# Appendix A - Statistical Tables

# Table A.1
# Table of Random Numbers

52285 53301 71193 18991 34854 61701 10262 41876 19487 06996
70189 45193 46899 90746 97060 46547 64523 16987 60706 51116
83441 17072 50243 83300 63817 07510 05828 95271 07689 29757
16254 51933 02155 93543 20033 88132 16695 58878 39877 32928
84056 70489 14252 94132 04605 38293 60501 20415 82886 04396

01398 76800 59510 70789 59184 72725 81987 60820 67407 06777
46083 69602 70703 54693 85747 69453 40158 84787 65193 07982
74331 57826 36074 65258 56350 67475 10856 05061 54175 45490
25271 57349 62441 93647 49612 26541 48268 02745 50788 47621
39202 76311 95471 81348 27869 51539 78557 98949 37103 52406

34579 78142 36414 51185 76519 93762 52903 80133 50426 60660
05815 46497 15942 97225 73766 07688 03995 63519 96185 79889
12256 07450 14703 66624 65096 05991 47792 58688 42390 72273
47813 55926 97304 99955 38725 88637 32076 00059 69312 80328
14387 76015 22189 48236 40675 56748 08779 05052 81391 91175

12871 38800 45132 29899 05541 97211 00228 37943 75643 24707
01790 16008 71773 70610 30776 99122 06371 95624 85352 10541
51382 87353 27396 55549 61991 35378 10592 91612 24863 82825
07971 05251 88080 17223 00214 02158 32327 39454 00488 12942
10387 42632 71863 15885 19797 33809 43331 23947 52006 49206

12011 45094 27304 69411 93673 44863 57021 27073 02530 05554
58606 04896 14194 50974 15115 47297 46172 09915 53970 45107
32011 56324 53923 94211 02603 31294 56761 62518 28402 94369
76525 54749 54816 27929 16543 25163 23840 82143 67689 79686
00885 78999 80525 67499 38229 07047 70281 27868 28125 70575

23897 16928 42382 97327 00010 12017 09077 55481 11533 67540
48352 74755 54420 95487 88860 15474 71007 71176 15957 55571
84380 89850 63181 12825 39798 64614 75342 59859 67318 25476
66567 06900 85623 22070 69356 51678 13675 05150 96915 45290
69947 95093 39012 02347 65222 14159 89423 39265 32780 51071

32141 18317 80061 61293 46152 44829 58578 36048 44759 64105
45368 48058 18536 63557 56784 04185 01216 22678 62302 24258
52706 41960 83558 74152 64791 79863 70243 23171 27735 78682
24526 79798 65401 66916 12948 49739 83458 02173 36878 75965
11143 51658 61071 92410 67354 71470 50351 49670 90852 53216

90436 16823 71004 28796 31809 88289 64149 41630 10620 71445
50695 16631 30928 17151 48671 89919 33993 24160 00432 22646
12661 40143 80449 48565 66144 02950 23605 14699 88568 94758
26095 37076 50570 47774 35325 53309 57110 81659 44003 47856
37401 08878 38040 40886 79776 17453 16386 21934 51879 98593

(*Continued Next Page* ➔)

# Table A.1
# Table of Random Numbers

(*Continued From Previous Page*)

73436 15235 68742 37408 54480 14276 19199 09247 51883 31558
52085 88482 90978 50213 81692 71534 55172 89050 75064 22917
74921 67644 43406 47325 74620 57386 21548 45531 23754 57318
11779 30845 04668 57815 86485 68987 65038 86689 37010 89742
12697 40295 38712 00802 12178 04126 79675 89010 56424 10949

01530 95937 54384 88243 69773 47554 87366 84193 32006 74961
85512 28259 62138 06750 74984 82732 39352 47103 66217 69987
56593 44037 62860 83469 59660 03227 30197 38752 01730 91522
48971 46910 05183 47636 38411 46134 05488 43152 50491 46963
80174 37041 68187 14965 08437 09184 78120 77283 85683 03000

10451 16818 20909 41817 22258 05721 89967 52817 48289 37249
15709 96644 07787 19769 89928 25008 75161 71145 87686 14697
80410 73637 91873 19172 31133 03062 47472 77656 89860 29345
78387 26525 83058 27609 39367 60404 17724 19381 09401 41116
80340 45802 51136 53479 02947 21568 38732 80795 32185 67188

19303 67335 12415 52422 88966 42252 39225 12063 66612 96193
65547 43596 26205 84974 19288 64531 30886 67894 59180 87874
55138 63666 92563 73142 49480 87045 90442 51276 24575 08896
41251 49346 92401 08983 37610 51640 32392 38966 94555 30858
34360 33395 50573 54086 76272 04358 49068 74122 65103 46740

28798 62591 88671 40094 20671 94535 13831 73727 23013 17250
28821 35325 49845 40713 10831 62889 31144 94793 47631 69475
61406 11022 10801 82661 96007 82777 10886 50354 80586 36537
53064 03232 18298 42549 17696 90115 73195 46877 43756 65747
10340 07841 84279 96642 43454 32981 36294 62135 87647 37954

75471 44635 36918 78946 58286 46874 08289 02970 45582 97166
54595 16847 31134 89115 09788 97384 69642 64739 60784 08725
46054 66831 76812 76767 33350 66654 32282 46201 38030 61321
64056 31307 94018 92901 18269 76377 77698 36684 01007 31710
48772 39634 51600 09518 90956 87022 30606 24204 42723 99132

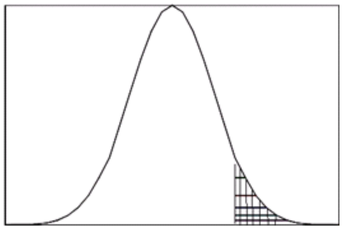
71878 37326 75740 56392 33145 48232 04240 85284 24372 70326
52795 28840 07950 09409 59846 58692 84039 66761 14916 74160
94307 80909 98649 46434 65594 18673 27853 77889 54909 49947
36496 61287 09743 69322 00658 57232 68305 88356 10208 65712
93837 94788 28566 25575 69803 02395 80901 40244 25023 58347

62769 19152 42725 06747 32435 50598 47708 66061 26076 77413
01441 77154 23681 26553 06565 60362 97591 65225 55668 47806
52357 67042 87617 05415 84880 38953 67029 58816 03215 41258
91948 81731 28846 88081 38023 26118 25129 69856 67321 65109
49574 96113 07275 51855 73484 97206 38430 93330 87042 50463

# Table A.2
# Standard Normal Distribution

Areas reported below correspond to the shaded region in the figure on the right.

$$z = \frac{x - \mu}{\sigma}$$




| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2297 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| 1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| 2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| 2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| 2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| 2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| 2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| 2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| 3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |

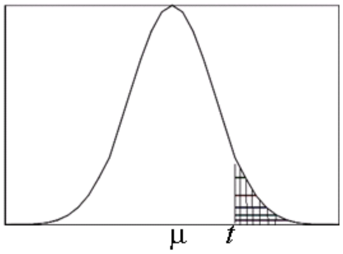
$z_{0.10} = 1.282$ $z_{0.05} = 1.645$ $z_{0.025} = 1.960$
$z_{0.01} = 2.326$ $z_{0.005} = 2.576$ $z_{0.0005} = 3.291$

# Table A.3
# Student's *t* Distributions

Rows are labeled by *df* (degrees of freedom).

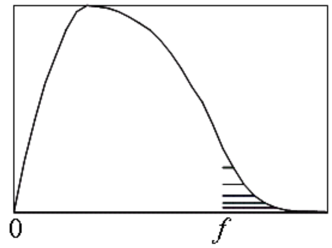
Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are *t* values.




| *df* | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 636.619 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.598 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.921 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.859 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.595 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.405 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.767 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.460 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.373 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 |

# Table A.4
# Fisher's *f* Distributions




Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure on the right. First, second, third, and fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

**Denominator Degrees of Freedom**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 | 60.2 |
| | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 |
| | 4052 | 5000 | 5403 | 5625 | 5764 | 5859 | 5928 | 5981 | 6022 | 6056 |
| | ——— | ——— | ——— | ——— | ——— | ——— | ——— | ——— | ——— | ——— |
| 2 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 |
| | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 |
| | 98.5 | 99.0 | 99.2 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 |
| | 999 | 999 | 999 | 999 | 999 | 999 | 999 | 999 | 999 | 999 |
| 3 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 |
| | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 |
| | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 |
| | 167 | 148 | 141 | 137 | 135 | 133 | 132 | 131 | 130 | 129 |
| 4 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 |
| | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 |
| | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 |
| | 74.1 | 61.2 | 56.2 | 53.4 | 51.7 | 50.5 | 49.7 | 49.0 | 48.5 | 48.1 |
| 5 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 |
| | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 |
| | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 |
| | 47.2 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 27.2 | 26.9 |
| 6 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 |
| | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 |
| | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 |
| | 35.5 | 27.0 | 23.7 | 21.9 | 20.8 | 20.0 | 19.5 | 19.0 | 18.7 | 18.4 |
| 7 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 |
| | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 |
| | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 |
| | 29.2 | 21.7 | 18.8 | 17.2 | 16.2 | 15.5 | 15.0 | 14.6 | 14.3 | 14.1 |

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# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

**Denominator Degrees of Freedom**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 |
| | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 |
| | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 |
| | 25.4 | 18.5 | 15.8 | 14.4 | 13.5 | 12.9 | 12.4 | 12.0 | 11.8 | 11.5 |
| 9 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 |
| | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 |
| | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 |
| | 21.0 | 16.4 | 12.6 | 11.3 | 10.5 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 |
| 10 | 3.92 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 |
| | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 |
| | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 |
| | 21.0 | 14.9 | 12.6 | 11.3 | 10.5 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 |
| 11 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 |
| | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 |
| | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 |
| | 19.7 | 13.8 | 11.6 | 10.3 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 | 7.92 |
| 12 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 |
| | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 |
| | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 |
| | 18.6 | 13.0 | 10.8 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 | 7.29 |
| 13 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 |
| | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 |
| | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 |
| | 17.8 | 12.3 | 10.2 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 | 6.80 |
| 14 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 |
| | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 |
| | 8.88 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 |
| | 17.1 | 11.8 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 | 6.40 |

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# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

| **Denominator Degrees of Freedom** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 |
| | 4.54 | 3.68 | 3.29 | 3.05 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 |
| | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 |
| | 16.6 | 11.3 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 |
| 16 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 |
| | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 |
| | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 |
| | 16.1 | 11.0 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 | 5.81 |
| 17 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 |
| | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 |
| | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 |
| | 15.7 | 10.7 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 | 5.58 |
| 18 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.98 |
| | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 |
| | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 |
| | 15.4 | 10.4 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 | 5.39 |
| 19 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 |
| | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 |
| | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 |
| | 15.1 | 10.2 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 | 5.22 |
| 20 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 |
| | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 |
| | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 |
| | 14.8 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 |
| 21 | 2.96 | 2.57 | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 | 1.92 |
| | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 |
| | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 |
| | 14.6 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 | 4.95 |

(*Continued Next Page* ➔)

# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

**Denominator Degrees of Freedom**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 22 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 |
| | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 |
| | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 |
| | 14.4 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 | 4.83 |
| 23 | 2.94 | 2.55 | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 | 1.89 |
| | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 |
| | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 |
| | 14.2 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 | 4.73 |
| 24 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 |
| | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 |
| | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 |
| | 14.0 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 | 4.64 |
| 25 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 | 1.87 |
| | 4.24 | 3.39 | 2.99 | 2.75 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 |
| | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 |
| | 13.9 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 |
| 26 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 |
| | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 |
| | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 |
| | 13.7 | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.38 | 4.64 | 4.48 |
| 27 | 2.90 | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 | 1.85 |
| | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 |
| | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 |
| | 13.6 | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 | 4.41 |
| 28 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 |
| | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 |
| | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 |
| | 13.5 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 | 4.35 |

(*Continued Next Page* ➔)

# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

| Denominator Degrees of Freedom | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 29 | 2.89 | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 | 1.83 |
| | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 |
| | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 |
| | 13.4 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 | 4.29 |
| 30 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 |
| | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 |
| | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 |
| | 13.3 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 |
| 32 | 2.87 | 2.48 | 2.26 | 2.13 | 2.04 | 1.97 | 1.91 | 1.87 | 1.83 | 1.81 |
| | 4.15 | 3.29 | 2.90 | 2.67 | 2.51 | 2.40 | 2.31 | 2.24 | 2.19 | 2.14 |
| | 7.50 | 5.34 | 4.46 | 3.97 | 3.65 | 3.43 | 3.26 | 3.13 | 3.02 | 2.93 |
| | 13.1 | 8.64 | 6.94 | 6.01 | 5.43 | 5.02 | 4.72 | 4.48 | 4.30 | 4.14 |
| 34 | 2.86 | 2.47 | 2.25 | 2.12 | 2.02 | 1.96 | 1.90 | 1.86 | 1.82 | 1.79 |
| | 4.13 | 3.28 | 2.88 | 2.65 | 2.49 | 2.38 | 2.29 | 2.23 | 2.17 | 2.12 |
| | 7.44 | 5.29 | 4.42 | 3.93 | 3.61 | 3.39 | 3.22 | 3.09 | 2.98 | 2.89 |
| | 13.0 | 8.52 | 6.83 | 5.92 | 5.34 | 4.93 | 4.63 | 4.40 | 4.22 | 4.06 |
| 36 | 2.85 | 2.46 | 2.24 | 2.11 | 2.01 | 1.94 | 1.89 | 1.85 | 1.81 | 1.78 |
| | 4.11 | 3.26 | 2.87 | 2.63 | 2.48 | 2.36 | 2.28 | 2.21 | 2.15 | 2.11 |
| | 7.40 | 5.25 | 4.38 | 3.89 | 3.57 | 3.35 | 3.18 | 3.05 | 2.95 | 2.86 |
| | 12.8 | 8.42 | 6.74 | 5.84 | 5.26 | 4.86 | 4.56 | 4.33 | 4.14 | 3.99 |
| 38 | 2.84 | 2.45 | 2.23 | 2.10 | 2.01 | 1.94 | 1.88 | 1.84 | 1.80 | 1.77 |
| | 4.10 | 3.24 | 2.85 | 2.62 | 2.46 | 2.35 | 2.26 | 2.19 | 2.14 | 2.09 |
| | 7.35 | 5.21 | 4.34 | 3.86 | 3.54 | 3.32 | 3.15 | 3.02 | 2.92 | 2.83 |
| | 12.7 | 8.33 | 6.66 | 5.76 | 5.19 | 4.79 | 4.49 | 4.26 | 4.08 | 3.93 |
| 40 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 |
| | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 |
| | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 |
| | 12.6 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 |

(*Continued Next Page* ➔)

# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

| Denominator Degrees of Freedom | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 42 | 2.83 | 2.43 | 2.22 | 2.08 | 1.99 | 1.92 | 1.86 | 1.82 | 1.78 | 1.75 |
| | 4.07 | 3.22 | 2.83 | 2.59 | 2.44 | 2.32 | 2.24 | 2.17 | 2.11 | 2.06 |
| | 7.28 | 5.15 | 4.29 | 3.80 | 3.49 | 3.27 | 3.10 | 2.97 | 2.86 | 2.78 |
| | 12.5 | 8.18 | 6.53 | 5.64 | 5.07 | 4.68 | 4.38 | 4.16 | 3.97 | 3.83 |
| 44 | 2.82 | 2.43 | 2.21 | 2.08 | 1.98 | 1.91 | 1.86 | 1.81 | 1.78 | 1.75 |
| | 4.06 | 3.21 | 2.82 | 2.58 | 2.43 | 2.31 | 2.23 | 2.16 | 2.10 | 2.05 |
| | 7.25 | 5.12 | 4.26 | 3.78 | 3.47 | 3.24 | 3.08 | 2.95 | 2.84 | 2.75 |
| | 12.4 | 8.12 | 6.48 | 5.59 | 5.02 | 4.63 | 4.34 | 4.11 | 3.93 | 3.78 |
| 46 | 2.82 | 2.42 | 2.21 | 2.07 | 1.98 | 1.91 | 1.85 | 1.81 | 1.77 | 1.74 |
| | 4.05 | 3.20 | 2.81 | 2.57 | 2.42 | 2.30 | 2.22 | 2.15 | 2.09 | 2.04 |
| | 7.22 | 5.10 | 4.24 | 3.76 | 3.44 | 3.22 | 3.06 | 2.93 | 2.82 | 2.73 |
| | 12.4 | 8.06 | 6.42 | 5.54 | 4.98 | 4.59 | 4.30 | 4.07 | 3.89 | 3.74 |
| 48 | 2.81 | 2.42 | 2.20 | 2.07 | 1.97 | 1.90 | 1.85 | 1.80 | 1.77 | 1.73 |
| | 4.04 | 3.19 | 2.80 | 2.57 | 2.41 | 2.29 | 2.21 | 2.14 | 2.08 | 2.03 |
| | 7.19 | 5.08 | 4.22 | 3.74 | 3.43 | 3.20 | 3.04 | 2.91 | 2.80 | 2.71 |
| | 12.3 | 8.00 | 6.38 | 5.50 | 4.94 | 4.55 | 4.26 | 4.03 | 3.85 | 3.70 |
| 50 | 2.81 | 2.41 | 2.20 | 2.06 | 1.97 | 1.90 | 1.84 | 1.80 | 1.76 | 1.73 |
| | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 |
| | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 |
| | 12.2 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 | 3.67 |
| 60 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 |
| | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 |
| | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 |
| | 12.0 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 |
| 70 | 2.78 | 2.38 | 2.16 | 2.03 | 1.93 | 1.86 | 1.80 | 1.76 | 1.72 | 1.69 |
| | 3.93 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 |
| | 7.01 | 4.92 | 4.07 | 3.60 | 3.29 | 3.07 | 2.91 | 2.78 | 2.67 | 2.59 |
| | 11.8 | 7.64 | 6.06 | 5.20 | 4.66 | 4.28 | 3.99 | 3.77 | 3.60 | 3.45 |

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# Table A.4
# Fisher's *f* Distributions

(*Continued From Previous Page*)

Rows and columns are labeled respectively by numerator degrees of freedom and denominator degrees of freedom.

Table entries are *f* values each corresponding to an upper tail probability of the type displayed by the shaded region in the figure at the beginning of the table. First to fourth rows in each section correspond respectively to tail probabilities 0.10, 0.05, 0.01, 0.001.

**Numerator Degrees of Freedom**

**Denominator Degrees of Freedom**

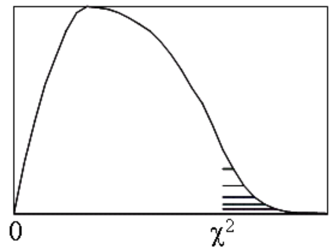
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 2.77 | 2.37 | 2.15 | 2.02 | 1.92 | 1.85 | 1.79 | 1.75 | 1.71 | 1.68 |
| | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 |
| | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.64 | 2.55 |
| | 11.7 | 7.54 | 5.97 | 5.12 | 4.58 | 4.20 | 3.92 | 3.70 | 3.53 | 3.39 |
| 90 | 2.76 | 2.36 | 2.15 | 2.01 | 1.91 | 1.84 | 1.78 | 1.74 | 1.70 | 1.67 |
| | 3.95 | 3.10 | 2.71 | 2.47 | 2.32 | 2.20 | 2.11 | 2.04 | 1.99 | 1.94 |
| | 6.93 | 4.85 | 4.01 | 3.53 | 3.23 | 3.01 | 2.84 | 2.72 | 2.61 | 2.52 |
| | 11.6 | 7.47 | 5.91 | 5.06 | 4.53 | 4.15 | 3.87 | 3.65 | 3.48 | 3.34 |
| 100 | 2.76 | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 |
| | 3.94 | 3.09 | 2.70 | 2.45 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 |
| | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 |
| | 11.5 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 | 3.30 |
| 1000 | 2.71 | 2.31 | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 |
| | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 |
| | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 |
| | 10.9 | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 | 2.99 |

# Table A.5
# Chi-Square ($\chi^2$) Distributions

Rows are labeled by *df* (degrees of freedom).

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are $\chi^2$ values.




| *df* | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
|---|---|---|---|---|---|---|
| 1 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.828 |
| 2 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 13.816 |
| 3 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 | 16.266 |
| 4 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 | 18.467 |
| 5 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 |
| 6 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 |
| 7 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 |
| 8 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.125 |
| 9 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 |
| 10 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 |
| 11 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 31.264 |
| 12 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 32.909 |
| 13 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 34.528 |
| 14 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 36.123 |
| 15 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 37.697 |
| 16 | 23.542 | 26.269 | 28.845 | 32.000 | 34.267 | 39.252 |
| 17 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 40.790 |
| 18 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 42.312 |
| 19 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 43.820 |
| 20 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 45.315 |
| 21 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 | 46.797 |
| 22 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 | 48.268 |
| 23 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 | 49.728 |
| 24 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 | 51.179 |
| 25 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 | 52.620 |
| 26 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 | 54.052 |
| 27 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 | 55.476 |
| 28 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 | 56.829 |
| 29 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 | 58.302 |
| 30 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 59.703 |
| 40 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 73.402 |
| 50 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 86.661 |
| 60 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 | 99.607 |
| 80 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 | 124.839 |
| 100 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 | 149.449 |

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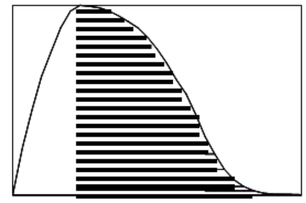
# Table A.5
# Chi-Square ($\chi^2$) Distributions

(*Continued From Previous Page*)

Rows are labeled by *df* (degrees of freedom).

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are $\chi^2$ values.




| *df* | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 |
|---|---|---|---|---|---|
| 1 | 0.0001 | 0.0002 | 0.001 | 0.004 | 0.016 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 |
| 16 | 5.412 | 5.812 | 6.908 | 7.962 | 9.312 |
| 17 | 5.679 | 6.408 | 7.564 | 8.672 | 10.085 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.144 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.501 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 |

# Appendix B - Derivations and Formulas

Section B.1
The Multiplication Principle

Section B.2
Calculation of the Two Sample *t* Test Statistics

Section B.3
Calculation of Fisher's One-Way ANOVA *f* Statistic

# Section B.1
# The Multiplication Principle

The *multiplication principle* gives us the number of results which are possible with an ordered sequence of actions. Suppose you are about to order a quick lunch at a cafeteria, and you must first choose one of 5 sandwiches (hamburger, hot dog, grilled cheese, hoagie, tuna fish) after which you must choose one of 3 drinks (soda, juice, milk). How many different combinations are there for lunch? If you match each one of the 5 sandwich choices with each one of the 3 drink choices, you find that there are (5)(3) = 15 lunch combinations. (Go ahead and write down all the combinations to see that this is true!) The multiplication principle states that if there are $w_1$ possible results for a first action and $w_2$ possible results for a second action, then there are $w_1w_2$ different results for both actions together performed in sequence.

The multiplication principle can be extended to any number of actions performed in sequence. Suppose that for your quick lunch in the cafeteria, you must first choose one of the 5 sandwiches (hamburger, hot dog, grilled cheese, hoagie, tuna fish), you must then choose one of the 3 drinks (soda, juice, milk), and finally you must choose one of 4 desserts (pie, ice cream, cheesecake, jello). How many different combinations are there for lunch? If you match each one of the 5 sandwich choices with each one of the 3 drink choices, then match each of these pairs with each one of the 4 dessert choices, you find that there are (5)(3)(4) = 60 lunch combinations. (It may take a few minutes, but you can write down all the combinations to see that this is true!) In general, the multiplication principle states that if there are $w_1$ possible results for a first action, $w_2$ possible results for a second action, ..., $w_k$ possible results for a $k$th action, then there are $w_1w_2...w_k$ different results for all $k$ actions performed together in sequence.

Now, let us consider a cube with four of the sides each painted red and the other two sides each painted blue. In order to use the multiplication principle to obtain a list of all the possible samples of size $n$ = 3, we first observe that on each roll there are 4 red sides which can be facing up and 2 blue sides which can be facing up. To enumerate the number of possible samples, we ask the following eight questions: How many samples will result in

(i) a red side facing up for all three rolls, which we designate as RRR?

(ii) a blue side facing up on the first roll, and a red side facing up for the second and third rolls, which we designate as BRR?

(iii) a red side facing up for the first and third rolls, and a blue side facing up on the second roll, which we designate as RBR?

(iv) a red side facing up for the first and second rolls, and a blue side facing up on the third roll, which we designate as RRB?

(v) a red side facing up on the first roll, and a blue side facing up for the second and third rolls, which we designate as RBB?

(vi) a blue side facing up for the first and third rolls, and a red side facing up on the second roll, which we designate as BRB?

(vii) a blue side facing up for the first and second rolls, and a red side facing up on the third roll, which we designate as BBR?

(viii) a blue side facing up for all three rolls, which we designate as BBB?

We can use the multiplication principle to answer each question by realizing that on each roll, there are four ways a red side can be facing up and two ways a blue side can be facing up.

Since there are 4 ways to obtain a red side on the first roll, 4 ways to obtain a red side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be (4)(4)(4) = 64 ways to obtain the result designated as RRR; this answers question (i). This result is recorded in the first row of Table 18-1, where we also find that the sample proportion of red is 3/3 = 1.000.

Since there are 2 ways to obtain a blue side on the first roll, 4 ways to obtain a red side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be (2)(4)(4) = 32 ways to obtain the result designated as BRR; this answers question (ii). This result is recorded in the second row of Table 18-1, where we also find that the sample proportion of red is 2/3 = 0.667.

Since there are 4 ways to obtain a red side on the first roll, 2 ways to obtain a blue side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be (4)(2)(4) = 32 ways to obtain the result designated as RBR; this answers question (iii). This result is recorded in the third row of Table 18-1, where we also find that the sample proportion of red is 2/3 = 0.667.

Since there are 4 ways to obtain a red side on the first roll, 4 ways to obtain a red side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be (4)(4)(2) = 32 ways to obtain the result designated as RRB; this answers question (iv). This result is recorded in the fourth row of Table 18-1, where we also find that the sample proportion of red is 2/3 = 0.667.

Since there are 4 ways to obtain a red side on the first roll, 2 ways to obtain a blue side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be (4)(2)(2) = 16 ways to obtain the result designated as RBB; this answers question (v). This result is recorded in the fifth row of Table 18-1, where we also find that the sample proportion of red is 1/3 = 0.333.

Since there are 2 ways to obtain a blue side on the first roll, 4 ways to obtain a red side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be (2)(4)(2) = 16 ways to obtain the result designated as BRB; this answers question (vi). This result is recorded in the sixth row of Table 18-1, where we also find that the sample proportion of red is 1/3 = 0.333.

Since there are 2 ways to obtain a blue side on the first roll, 2 ways to obtain a blue side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be (2)(2)(4) = 16 ways to obtain the result designated as BBR; this answers question (vii). This result is recorded in the seventh row of Table 18-1, where we also find that the sample proportion of red is 1/3 = 0.333.

Since there are 2 ways to obtain a blue side on the first roll, 2 ways to obtain a blue side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be (2)(2)(2) = 8 ways to obtain the result designated as BBB; this answers question (viii). This result is recorded in the eighth row of Table 18-1, where we also find that the sample proportion of red is 0/3 = 0.000.

# Section B.2
# Calculation of the Two Sample *t* Test Statistics

<table>
<tr><th>Notation & Formulas</th><th>Example</th></tr>
<tr><td>
Data consisting of two independent samples of sizes $n_1$ and $n_2$ can be represented as follows:<br><br>
Sample #1: $x_{11}, x_{12}, \ldots, x_{1n_1}$<br>
Sample #2: $x_{21}, x_{22}, \ldots, x_{2n_2}$<br><br>
(The first subscript indicates which sample the observation is from, and the second subscript distinguishes between different observations in the same sample.)<br><br>
$\sum x_1$ and $\sum x_2$ respectively represent the sum of the observations in sample #1 and the sum of the observations in sample #2, from which the sample means can be obtained as follows:<br>
$\bar{x}_1 = \dfrac{\Sigma x_1}{n_1}$ and $\bar{x}_2 = \dfrac{\Sigma x_2}{n_2}$ .<br><br>
$\sum(x_1 - \bar{x}_1)^2$ and $\sum(x_2 - \bar{x}_2)^2$ respectively represent the sum of the squared deviations from the mean in sample #1 and the sum of the squared deviations from the mean in sample #2, from which the sample variances can be obtained as follows:<br>
$s_1^2 = \dfrac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1}$ and $s_2^2 = \dfrac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1}$.<br>
Note that if the sample variances $s_1^2$ and $s_2^2$ are available, then the sums of squared deviations can be obtained from<br>
$\sum(x_1 - \bar{x}_1)^2 = (n_1 - 1)s_1^2$ and<br>
$\sum(x_2 - \bar{x}_2)^2 = (n_2 - 1)s_2^2$ .
</td><td>
The hours of lifetime is recorded for each of four bulbs from the brand named Sunn and three bulbs from the brand named Brighto, resulting in the following data consisting of two independent samples:<br>
<u>Sunn</u>: 575 611 572 602 ($n_S = 4$)<br>
<u>Brighto</u>: 568 569 528 ($n_B = 3$)<br><br>
$\sum x_S = 575 + 611 + 572 + 602 = 2360$<br><br>
$\sum x_B = 568 + 569 + 528 = 1665$<br><br>
$\bar{x}_S = \dfrac{2360}{4} = 590$ and $\bar{x}_B = \dfrac{1665}{3} = 555$<br><br>
$\sum(x_S - \bar{x}_S)^2 = (575 - 590)^2 + (611 - 590)^2 + (572 - 590)^2 + (602 - 590)^2 = 1134$<br><br>
$\sum(x_B - \bar{x}_B)^2 = (568 - 555)^2 + (569 - 555)^2 + (528 - 555)^2 = 1094$<br><br>
$s_S^2 = \dfrac{1134}{4-1} = 378$<br><br>
$s_B^2 = \dfrac{1094}{3-1} = 547$
</td></tr>
</table>

(*continued next page*)

# Section B.2 (*continued*)

The pooled two sample $t$ statistic requires the calculation of the pooled sample variance

$$s_p^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2} .$$

The pooled two sample $t$ statistic, with $df = n_1 + n_2 - 2$ , is

$$t_{n_1 + n_2 - 2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} .$$

$$s_p^2 = \frac{\sum (x_S - \bar{x}_S)^2 + \sum (x_B - \bar{x}_B)^2}{n_S + n_B - 2} = \frac{1134 + 1094}{4 + 3 - 2} = \frac{2228}{5} = 445.6$$

$$t_5 = \frac{\bar{x}_S - \bar{x}_B}{\sqrt{s_p^2\left(\frac{1}{n_S} + \frac{1}{n_B}\right)}} = \frac{590 - 555}{\sqrt{445.6\left(\frac{1}{4} + \frac{1}{3}\right)}} = 2.171$$

The separate two sample $t$ statistic requires the calculation of the degrees of freedom

$$v = \frac{(n_1 - 1)(n_2 - 1)\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{(n_2 - 1)\left(\frac{s_1^2}{n_1}\right)^2 + (n_1 - 1)\left(\frac{s_2^2}{n_2}\right)^2} .$$

The separate two sample $t$ statistic is

$$t_v = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} .$$

$$v = \frac{(n_S - 1)(n_B - 1)\left(\frac{s_S^2}{n_S} + \frac{s_B^2}{n_B}\right)^2}{(n_B - 1)\left(\frac{s_S^2}{n_S}\right)^2 + (n_S - 1)\left(\frac{s_B^2}{n_B}\right)^2} = 3.9 \approx 4$$

$$t_4 = \frac{\bar{x}_S - \bar{x}_B}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_B^2}{n_B}}} = \frac{590 - 555}{\sqrt{\frac{378}{4} + \frac{547}{3}}} = +2.104$$

# Section B.3
# Calculation of Fisher's One-Way ANOVA *f* Statistic

**Notation & Formulas**

Data consisting of $k$ independent samples of sizes $n_1$ , $n_2$ , … , $n_k$ can be represented as follows:

| Sample #1 | … | Sample #$k$ |
|---|---|---|
| $x_{11}, x_{12}, \ldots, x_{1\,n_1}$ | … | $x_{k1}, x_{k2}, \ldots, x_{k\,n_k}$ |

(The first subscript indicates which sample the observation is from, and the second subscript distinguishes between different observations in the same sample.)

$\sum x_i$ represents the sum of the observations in sample #$i$ , from which the sample means can be obtained as follows:

$\bar{x}_i = \dfrac{\Sigma x_i}{n_i}$ is the sample mean for sample # $i$.

$n_\bullet = n_1 + n_2 + \ldots + n_k$ = total sample size

$\bar{x}_\bullet = \dfrac{\Sigma x_1 + \Sigma x_2 + \ldots + \Sigma x_k}{n_\bullet}$ = grand mean

$SSB = \sum n_i(\bar{x}_i - \bar{x}_\bullet)^2$ and $MSB = \dfrac{SSB}{k-1}$

$\sum(x_i - \bar{x}_i)^2$ represents the sum of the squared deviations from the mean in sample #$i$, from which $SSE$ can be obtained:

$SSE =$

$\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2 + \ldots + \sum(x_k - \bar{x}_k)^2$

and $MSB = \dfrac{SSE}{n_\bullet - k}$ .

Note that if the sample variances $s_1^2$ , $s_2^2$ … , $s_k^2$ are available, then the sums of squared deviations for sample #$i$ can be obtained from $\sum(x_i - \bar{x}_i)^2 = (n_i - 1)s_i^2$ .

$$f_{k-1,\ n_* - k} = \frac{MSB}{MSE}$$

**Example**

Breaking strength is recorded for each of three random samples of rope, one for rope brand Deluxe, one for rope brand Econ, and one for rope brand Nogood, resulting in the following data consisting of three independent samples:

| | | | | | |
|---|---|---|---|---|---|
| Deluxe | 165 | 162 | 159 | 162 | ($n_D = 4$) |
| Econ | 156 | 163 | 158 | | ($n_E = 3$) |
| Nogood | 151 | 154 | 160 | | ($n_N = 3$) |

$\sum x_D = 165 + 162 + 159 + 162 = 648$

$\sum x_E = 156 + 163 + 158 = 477$

$\sum x_N = 151 + 154 + 160 = 465$

$\bar{x}_D = \dfrac{648}{4} = 162$ , $\bar{x}_E = \dfrac{477}{3} = 159$ ,

$\bar{x}_N = \dfrac{465}{3} = 155$

$n_\bullet = n_D + n_E + n_N = 4 + 3 + 3 = 10$

$\bar{x}_\bullet = \dfrac{648 + 477 + .465}{10} = \dfrac{1590}{10} = 159$

$SSB = (4)(162 - 159)^2 + (3)(159 - 159)^2 + (3)(155 - 159)^2 = 84$

$MSB = \dfrac{84}{3-1} = 42$

$\sum(x_S - \bar{x}_S)^2 = (165 - 162)^2 + (162 - 162)^2 + (159 - 162)^2 + (162 - 162)^2 = 18$

$\sum(x_E - \bar{x}_E)^2 = (156 - 159)^2 + (163 - 159)^2 + (158 - 159)^2 = 26$

$\sum(x_N - \bar{x}_N)^2 = (151 - 155)^2 + (154 - 155)^2 + (160 - 155)^2 = 42$

$SSE = 18 + 26 + 42 = 86$

$MSB = \dfrac{86}{10-3} = 12.2857$

$f_{2,7} = \dfrac{42}{12.2857} = 3.42$