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Mathematical control theory: An introduction, by Jerzy Zabczyk. Birkhäuser, Boston, 1992, viii+260 pp., \$69.50. ISBN 3-7643-3645-5

The tradition of book reviews for the *Bulletin* usually involves an exposition of the subject as well as a review of the book. In the present book Zabczyk discusses almost all of deterministic control theory, so a complete exposition is difficult. I will mention a few of the concepts and results from the part of the book on linear systems to try to give a flavor of the material.

A linear control system is modeled by an n-dimensional vector differential equation

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = a \tag{1}$$

describing the evolution of the n-dimensional state $x(t)$ of the system. The control $u(t)$ is an m-dimensional vector function of time which must be chosen to make the system behave in a desired manner. A and B are $n \times n$ and $n \times m$ matrices, and a is the initial state of the system.

In many situations it is desired to keep the system near zero. A square matrix A is called a stable matrix if all its eigenvalues have negative real parts. If A is a stable matrix, solutions of

$$\dot{x}(t) = Ax(t), \qquad x(0) = a \tag{2}$$

satisfy

$$|x(t)| \le Me^{-\omega t}|a| \tag{3}$$

for positive constants M and ω. Thus, in this case, all solutions of (2) decay exponentially to zero. The system (2) is said to be exponentially stable when (3) holds.

A control $u(t)$ is given by a linear feedback if it has the form

$$u(t) = Kx(t) \tag{4}$$

where K is an $m \times n$ matrix and $x(t)$ is the solution of (1) with this control. The problem of exponential stabilization of the linear system (1) is the problem of finding a matrix K so that $A + BK$ is a stable matrix. Thus when the linear feedback control (4) with this K is used in (1), the system will be exponentially stable.

The linear system (1) is said to be controllable if for any two states a and b there is a control $u(t)$ which drives the system from the initial state a to the state b. An algebraic condition can be given for controllability. The system will be controllable if and only if the matrix

$$[B, AB, A^2B, \dots, A^nB]$$

has rank n.

A fundamental result of linear control theory is that the three following concepts are equivalent:

(a) controllability,
(b) existence of a matrix K so that $A + BK$ has arbitrarily prescribed eigenvalues,
(c) exponential stabilizability of (1) with (3) holding for any positive constant w.

The system (1) may also be exponentially stabilizable in cases in which it is not controllable. If (1) is not controllable, it can be shown that there is a change of coordinates so that in the new coordinates the system has the form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \end{pmatrix} u(t) \tag{5}$$

and the system

$$\dot{x}_1 = A_{11}x_1 + B_1u(t)$$

is controllable. This reduces the system to a controllable part x_1 and an uncontrollable part x_2. This decomposition, the previous equivalence result, and some estimation can be used to show system (5) will be exponentially stabilizable if and only if A_{22} is a stable matrix.

This book is designed as a graduate text on the mathematical theory of deterministic control. It covers a remarkable number of topics. The book is divided into four parts: linear systems, nonlinear systems, optimal control, and infinite-dimensional linear systems. The book includes material on the realization of both linear and nonlinear systems, impulsive control, and positive linear systems—subjects not usually covered in an "introductory" book. Since there are individual books devoted to each of the four parts, it is natural to ask, "Does Zabczyk succeed in presenting these all in one thin two-hundred-fifty-page book?" I feel the answer is, yes indeed!

Most all of the material covered in a standard course in linear systems is covered in Chapters 1 and 2 of Part I, the first fifty pages of the book. The two highlight theorems, that controllability implies a linear feedback can be selected so that the system has arbitrary poles, and that if only linear measurements of the systems states are available and if the system is detectable and stabilizable, there is a dynamic observer and a feedback under which the system is stable are there.

Local controllability and observability and various types of stabilizability are discussed for nonlinear systems in Chapters 1 and 2 of Part II. A major portion of this is concerned with the relationship of concepts for the nonlinear system and its linearization resulting from Lyapunov's first method. A typical result is "The nonlinear system is locally exponentially stabilizable if and only if its linearization is stabilizable."

In Chapter 1 of Part III, dynamic programming for fixed-time free endpoint optimal control problems is treated, and an extensive discussion applying this to the linear regulator problem is presented. Chapter 2 discusses dynamic programming for impulse control problems (again without terminal conditions). Chapter 3 discusses maximum principles for the fixed-time free endpoint optimal control problem, the impulse control problem, and the time-optimal control for a linear system to hit a terminal state.

Part IV is a good introduction to the formidable mathematical machinery necessary to discuss infinite-dimensional linear systems. The problems discussed center on controllability of infinite-dimensional linear systems and extending the linear regulator problem to infinite-dimensional linear systems.

To get so much material in such a short space, the pace of the presentation is brisk. However, the exposition is excellent, and the book is a joy to read. A novel one-semester course covering both linear and nonlinear systems could be given using Chapters 0, 1, and 2 of Part I and Chapters 1 and 2 of Part II. If time permitted, Chapter 3 of Part I and Chapter 3 of Part II on realizations of respectively linear and nonlinear systems could be added. The book is an excellent one for introducing a mathematician to control theory. The book presents a large amount of material very well, and its use is highly recommended.

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