Chapter 5

Forces and Motion II

5.1 The Important Stuff

5.1.1 Friction Forces

Forces which are known collectively as "friction forces" are all around us in daily life. In elementary physics we discuss the friction force as it occurs between two objects whose surfaces are in contact and which slide against one another.

If in such a situation, a body is *not moving* while an applied force **F** acts on it, then **static friction** forces are opposing the applied force, resulting in zero net force. Empirically, one finds that this force can have a *maximum* value given by:

$$f_s^{\text{max}} = \mu_s N \tag{5.1}$$

where μ_s is the **coefficient of static friction** for the two surfaces and N is the normal (perpendicular) force between the two surfaces.

If one object is in motion relative to the other one (i.e. it is sliding on the surface) then there is a force of **kinetic friction** between the two objects. The direction of this force is such as to oppose the sliding motion and its magnitude is given by

$$f_k = \mu_k N \tag{5.2}$$

where again N is the normal force between the two objects and μ_k is the **coefficient of kinetic friction** for the two surfaces.

5.1.2 Uniform Circular Motion Revisited

Recall the result given in Chapter 3: When an object is in uniform circular motion, moving in a circle of radius r with speed v, the acceleration is directed toward the center of the circle and has magnitude

$$a_{\text{cent}} = \frac{v^2}{r} \ .$$

Therefore, by Newton's Second Law of Motion, the *net force* on this object must also be directed toward the center of the circle and have magnitude

$$F_{\text{cent}} = \frac{mv^2}{r} \ .$$
(5.3)

Such a force is called a **centripetal force**, as indicated in this equation.

5.1.3   Newton's Law of Gravity (Optional for Calculus–Based)

The force of gravity is one of the fundamental forces in nature. Although in our first physics examples we only dealt with the fact that the *earth* pulls downward on all masses, in fact all masses exert an attractive gravitational force on each other, but for most objects the force is so small that we can ignore it.

   Newton's Law of Gravity says that for two masses m_1 and m_2 separated by a distance r, the magnitude of the (attractive) gravitational force is

$$F = G\frac{m_1 m_2}{r^2} \qquad \text{where} \quad G = 6.67 \times 10^{-11} \ \tfrac{\text{N·m}^2}{\text{kg}^2}$$
(5.4)

While the law as given really applies to *point* (i.e. small) masses, it can be used for *spherical* masses as long as we take r to be the distance between the centers of the two masses.

5.2   Worked Examples

5.2.1   Friction Forces

1. An ice skater moving at $12 \frac{\text{m}}{\text{s}}$ coasts to a halt in $95\,\text{m}$ on an ice surface. What is the coefficient of (kinetic) friction between ice and skates? [Ser4 5-51]

   The information which we are given about the skater's (one-dimensional) motion is shown in Fig. 5.1(a). We know that the skater's notion is one of constant acceleration so we can use the results in Chapter 2. In particular we know the initial and final velocities of the skater:

$$v_0 = 12 \tfrac{\text{m}}{\text{s}} \qquad v = 0$$

and we know the displacement for this interval:

$$x - x_0 = 95\,\text{m}$$

we can use 2.8 to find the (constant) acceleration a. We find:

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad \Longrightarrow \qquad a_x = \frac{(v_x^2 - v_{0x}^2)}{2(x - x_0)}$$
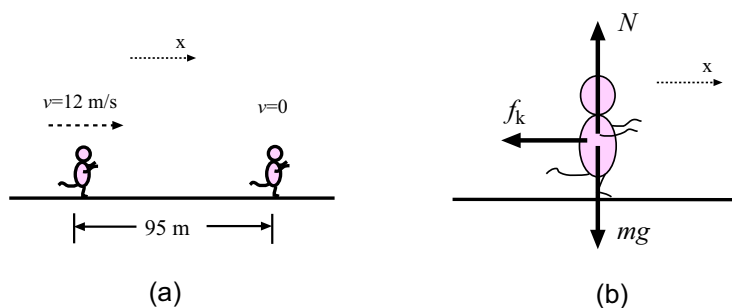
Figure 5.1: Skater slowed to a halt by friction in Example 1. Motion is shown in (a); forces acting on the skater are shown in (b).

Substituting, we get:

$$a_x = \frac{((0\,\frac{m}{s})^2 - (12\,\frac{m}{s})^2)}{2(95\,\text{m})} = -0.76\,\frac{m}{s^2} \ .$$

Next, think about the forces acting on the skater; these are shown in Fig. 5.1(b). If the mass of the skater is m then gravity has magnitude mg and points downward; the ice exerts a normal force N upward. It also exerts a frictional force f_k in a direction opposing the motion. Since the skater has no motion in the vertical direction, the vertical forces must sum to zero so that $N = mg$. Also, since the *magnitude* of the force of kinetic friction is given by $f_k = \mu_k N$ we have:

$$f_k = \mu_k N = \mu_k mg \ .$$

So the net force in the x direction is $F_x = -\mu_k mg$.

Newton's law tells us: $F_{x,\,\text{net}} = ma_x$. Using the results we have found, this gives us:

$$-\mu_k mg = m(-0.76\,\tfrac{m}{s^2})$$

From which the m cancels to give:

$$\mu_k = \frac{(0.76\,\frac{m}{s^2})}{g} = \frac{(0.76\,\frac{m}{s^2})}{(9.80\,\frac{m}{s^2})} = 7.7 \times 10^{-2}$$

The coefficient of kinetic friction between ice and skates is 7.7×10^{-2}. (Note, the coefficient of friction is *dimensionless*.)

Recall that we were not given the mass of the skater. That didn't matter, because it cancelled out of our equations. But we did have to *consider* it in writing down our equations.

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2. Block B in Fig. 5.2 weighs $711\,\text{N}$. The coefficient of static friction between block and table is 0.25; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.
[HRW6 6-19]

We need to look at the forces acting at the knot (the junction of the three cables). These are shown in Fig. 5.3(a). The vertical cord must have a tension equal to the weight of block
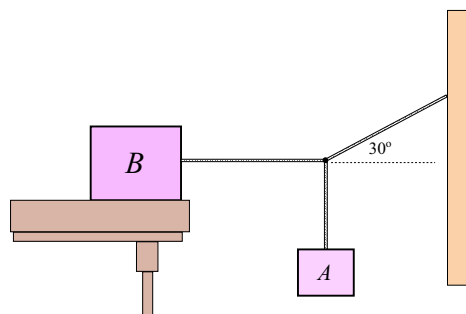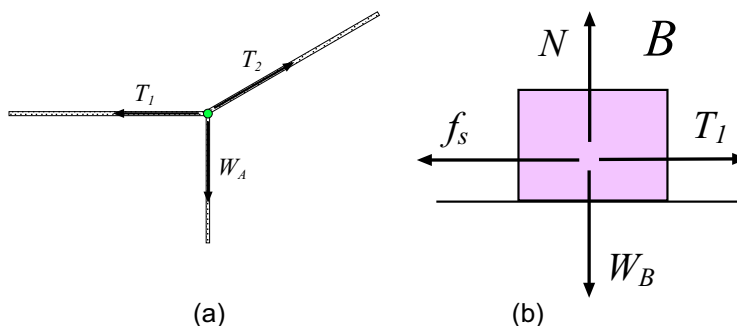
Figure 5.2: Diagram for Example 2.

(a)   (b)

Figure 5.3: (a) Forces acting at the knot in Example 2. (b) Forces acting on block B in Example 2.

A (which we'll call W_A) because at its other end this cord is pulling *up* on A so as to support it. Let the tensions in the other cords be T_1 for the horizontal one and T_2 for the one that pulls at 30° above the horizontal. The knot is in equilibrium so the forces acting on it add to zero. In particular, the vertical components of the forces add to zero, giving:

$$T_2 \sin \theta - W_A = 0 \qquad \text{or} \qquad T_2 \sin \theta = W_A$$

(where $\theta = 30°$) and the horizontal forces add to zero, giving:

$$-T_1 + T_2 \cos \theta = 0 \qquad \text{or} \qquad T_1 = T_2 \cos \theta \ .$$

Now look at the forces acting on the block which rests on the table; these are shown in Fig. 5.3(b). There is the force of gravity pointing down, with magnitude W_B (that is, the weight of B, equal to $m_B g$). There is a normal force from the table pointing upward; there is the force from the cable pointing to the right with magnitude T_1, and there is the force of static friction pointing to the left with magnitude f_s. Since the vertical forces add to zero, we have

$$N - W_B = 0 \qquad \text{or} \qquad N = W_b$$

The horizontal forces on the block also sum to zero giving

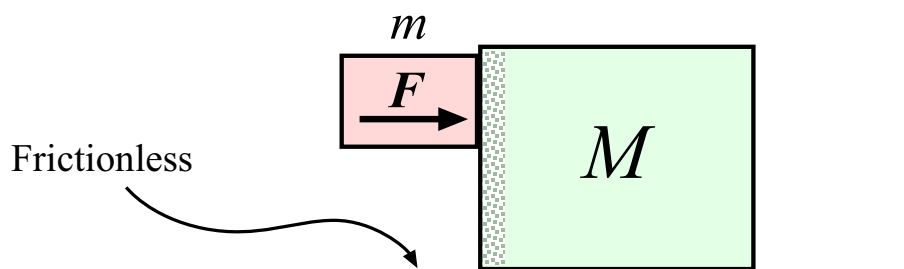$$T_1 - f_s = 0 \qquad \text{or} \qquad T_1 = f_s$$

Figure 5.4: Diagram for Example 3.

Now, the problem states that the value of W_A we're finding is the *maximum* value such that the system is stationary. This means that at the value of W_A we're finding, block B is *just about to slip*, so that the friction force f_s takes on its maximum value, $f_s = \mu_s N$. Since we also know that $N = W_B$ from the previous equation, we get:

$$T_1 = f_s = \mu_s N = \mu_s W_B$$

From before, we had $T_1 = T_2 \cos\theta$, so making this substitution we get

$$T_2 \cos\theta = \mu_s W_B$$

Almost done! Our very first equation gave $T_2 = \frac{W_A}{\sin\theta}$, so substituting for T_2 gives:

$$\left(\frac{W_A}{\sin\theta}\right)\cos\theta = \mu_s W_B \qquad \text{or} \qquad W_A \cot\theta = \mu_s W_B$$

Finally, we get:
$$W_A = \mu_s W_B \tan\theta$$

Now just plug in the numbers:

$$W_A = (0.25)(711\,\text{N})\tan\,30° = 103\,\text{N}$$

Since we solved for W_A under the condition that block B was about to slip, this is the maximum possible value for W_A so that the system is stationary.

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3. The two blocks (with $m = 16\,$kg and $M = 88\,$kg) shown in Fig. 5.4 are not attached. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath M is frictionless. What is the minimum magnitude of the horizontal force F required to hold m against M? [HRW5 6-38]

Having understood the basic set-up of the problem, we *immediately* begin thinking about the the forces acting on each mass so that we can draw free–body diagrams. The forces on mass m are: (1) The force of gravity mg which points downward. (2) The applied force F which points to the right. (3) The *normal* force with which block M pushes on m. This
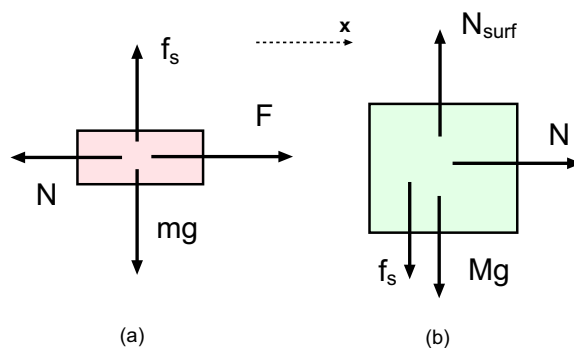
Figure 5.5: Free–body diagrams for the blocks described in Example 3.

force necessarily points to the left. (4) The frictional force which block M exerts on m. This is to be a *static* friction force, so we have to think about its direction... in this case, it must clearly oppose the force of gravity to keep the block m from falling. So we include a force f_{s} pointing *up*. These forces are shown in Fig. 5.5.

Next we diagram the forces acting on M. There is the force of gravity, with magnitude Mg, pointing down; the surface beneath M exerts a normal force N pointing upward. Since this surface is frictionless, it does not exert a horizontal force on M. The mass m will exert forces on M and these will be *equal and opposite to the forces which M exerts on m*. So there is a force N on mass M pointing to the *right* and a frictional force f_s pointing *downward*.

Now that we have shown *all* the forces acting on all the masses we can start to discuss the accelerations of the masses and apply Newton's Second Law.

The problem says that mass m is not slipping downward during its motion. This must mean that the forces of friction and gravity balance:

$$f_{\mathrm{s}} = mg \ .$$

But this does us little good until we have an expression for f_{s}. Now, in this problem we are being asked about a *critical condition* for the slippage of m. We can reasonably guess that here the force of static friction takes on its *maximum* value, namely

$$f_{\mathrm{s}} = \mu_{\mathrm{s}} N \ ,$$

N being the normal force between the two surfaces. This is an important bit of information, because combining that last two equations we get:

$$mg = \mu_{\mathrm{s}} N \ .$$

Let's consider the horizontal motion of both of the masses. Now, since the masses are always touching, their displacements, velocities and accelerations are always the same. Let the x acceleration of both masses be a. Then for mass m, Newton's Second Law gives us:
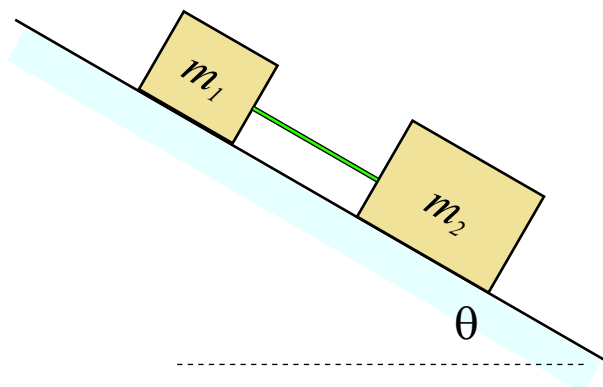
$$\sum F_x = F - N = ma$$

Figure 5.6: Two blocks joined by a rod slide down a slope with friction (coefficient of friction is different for the two blocks).

while for mass M, we get

$$N = Ma$$

Combining these last two equations gives

$$F - Ma = ma \qquad \Longrightarrow \qquad F = (M + m)a \qquad \Longrightarrow \qquad a = \frac{F}{(M + m)}$$

which tells us the force N:

$$N = Ma = \frac{MF}{(M + m)}$$

Putting this result for N into our result involving the friction force gives

$$mg = \mu_s N = \mu_s \frac{MF}{(M + m)}$$

which lets us solve for F:

$$F = \frac{(M + m)m}{M \mu_s} g$$

And now we can substitute the given values:

$$F = \frac{(M + m)m}{M \mu_s} g = \frac{(16\,\text{kg} + 88\,\text{kg})(16\,\text{kg})}{(88\,\text{kg})(0.38)}(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 488\,\text{N}$$

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4. In Fig. 5.6 a box of mass $m_1 = 1.65\,\text{kg}$ and a box of mass $m_2 = 3.30\,\text{kg}$ slide down an inclined plane while attached by a massless rod parallel to the plane. The angle of incline is $\theta = 30°$. The coefficient of kinetic friction between m_1 and the incline is $\mu_1 = 0.226$; that between m_2 and the incline is $\mu_2 = 0.113$. Compute (a) the tension in the rod and (b) the common acceleration of the two boxes. c) How would the answers to (a) and (b) change if m_2 trailed m_1?   [HRW6 6-26]
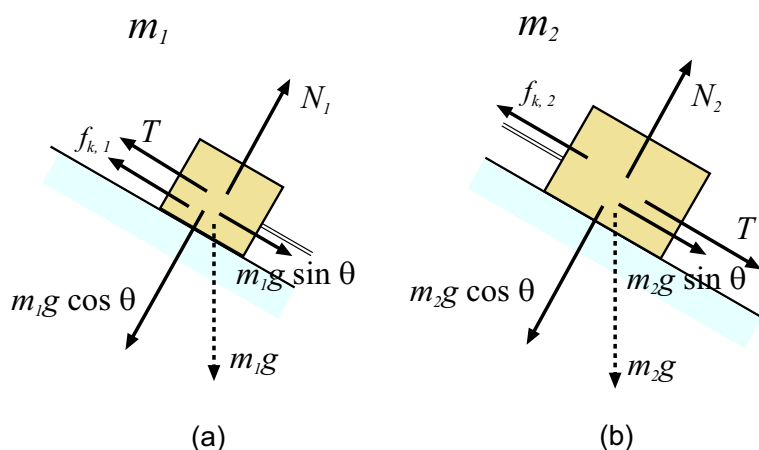
Figure 5.7: (a) Forces acting on block 1 in Example 4. We have assumed that the rod pushes outward; if that is wrong, then T will turn out to be negative. The force of gravity has be split up into components. (b) Forces acting on block 2 in Example 4.

(a) We will shortly be drawing force diagrams for the two masses, but we should first pause and consider the force which comes from the rod joining the two masses. A "rod" differs from a "cord" in our problems in that it can pull inward on either end or else *push outward*. (Strings can only pull inward.) For the purpose of writing down our equations we need to make some assumption about what is happening and so here I will assume that the rod is pushing outward with a force of magnitude T, i.e. the rod is compressed. Should it arise in our solution that we get a negative number for T, all is not lost; we will then know that the rod is really *pulling inward* with a force of magnitude $|T|$ and so the rod is being stretched.

With that in mind, we draw a diagram for the forces acting on block 1 and there are a lot of them, as shown in Fig. 5.7(a). Rod tension T and the force of kinetic friction on block 1 (to oppose the motion) point up the slope. The "slope" component of gravity $m_1 g \sin \theta$ points down the slope. The normal component of gravity $m_1 g \cos \theta$ points into the surface and the normal force N_1 from the slope points out of the surface.

As there is no acceleration perpendicular to the slope, those force components sum to zero, giving:

$$N_1 - m_1 g \cos \theta = 0 \qquad \text{or} \qquad N_1 = m_1 g \cos \theta$$

The sum of force components in the down–the–slope direction gives $m_1 a$, where a is the down–the–slope acceleration common to both masses. So then:

$$m_1 g \sin \theta - T - f_{k,1} = m_1 a$$

We can substitute for $f_{k,1}$, since $f_{k,1} = \mu_1 N_1 = \mu_1 m_1 g \cos \theta$. That gives:

$$m_1 g \sin \theta - T - \mu_1 m_1 g \cos \theta = m_1 a \tag{5.5}$$

We have a fine equation here, but T and a are both unknown; we need another equation!

The forces acting on block 2 are shown in Fig. 5.7(b). The force of kinetic friction $f_{k,2}$ points up the slope. The rod tension T and the "slope" component of gravity $m_2 g \sin \theta$ point

down the slope. The normal component of gravity $m_2 g \cos \theta$ points into the surface and the normal force of the surface on 2, N_2, points out of the slope.

Again there is no net force perpendicular to the slope, so

$$N_2 - m_2 g \cos \theta = 0 \qquad \text{or} \qquad N_2 = m_2 g \cos \theta .$$

The sum of the down–the–slope forces on m_2 gives $m_2 a$, so:

$$m_2 g \sin \theta + T - f_{k,2} = m_2 a$$

We can substitute for the force of kinetic friction here, with $f_{k,2} = \mu_2 N_2 = \mu_2 m_2 g \cos \theta$. Then:

$$m_2 g \sin \theta + T - \mu_2 m_2 g \cos \theta = m_2 a \tag{5.6}$$

Two equations (5.5 and 5.5) and tow unknowns (T and a). The physics is done, the rest is math!

In solving the equations I will go for an analytic (algebraic) solution, then plug in the numbers at the end. Aside from giving us some good practice with algebra, it will be useful in answering part (c).

We note that if we add Eqs. 5.5 and 5.5, T will be eliminated and we can then find a. When we do this, we get:

$$m_1 g \sin \theta + m_2 g \sin \theta - \mu_1 g \cos \theta - \mu_2 g \cos \theta = m_a + m_2 a$$

Lots of factoring to do here! Pulling out some common factors, this is:

$$g \left[(m_1 + m_2) \sin \theta - \cos \theta (\mu_1 m_1 + \mu_2 m_2) \right] = (m_1 + m_2) a$$

and then we get a:

$$a = \frac{g \left[(m_1 + m_2) \sin \theta - \cos \theta (\mu_1 m_1 + \mu_2 m_2) \right]}{(m_1 + m_2)} \tag{5.7}$$

But it's really T we want in part (a). We can eliminate a by multiplying Eq. 5.5 by m_2:

$$m_1 m_2 g \sin \theta - m_2 T - \mu_1 m_1 m_2 g \cos \theta = m_1 m_2 a$$

and Eq. 5.6 by m_2:

$$m_1 m_2 g \sin \theta + m_1 T - \mu_2 m_1 m_2 g \cos \theta = m_1 m_2 a$$

and then subtracting the second from the first. Some terms cancel, and this gives:

$$-m_2 T - m_1 T - \mu_1 m_1 m_2 g \cos \theta + \mu_2 m_1 m_2 g \cos \theta = 0$$

Factor things:

$$-T(m_1 + m_2) = m_1 m_2 g \cos \theta (\mu_1 - \mu_2)$$

and finally get an expression for T:

$$T = \frac{m_1 m_2 g \cos\theta(\mu_2 - \mu_1)}{(m_1 + m_2)} \tag{5.8}$$

Hey, that algebra wasn't so bad, was it? Now we have general expressions for T and a. Plugging numbers into Eq. 5.8, we get:

$$
\begin{aligned}
T &= \frac{(1.65\,\text{kg})(3.30\,\text{kg})(9.80\,\frac{\text{m}}{\text{s}^2})\cos 30°(0.113 - 0.226)}{(1.65\,\text{kg} + 3.30\,\text{kg})} \\
&= -1.05\,\text{N}
\end{aligned}
$$

Oops! T came out negative! What we find from this is that the assumption was wrong and the rod is really being *stretched* as the blocks slide down the slope, and the magnitude of the rod's tension is $1.05\,\text{N}$.

(b) To find the acceleration of the blocks, plug numbers into Eq. 5.7:

$$
\begin{aligned}
a &= \frac{(9.80\,\frac{\text{m}}{\text{s}^2})\left[(1.65\,\text{kg} + 3.30\,\text{kg})\sin 30° - \cos 30°((0.226)(1.65\,\text{kg}) + (0.113)(3.30\,\text{kg}))\right]}{(1.65\,\text{kg} + 3.30\,\text{kg})} \\
&= 3.62\,\tfrac{\text{m}}{\text{s}^2}
\end{aligned}
$$

The (common) acceleration of the blocks is $3.62\,\frac{\text{m}}{\text{s}^2}$.

(c) Now we ask: What would the answers to (a) and (b) be if the blocks had slid down the slope with m_1 in the lead? Would it make any difference at all? It might, since the friction forces on the masses come from two different μ's. In any case, with our analytic results Eqs. 5.7 and 5.8 we can find the results of switching the labels "1" and "2", since that is all we would get from having the blocks switch positions.

If we switch "1" and "2" in Eq. 5.7, we can see that the result for a will not change at all because the sums within that expression are not affected by the switch. So the connected blocks will slide down the slope with the *same* acceleration, namely $3.62\,\frac{\text{m}}{\text{s}^2}$ for the given values.

What about T? From Eq. 5.8 we see that switching "1" and "2" gives an overall *change in sign* because of the factor $(\mu_2 - \mu_1)$. (The other factors don't change for this switch.) So we know that plugging in the numbers for the case where blocks 1 leads would give $T = +1.05\,\text{N}$, and since this is a positive number, the assumption about the rod being compressed (and as a result pushing outward) would be correct. So for the case where m_1 leads, the *magnitude* of the rod's tension is the same $(1.05\,\text{N})$, but now it pushing *outward*.

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5. A $3.0 - \text{kg}$ **block starts from rest at the top of a** $30.0°$ **incline and slides** $2.0\,\text{m}$ **down the incline in** $1.5\,\text{s}$**. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between the block and the plane, (c) the frictional force acting on the block and (d) the speed of the block after it has slid** $2.0\,\text{m}$**.** [Ser4 5-57]
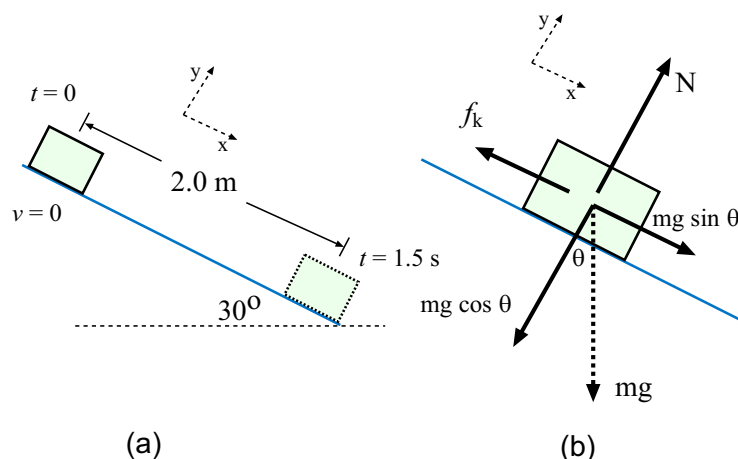
Figure 5.8: (a) Block slides down rough slope in Example 5. (b) Forces acting on the block.

(a) The basic information about the motion of the block is summarized in Fig. 5.8(a). We use a coordinate system where x points down the slope and y is perpendicular to the slope. We'll put the origin of the coordinate system at the place where the block begins its motion.

The block's motion down the slope is one of constant acceleration. (This must be so, since all of the forces acting on the block are constant.) Of course, this is an acceleration in the x direction, as there is no y motion. It begins its slide starting from rest ($v_{0x} = 0$) and so the block's motion is given by:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}a_x t^2 \ .$$

We are told that at $t = 1.5\,\text{s}$, $x = 2.0\,\text{m}$. Substitute and solve for a_x:

$$2.0\,\text{m} = \tfrac{1}{2}a_x(1.5\,\text{s})^2 \qquad \Longrightarrow \qquad a_x = \frac{2(2.0\,\text{m})}{(1.5\,\text{s})^2} = 1.78\,\tfrac{\text{m}}{\text{s}^2}$$

The magnitude of the block's acceleration is $1.78\,\tfrac{\text{m}}{\text{s}^2}$.

(b) We must now think about the forces which act on the block. They are shown in Fig. 5.8(b). Gravity pulls downward with a force mg, which we decompose into its components along the slope and perpendicular to it. The surface exerts a normal force N. There is also a force of kinetic friction from the slope. Since the block is moving down the slope, the friction force must point *up* the slope.

The block moves only along the x axis; the forces in the y direction must sum to zero. Referring to Fig. 5.8(b), we get:

$$\sum F_y = N - mg\cos\theta = 0 \qquad \Longrightarrow \qquad N = mg\cos\theta \ .$$

This gives us the normal force of the surface on the block; here, $\theta = 30.0°$.

The block *does* have an acceleration in the x direction, which we've already found in part (a). The sum of the forces in the $+x$ direction gives ma_x:

$$\sum F_x = mg\sin\theta - f_{\text{k}} = ma_x$$

Now we use the formula for the force of kinetic friction: $f_k = \mu_k N$. Using our expression for the normal force gives us:

$$f_k = \mu_k N = \mu_k mg \cos \theta$$

and using this result in the last equation gives

$$mg \sin \theta - \mu_k mg \cos \theta = ma_x \ .$$

Here, the *only* unknown is μ_k, so we find it with a little algebra: First off, we can cancel the common factor of m that appears in all terms:

$$g \sin \theta - \mu_k g \cos \theta = a_x$$

and then solve for μ_k:

$$
\begin{aligned}
\mu_k g \cos \theta &= g \sin \theta - a_x \\
&= (9.80 \tfrac{m}{s^2}) \sin 30.0° - (1.78 \tfrac{m}{s^2}) = 3.12 \tfrac{m}{s^2}
\end{aligned}
$$

So we get:

$$\mu_k = \frac{(3.12 \tfrac{m}{s^2})}{(9.80 \tfrac{m}{s^2})(\cos 30.0°)} = 0.368$$

(c) As we have seen in part (b), the magnitude of the (kinetic) friction force on the mass is

$$
\begin{aligned}
f_k &= \mu_k mg \cos \theta \\
&= (0.368)(3.0 \, \text{kg})(9.80 \tfrac{m}{s^2}) \cos 30.0° \\
&= 9.4 \, \text{N}
\end{aligned}
$$

The force of friction is $9.4 \, \text{N}$.

(d) We know the acceleration of the block, its initial velocity $(v_{0x} = 0)$ and the time of travel to slide $2.0 \, \text{m}$; its final velocity is

$$v = v_{0x} + a_x t = 0 + (1.78 \tfrac{m}{s^2})(1.50 \, \text{s}) = 2.67 \tfrac{m}{s}$$

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6. Three masses are connected on a table as shown in Fig. 5.9. The table has a coefficient of sliding friction of 0.35. The three masses are $4.0 \, \text{kg}$, $1.0 \, \text{kg}$, and $2.0 \, \text{kg}$, respectively and the pulleys are frictionless. (a) Determine the acceleration of each block and their directions. (b) Determine the tensions in the two cords.
[Ser4 5-59]

(a) First, a little thinking about what we expect to happen. Surely, since the larger mass is hanging on the left side we expect the $4.0 \, \text{kg}$ mass to accelerate downward, the 1.0 [kg block to accelerate to the right and the $2.0 \, \text{kg}$ block to accelerate upward. Also, since the masses are connected by strings as shown in the figure, the *magnitudes* of all three accelerations
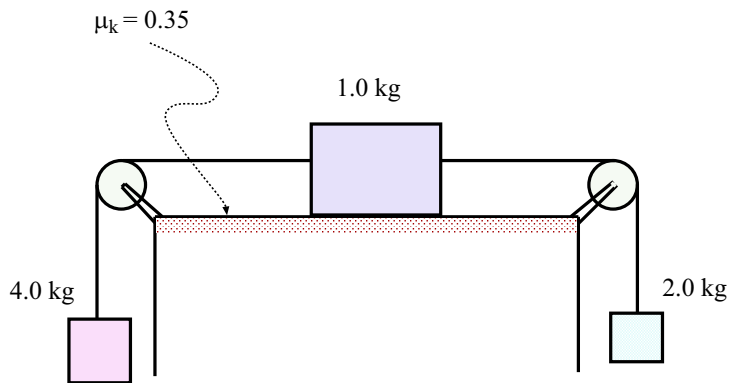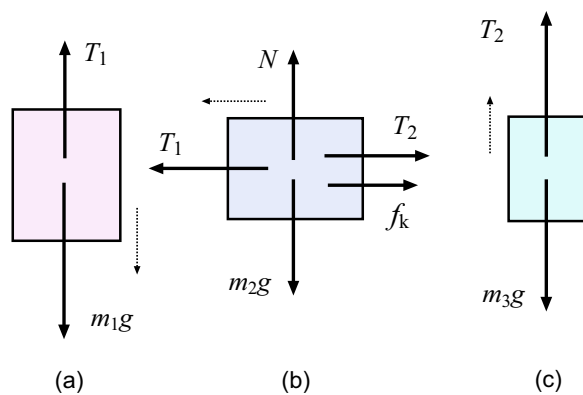
Figure 5.9: System for Example 6

Figure 5.10: Free–body diagrams for the three masses in Example 6. (a) Forces on the mass $m_1 = 4.0$ kg. (b) Forces on the mass $m_2 = 1.0$ kg. (c) Forces on the mass $m_3 = 2.0$ kg. The directions of motion assumed for each mass are also shown.

must be the same, because in any time interval the magnitudes of their *displacements* will always be the same. So each mass will have an acceleration of magnitude a with the direction appropriate for each mass.

Now we consider the forces acting on each mass. We draw free–body diagrams! If the tension in the left string is T_1 then the forces on the 4.0 kg mass are as shown in Fig. 5.10(a). The string tension T_1 pulls upward; gravity pulls downward with a force $m_1 g$.

The forces acting on m_2 are shown in Fig. 5.10(b). We have more of them to think about; gravity pulls with a force $m_2 g$ downward. The table pushes upward with a normal force N. It also exerts a frictional force on m_2 which opposes its motion. Since we think we know which way m_2 is going to go (left!), the friction force f_k must point to the right. There are also forces from the strings. There is a force T_1 to the left from the tension in the first string and a force T_2 pointing to the right from the tension in the other string. (Note, since these are *two different pieces of string*, they can have *different* tensions.)

The forces on m_3 are shown in Fig. 5.10(c). There is a string tension T_2 pulling up and gravity $m_3 g$ pulling down.

All right, lets write down some equations! By Newton's Second Law, the sum of the *downward* forces on m_1 should give $m_1 a$. (We agreed that its acceleration would be downward.) This gives:

$$m_1 g - T_1 = m_1 a \tag{5.9}$$

Moving on to mass m_2, the vertical forces must cancel, giving

$$N = m_2 g \ .$$

Newton tells us that the sum of the *left–pointing* forces must give $m_2 a$ (we decided that its acceleration would be of magnitude a, toward the left) and this gives:

$$T_1 - f_k - T_2 = m_2 a$$

But since

$$f_k = \mu_k N = \mu_k m_2 g \ ,$$

this becomes

$$T_1 - \mu_k m_2 g - T_2 = m_2 a \ . \tag{5.10}$$

Finally, the sum of the *upward* forces on m_3 must give $m_3 a$. So:

$$T_2 - m_3 g = m_3 a \tag{5.11}$$

Having done this work in writing down these wonderful equations we stand back, admire our work and ask if we can go on to solve them. We note that there are three unknowns (a, T_1 and T_2) and we have three equations. We *can* find a solution. The *physics* is *done*... only the algebra remains.

We can do the algebra in the following way: If we just add Eqs. 5.9, 5.10 and 5.11 together (that is, add all the left–hand–sides together and the right–hand–sides together) we find that both T's cancel out. We get:

$$m_1 g - T_1 + T_1 - \mu_k m_2 g - T_2 +_2 - m_3 g = m_1 a + m_2 a + m_3 a$$

which simplifies to:

$$m_1 g - \mu_k m_2 g - m_3 g = (m_1 + m_2 + m_3)a$$

Now we can easily find a:

$$
\begin{aligned}
a &= \frac{(m_1 - \mu_k m_2 - m_3)g}{(m_1 + m_2 + m_3)} \\
&= \frac{[(4.0\,\text{kg}) - (0.35)(1.0\,\text{kg}) - (2.0\,\text{kg})](9.80\,\frac{\text{m}}{\text{s}^2})}{(4.0\,\text{kg} + 1.0\,\text{kg} + 2.0\,\text{kg})} \\
&= \frac{(1.65\,\text{kg})(9.80\,\frac{\text{m}}{\text{s}^2})}{(7.0\,\text{kg})} = 2.31\,\frac{\text{m}}{\text{s}^2}
\end{aligned}
$$

So our *complete* answer to part (a) is: m_1 accelerates at $2.31\,\frac{\text{m}}{\text{s}^2}$ downward; m_2 accelerates at $2.31\,\frac{\text{m}}{\text{s}^2}$ to the left; m_3 accelerates at $2.31\,\frac{\text{m}}{\text{s}^2}$ upward.

(b) Finding the tensions in the strings is now easy; just use the equations we found in part (a).

To get T_1, we can use Eq. 5.9, which gives us:

$$T_1 = m_1 g - m_1 a = m_1(g - a) = (4.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2} - 2.31\,\tfrac{\text{m}}{\text{s}^2}) = 30.0\,\text{N} \ .$$

To get T_2 we can use Eq. 5.11 which gives us:

$$T_2 = m_3 g + m_3 a = m_3(g + a) = (2.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2} + 2.31\,\tfrac{\text{m}}{\text{s}^2}) = 24.2\,\text{N} \ .$$

The tension in the string on the left is $30.0\,\text{N}$. The tension in the string on the right is $24.2\,\text{N}$.

---

7. A block is placed on a plane inclined at $35°$ relative to the horizontal. If the block slides down the plane with an acceleration of magnitude $g/3$, determine the coefficient of kinetic friction between block and plane. [Ser4 5-61]

The forces acting on the block (which has mass m) as it slides down the inclined plane are shown in Fig. 5.11. The force of gravity has magnitude mg and points straight down; here it is split into components normal to the slope and down the slope, which have magnitudes $mg\cos\theta$ and $mg\sin\theta$, respectively, with $\theta = 35°$. The surface exerts a normal force N and a force of kinetic friction, f_k, which, since the block is moving *down* the slope, points *up* the slope.

The block can only accelerate along the direction of the slope, so the forces perpendicular to the slope must add to zero. This gives us:

$$N - mg\cos\theta = 0 \qquad \Longrightarrow \qquad N = mg\cos\theta$$

The acceleration of the block down the slope was given to us as $a = g/3$. Then summing the forces which point along the slope, we have
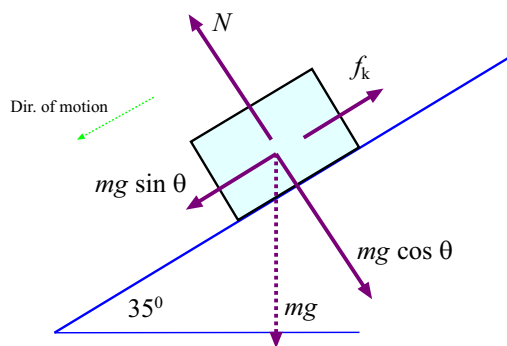
$$mg\sin\theta - f_\text{k} = ma = mg/3$$

Figure 5.11: Forces on the block in Example 7.

The force of kinetic friction is equal to $\mu_k N$, and using our expression for N we have

$$f_k = \mu_k N = \mu_k mg \cos \theta$$

and putting this into the previous equation gives:

$$mg \sin \theta - \mu_k mg \cos \theta = mg/3 \ .$$

Fortunately, the mass m cancels from this equation; we get:

$$g \sin \theta - \mu_k g \cos \theta = g/3$$

And now the only unknown is μ_k which we solve for:

$$\mu_k g \cos \theta = g \sin \theta - \frac{g}{3} = g(\sin \theta - \tfrac{1}{3})$$

Here we see that g also cancels, although we always knew the value of g! We then get:

$$\mu_k = \frac{(\sin \theta - \tfrac{1}{3})}{\cos \theta} = \frac{(\sin 35° - \tfrac{1}{3})}{\cos 35°} = 0.293$$

So the coefficient of kinetic friction between block and slope is 0.293.

---

8. A $2.0\,\text{kg}$ **block is placed on top of a** $5.0\,\text{kg}$ **as shown in Fig. 5.12. The coefficient of kinetic friction between the** $5.0\,\text{kg}$ **block and the surface is** 0.20**. A horizontal force F is applied to the** $5.0\,\text{kg}$ **block. (a) Draw a free–body diagram for each block. What force accelerates the** $2.0\,\text{kg}$ **block? (b) Calculate the magnitude of the force necessary to pull both blocks to the right with an acceleration of** $3.0\,\frac{\text{m}}{\text{s}^2}$**. (c) find the minimum coefficient of static friction between the blocks such that the** $2.0\,\text{kg}$ **block does not slip under an acceleration of** $3.0\,\frac{\text{m}}{\text{s}^2}$**.** [Ser4 5-73]

(a) What forces act on each block?

On the big block (with mass $M = 5.0\,\text{kg}$, let's say) we have the applied force **F** which pulls to the right. There is the force of gravity, Mg downward. The surface exerts a normal
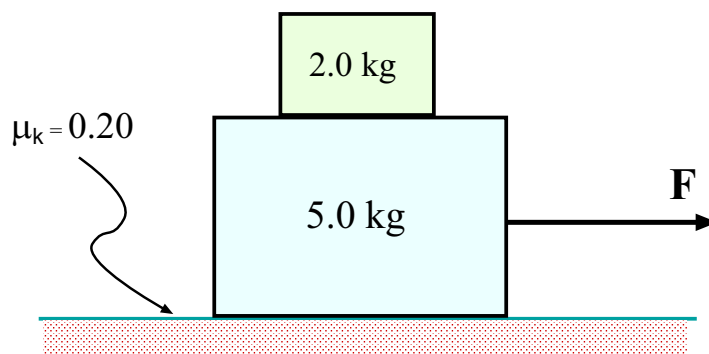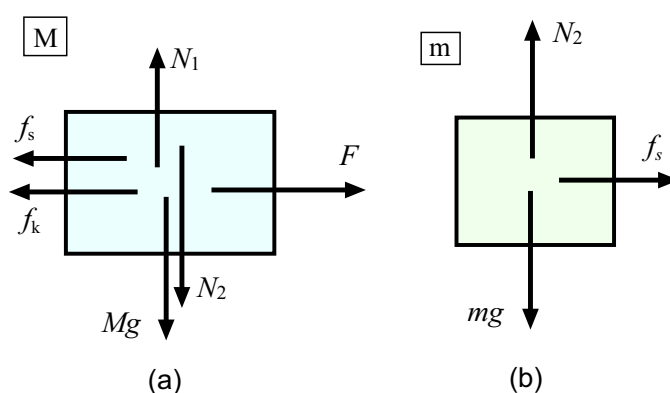
Figure 5.12: Figure for Example 8.

Figure 5.13: (a) Forces acting on the large block, $M = 5.0\,\text{kg}$. (b) Forces acting on the small block, $m = 2.0\,\text{kg}$.

force N_1 upward. There is a friction force from the surface, which is directed to the left. The small mass will also exert forces on mass M; it exerts a normal force N_2 which is directed *downward*; we know this because M is pushing upward on m. Now, M is exerting a force of static friction f_s on m which goes to the right; so m must exert a friction force f_s on M which points to the left.

These forces are diagrammed in Fig. 5.13(a).

On the small block we have the force of gravity, mg downward. Mass M exerts an upward normal force N_2 on it, and also a force of static friction f_s on it, pointing to the right. It is this force which accelerates m as it moves along with M (without slipping). These forces are diagrammed in Fig. 5.13(b).

Notice how the forces between M and m, namely N_2 (normal) and f_s, have the same magnitude but opposite directions, in accordance with Newton's Third Law. They are so–called "action–reaction pairs".

(b) The blocks will have a horizontal acceleration but no vertical motion, so that allows us to solve for some of the forces explained in part (a). The vertical forces on m must sum to zero, giving us:

$$N_2 - mg = 0 \quad \Longrightarrow \quad N_2 = mg = (2.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 19.6\,\text{N}$$

and the vertical forces on M must sum to zero, giving us:

$$N_1 - N_2 - Mg = 0 \qquad \Longrightarrow \qquad N_1 = N_2 + Mg = 19.6\,\text{N} + (5.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 68.6\,\text{N}$$

We are given that the acceleration of *both* blocks is $3.0\,\tfrac{\text{m}}{\text{s}^2}$. Applying Newton's Second Law to mass m we find:

$$\sum F_x = f_s = ma_x = (2.0\,\text{kg})(3.0\,\tfrac{\text{m}}{\text{s}^2}) = 6.0\,\text{N}$$

While applying it to M gives

$$\sum F_x = F - f_k - f_s = Ma_x = (5.0\,\text{kg})(3.0\,\tfrac{\text{m}}{\text{s}^2}) = 15.0\,\text{N}$$

We found f_s above; we do know the force of kinetic friction (from M's sliding on the surface) because we know the coefficient of kinetic friction and the normal force N_1:

$$f_k = \mu_k N_1 = (0.20)(68.6\,\text{N}) = 13.7\,\text{N}$$

Now we can solve for F:

$$\begin{aligned} F &= 15.0\,\text{N} + f_k + f_s \\ &= 15.0\,\text{N} + 13.7\,\text{N} + 6.0\,\text{N} \\ &= 34.7\,\text{N} \end{aligned}$$

To pull the blocks together to the right with an acceleration $3.0\,\tfrac{\text{m}}{\text{s}^2}$ we need an applied force of 34.7 N.

(c) As we've seen, mass m accelerates because of the friction force f_s (from M's surface) which acts on it. Forces of static friction have a maximum value; here we know that we must have

$$f_s \le \mu_s N_2$$

in order for m not to slip on M. Here, we have $f_s = 6.0\,\text{N}$ and $N_2 = 19.6\,\text{N}$. So the critical value of μ_s for our example is

$$\mu_s = \frac{f_2}{N_2} = 0.306$$

If μ_s is less than this value, static friction cannot supply the force needed to accelerate m at $3.0\,\tfrac{\text{m}}{\text{s}^2}$. So $\mu_s = 0.306$ is the minimum value of the coefficient of static friction so that the upper block does not slip.

5.2.2   Uniform Circular Motion Revisited

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9. A toy car moving at constant speed completes one lap around a circular track (a distance of $200\,\text{m}$**) in** $25.0\,\text{s}$**. (a) What is the average speed? (b) If the mass of the car is** $1.50\,\text{kg}$**, what is the magnitude of the central force that keeps it in a circle?** [Ser4 6-1]
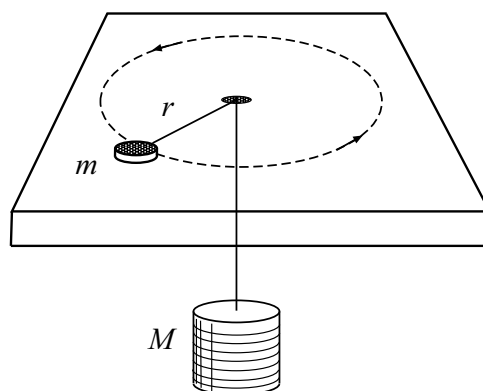
Figure 5.14: Mass m moves; mass M hangs!

(a) If a lap around the circular track is of length $200\,\text{m}$ then the (average) speed of the car is

$$v = \frac{d}{t} = \frac{200\,\text{m}}{25.0\,\text{s}} = 8.00\,\tfrac{\text{m}}{\text{s}}$$

(b) The car undergoes uniform circular motion, moving in a circle of radius r with speed v. The net force on the car points toward the *center of the circle* and has magnitude

$$F_{\text{cent}} = \frac{mv^2}{r}$$

Actually, we haven't found r yet. We are given the circumference of the circle, and from $C = 2\pi r$ we find

$$r = \frac{C}{2\pi} = \frac{200\,\text{m}}{2\pi} = 31.8\,\text{m}$$

So the net force on the car has magnitude

$$F_{\text{cent}} = \frac{mv^2}{r} = \frac{(1.50\,\text{kg})(8.00\,\tfrac{\text{m}}{\text{s}})^2}{(31.8\,\text{m})} = 3.02\,\text{N}$$

The net force on the car has magnitude $3.02\,\text{N}$; its direction is always inward, keeping the car on a circular path.

---

10. A mass M on a frictionless table is attached to a hanging mass M by a cord through a hole in the table, as shown in Fig. 5.14. Find the speed with which m must move in order for M to stay at rest. [HRW5 6-57]

Taking mass M to be at rest, we see that mass m must be moving in a circle of constant radius r. It is moving at (constant) speed v; so mass m undergoes uniform circular motion. So the net force on m points toward the center of the circle and has magnitude $F_{\text{cent}} = mv^2/r$. The free–body diagram for m is shown in Fig. 5.15(a). The *only* force on m is the string
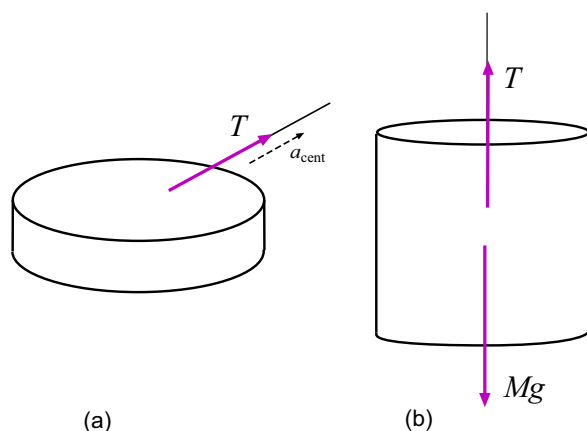
Figure 5.15: (a) Force on mass m and the direction of its acceleration. (There are also vertical forces, gravity and the table's normal force, which cancel; these are not shown.) (b) Forces acting on hanging mass M.

tension (pointing toward the center of the circle). This gives us:

$$T = \frac{mv^2}{r}$$

Next consider the forces acting on M and its motion. The force diagram for M is shown in Fig. 5.15(b). Since mass M is at rest, the net force on it is zero, which gives:

$$T = Mg$$

Combining these two results, we get:

$$\frac{mv^2}{r} = Mg$$

Solving for v, we get:

$$v^2 = \frac{Mgr}{m} \qquad \Longrightarrow \qquad v = \sqrt{\frac{Mgr}{m}}$$

---

11. A stuntman drives a car over the top of a hill, the cross section of which can be approximated by a circle of radius $250\,\mathrm{m}$, as in Fig. 5.16. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill? [HRW5 6-58]

We begin by thinking about the forces acting on the car and its acceleration when it is at the top of the hill.

At the top of the hill, the car is moving in a circular path of radius $r = 250\,\mathrm{m}$ with some speed v. Then the car has a centripetal acceleration of magnitude v^2/r which is directed downward. (For all we know, it may also have a tangential acceleration as well, but the
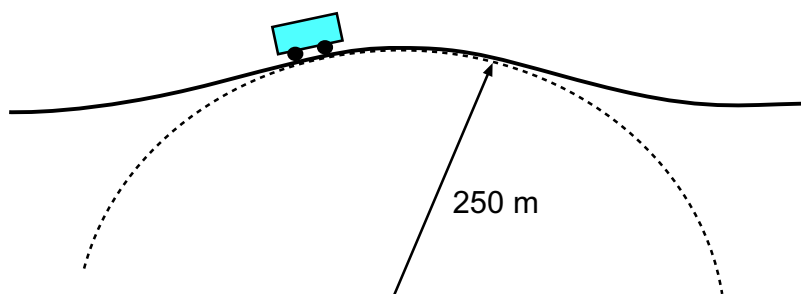
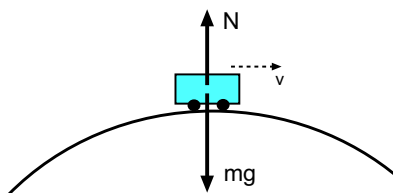Figure 5.16: Car drives over the top of a hill in Example 11.

Figure 5.17: Forces acting on the car in Example 11 when it is at the top of the hill.

problem gives no information on it, and it won't be relevant for the problem.) By Newton's Second Law, the net (vertical) force on the car must have magnitude mv^2/r and must be directed *downward*.

The forces acting on the car are shown in Fig. 5.17. Then the force of gravity is mg downward. The road exerts a normal force of magnitude N upward. One may ask how we know the road's force goes upward. This is because there is no physical way in which a road can pull downward on a car driving over it. But it can push *up*.

We combine the results from the last two paragraphs. The net *downward* force must equal mv^2/r. This gives us:

$$mg - N = \frac{mv^2}{r} \ .$$

however without knowing anything more, we can't solve for v in this equation because we don't know N (or, for that matter, m).

We have not yet used the condition that the car is on the verge of leaving the road at the top of the hill. What does this condition give us? If we use the last equation to find the normal force:

$$N = mg - \frac{mv^2}{r}$$

we see that if we increase v there comes a point at which N must be negative in order for the car to stay on the road moving on its circular arc. But as discussed above, N *can't* be negative. But it can be zero, and it is for this speed that the car is on the verge of leaving the road at the top of the hill. The critical case has $N = 0$, and this gives us:

$$0 = mg - \frac{mv^2}{r} \qquad \Longrightarrow \qquad \frac{mv^2}{r} = mg \ .$$
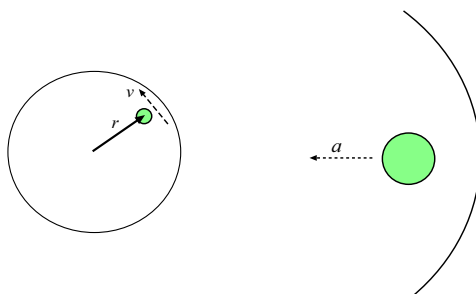
Figure 5.18: Coin moves with a rotating turntable

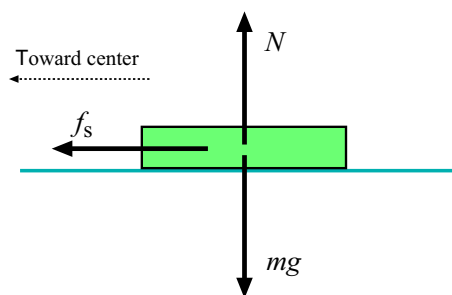

Figure 5.19: Forces acting on the coin in Example 12

Note that the mass m cancels out of this equation so we don't need to know m. We get:

$$v^2 = rg = (250\,\text{m})(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 2.45 \times 10^3\,\tfrac{\text{m}^2}{\text{s}^2}$$

and finally

$$v = 49.5\,\tfrac{\text{m}}{\text{s}}\ .$$

The car may be driven as fast as $49.5\,\tfrac{\text{m}}{\text{s}}$ and it will stay on the road.

---

12. A coin placed $30.0\,\text{cm}$ **from the center of a rotating, horizontal turntable slips when its speed is** $50.0\,\tfrac{\text{cm}}{\text{s}}$**. (a) What provides the central force when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between the coin and turntable?** [Ser4 6-13]

(a) See Fig. 5.18 for a fine illustration of the problem.

As the coin executes uniform circular motion (before it slips) it is accelerating toward the center of the turntable! So there *must* be a force (or forces) on the coin causing it to do this. This force can only come from its contact interaction with the turntable, i.e. from friction. Here, since we are dealing with the case where the coin is not sliding with respect to the surface, it is the force of *static* friction. Furthermore, the force of static friction is directed *toward the center of the turntable*.

(b) A view of the forces acting on the coin is given in Fig. 5.19. If the mass of the coin is m then gravity exerts a force mg downward, the turntable exerts a normal force N upward

and there is a force of static friction, which as we discussed in part (a) *must* point toward the center of the turntable.

The acceleration of the coin points toward the center of the circle and has magnitude v^2/r, (r being the distance of the coin from the center). So the vertical forces must cancel, giving us $N = mg$. The net force points inward and has magnitude mv^2/r, so that $f_s = mv^2/r$.

Now for the conditions at which the coin starts to slip, the force of static friction has reached its maximum value, i.e.

$$f_s = \mu_s N$$

but here we can use our results to substitute for f_s and for N. This give us:

$$\frac{mv^2}{r} = \mu_s mg$$

which lets us solve for μ:

$$\mu_s = \frac{v^2}{rg} = \frac{(50.0\,\frac{cm}{s})^2}{(30.0\,cm)(9.80\,\frac{m}{s^2})} = \frac{(0.500\,\frac{m}{s})^2}{(0.300\,m)(9.80\,\frac{m}{s^2})} = 8.50 \times 10^{-2}$$

So the coefficient of static friction for the turntable and coin is $\mu_s = 8.50 \times 10^{-2}$.

We were never given the mass of the coin, but we did not need it because it cancelled out of our equations just before the final answer.

---

13. A Ferris wheel rotates four times each minute; it has a diameter of $18.0\,$m.
(a) What is the centripetal acceleration of a rider? What force does the seat exert on a $40.0-$ kg **rider (b) at the lowest point of the ride and (c) at the highest point of the ride? (d) What force (magnitude and direction) does the seat exert on a rider when the rider is halfway between top and bottom?** [Ser4 6-51]

(a) First, calculate some numbers which we know are important for circular motion! The wheel turns around 4 times in one minute, so the time for *one* turn must be

$$T = \frac{1.0\,min}{4} = \frac{60.0\,s}{4} = 15\,s\ .$$

Also, since the radius of the wheel is $R = D/2 = 18.0\,m/2 = 9.0\,m$, the circumference of the wheel is

$$C = 2\pi R = 2\pi(9.0\,m) = 57\,m$$

and then the speed of a rider is

$$v = \frac{C}{T} = \frac{57\,m}{15\,s} = 3.8\,\frac{m}{s}\ .$$

The rider moves at constant speed in a circular path of radius R. So the rider's acceleration is always *directed toward the center of the circle* and it has magnitude

$$a_{cent} = \frac{v^2}{R} = \frac{(3.8\,\frac{m}{s})^2}{(9.0\,m)} = 1.6\,\frac{m}{s^2}$$
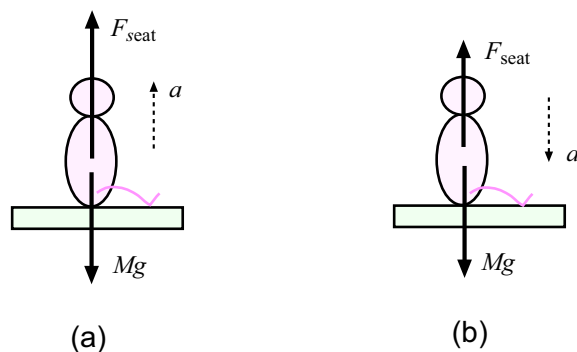
Figure 5.20: Forces acting on the Ferris wheel rider (a) at the lowest point of the ride and (b) at the highest point of the ride.

The centripetal acceleration of the rider is $1.6 \, \frac{m}{s^2}$.

(b) Consider what is happening when rider is at the lowest point of the ride. His acceleration is *upward* (toward the center of the circle!) and has magnitude $1.6 \, \frac{m}{s^2}$.

What are the forces acting on the rider (who has mass M, let's say) at this point? These are shown in Fig. 5.20(a). Gravity pulls down on the rider with a force of magnitude Mg, and the seat pushes upward on the rider with a force F_{seat}. (Usually seats *can't* pull downward; also, the force of the seat can't have any sideways component because here the net force must point *upward*). Since the net force points upward and has magnitude $F_{\text{cent}} = Mv^2/R$, Newton's Second Law gives us:

$$F_{\text{seat}} - Mg = \frac{Mv^2}{R}$$

Since $M = 40.0 \, \text{kg}$, we get:

$$F_{\text{seat}} = Mg + \frac{Mv^2}{R} = (40.0 \, \text{kg})(9.80 \, \tfrac{m}{s^2}) + \frac{(40.0 \, \text{kg})(3.8 \, \frac{m}{s})^2}{(9.0 \, \text{m})} = 460 \, \text{N}$$

The seat pushes upward on the rider with a force of magnitude $460 \, \text{N}$. We might say that when the rider at the lowest point, the rider has an *apparent weight* of $460 \, \text{N}$, since that is the force of the surface on which the rider rests. Here, the apparent weight is *greater than* the true weight Mg of the rider.

(c) When the rider is at the highest point of the wheel, his acceleration is *downward*. The forces acting on the rider are shown in Fig. 5.20(b); these are the same forces as in part (a) but now the net force points *downward* and has magnitude $F_{\text{cent}} = Mv^2/R$. Adding up the *downward* forces, Newton's Second Law now gives us:

$$Mg - F_{\text{seat}} = \frac{Mv^2}{R}$$

which now gives us

$$F_{\text{seat}} = Mg - \frac{Mv^2}{R} = (40.0 \, \text{kg})(9.80 \, \tfrac{m}{s^2}) - \frac{(40.0 \, \text{kg})(3.8 \, \frac{m}{s})^2}{(9.0 \, \text{m})} = 330 \, \text{N} \ .$$
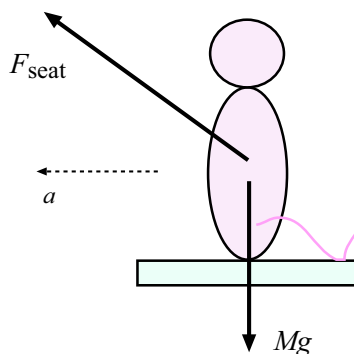
Figure 5.21: Forces on the rider when he is halfway between top and bottom.

The seat pushes upward on the rider with a force of magnitude $330\,\mathrm{N}$. We would say that at the top of the ride, the apparent weight of the rider is $330\,\mathrm{N}$. This time the apparent weight is *less than* the true weight of the rider.

(d) When the rider is halfway between top and bottom, the net force on him still points toward the center of the circle (and has magnitude Mv^2/R), but in this case the direction is *horizontal*, as indicated in Fig. 5.21. (In this picture the rider is on the *right* side of the Ferris wheel, as we look at it face–on.) The forces acting on the rider are also shown in this picture. The force of gravity, Mg can only pull downward. The only other force on the rider, namely that of the seat does *not* push straight upward in this case. We know that it can't, because the sum of the two forces must point horizontally (to the right). The force of the seat must also have a horizontal component; it must point as shown in Fig. 5.21.

Without being overly formal about the mathematics we can see that the vertical component of $\mathbf{F}_{\mathrm{seat}}$ must be Mg so as to cancel the force of gravity. The vertical component of $\mathbf{F}_{\mathrm{seat}}$ must have magnitude

$$F_{\mathrm{seat,\ vert}} = Mg = (40.0\,\mathrm{kg})(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 392\,\mathrm{N}$$

The horizontal component of $\mathbf{F}_{\mathrm{seat}}$ must equal Mv^2/R since as we've seen, that is the net force on the rider. So:

$$F_{\mathrm{seat,\ horiz}} = \frac{Mv^2}{R} = \frac{(40.0\,\mathrm{kg})(3.8\,\tfrac{\mathrm{m}}{\mathrm{s}})^2}{(9.0\,\mathrm{m})} = 64\,\mathrm{N}$$

Now we can find the magnitude of the force of the seat:

$$
\begin{aligned}
F_{\mathrm{seat}} &= \sqrt{F_{\mathrm{seat,\ vert}}{}^2 + F_{\mathrm{seat,\ horiz}}{}^2} \\
&= \sqrt{(392\,\mathrm{N})^2 + (64\,\mathrm{N})^2} \\
&= 397\,\mathrm{N}
\end{aligned}
$$

and this force is directed at an angle θ above the horizontal, where θ is given by

$$\theta = \tan^{-1}\left(\frac{F_{\mathrm{seat,\ vert}}}{F_{\mathrm{seat,\ horiz}}}\right) = \tan^{-1}\left(\frac{392\,\mathrm{N}}{64\,\mathrm{N}}\right) = 81°$$

The force of the seat has magnitude $397\,\mathrm{N}$ and is directed at $81°$ above the horizontal.
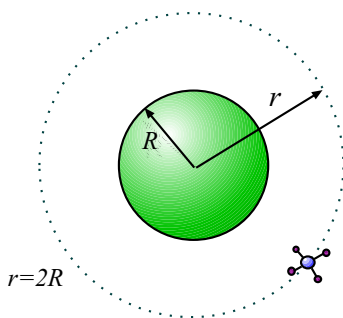
Figure 5.22: Satellite orbits the Earth in Example 14.

5.2.3   Newton's Law of Gravity (Optional for Calculus–Based)

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14. A satellite of mass $300\,\mathrm{kg}$ is in a circular orbit around the Earth at an altitude equal to the Earth's mean radius (See Fig. 5.22.) Find (a) the satellite's orbital speed, (b) the period of its revolution, and (c) the gravitational force acting on it. Use: $R_{\mathrm{Earth}} = 6.37 \times 10^6\,\mathrm{m}$ and $M_{\mathrm{Earth}} = 5.98 \times 10^{24}\,\mathrm{kg}$. [Ser4 6-6]

(a) (Comment: This was the way the problem was originally stated. I will find the answers in a different order!)

We are told that the height of satellite *above the surface of the Earth* is equal to the Earth's radius. This means that the radius of the satellite's orbit is equal to *twice* the radius of the Earth. Since the mean radius of the Earth is $R = 6.37 \times 10^6\,\mathrm{m}$, then the orbit radius is

$$r = 2R = 2(6.37 \times 10^6\,\mathrm{m}) = 1.27 \times 10^7\,\mathrm{m}\ .$$

The satellite is always at this distance from the center of the Earth; Newton's law of gravitation tells us the force which the Earth exerts on the satellite:

$$
\begin{aligned}
F_{\mathrm{grav}} &= G\frac{m_{\mathrm{sat}}M_{\mathrm{Earth}}}{r^2} \\
&= \left(6.67 \times 10^{-11}\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{(300\,\mathrm{kg})(5.98 \times 10^{24}\,\mathrm{kg})}{(1.27 \times 10^7\,\mathrm{m})^2} \\
&= 7.42 \times 10^2\,\mathrm{N}
\end{aligned}
$$

This force is always directed toward the center of the Earth. Since this is the only force which acts on the satellite, it is also the (net) centripetal force on it:

$$F_{\mathrm{cent}} = \frac{mv^2}{r} = 7.42 \times 10^2\,\mathrm{N}$$

We can now find the speed of the satellite. It is:

$$v^2 = \frac{rF_{\mathrm{cent}}}{m} = \frac{(1.27 \times 10^7\,\mathrm{m})(7.42 \times 10^2\,\mathrm{N})}{(300\,\mathrm{kg})} = 3.14 \times 10^7\ \tfrac{\mathrm{m^2}}{\mathrm{s^2}}$$

which gives
$$v = 5.60 \times 10^3 \, \tfrac{\text{m}}{\text{s}} \ .$$

So the satellite's orbital speed is $5.60 \times 10^3 \, \tfrac{\text{m}}{\text{s}}$.

(b) Recall that the speed of an object in uniform circular motion is related to the period and radius by:
$$v = \frac{2\pi r}{T}$$

From this we get the period of the satellite's orbit:

$$
\begin{aligned}
T &= \frac{2\pi r}{v} \\
&= \frac{2\pi (1.27 \times 10^7 \, \text{m})}{(5.60 \times 10^3 \, \tfrac{\text{m}}{\text{s}})} \\
&= 1.42 \times 10^4 \, \text{s} = 3.96 \, \text{hr}
\end{aligned}
$$

The period of the satellite is $3.96 \, \text{hr}$.

(c) The answer to this part has been found already! The gravitational force acting on the satellite is $7.42 \times 10^2 \, \text{N}$.