prolonged settlement due to secondary compression that is relatively larger in magnitude than the primary consolidation. This is particularly the case when the organic content of the soil deposit is significant. Therefore, foundation designers, who do not recommend the removal of organic soils from potential building sites, must alternatively use specific analytical techniques to estimate the expected secondary compression component that predominates the total settlement of the foundation. The following analytical treatise is presented to address this need.

The organic content of a soil (oc) is defined as

$$\text{oc} = \frac{W_O}{W_S} \times 100 (\%) \quad (3.32)$$

where

W_O = weight of organic matter in the soil sample (usually determined based on the loss of weight of the sample on combustion)

W_S = total weight of the solids in the soil sample

Many researchers (Andersland and Al-Khafaji, 1980; Gunaratne et al., 1998) have discovered linear relationships between the organic content of organic soils and their initial void ratios and water contents. Gunaratne et al. (1998) determined the following specific relationships for Florida organic soils, based on an extensive laboratory testing program:

$$e_{\infty} = 0.46 + 1.55(\text{oc}) \quad (3.33)$$

$$\text{oc} = w * 0.136 + 2.031 \quad (3.34)$$

where e_{∞} and w are the ultimate void ratio and the water content, respectively.

The ultimate 1-D compressibility of organic soils (vertical strain per unit load increment) constitutes a primary compressibility component, a , and a secondary compressibility component, b , as expressed below.

$$\epsilon_{\text{ult}} = \Delta\sigma [a + b] \quad (3.35)$$

The a and b parameters specific to any organic soil can be expressed in terms of the primary and secondary void ratio components (e_p and e_s , respectively) of the initial void ratio, e_0 , as illustrated in Equations 3.36a and 3.36b

$$a = -\frac{1}{(1+e)} \frac{\partial e_p}{\partial \sigma} \quad (3.36a)$$

$$b = -\frac{1}{(1+e)} \frac{\partial e_s}{\partial \sigma} \quad (3.36b)$$

Based on observed linear relationships such as that in Equation 3.33, Gunaratne et al. (1998) also determined that

$$a = -\frac{1}{(1+e)} \left[\frac{d}{d\sigma} \overline{I_p(\sigma)} + oc \frac{d}{d\sigma} \overline{M_p(\sigma)} \right] \tag{3.37a}$$

$$b = -\frac{1}{(1+e)} \left[\frac{d}{d\sigma} \overline{I_s(\sigma)} + oc \frac{d}{d\sigma} \overline{M_s(\sigma)} \right], \tag{3.37b}$$

where $[\overline{I_p(\sigma)}, \overline{M_p(\sigma)}]$ and $[\overline{I_s(\sigma)}, \overline{M_s(\sigma)}]$ are stress-dependent functions associated with primary and secondary compressibilities, respectively. Finally, by using Equations 3.37a and 3.37b, Gunaratne et al. (1998) derived the following specific relationships for Florida organic soils:

$$a = \frac{97.79}{(1.27\sigma + 97.79)^2} + \frac{23.13 oc}{(0.16\sigma + 23.13)^2} \tag{3.38a}$$

$$b = \frac{360.17}{(1.86\sigma + 360.17)^2} + \frac{40.61 oc}{(0.52\sigma + 40.61)^2} \tag{3.38b}$$

where $F(oc, \sigma) = \left[2.79 - \frac{\sigma}{(0.78\sigma + 74.28)} \right] + oc \left[9.72 - \frac{\sigma}{(0.12\sigma + 15.33)} \right]$

The a and b parameters specific for a field organic soil deposit would be dependent on the depth of location, z , because of their strong stress dependency.

The vertical strain in a layer of thickness, Δz , can be expressed in terms of its total (primary and secondary) 1-D settlement, Δs_{p+s} , as in Equation 3.39

$$\epsilon_{ult} = \frac{\Delta s_{p+s}}{\Delta z} \tag{3.39}$$

Hence, the total 1-D settlement can be determined as

$$s_{p+s} = \int_0^H [a(z) + b(z)] (\Delta \sigma_z) dz, \tag{3.40}$$

where $a(z)$ and $b(z)$ are a and b parameters in Equations 3.38a and 3.38b expressed in terms of the average current stress [initial overburden stress, $\sigma'_{vo} + 1/2$ stress increment, $\Delta \sigma_z$, produced due to the footing at depth z]. $\Delta \sigma_z$ can be determined using the Bousinesq's distribution (Equation 3.27) or any other appropriate stress attenuation such as the 2:1 distribution (Equation 3.28) commonly used in foundation design.

Because of the complex nature of a and b functions (Equations 3.38a and 3.38b), one can numerically integrate Equation 3.40 to estimate the total settlement of an organic soil layer due to a finite stress increment imposed by a foundation.

Example 3.6

Assume that, based on laboratory consolidation tests, one wishes to predict the ultimate 1-D settlement expected in a 1-m-thick organic soil layer ($oc = 50\%$) and the current overburden pressure of 50 kPa due to an extensively placed surcharge of 50 kPa.

Solution

Because there is no significant stress attenuation within 1 m due to an extensive surcharge, the final pressure would be

$$\sigma_v + \Delta\sigma_z = 50 + 50 = 100 \text{ kPa}$$

throughout the organic layer. Then by applying Equation 3.35

$$\epsilon_{ult} = \int_{50}^{100} [a(\sigma) + b(\sigma)](d\sigma),$$

where $a(\sigma)$ and $b(\sigma)$ are obtained from Equation 3.38 using an oc and σ values of 0.5 and 50 kPa, respectively.

Finally, on the performing the integration numerically, one obtains primary and secondary compressions of 0.107 and 0.041 m, which produces a total settlement of 0.148 m.

Fox and Edil (1992) used the C_α/C_c concept to predict the secondary settlement of organic soils.

3.4 Load and Resistance Factor Design Criteria

The two design philosophies commonly used in design of foundations are

1. Allowable stress design (ASD)
2. LRFD

Of the above, the more popular and historically successful design philosophy is the ASD, which has been adopted in this chapter so far. ASD can be summarized by the following generalized expression:

$$R_n / FS \geq \sum Q_i \quad (3.41)$$

where

- R_n = nominal resistance
- Q_i = load effect
- FS = factor of safety

The main disadvantages of the ASD methods are (1) factor of safety (FS) is applied only to the resistance part of the equation without heeding the fact that varying types of loads have different levels of uncertainty, (2) FS is only based on judgment and experience, and (3) no quantitative measure of risk is incorporated in FS.

The design of spread footings using LRFD requires evaluation of the footing performance at various Limit States. The primary Limit States for spread footing design include strength limits such as bearing capacity failure or sliding failure and service limits such as excessive settlements or vibration.

The goal of LRFD is to design, without being conservative as to be wasteful of resources, a foundation that serves its function without reaching the *Limit States*.

3.4.1 LRFD Philosophy

LRFD-based evaluation of strength limit state can be summarized as

$$\phi R_n \geq \eta \sum \gamma_i Q_i \quad (3.42)$$

where

- ϕ = resistance factor
- γ_i = load factors
- η = load modifier

Load factors account for the uncertainties in magnitude and direction of loads, location of application of loads, and combinations of loads.

On the other hand, resistance factors can be made to incorporate variability of soil properties, reliability of predictive equations, quality control of construction, extent of soil exploration, and even the consequences of failure. The main advantages of LRFD are that it accounts for variability in both resistance and loads and provides a qualitative measure of risk related to the probability of failure. However, LRFD also has the limitation of not facilitating the selection of appropriate resistance factors to suit the design of different foundation types. The LRFD-based evaluation of service limit state can be described by the same Equation 3.1b in Section 3.2.

Three different methods are adopted to select the resistance and load factors (FHWA, 1998):

1. Calibration by judgment requires extensive experience
2. Calibration by fitting to ASD
3. Calibration by the theory of reliability

The procedure used for the selection of load and resistance factors is known as the calibration of LRFD. The two latter procedures will be discussed in this chapter.

3.4.2 Calibration by Fitting to ASD

Using Equations 3.41 and 3.42 and assuming $\eta = 1.0$

$$\phi = \frac{\gamma_D Q_D + \gamma_L Q_L}{FS(Q_D + Q_L)} \quad (3.43)$$

where Q_D denotes dead load and Q_L denotes live load.

If one assumes a dead load/live load ratio ($= Q_D/Q_L$) of 3.0, $FS = 2.5$, and load factors of $\gamma_D = 1.25$ and $\gamma_L = 1.75$, then

$$\phi = \frac{1.25(3.0) + 1.75(1.0)}{2.5(3.0 + 1.0)} = 0.55$$

Hence, the resistance factor, ϕ , corresponding to an ASD safety factor of 2.5 and a dead/live load ratio of 3 is 0.55. Similarly, one can estimate the ϕ values corresponding to other FS and Q_D/Q_L values as well.

3.4.3 Calibration by Reliability Theory

In the LRFD calibration using the theory of reliability, the foundation resistance and the loads are considered random variables. Therefore, the resistance and the loads are incorporated in the design using their statistical distributions. The statistical concepts relevant to the calibration procedure are discussed in the next section.

3.4.3.1 Variability of Soil Data

A quantitative measure of the variability of site soil can be provided by the coefficient of variation (COV) of a given soil property, X , defined as follows:

$$\text{COV}(x) = \frac{\sigma}{\mu}, \quad (3.44)$$

where

μ = mean of the entire population of X at the site

σ = standard deviation of the entire population of X at the site

However, both μ and σ can be estimated by their respective sample counterparts \bar{x} and s obtained from an unbiased finite sample of data (on X) of size n , obtained at the same site using the following expressions:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad (3.45)$$

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1} \quad (3.46)$$

Using data from Teng et al. (1992) (Figure 3.18), it can be illustrated how the sample standard deviation is related to the population standard deviation. Figure 3.18 shows that in the estimation of the undrained shear strength (S_u) of clay at a particular site using three different methods: (1) CPT, (2) vane shear test (VST), and (3) preconsolidation pressure

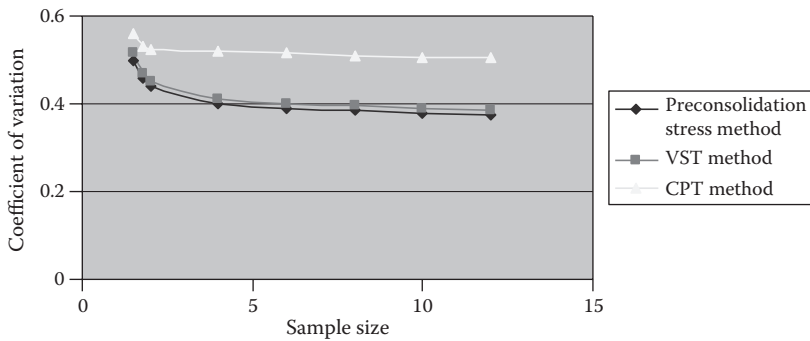


FIGURE 3.18 Reliability variation of undrained strength prediction with sample size. (From Teng, W.H. et al., *Soils Found.*, 32, 4, 107–116, 1992.)

(σ'_p) based on the laboratory consolidation tests. Figure 3.18 shows that, in each case, the estimation can be improved by increasing the sample size up to an optimum size of about 7. The corresponding standard deviation estimate can be possibly interpreted as the population standard deviation. However, the best estimate of the standard deviation that one can make varies with the specific technique used in the estimation. Moreover, Figure 3.18 also shows that, based on the laboratory prediction method, VST provides a much more accurate estimate of the “true” standard deviation of the undrained shear strength (S_u) of a clayey site soil. Alternatively, the information contained in research findings such as that shown in Figure 3.18 can be used in planning subsurface investigations. Intuitively, one also realizes that the standard deviation estimates obtained from a given evaluation method correlate well with reliability of the evaluation method, that is, a relatively higher standard deviation indicates a less reliable evaluation method.

The typical variability associated with soil index tests and strength tests as reported by Phoon et al. (1995) are shown in Tables 3.10 and 3.11, respectively. For analytical purposes, one can describe a random variable completely using an appropriate probability density function (in the case of a continuous random variable) or probability mass function (in the case of a discrete random variable) that satisfies the statistics of that particular random

TABLE 3.10

Soil Variability in Index Tests

Property	Soil Type	Inherent Soil Variability (COV)	Measurement Variability (COV)
Natural water content	Fine grained	0.18	0.08
Liquid limit	Fine grained	0.18	0.07
Plastic limit	Fine grained	0.16	0.1
Plasticity index	Fine grained	0.29	0.24
Bulk density	Fine grained	0.09	0.01
Dry density	Fine grained	0.07	—
Relative density—direct	Sand	0.19	—
Relative density—indirect	Sand	0.61	—

Source: Phoon, K. et al., Reliability-based design of foundations for transmission line structures, TR 105000 Final report, report prepared by Cornell University for the Electric Power Research Institute, Palo Alto, CA, 1995. With permission.

TABLE 3.11

Soil Variability in Strength Tests

Property	Soil Type	Inherent Soil Variability (COV)	Measurement Variability (COV)
Undrained strength (unconfined compression testing)	Fine grained	0.33	—
Undrained strength (unconsolidated undrained triaxial testing)	Clay, silt	0.22	—
Undrained strength (preconsolidation pressure from consolidated undrained triaxial testing)	Clay	0.32	0.19
Tan ϕ (triaxial compression)	Clay, silt	0.20	—
	Sand, silt	—	0.08
Tan ϕ (direct shear)	Clay, silt	0.23	—
	Clay	—	0.14

Source: Phoon, K. et al., Reliability-based design of foundations for transmission line structures, TR 105000 Final report, report prepared by Cornell University for the Electric Power Research Institute, Palo Alto, CA, 1995. With permission.

variable. The distribution that satisfies all the statistical properties of the random variable would obviously be its own histogram. However, what is assumed in many instances is a mathematical function that would closely “model” the statistical properties of the considered random variable. When selecting an appropriate mathematical distribution for a given variable, it is most common to match only the mean and the standard deviation of that variable with the corresponding quantities that are computed using the mathematical equation of the considered distribution as follows:

$$\mu = E(x) = \int_{-\infty}^{\infty} xf(x)dx = \text{moment of area of the distribution about the origin of the } X \text{ axis} = \text{centroidal location} \tag{3.47}$$

$E(x)$ = expected value of x

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx = \text{second moment of area of the distribution about the centroidal location (mean)} \tag{3.48}$$

The following statistical relations can be derived between two different random variables, a and b

$$E(a + b) = E(a) + E(b), \tag{3.49a}$$

where E indicates the expected value or the mean.

$$\sigma^2(a + b) = \sigma^2(a) + \sigma^2(b) \tag{3.49b}$$

Two very commonly used distributions that merely satisfy the above-mentioned mean and the standard deviation criteria (Equations 3.47 and 3.48) only are the normal and the lognormal distributions. However, in the case of a given variable, if the analyst is forced to select a probability distribution that would represent the random variation of that variable more accurately, then in addition to the mean and the standard deviation estimates, one could also compute the coefficients of skewness and kurtosis (flatness) computed from the sample data (Harr, 1977). It must be noted that the coefficients of skewness and kurtosis for the population can be related to the third and the fourth moments of the area of the probability distribution about the mean, respectively.

3.4.3.2 Normal (Gaussian) Distribution

If a continuous random variable X is normally distributed, its probability density function is given by Equation 3.49

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left\{\frac{x-\mu}{\sigma}\right\}^2\right] \tag{3.50}$$

It can be shown that Equation 3.50 automatically satisfies the conditions imposed by Equations 3.47 and 3.48.

3.4.3.3 Lognormal Distribution

If a continuous random variable X is lognormally distributed, then the natural logarithm of x , $\ln(x)$, is normally distributed and its probability density function is given by Equation 3.51a

$$f(x) = \frac{1}{\xi\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left\{\frac{\ln(x)-\lambda}{\xi}\right\}^2\right] \tag{3.51a}$$

where λ and ξ are the mean and the standard deviation of $\ln(x)$, respectively. The statistics of $\ln(x)$ can be expressed by those of x as

$$\lambda = \ln\left[\frac{\mu}{\sqrt{1+\text{COV}^2(X)}}\right] \tag{3.51b}$$

and

$$\xi = \sqrt{\ln(1+\text{COV}^2(X))} \tag{3.51c}$$

Furthermore, it can be shown that when the random variable X exhibits a variation within a relatively minor range, that is, when the $\text{COV}(X)$ is relatively small (<0.2), the above expressions simplify to

$$\lambda = \ln(\mu) \tag{3.51d}$$

and

$$\xi = \text{COV}(X) \quad (3.51e)$$

3.4.3.4 Estimation of Probabilities

A primary use of mathematically expressed probability distributions such as the normal or the lognormal distribution is the convenience that such a distribution provides in the computation of probability estimates. Similar computations also significantly enhance the assessment of reliability estimates in the design procedures that incorporate the random characteristics of loads applied on earthen structures and the relevant geotechnical parameters of the foundation soil. Accordingly, if X is a random variable that assumes values in the range of $[a, b]$, then the probability of finding values of X less than c ($< b$) can be expressed in terms of its probability distribution as

$$P(X < c) = \int_a^c f(x) dx \quad (3.52)$$

3.4.3.5 Reliability of Design

If the effect of a load applied on a substructure such as a foundation, and the resistance provided by the shear strength of the foundation soil are expressed in terms of random variables Q and R , respectively, then the reliability of the design can be expressed as

$$Re = P(R \geq Q) \quad (3.53)$$

In order to compute the reliability of a design that involves randomly distributed load effects, Q , and soil resistance, R , it is convenient to express the interaction between R and Q in terms of the combined random variable $g(R, Q) = (R - Q)$.

The Central axis theorem of statistics (Harr, 1977) states that, if both R and Q are normally distributed, that is, normal variates, then $g(R, Q)$ would be normally distributed as well. Therefore, it follows that, if both R and Q are lognormally distributed, that is, lognormal variates, then $g'(R, Q) = \ln R - \ln Q$ would be *normally distributed* as well.

Based on Equation 3.49, $g'(R, Q)$ would have the following characteristics:

$$\text{mean}[g'(R, Q)] = \text{mean}(\ln R) - \text{mean}(\ln Q) \quad (3.54a)$$

$$\text{standard deviation } [g'(R, Q)] = \sqrt{\sigma_{\ln(R)}^2 + \sigma_{\ln(Q)}^2} \quad (3.54b)$$

Using Equation 3.51b,

$$\text{mean}[g'(R, Q)] = \ln \left[\frac{\bar{R}}{\sqrt{(1 + \text{COV}_R^2)}} \right] - \ln \left[\frac{\bar{Q}}{\sqrt{(1 + \text{COV}_Q^2)}} \right]$$

$$\mu = \ln \left[\frac{\bar{R}}{\bar{Q}} \frac{\sqrt{(1 + \text{COV}_Q^2)}}{\sqrt{(1 + \text{COV}_R^2)}} \right] \tag{3.55a}$$

Similarly, using Equations 3.51c

$$\text{Standard deviation } [g'(R, Q)] = \sqrt{[\ln(1 + \text{COV}_R^2)] + [\ln(1 + \text{COV}_Q^2)]}$$

$$\sigma = \sqrt{\ln \left[(1 + \text{COV}_R^2)(1 + \text{COV}_Q^2) \right]} \tag{3.55b}$$

Then, the reciprocal of the coefficient of variation of $g'(R, Q)$ can be expressed as

$$\beta = \frac{\mu}{\sigma} = \frac{\ln \left[\frac{\bar{R}}{\bar{Q}} \frac{\sqrt{(1 + \text{COV}_Q^2)}}{\sqrt{(1 + \text{COV}_R^2)}} \right]}{\sqrt{\ln \left[(1 + \text{COV}_R^2)(1 + \text{COV}_Q^2) \right]}} \tag{3.55c}$$

If one expresses the mathematical expression for the normal distribution (Equation 3.50) in terms of the standard normal variate z , where

$$R - Q = X \tag{3.56a}$$

$$z = \frac{x - \mu}{\sigma}, \tag{3.56b}$$

then Equation 3.50 simplifies to

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} z^2 \right] \tag{3.57}$$

Then, from the differential form of Equation 3.56b,

$$\sigma(dz) = dx \tag{3.58}$$

Therefore, the estimation of probability in Equation 3.52 would be simplified as follows

$$P(X < c) = \int_{-\infty}^c f(x) dx$$

$$P(X < c) = P \left(Z < \frac{c - \mu}{\sigma} \right) = \int_{-\infty}^{\frac{c - \mu}{\sigma}} f(z) dz \tag{3.59}$$

Substituting from Equations 3.57 and 3.58,

$$\begin{aligned}
 P(X < c) &= \int_{-\infty}^{\frac{c-\mu}{\sigma}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}z^2\right] \sigma(dz) \\
 &= \int_{-\infty}^{\frac{c-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}z^2\right] dz \\
 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{c-\mu}{\sigma}} \exp\left[-\frac{1}{2}z^2\right] dz
 \end{aligned} \tag{3.60}$$

3.4.3.6 Reliability Index

The reliability of the design can be computed using Equations 3.56a and 3.52 as

$$Re = P(R \geq Q) = P(R - Q \geq 0) = 1 - P[(R - Q) < 0] = 1 - P(X < 0)$$

Then, setting the arbitrary value $c = 0$ in Equation 3.60,

$$\begin{aligned}
 P(X < 0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{-\mu}{\sigma}} \exp\left[-\frac{1}{2}z^2\right] dz \\
 1 - Re &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{-\mu}{\sigma}} \exp\left[-\frac{1}{2}z^2\right] dz
 \end{aligned}$$

Since $\beta = \mu/\sigma$ (Equation 3.55c),

$$1 - Re = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\beta} \exp\left[-\frac{1}{2}z^2\right] dz$$

If one defines the above integral in terms of the error function (erf), which is conveniently tabulated in the standard normal distribution tables, as follows:

$$F(-\beta) = \frac{1}{\sqrt{2\pi}} \operatorname{erf}(-\beta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\beta} \exp\left[-\frac{1}{2}z^2\right] dz \tag{3.61a}$$

Then, the reliability of the design,

$$Re = 1 - F(-\beta), \tag{3.61b}$$

where the reliability index β is defined in terms of the load and resistance statistics in Equation 3.55c as

$$\beta = \frac{\ln \left[\frac{\bar{R} \sqrt{(1 + \text{COV}_Q^2)}}{\bar{Q} \sqrt{(1 + \text{COV}_R^2)}} \right]}{\sqrt{\ln \left[(1 + \text{COV}_R^2)(1 + \text{COV}_Q^2) \right]}} \tag{3.55c}$$

3.4.3.7 Resistance Statistics

The measured resistance R_m can be expressed in terms of the predicted (nominal) resistance, R_n , as

$$R_m = \lambda_R R_n \tag{3.62}$$

where λ_R represents the bias factor for resistance. The bias factor includes the net effect of various sources of error such as the tendency of a particular method (e.g., Hansen’s bearing capacity) to underpredict foundation resistance, energy losses in the equipment in obtaining SPT blow counts, and soil borings in strata not being representative of the site. For n number of sources of error with individual factors affecting the strength of resistance prediction procedure, the mean bias factor can be expressed as follows:

$$\lambda_R = \lambda_1 \lambda_2 \dots \lambda_n \tag{3.63a}$$

Then based on the principles of statistics, the coefficient of variation of λ_R is given by

$$\text{COV}_R^2 = \text{COV}_1^2 + \text{COV}_2^2 + \dots + \text{COV}_n^2 \tag{3.63b}$$

Table 3.12 indicates the values recommended by FHWA (1998) for λ_R and COV_R .

TABLE 3.12

Resistance Statistics

Correction	Statistics for Correction Factors	
	λ_i	COV_i
Model error	1.3	0.5
Equipment/procedure used in SPT	1.0	0.15–0.45 (use 0.3)
Inherent spatial variability	1.0	$(0.44/L)^{0.5}$

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

Note: L = length of pile.

TABLE 3.13

Bias Factors and Coefficients of Variation for Bridge Foundation Dead Loads

Component	λ_{QD}	COV_{QD}
Factory made	1.03	0.08
Cast-in-place	1.05	0.10
Asphaltic wearing surface	1.00	0.25
Live load	(1.15 = λ_{QL})	(0.18 = COV_{QL})

Source: Federal Highway Administration, 1998, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

3.4.3.8 Load Statistics

Similarly for the measured load, one can write

$$Q_m = \lambda_{QD}Q_D + \lambda_{QL}Q_L \quad (3.64)$$

where the load bias factor includes various uncertainties associated with dead and live loads. λ_{QD} values for commonplace materials are found in Table 3.13. On the other hand, AASHTO (American Association for State Highway and Transportation Officials) LRFD live load model specifies $\lambda_{QL} = 1.15$ and $COV_{QL} = 0.18$ for the live loads. If there are two significant sources of bias for dead loads in a given design situation, then from Equations 3.63a and 3.63b

$$\lambda_{QD} = \lambda_1 \lambda_2 \dots \lambda_i \quad (3.65a)$$

$$COV_{QD}^2 = COV_1^2 + COV_2^2 + \dots + COV_n^2 \quad (3.65b)$$

3.4.3.9 Determination of Resistance Factors

By rearranging Equation 3.55c, one obtains

$$\bar{R} = \bar{Q} \exp \left\{ \beta_T \sqrt{\ln \left[(1 + COV_R^2)(1 + COV_Q^2) \right]} \right\} \frac{\sqrt{(1 + COV_R^2)}}{\sqrt{(1 + COV_Q^2)}} \quad (3.66a)$$

where $\bar{Q} = Q_m$.

Using Equation 3.62

$$\bar{R} = R_m = \lambda_n R_n \quad (3.66b)$$

From Equation 3.42

$$\phi_R R_n = \gamma Q_n = \gamma_D Q_D + \gamma_L Q_L \quad (3.66c)$$

By eliminating R_n from Equations 3.66b and 3.66c, and using the relation

$$COV_Q^2 = COV_{QD}^2 + COV_{QL}^2 \quad (3.66d)$$

the resistance factor can be derived as

$$\phi_R = \frac{\lambda_R[\gamma_D Q_D + \gamma_L Q_L] \sqrt{\frac{(1 + COV_{QD}^2 + COV_{QL}^2)}{(1 + COV_R^2)}}}{Q_m \exp\left\{\beta_T \sqrt{\ln\left[\frac{(1 + COV_{QD}^2 + COV_{QL}^2)(1 + COV_R^2)}{1}\right]}\right\}} \tag{3.67}$$

where

$$Q_m = \lambda_{QD} Q_D + \lambda_{QL} Q_L \tag{3.67a}$$

and β_T is the target reliability index evaluated from Table 3.14.

Finally, the resistance factors suggested by FHWA (1998) for SPT and CPT results based on selected reliability indices are provided in Table 3.15.

Table 3.16 outlines the suggested resistance factors for a variety of foundation strength prediction methods in common use.

TABLE 3.14

Relationship between Probability of Failure and Reliability Index for Lognormal Distribution

Reliability Index	Probability of Failure
2.0	0.85×10^{-1}
2.5	0.99×10^{-2}
3.0	1.15×10^{-3}
3.5	1.34×10^{-4}
4.0	1.56×10^{-5}
4.5	1.82×10^{-6}
5.0	2.12×10^{-7}
5.5	2.46×10^{-8}

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

TABLE 3.15

Bias Factors and Coefficients of Variation for Soil Strength Measurements

Test	λ_R	COV_R
SPT	1.3	0.6–0.8
CPT	1.0	0.4
Angle of friction (ϕ)	1.0	0.1
Cohesion	1.0	0.4
Wall friction (δ)	1.0	0.2
Earth pressure coefficient (K)	1.0	0.15

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

TABLE 3.16

Resistance Factors for Geotechnical Strength Limit State for Shallow Foundations

Method/Soil/Condition	Resistance Factor
<i>Sand</i>	
Semiempirical procedure using SPT data	0.45
Semiempirical procedure using CPT data	0.55
Rational method using shear strength (N) from	
SPT data	0.35
CPT data	0.45
<i>Clay</i>	
Semiempirical procedure using CPT data	0.5
Rational method using shear strength (S_u) from	
CPT data	0.5
Laboratory test (UU triaxial)	0.6
Field vane shear tests	0.6
<i>Rock</i>	
Semiempirical procedure	0.6
Plate load test	0.55

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

3.4.3.10 Determination of the Simplified Resistance Factor

The denominator of Equation 3.55c can be simplified as follows:

$$\ln \left[(1 + \text{COV}_R^2)(1 + \text{COV}_Q^2) \right] = \ln(1 + \text{COV}_R^2) + \ln(1 + \text{COV}_Q^2) \tag{3.68}$$

Using the Taylor series expansion, for relatively small values of COV (e.g., <0.3), Equation 3.68 can be written as

$$\ln \left[(1 + \text{COV}_R^2)(1 + \text{COV}_Q^2) \right] \approx \text{COV}_R^2 + \text{COV}_Q^2$$

Similarly, the numerator of Equation 3.55c can be simplified as

$$\ln \left[\frac{\bar{R}}{\bar{Q}} \frac{\sqrt{(1 + \text{COV}_Q^2)}}{\sqrt{(1 + \text{COV}_R^2)}} \right] = \ln \frac{\bar{R}}{\bar{Q}} + \ln \sqrt{(1 + \text{COV}_Q^2)} - \ln \sqrt{(1 + \text{COV}_R^2)}$$

For relatively small values of COV (e.g., <0.3), the above expression can be simplified to

$$\ln \frac{\bar{R}}{\bar{Q}} + \ln \sqrt{(1 + \text{COV}_Q^2)} - \ln \sqrt{(1 + \text{COV}_R^2)} \approx \ln \frac{\bar{R}}{\bar{Q}}$$

Hence, the expression for β can be simplified to

$$\beta = \frac{\ln \frac{\bar{R}}{\bar{Q}}}{\sqrt{\text{COV}_R^2 + \text{COV}_Q^2}} \tag{3.69}$$

By defining the α factor as follows

$$\text{COV}_R^2 + \text{COV}_Q^2 = \alpha(\text{COV}_R + \text{COV}_Q)$$

and rearranging terms in Equation 3.69, one obtains

$$\frac{\bar{R}}{\bar{Q}} = e^{\alpha\beta(\text{COV}_R + \text{COV}_Q)} \tag{3.70a}$$

Separately combining R and Q terms, one obtains

$$\bar{R}e^{-\alpha\beta\text{COV}_R} = \bar{Q}e^{\alpha\beta\text{COV}_Q} \tag{3.70b}$$

Using the definitions of the nominal resistance and load in Equations 3.62 and 3.64, respectively, one can obtain

$$\lambda_R R_n e^{-\alpha\beta\text{COV}_R} = \lambda_n Q_n e^{\alpha\beta\text{COV}_Q} \tag{3.70c}$$

Recalling Equation 3.42,

$$\phi R_n = \gamma Q_n \tag{3.42}$$

From Equation 3.70c, it is seen that the load and resistance factors, γ and ϕ , respectively, depend on the statistics of each other (COV_R and COV_Q) as well. However, for convenience, if one assumes that the resistance and load factors are independent of each other's statistics, then comparison of Equation 3.70c with Equation 3.42 yields a convenient and approximate method to express the resistance and load factors as follows:

$$\phi = \lambda_R e^{-\alpha\beta_T \text{COV}_R} \tag{3.70d}$$

$$\gamma = \lambda_n e^{\alpha\beta_T \text{COV}_Q} \tag{3.70e}$$

where β_T is the target reliability. Resistance factors corresponding to a target reliability of 3.5 are listed in Table 3.17.

Example 3.7

Estimate a suitable resistance factor for a bridge footing that is to be designed based on SPT tests.

Solution

From Table 3.13 and Equation 3.65,

$$\lambda_{QD} = 1.03(1.05)(1.00) = 1.08$$

$$COV_{QD} = \sqrt{(0.08^2 + (0.1)^2 + (0.25)^2)} = 0.289$$

From Table 3.15, for SPT, $\lambda_R = 1.3$, $COV_R = 0.7$.

Also, since it is recommended that $\lambda_{QL} = 1.15$, $COV_{QL} = 0.18$ (FHWA, 1998; Table 3.13) and assuming that $\gamma_L = 1.75$ and $\gamma_D = 1.25$ (Table 3.18).

TABLE 3.17

Resistance Factors for Semiempirical Evaluation of Bearing Capacity for Spread Footings on Sand Using Reliability-Based Calibration

Estimation Method	Factor of Safety, FS	Average Reliability Index, β	Target Reliability Index, β_T	Span (m)	Resistance Factor		
					Fitting with ASD	Reliability Based	Selected Φ
SPT	4.0	4.2	3.5	10	0.37	0.49	0.45
				50	0.37	0.53	0.45
CPT	2.5	3.2	3.5	10	0.60	0.52	0.55
				50	0.60	0.57	0.55

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

TABLE 3.18

Load Factors for Permanent Loads

Type of Load	Load Factor	
	Maximum	Minimum
Components and attachments	1.25	0.90
Downdrag	1.8	0.45
Wearing surfaces and utilities	1.5	0.65
Horizontal earth pressure		
Active	1.5	0.9
At rest	1.35	0.9
Vertical earth pressure		
Overall stability	1.35	N/A
Retaining structure	1.35	1.00
Rigid buried structure	1.30	0.90
Rigid frames	1.35	0.90
Flexible buried structure	1.95	0.90
Flexible metal box	1.50	0.90
Culverts		
Earth structures	1.50	0.75

Source: Federal Highway Administration, *Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures*, Washington, DC, 1998. With permission.

TABLE 3.19

Variation of Resistance Factor Φ_R with Q_D/Q_L and Required Reliability

$\frac{Q_D}{Q_L}$	Probability of Failure			
	0.085	0.0099	0.00115	0.000134
	$\beta = 2$	$\beta = 2.5$	$\beta = 3$	$\beta = 3.5$
1.0	0.554	0.432	0.336	0.262
2.0	0.529	0.412	0.321	0.2250
3.0	0.516	0.402	0.313	0.244

Applying Equation 3.67,

$$\phi_R = \frac{1.3 \left[1.25 \frac{Q_D}{Q_L} + 1.75 \right] \sqrt{\frac{(1 + (0.289)^2 + (0.18)^2)}{(1 + (0.7)^2)}}}{\left[1.08 \frac{Q_D}{Q_L} + 1.15 \right] \exp \left\{ \beta_T \sqrt{\ln[(1 + (0.289)^2 + (0.18)^2)((1 + (0.7)^2)]} \right\}}$$

$$\phi_R = \frac{1.13 \left[1.25 \frac{Q_D}{Q_L} + 1.75 \right]}{\left[1.08 \frac{Q_D}{Q_L} + 1.15 \right] \exp \{ 0.5 \beta_T \}}$$

Using Equation 3.67 and Table 3.14, the resistance factor can be expressed in terms of the probability of failure and the dead load/live load ratio (Table 3.19).

Example 3.8

For the column shown in Figure 3.19, use *LFRD concepts* to design a suitable footing to carry a column load of 400 kN. The subsoil can be considered as a homogenous

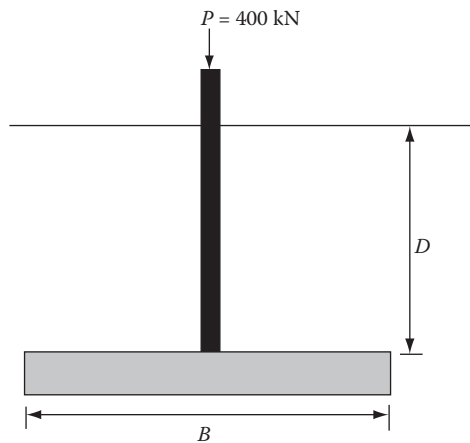


FIGURE 3.19
Illustration for Example 3.7.

silty clay with the following properties (assume that the groundwater table is not in the vicinity):

Unit weight (γ) = 17 kN/m³

Internal friction (ϕ) = 15°

Unit cohesion (c) = 20 kPa

Assume resistance factors ϕ_R of 0.6 and 0.6 (Table 3.16) for $\tan \phi$ and c , respectively.

$$\phi' = \tan^{-1} [0.6 \tan \phi] = 9^\circ$$

$$c' = 20(0.6) = 12 \text{ kPa}$$

Table 3.5 indicates the bearing capacity parameters
Using Hansen's bearing capacity expression (Equation 3.5),

$N_c = 8$	$N_q = 2$	$N_\gamma = 0.4$
$s_c = 1.359$	$s_q = 1.26$	$s_\gamma = 0.6$
$d_c = 1.4$	$d_q = 1.294$	$d_\gamma = 1.0$

The vertical effective stress at the footing base level (q) = (17)(depth) = 17B.

Then, the following expressions can be written for the ultimate bearing capacity:

$$\begin{aligned} q_{\text{ult}} &= (12)(8)(1.359)(1.4) + (17B)(2)(1.26)(1.294) + 0.5(17)(B)(0.4)(0.6)(1.0) \\ &= 182.65 + 57.47B \end{aligned}$$

Factored contact stress at the foundation level = $1.25 \times 4 \times 400/(\pi B^2) + (1.0)17B$.

The load factor for the dead load is obtained from Table 3.18. It must be noted that the recommended load factor for recompacted soil is 1.0.

By applying $\phi R_n = \eta \sum \gamma_i Q_i$ with no load modifier ($\eta = 1.0$)

$$q_{\text{ult}} = 1.25 \times 4 \times 400/(\pi B^2) + 17B$$

From Hansen's expression

$$637/B^2 + 17B = 182.65 + 57.47B$$

$$637/B^2 = 182.65 + 40.47B$$

$$B = 1.6 \text{ m}$$

When one compares the above footing width with $B = 1.55 \text{ m}$ obtained from the ASD method (Example 3.1), the limit state design is seen to be slightly more conservative.

3.5 Design of Footings to Withstand Vibrations

Foundations subjected to dynamic loads such as that due to operating machines, wave loadings, etc., have to satisfy special design criteria in addition to the regular bearing

capacity and settlement criteria. Table 3.20 lists a number of additional criteria that may be considered during the design of a foundation that would be subjected to vibrations. However, the main design criteria are related to the limiting amplitude of vibration and the limiting acceleration for a given operating frequency. Figure 3.20 indicates the order of magnitudes of vibration corresponding to different levels of severity.

For steady-state harmonic oscillations, the limiting accelerations can be deduced from the limiting amplitudes in terms of the frequency of oscillation (ω) as

$$\text{acceleration}_{\text{limit}} = \text{displacement}_{\text{limit}}\omega^2 \tag{3.71}$$

TABLE 3.20

List of Criteria for Design of Vibrating Footings

I. Functional considerations of installation
A. Modes of failure and the design objectives
B. Causes of failure
C. Total operational environment
D. Initial cost and its relation to item A
E. Cost of maintenance
F. Cost of replacement
II. Design considerations for installations in which the equipment produces exciting forces
A. Static bearing capacity
B. Static settlement
C. Bearing capacity: static + dynamic loads
D. Settlement: static + repeated dynamic loads
E. Limiting dynamic conditions
1. Vibration amplitude at operating frequency
2. Velocity
3. Acceleration
F. Possible modes of vibration—coupling effects
G. Fatigue failures
1. Machine components
2. Connections
3. Supporting structure
H. Environmental demands
1. Physiological effect on persons
2. Psychological effect on persons
3. Sensitive equipment nearby
4. Resonance of structural components
III. Design considerations for installation of sensitive equipment
A. Limiting displacement, velocity, or acceleration amplitudes
B. Ambient vibrations
C. Possible changes in ambient vibrations
1. By construction
2. By new equipment
D. Isolation of foundations
E. Local isolation of individual machines

Source: Richart, F.E. et al., *Vibrations of Soils and Foundations*, Prentice Hall, Englewood Cliffs, NJ, 1970. With permission.

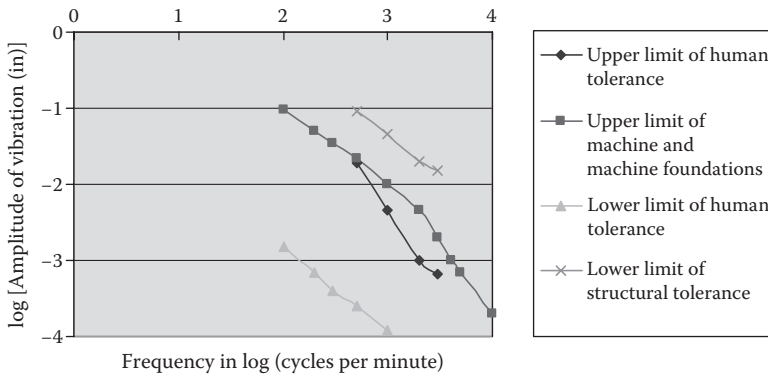


FIGURE 3.20

General limits of displacement amplitude for a particular frequency of vibration (note: 1 in = 25.4 mm). (From Richart, F.E. et al., *Vibrations of Soils and Foundations*, Prentice Hall, Englewood Cliffs, NJ, 1970. Reproduced by author.)

On the other hand, the maximum amplitudes (or accelerations) undergone by a given vibrating foundation can be determined by the principles of soil dynamics. Analytical formulations available from such analyses are provided in the ensuing sections for a number of different modes of vibration.

3.5.1 Vertical Steady-State Vibrations

The equation of motion for a rigid foundation of mass m subjected to a vertical steady-state constant amplitude simple harmonic force can be written as (Lysmer and Richart, 1966)

$$m\ddot{z} + c_z\dot{z} + k_z z = P = P_0 e^{i\omega t} = P_0 [\cos(\omega t) + i \sin(\omega t)] \tag{3.72}$$

If the foundation is circular, the spring and the damping constants are given by

$$k_z = \frac{2G_s B}{1 - \nu_s} \tag{3.73a}$$

and

$$c_z^2 = \frac{0.85B^2}{1 - \nu_s} \sqrt{G_s \rho_s} \tag{3.73b}$$

respectively, where B is the equivalent footing diameter, G_s , ρ_s , and ν_s denote the shear modulus, mass density, and Poisson's ratio of the foundation soil, respectively (Figure 3.21).

Then, the following important parameters that relate to the vibratory motion can be derived using the elementary theory of vibrations:

1. Natural frequency of vibration

$$f_n = \sqrt{1 - 2\nu_s^2} \frac{1}{2\pi} \sqrt{\frac{2G_s B}{1 - \nu_s} \frac{1}{m}} \tag{3.74a}$$

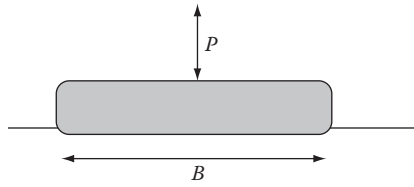


FIGURE 3.21
Footing subjected to vertical vibration.

2. Resonant frequency

For force-type excitation

$$f_r = \frac{1}{\pi B} \sqrt{\frac{G_s(B_z - 0.36)}{\rho_s B_z}} \text{ for } B_z > 0.3 \tag{3.74b}$$

where the modified dimensionless mass ratio B_z is given by

$$B_z = 2(1 - \nu_s) \frac{m}{\rho_s B^3} \tag{3.74c}$$

3. Damping ratio

Damping ratio = D = (damping constant)/(critical damping constant)

Critical damping constant = $2(k_z m)^{1/2}$

$$D = D_z = \frac{0.425}{\sqrt{B_z}} \tag{3.74d}$$

4. Amplitude of vibration

The amplitude of vibration can be expressed as follows:

$$A_z = \frac{P_0}{k_z} M = P_0 \frac{1 - \nu_s}{2G_s B} M, \tag{3.74e}$$

where M is the amplification factor, $\frac{A_z}{(P_0/k_z)}$, expressed in Equation 3.74f and plotted in Figure 3.22 against the nondimensional frequency or frequency ratio, ω/ω_n , and the damping ratio, D .

$$M = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2D \frac{\omega}{\omega_n}\right)^2}} \tag{3.74f}$$

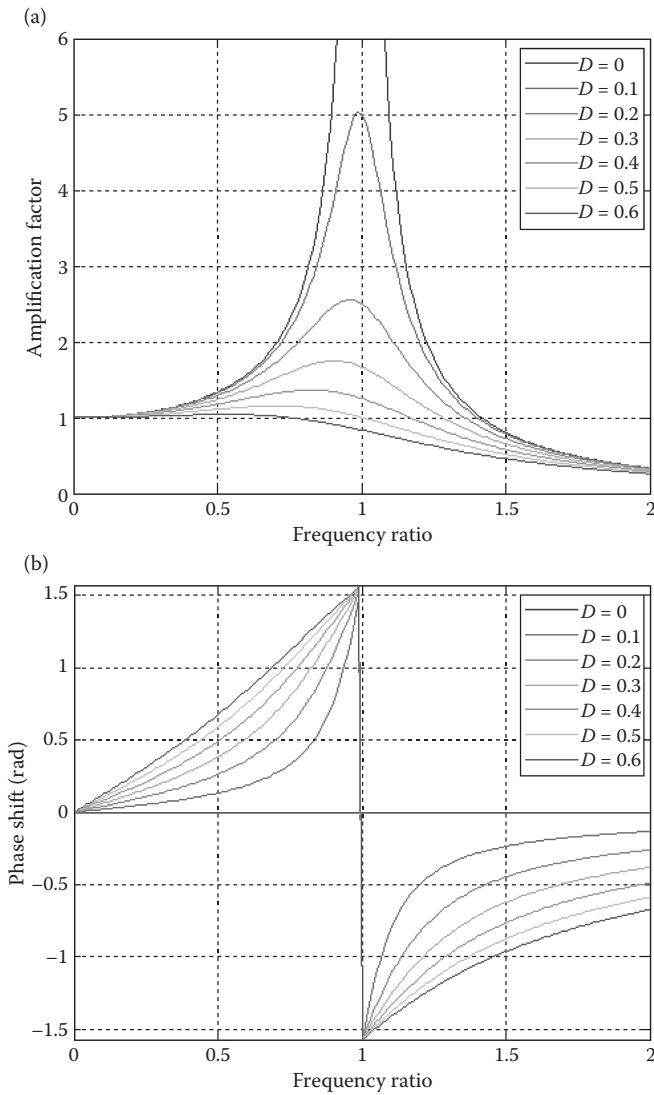


FIGURE 3.22 (a) Magnification factor (M) against frequency ratio (ω/ω_n). (b) Magnification factor (M) against frequency ratio (ω/ω_n).

where

$$\omega_n = 2\pi f_n = \sqrt{\frac{2G_s B}{1 - \nu_s} \frac{1}{m}} \tag{3.74a}$$

5. The phase lag ϕ can be determined from Equation 3.74g or Figure 3.22b.

$$\phi = \tan^{-1} \left(\frac{2\omega_n \omega D}{\omega_n^2 - \omega^2} \right) \tag{3.74g}$$

Example 3.9

A rigid circular concrete foundation supporting a machine is 4 m in diameter (Figure 3.23). The total weight of the machine and foundation is 700 kN. The machine imparts an equivalent vertical harmonic force of $25 \sin 20t$ kN on the footing. If the foundation soil is dense sand having the following properties; unit weight = 17 kN/m^3 and elastic modulus = 55 MPa . Determine (1) the resonant frequency, (2) amplitude of the vibration at the resonant frequency, and (3) amplitude of the vibration at the operating frequency.

Solution

For sandy soil, Poisson’s ratio can be assumed to be 0.33.

Hence, the shear modulus and the mass ratio can be computed as

$$G = \frac{E_s}{2(1 + \nu_s)} = (55)/2/1.33 = 20.7 \text{ MPa}$$

$$B_z = 2(1 - \nu_s) \frac{m}{\rho_s B^3} = 2(1 - 0.33)(700)(1000)/g(17)(1000/g/(4)^3) = 0.86$$

(Note that weight and the unit weight are used in the computation in place of the mass and mass density.)

(1) Resonant frequency

$$f_r = \frac{1}{\pi B} \sqrt{\frac{G_s(B_z - 0.36)}{\rho_s B_z}} = \frac{1}{\pi(4)} \sqrt{\frac{(20.7 \times 1,000,000)(0.86 - 0.36)}{(17 \times 1000/9.8)(0.86)}} = 6.63 \text{ cps}$$

(2) Natural frequency

$$f_n = \frac{1}{2\pi} \sqrt{\frac{2G_s B}{1 - \nu_s} \frac{1}{m}} = \frac{1}{2\pi} \sqrt{\frac{2(20.7)(1,000,000)(4)}{1 - 0.33} \frac{1}{700(1000)/(9.8)}} = 9.36 \text{ cps}$$

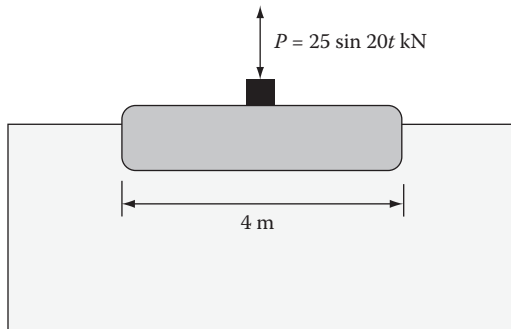


FIGURE 3.23
Illustration for Example 3.8.

(3) Operating frequency

$$f_o = 20/(2\pi) = 3.18 \text{ cps}$$

Hence, $f_m/f_o = 6.63/3.18 = 2.08 > 2$.

Thus, the operating frequency range is considered safe.

(4) Amplitude of vibration

$$f_m/f_n = 6.63/9.36 = 0.71$$

$$D = 0.425/B_z = 0.425/0.86 = 0.491$$

In Figure 3.21, the magnification factor, $M = 1.2$

$$A_z = \frac{P_0}{k_z} M = P_0 \frac{1 - \nu_s}{2G_s B} M = 25(1 - 0.33)(1.2)/(2 \times 20.7 \times 1000 \times 4) = 0.12 \text{ mm}$$

Based on an operating frequency of 3.18 cps or 191 cpm, the above amplitude of 0.27 mm or 0.011 in would fall below the upper limit of human tolerance in Figure 3.20.

3.5.2 Rocking Oscillations

The motion of a rigid foundation subjected to a steady-state constant amplitude harmonic rocking moment about the y axis, can be written as (Hall, 1967)

$$I_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = M_\theta e^{i\omega t} \tag{3.75a}$$

where

$$I_\theta = m \left(\frac{B^2}{16} + \frac{h^2}{3} \right) \tag{3.75b}$$

B and h are the diameter and height of the foundation, respectively (Figure 3.24).

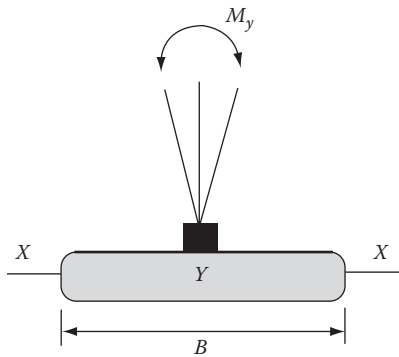


FIGURE 3.24
Footing subjected to rocking.

If the foundation is circular, the spring and the damping constants are given by

$$k_{\theta} = \frac{G_s B^3}{3(1-\nu_s)} \tag{3.76a}$$

and

$$c_{\theta} = \frac{0.05 B^4}{(1-\nu_s)(1+B_{\theta})} \sqrt{G_s \rho_s} \tag{3.76b}$$

respectively; B_{θ} , the inertia ratio, is given by

$$B_{\theta} = 12(1-\nu_s) \frac{I}{\rho B^5} \tag{3.76c}$$

Then, the following parameters relevant to the vibratory motion can be derived using the elementary theory of vibrations:

1. Natural frequency of vibration

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_{\theta}}{I_{\theta}}} \tag{3.77a}$$

2. Resonant frequency
Moment-type excitation

$$f_r = f_n \sqrt{1 - \frac{0.45}{B_{\theta}(1+B_{\theta})^2}} \tag{3.77b}$$

3. Damping ratio

$$D = D_{\theta} = \frac{0.15}{\sqrt{B_{\theta}(1+B_{\theta})}} \tag{3.77c}$$

4. Amplitude of vibration

The amplitude of vibration can be expressed as follows:

$$\theta = \frac{M_y}{k_{\theta}} M, \tag{3.77d}$$

where M is the magnification factor, $\frac{\theta}{(M_y/k_{\theta})}$, which is plotted in Figure 3.22a against the nondimensional frequency ω/ω_n ; where

$$\omega_n = 2\pi f_n = \sqrt{\frac{k_\theta}{I_\theta}} \tag{3.77a}$$

The phase lag ϕ can be determined from Equation 3.74g or Figure 3.22b.

The above relations can be applied to a rectangular footing (of the same height, h) using an equivalent B_e that is determined by equating the moment of area of the surface of the footing about the y axis (I_θ) to that of the equivalent circular footing.

Thus,

$$\frac{1}{64} \pi B_e^4 = \frac{1}{12} BL^3$$

3.5.3 Sliding Oscillations

A mass–spring–dashpot analog was developed by Hall (1967) to simulate the horizontal sliding oscillations of a rigid circular footing of mass m (Figure 3.25). This can be expressed by Equation 3.78

$$m\ddot{x} + c_x\dot{x} + k_x x = P_0 e^{i\omega t} \tag{3.78}$$

If the foundation is circular, the spring and the damping constants are given by

$$k_x = \frac{16(1 - \nu_s)G_s B}{7 - 8\nu_s} \tag{3.79a}$$

and

$$c_z = \frac{4.6(1 - \nu_s)B^2}{7 - 8\nu_s} \sqrt{G_s \rho_s} \tag{3.79b}$$

respectively.

Then, the following important parameters with respect to the above motion can be derived:

1. Natural frequency of vibration

$$f_n = \frac{1}{2\pi} \sqrt{\frac{16(1 - \nu_s)G_s B}{7 - 8\nu_s} \frac{1}{m}} \tag{3.80a}$$

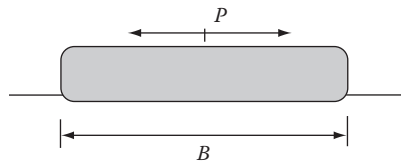


FIGURE 3.25
Footing subjected to sliding oscillation.

- 2. Resonant frequency
Moment-type excitation

$$f_r = f_n \sqrt{1 - \frac{0.166}{B_x}} \tag{3.80b}$$

where the modified dimensionless mass ratio B_x is given by

$$B_x = \frac{7 - 8v_s}{4(1 - v_s)} \frac{m}{\rho B^3} \tag{3.80c}$$

- 3. Damping ratio

$$D = D_x = \frac{0.288}{\sqrt{B_x}} \tag{3.80d}$$

- 4. Amplitude of vibration

The amplitude of vibration can be expressed as follows:

$$A_x = \frac{P_0}{k_x} M, \tag{3.80e}$$

where M is the amplification factor, P_0/k_x , which is also plotted in Figure 3.22 against the nondimensional frequency ω/ω_n , where

$$\omega_n = 2\pi f_n = \sqrt{\frac{16(1 - v_s)G_s B}{7 - 8v_s} \frac{1}{m}} \tag{3.80a}$$

The phase lag ϕ can be determined from Equation 3.74g or Figure 3.22b.

3.5.4 Foundation Vibrations due to Rotating Masses

If the foundation vibrations described in Sections 3.5.1 through 3.5.3 are created by unbalanced masses (m_1 with an eccentricity of e) rotating at an angular frequency of ω , then, the modifications in the following sections must be made to Equations 3.72, 3.75a, and 3.78. In all of the above cases, the new equations of motion have to be solved to determine the resonance frequencies and the amplitudes of vibrations. In keeping with the new solutions, the amplification factor in Equation 3.74f can be modified as

$$M_1 = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2D \frac{\omega}{\omega_n}\right)^2} \tag{3.81}$$

M_1 is plotted in Figure 3.26.

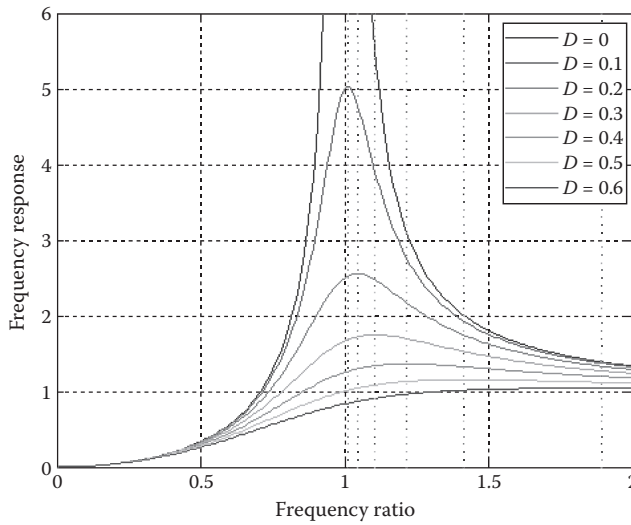


FIGURE 3.26
Magnification factor (M_1) against frequency ratio (ω/ω_n).

3.5.4.1 Translational Oscillations

$P_0 = m_1 e \omega^2$ must be substituted in Equation 3.72 for P_0

$$m\ddot{z} + c_z \dot{z} + k_z z = P = P_0 e^{i\omega t} = m_1 e \omega^2 [\cos(\omega t) + i \sin(\omega t)] \tag{3.82a}$$

The amplitude of vibration can be expressed by the modified Equation 3.74e as

$$A_z = \frac{m_1 e}{m} M_1 \tag{3.82b}$$

3.5.4.2 Rocking Oscillations

$M_y = m_1 e z \omega^2$ must be substituted in Equations 3.75a for M_y , where z = moment arm of the unbalanced force

$$I_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = m_1 e z \omega^2 e^{i\omega t} \tag{3.83a}$$

The amplitude of vibration can be expressed by the modified Equation 3.77d as

$$\theta = \frac{m_1 e z}{I_\theta} M_1 \tag{3.83b}$$

3.5.4.3 Sliding Vibrations

$P_0 = m_1 e \omega^2$ must be substituted in Equation 3.78 for P_0

$$m\ddot{x} + c_x \dot{x} + k_x x = m_1 e \omega^2 e^{i\omega t} \tag{3.84a}$$

The amplitude of vibration can be expressed by the modified Equation 3.80e as

$$A_x = \frac{m_1 e}{m} M_1 \tag{3.84b}$$

Further details are also found in Das (1993).

3.6 Additional Examples

Example 3.10

Predict the following settlement components for a circular footing with a 2 m diameter that carries a load of 200 kN as shown in Figure 3.27.

- (a) Average consolidation settlement of the footing in 5 years (use the 2:1 distribution method)
- (b) Maximum ultimate differential settlement
- (c) Elastic settlement from Schmertmann’s method
- (d) Total ultimate settlement of the center of the footing

Consolidation properties of the clay layer can be obtained from Figure 3.15. Assume its coefficient of consolidation to be $1 \times 10^{-8} \text{ m}^2/\text{s}$. Suitable elastic parameters of the sandy soil can be obtained from Chapter 1 (Tables 1.5 and 1.6).

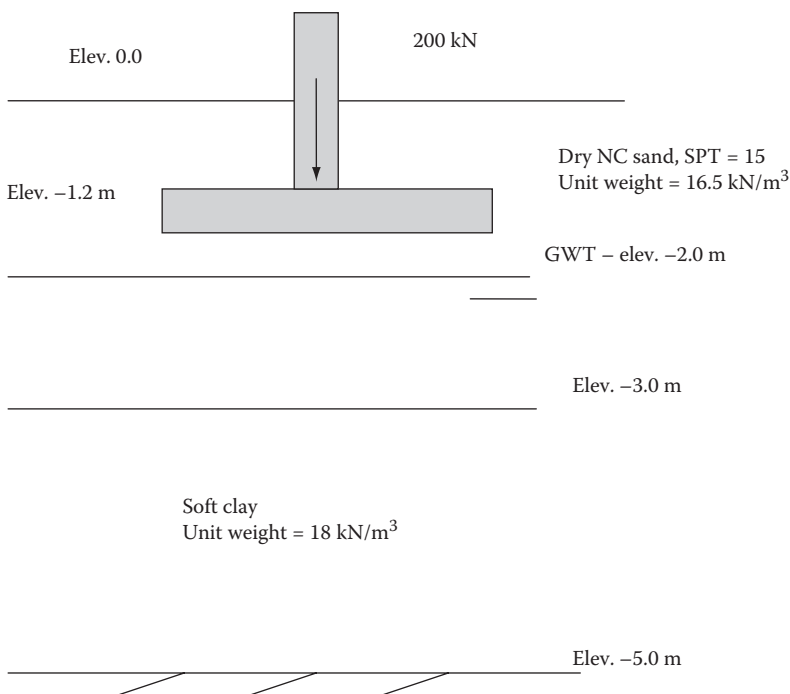


FIGURE 3.27
Illustration for Example 3.10.

Increase stress at the center of the soft clay (2-m-diameter footing),

$$\Delta\sigma = \frac{Q}{\pi/4(D+Z)^2}$$

To compute the increase in vertical stress at the mid-plane of the clay layer, apply Equation 3.28.

At mid-plane of clay

$$\Delta\sigma = \frac{200}{\pi/4(2+2.8)^2}$$

$$\Delta\sigma = 11.05 \text{ kPa}$$

From Figure 3.15, the preconsolidation pressure (P_c) = 60 kPa.

The average effective overburden pressure at the mid-plane of the soft clay,

$$\begin{aligned}\sigma'_{vo} &= (16.5)(2) + (17.5)(1) + (18)(1) - (9.8)(2) \\ &= 48.9 \text{ kPa} < P_c\end{aligned}$$

Thus, it is in the oc region

$$\sigma'_{vo} + \Delta\sigma = 48.9 + 11.05 = 59.95 \text{ kPa} < P_c \text{ (60 kPa)}$$

$$\begin{aligned}\therefore S_{\text{avg}} &= \left[\frac{H}{1+e_o} \right] \left[C_r \log \frac{\sigma'_{vo} + \Delta\sigma}{\sigma'_{vo}} \right] \\ &= \left[\frac{2}{1+1.06} \right] \left[0.064 \log \frac{59.95}{48.9} \right] = 5.5 \text{ mm}\end{aligned}$$

$$T = 5 \text{ years}, H_{\text{dr}} = 2 \text{ m}, C_u = 1 \times 10^{-8} \text{ m}^2/\text{s}$$

$$T = \frac{C_v t}{H_{\text{dr}}^2} = \frac{1 \times 10^{-8} (5 \times 365 \times 24 \times 60 \times 60)}{2^2} = 0.394$$

For $T = 0.394$, $U_{\text{avg}} = 0.69$

$$(a) \therefore S_{5\text{yrs}} = (5.5)(0.69) = 3.795 \text{ mm}$$

(b) Using Newmark's chart, $AB = 2.8 \text{ m}$ (Figure 3.16)

$$\text{footing radius} = 1 \text{ m} = \frac{1}{2.8} OQ = 0.36OQ$$

Placing the center of footing at the center of chart

$$N_{\text{center}} = 48 \times 4 = 192, \quad \Delta\sigma_{\text{center}} = 192 \times 0.001 \times \frac{200}{(\pi/4)(2)^2} = 12.2 \text{ kPa}$$

Placing the edge of footing at the center of chart

$$N_{\text{edge}} = 150$$

$$\Delta\sigma_{\text{edge}} = 150 \times 0.001 \times \frac{200}{(\pi/4)(2)^2} = 9.5 \text{ kPa}$$

$$\sigma'_{\text{vo}} + \Delta\sigma_{\text{center}} = 48.9 + 12.2 = 61.1 \text{ kPa} > P_c$$

$$\begin{aligned} S_{\text{center}} &= \left[\frac{H}{1+e_o} \right] \left[C_r \log \frac{P_c}{\sigma'_{\text{vo}}} + C_c \log \frac{P_c + \Delta\sigma}{P_c} \right] \\ &= \left[\frac{1}{1+1.06} \right] \left[0.064 \log \frac{60}{48.9} + 0.382 \log \frac{60+12.2}{60} \right] \\ &= 35.3 \text{ mm} \end{aligned}$$

$$\sigma'_{\text{vo}} + \Delta\sigma_{\text{edge}} = 48.9 + 9.5 = 58.4 < P_c$$

$$\begin{aligned} S_{\text{edge}} &= \left[\frac{H}{1+e_o} \right] \left[C_r \log \frac{\sigma'_{\text{vo}} + \Delta\sigma}{\sigma'_{\text{vo}}} \right] \\ &= \left[\frac{2}{1+1.06} \right] \left[0.064 \log \frac{48.9+9.5}{48.9} \right] \\ &= 4.79 \text{ mm} \\ \Delta S &= 35.3 - 4.79 = 30.5 \text{ mm} \end{aligned}$$

If it is assumed that the immediate settlement under the center is equal to that under the edge, the angular distortion of the footing can be computed as

$$30.5 \text{ mm}/1 \text{ m} (= \text{radius of the footing}) = 1/32$$

It is noted that the angular distortion is greater than 1/75 (= 0.0133), which is the limiting angular distortion needed for structural damage.

(c) Determination of the immediate settlement

$$\text{stress increase at the foundation level } (\Delta\sigma) = 200/\pi(2)^2 = 63.66 \text{ kPa}$$

$$\text{Initial effective overburden stress at the foundation level } (q) = 1.2 \times 16.5 = 19.8 \text{ kPa}$$

$$E \text{ for dry sand (Table 1.7)} = 500(N + 15) = 15,000 \text{ kPa}$$

$$E \text{ for saturated sand (Table 1.7)} = 250(N + 15) = 6750 \text{ kPa}$$

$$E \text{ for clays (Table 1.7)} = 300(N + 6) = 10,000 \text{ kPa}$$

$$\begin{aligned}
 S_c &= C_1 C_2 (\Delta\sigma - q) \sum_0^z \frac{I_z}{E_s} \Delta z \\
 &= \left[1 - 0.5 \frac{19.8}{63.66 - 19.8} \right] \left[1 + 0.2 \log \frac{0.1}{0.1} \right] \left[\frac{0.192}{15,000} + \frac{0.108 + 0.416}{6750} + \frac{0.48}{10,000} \right] (63.66 - 19.8) \\
 &= 4.7 \text{ mm}
 \end{aligned}$$

(d) Total ultimate settlement

$$\begin{aligned}
 &= \text{center consolidation} + \text{elastic settlement} \\
 &= 35.3 + 4.7 = 40.0 \text{ mm}
 \end{aligned}$$

Example 3.11

Assuming that the depth of embedment is 1.5 m, design a suitable *strip* footing for the wall that carries a load of 150 kN/m, as shown in Figure 3.28. The average corrected SPT value for the subsurface is 12. Suitable soil parameters for the site can be obtained in Chapter 2.

For SPT – $N' = 12$

<p>From Table 2.5,</p> <p>$\gamma_{\text{moist}} = 18.9 \text{ kN/m}^3$</p> <p>$\phi = 3^\circ$ (Assume $\phi = 0^\circ$)</p>	<p>medium stiff clay</p>	<p>From Table 2.6,</p> <p>$\gamma_{\text{sat}} = 18.9 \text{ kN/m}^3$</p> <p>$\gamma_{\text{sub}} = 9.1 \text{ kN/m}^3$</p> <p>$\phi = 0^\circ$</p> <p>$C = \frac{N}{T_i} = \frac{12}{8} \text{ ksf}$</p> <p>$= 1.5 \text{ ksf} = 0.0718 \text{ MPa}$</p>
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Using Meyerhoff's bearing capacity: Equation 3.3

$$q_{\text{ult}} = cN_c s_c d_c + qN_q s_q d_q + 0.5B_\gamma N_\gamma s_\gamma d_\gamma$$

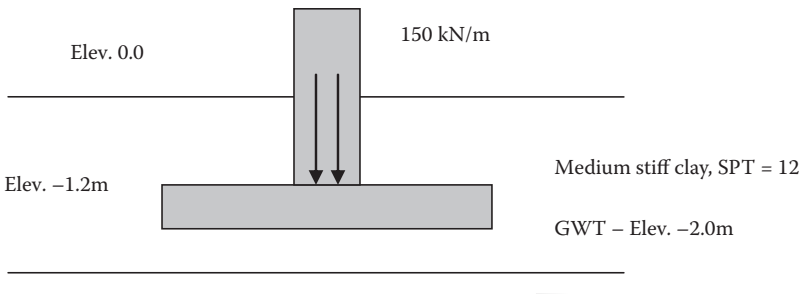


FIGURE 3.28
Illustration for Example 3.11.

From Table 3.1 (for $\phi = 0$, Meyerhoff)

$$N_c = 5.14, N_q = 1.0, N_\gamma = 0.0$$

From Table 3.2b (for $\phi = 0$) $K_p = \tan^2 \left(45 + \frac{\phi}{2} \right) = 1.0 \left(\frac{\phi}{2} \rightarrow 0 \right)$

$$s_c = 1 + 0.2(1) \frac{B}{1} = 1 + 0.2B$$

$$s_\gamma = 1.0$$

$$d_c = 1 + 0.2\sqrt{1} \left(\frac{1.5}{B} \right) = 1 + \frac{0.3}{B}$$

$$d_\gamma = 1.0$$

$$\therefore q_{ult} = (71.8)(5.14)(1 + 0.2B) \left(1 + \frac{0.3}{B} \right) + (18.9)(1.2)(1.0)(1.0)(1.0) + 0$$

$$\frac{P}{A} \leq \frac{q_{ult}}{F}$$

$$\frac{150}{(B \times 1)} \leq \frac{\left((369 + 73.8B) \left(1 + \frac{0.3}{B} \right) + 22.68 \right)}{2.5}$$

Therefore, $B \geq 1.0$ m

Example 3.12

A 5-kN horizontal load acts on the column shown in Figure 3.29 at a location of 1.5 m above the ground level. If the site soil is granular with an angle of friction 20° and a unit weight of 16.5 kN/m^3 , determine a suitable footing size. If the groundwater table subsides to a depth outside the foundation influence zone, what would be the factor of safety of the footing?

$$\phi = 20^\circ, \gamma = 16.5 \text{ kN/m}^3, K_p = \tan^2 (45 + \phi/2) = 2.04$$

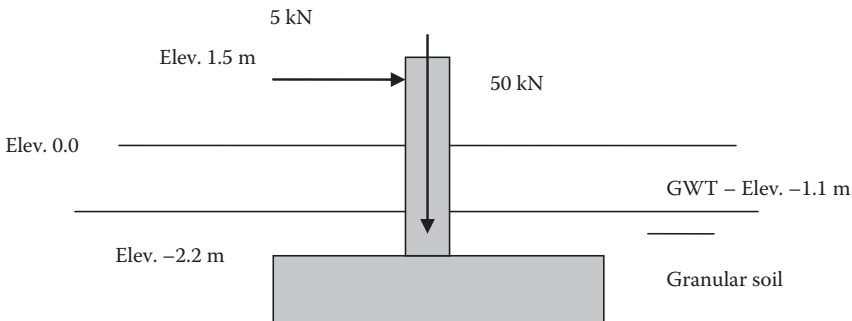


FIGURE 3.29
Illustration for Example 3.12.

Meyerhoff's bearing capacity expression:

$$q_{ult} = c N_c s_c d_c i_c + q N_q s_q d_q i_q + 0.5 B \gamma' N_\gamma s_\gamma d_\gamma i_\gamma$$

From Table 3.1, for $\phi = 20^\circ$, $N_q = 6.4$, $N_\gamma = 2.9$

From Table 3.2b

$$S_q = S_\gamma = 1 + 0.1(2.04) \frac{B}{B} = 1.204 \quad (\text{Assume circular or square footing})$$

$$d_q = d_\gamma = 1 + 0.2 \sqrt{2.04} \frac{D}{B} = + \frac{0.314}{B} \quad (\text{since } D = 1.1 \text{ m})$$

$$\tan \theta = 5/50; \theta = 5.71^\circ$$

$$i_q = \left(1 - \frac{5.71}{90}\right)^2 = 0.877, \quad i_\gamma = \left(1 - \frac{5.71}{20}\right)^2 = 0.51$$

$$q = (16.5)(1.1) + (16.5 - 9.8)(1.1) = 25.52 \text{ kPa}$$

$$\begin{aligned} q_{ult} &= (25.52)(6.4) \left(1 + \frac{0.314}{B}\right) (1.204)(0.877) + (0.5)(B)(16.5 - 9.8)(2.9)(1.204) \left(1 + \frac{0.314}{B}\right) (0.51) \\ &= 172.46 + \frac{54.15}{B} + 5.97B + 1.87 \end{aligned}$$

or

$$q_{ult} = 174.33 + \frac{54.15}{B} + 5.97B$$

$$q_d \leq \frac{q_{ult}}{F}$$

$$\text{Moment} = 5 \times 3.7 = 18.5$$

$$q_d = \frac{P}{A} + \gamma D + \frac{Mc}{I} = \frac{50}{B \times B} + 25.52 + \frac{18.5 \times \frac{B}{2}}{\frac{B \times B^2}{12}} \leq \frac{174.33 + \frac{54.15}{B} + 5.97B}{2.5}$$

$$B = 1.65 \text{ m}$$

With no water within influence zone:

$$B = 1.65 \text{ m}$$

$$Q = (16.5)(2.2) = 36.3 \text{ kPa}$$

$$q_d = \frac{P}{A} + \gamma D + \frac{Mc}{I} = \frac{50}{B^2} + 36.3 + \frac{185. \times \frac{B}{2}}{\frac{B \times B^3}{12}} = 95.5 \text{ kPa}$$

$$q_{ult} = (36.3)(6.4) \left(1 + \frac{0.314}{B} \right) (1.204)(0.877) + (0.5)(B)(16.5)(2.9)(1.204) \left(1 + \frac{0.314}{B} \right) (0.51)$$

$$= 320 \text{ kPa}$$

$$F = \frac{320}{95.5} = 3.35$$

Therefore, the safety factor increases from 2.5 to 3.35.

Example 3.13

A concrete machine foundation shown in Figure 3.30a is subjected to a periodic force that can be represented by

$$P(t) = 5 \sin(45t) + 10 \sin(90t) + 2.5 \sin(135t) \text{ kN}$$

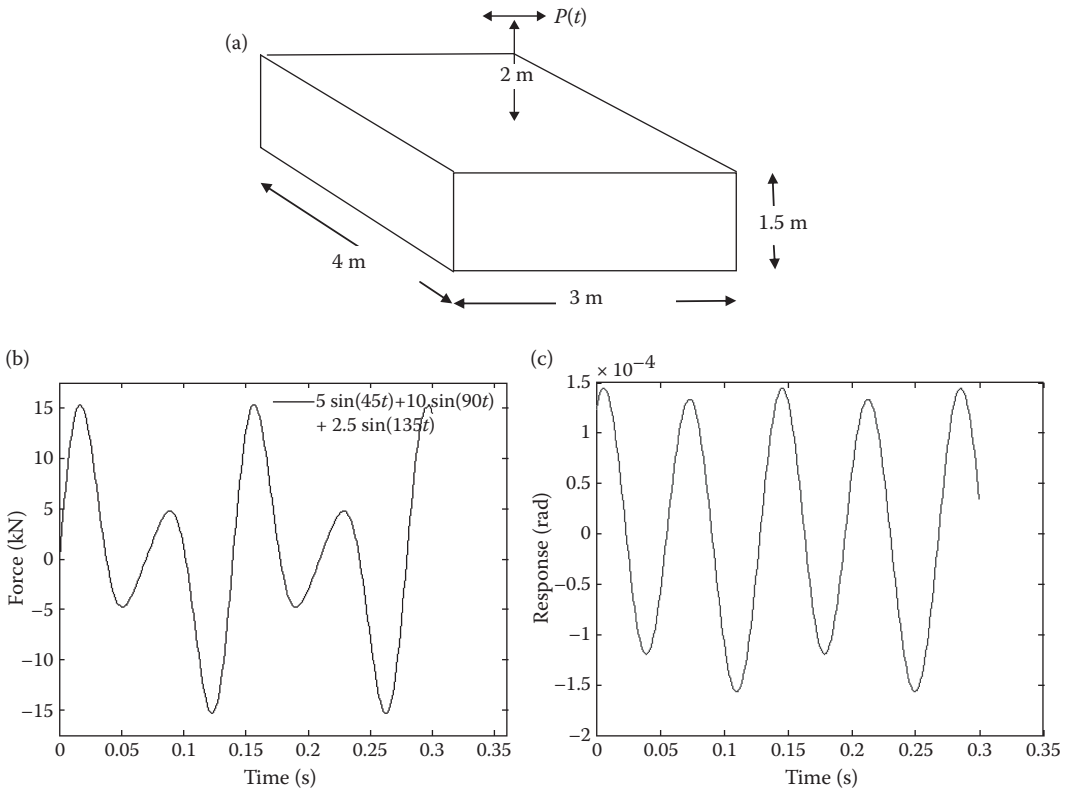


FIGURE 3.30 (a) Illustration for Example 3.13. (b) Forcing function $P(t)$. (c) Resultant rocking response.

- Plot $P(t)$ in components and as a *resultant force* by selecting an appropriate time scale to show at least two cycles of the least frequency component of $P(t)$.
- Neglecting the horizontal force components, estimate the amplitudes of rocking vibration corresponding to all three frequency components of $P(t)$.
- Estimate the phase shifts associated with all three frequency components of $P(t)$.
- Express the rocking response in the form $\theta(t) = \sum_1^3 A_i \sin(\omega_i t - \phi_i)$, tabulate A_i , ω_i , and ϕ_i , and plot the *resultant rocking response*.

Assume the unit weight of concrete to be 23 kN/m^3 .
The mass moment of inertia of the foundation is

$$I_{0(\text{foundation})} = \frac{W_0}{g} \left(\frac{r_0^2}{4} + \frac{h^2}{3} \right) = \frac{423,000}{9.81} \left(\frac{1.84^2}{4} + \frac{1.5^2}{3} \right) = 83,178 \text{ kg m}^2/\text{rad}$$

where

$W_0 = BLh\gamma_{\text{conc}} = 4(3)(1.5)(23,000) = 414,000 \text{ N}$ is the weight of the foundation
 $g = 9.81 \text{ m/s}^2 =$ acceleration due to gravity

$$r_0 = \sqrt[4]{\frac{BL^3}{3\pi}} = \sqrt[4]{\frac{4(3^3)}{3\pi}} = 1.84 \text{ m} = \text{equivalent radius}$$

$h = 1.5 \text{ m} =$ height of the foundation

It will be assumed that the machine mass moment of inertia is negligibly small with respect to those of the foundation and the vibrating soil, $I_{0(\text{machine})} \approx 0$.

Using Equations 3.76a through 3.76c, the static spring constant is

$$k_\theta = \frac{8Gr_0^3}{3(1-\mu)} = \frac{8(30 \times 10^6)(1.84^3)}{3(1-0.2)} = 6.23 \times 10^8 \text{ N m/rad}$$

The dashpot coefficient is

$$c_\theta = \frac{0.8r_0^4 \sqrt{G\rho}}{(1-\mu)(1+B_\theta)} = \frac{0.8(1.84^4) \sqrt{30 \times 10^6 \times 1700}}{(1-0.2)(1+0.696)} = 1.53 \times 10^7 \frac{\text{N m}}{\text{rad/s}}$$

where

$$B_\theta = \frac{3(1-\mu)}{8} \frac{I_0}{\rho r_0^5} = \frac{3(1-0.2)}{8} \frac{83,178}{1700(1.84^5)} = 0.696$$

is the inertia ratio.

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_\theta}{I_0}} = \sqrt{\frac{6.23 \times 10^8}{83,178}} = 86.54 \text{ rad/s}$$

The frequency ratios corresponding to the components of the horizontal force (kN) $5 \sin(45t)$, $10 \sin(90t)$, and $2.5 \sin(135t)$ are

$$\frac{\omega_1}{\omega_n} = \frac{45}{86.54} = 0.52$$

$$\frac{\omega_2}{\omega_n} = \frac{90}{86.54} = 1.04$$

$$\frac{\omega_3}{\omega_n} = \frac{135}{86.54} = 1.56$$

The damping ratio is

$$D_\theta = \frac{c_\theta}{c_{\theta 0}} = \frac{0.15}{\sqrt{B_\theta(1+B_\theta)}} = \frac{0.15}{\sqrt{0.696(1+0.696)}} = 0.106$$

From Figure 3.22a,

$$\frac{\Theta_1}{M_{y1}/k_\theta} = 1.4 \Rightarrow \Theta_1 = \frac{1.4(5000 \times 2)}{6.23 \times 10^8} = 2.25 \times 10^{-5} \text{ rad}$$

$$\frac{\Theta_2}{M_{y2}/k_\theta} = 4.3 \Rightarrow \Theta_2 = \frac{4.3(10,000 \times 2)}{6.23 \times 10^8} = 1.38 \times 10^{-4} \text{ rad}$$

$$\frac{\Theta_3}{M_{y3}/k_\theta} = 0.7 \Rightarrow \Theta_3 = \frac{0.7(2500 \times 2)}{6.23 \times 10^8} = 5.62 \times 10^{-6} \text{ rad}$$

From Figure 3.22b,

$$\phi_1 = 0.15 \text{ rad}$$

$$\phi_2 = -1.15 \text{ rad}$$

$$\phi_3 = -0.2 \text{ rad}$$

The rocking response is then

$$\theta(t) = (2.25 \times 10^{-5}) \sin(45t - 0.15) + (1.38 \times 10^{-4}) \sin(90t - 1.15) + (5.62 \times 10^{-6}) \sin(1.35t + 0.2)$$

θ_i (rad)	ω_i (rad)	ϕ_i (rad)
2.25×10^{-5}	45	0.15
1.38×10^{-4}	90	-1.15
5.62×10^{-6}	135	-0.2

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