Measure and Integration
A First Course

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2013

Preface

This book gives a short introduction to the theory of measure and integration. It is essentially an updated version of the notes which the author has been using for teaching courses on measure and integration many times for the last 23 years. The topics covered in this book are standard ones. However, the reader will definitely find that the presentation of the concepts and topics are different from the standard texts.

It starts by introducing the necessity of the concept of integration of functions that are more general than those allowed within the theory of Riemann integration, and then introduces the concept of Lebesgue measurable sets that is more general than the concept of intervals. Once we have this family of measurable sets, and the concept of a Lebesgue measure, it becomes almost obvious that one need not restrict the theory of integration to the subsets of the real line, but can be developed on any set together with a *sigma algebra* on it. Thus, the concept of a measure on a *measurable space* allows us to have a theory of integration in a very general setting which has immense potential for application to diverse areas of mathematics and its applications.

Although the theory of integration is very vast, the attempt in this book is to introduce the students to this modern subject in a simple and natural manner so that they can pursue the subject further with confidence, and also apply the concepts in other branches of mathematics, specially in the theory of differential and integral equations, Fourier analysis etc.

This book can be used for a one semester course of about 40 lectures for the first or second semester of a post graduate programme. As Lebesgue measure is introduced in the beginning, no pre-requisites sought, except the mathematical maturity to appreciate and grasp concepts in analysis.

January-May 2013/2014 M. Thamban Nair

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