The Mathematics of Options

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Quantifying Derivative Price, Payoff, Probability, and Risk

palgrave
macmillan

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ISBN 978-3-319-56634-4 ISBN 978-3-319-56635-1 (eBook)
DOI 10.1007/978-3-319-56635-1

Library of Congress Control Number: 2017943652

Cover design: Fatima Jamadar

Printed on acid-free paper

This Palgrave Macmillan imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Foreword

Writing a book—any book—looks easy after it is completed. However, the process itself is far from simple. An author must remember the proper tone for a target audience and decide what to include and exclude in every chapter. Finally, the author has to make sure that the essential message is communicated both clearly and accurately.

I have struggled in my own career with the challenges of communication. This is an unending process in my current roles as lecturer of energy trading and risk management for Fitch Learning's *Certificate in Quantitative Finance Programme* at Wall Street, energy trading lecturer at the Freeman School of Business at Tulane University in New Orleans, and author of three books, and in my development of www.MathQED.com—a mathematics homework platform for K-12 and college students, teachers, and parents.

Just as I am gratified when I see a student's face lights up at a realization of an underlying concept, teaching energy trading and applied mathematical topics at every level involves bridging that gap between theory and real-world applications. *The Mathematics of Options* accomplishes the goal of leading the reader through a maze of concepts and terminology to arrive at a rather simple result, the appreciation of concepts needed to become a successful options trader.

Too often, the mathematical aspects of options are made overly complex and unnecessarily obscure. Emphasis on theoretical ideas, such as options pricing models or estimated future volatility, is of no use to a trader who needs a few basic tools: An understanding of risk and probability, a means to visualize payoffs and volatility, and identification of a range of possible trading outcomes. In Thomsett's book, risk and probability are

expressed in practical terms. Historical volatility is explained and demonstrated as an effective means for timing trades. In addition, the outcomes (profit, breakeven, or loss) are summarized mathematically and illustrated so that every trader can simplify the process of selecting trades, rather than complicating it.

One truth I learned when I wrote my second book, *Energy Trading and Risk Management: A Practical Approach to Hedging, Trading and Portfolio Diversification* (John Wiley & Sons, 2014), is that an outside observer might assume that writing about options, hedging, and diversification comes easily to an expert. It does not! Even with a Harvard Ph.D. in Applied Mathematics, a London Business School Executive MBA, and experience as a faculty member at MIT, I discovered a challenge in articulating what readers needed to know about financial and energy derivatives. It does not matter how much knowledge the author possesses, or how well the author can communicate knowledge through the written word.

With this in mind, I applaud any author who is also an effective teacher, and who can explain the essential message in a book. *The Mathematics of Options* tackles a very big topic, indeed, and identifies not only what an options trader or professional needs to know, but also what can be rejected as nonessential. Thomsett's book demonstrates what works on a practical level, and what does not work. That makes this unusual, not only as a book of mathematics but also as a book of options trading.

Having worked at several universities, energy and financial institutions in the USA, London, and Asia, I have seen a repetitive challenge in each of my work environments. Complexity does not produce results; simplicity does. We need to strip education back to basics. Math, especially, too often is not taught effectively and, as a consequence, many intelligent and capable students are left behind when energetic and effective educational methods would have made all the difference. Taking on the challenge of writing a mathematics book—especially one involving the options world—is a daunting task, but not impossible, as Thomsett's book demonstrates.

Challenges, in fact, define everyone's degree of success in their profession. As a derivatives and applied math educator from very humble beginnings, I have discovered a great truth about how the world works: Math, such as reading, opens many doors. It is the gatekeeper subject to so many interesting and even lucrative professions, such as engineering, science, technology, computer science, trading, medicine, etc. Hence, sadly, we limit ourselves *only* by what we fail to pursue.

For anyone interested in improving their knowledge of math and options trading, the only limitation to progress is internal. This book, in its ease of

communication of a complex topic, results from Thomsett having identified the target audience, sets the appropriate tone, and precisely determined the information to include or exclude.

New Orleans, USA
March 2017

Iris Mack, Ph.D.

Preface

The Misunderstood Options

Options trading may have vastly different appearances, depending on the observer. The speculator treats options as efficient forms of leverage, and the conservative portfolio manager or equity investor seeks hedging through options to manage, reduce, or eliminate risk.

This book presents a range of information about specific groupings of strategies, all quantified mathematically in terms of profit, breakeven, or loss. The concept here is that even the most sophisticated and experienced options trader is likely to benefit from awareness of these all-important benchmarks for every trade. In setting price or profit goals for exit, these calculations are especially useful. Every trader is able to set goals for taking profits or accepting losses. That is the easy party. Actually taking action when those levels are reached is far more difficult.

Beyond the highly detailed chapters illustrating the appearance of profit and loss for many types of trades and applying these formulas, the book contains many additional chapters of interest. Among the observations in the book is a questioning of implied volatility as a useful tool. As a measurement of probability for favorable outcomes, historical volatility may offer an equal or better yardstick for identifying risk and opportunity. This is a topic of passionate debate, and both sides—those who believe in implied volatility and those who reject it—have their rationale.

Another observation in the book questions the methods used in the market for calculating probability. In Chap. 1, the comparison between

two separate methods is traced back to a chronic gambler in 175th-century France, the Chevalier de Mere. He realized that his carefully constructed probability of favorable outcomes was not being matched in actual play. He consulted with his friend, the famous Blaise Pascal who, in turn, consulted with Pierre de Fermat, a gifted mathematician. These two discovered that probability of getting a favorable outcome was *not* accurate. The more reliable system was to first calculate the probability of not getting the favorable outcome, and then subtracting the result from one.

This paradox—the difference between additive and multiplicative probability—is profound and has importance in modern options trading. So many examples of what is broadly lumped in together under the umbrella of "probability" involve the less reliable additive method of calculation. With this in mind, probability itself has to be studied by options traders not as a system of guarantees, but more as a system for better understanding the potential within a range of expected outcomes.

Just as de Mere was a chronic gambler in his day, many options traders are equally attracted to the law of averages. But many are further puzzled because outcomes do not always conform to expectations. Albert Einstein cautioned that in his opinion, God does not play dice. This is not a rejection of *risk* by any means, but an observation that probability involves only estimates of possible outcomes. This often is misunderstood by traders who find comfort in applying an online calculator and discovering that the odds of a particular options ending up in the money are 80%. But how is that probability calculated? The online source does not reveal its methods nor its assumptions, so traders are expected to grant significant trust in a source that does not disclose any details.

These examples of topics and their treatment within this book are only part of a broader set of assumptions concerning what may be termed *practical* mathematical application. This means that formulations involving a lot of theory and the exponential uncertainties of multiple variables are not used to set a basis for the premise underlying the book: Traders want to know the levels of profit, breakeven, and loss, and they want to be able to quantify risk. It is that simple.

On a theoretical level, much has been written about the wonders of pricing models, especially the best-known among these, the Black-Scholes pricing model. This book attempts to base a system on historical volatility, technical analysis, and fundamental analysis as the means for selecting underlying securities and their options. But why reject Black-Scholes? Actually, there are numerous reasons. Fischer Black himself wrote several years after publication of the original formula that the assumptions used by

himself and Myron Scholes were, in fact, deeply flawed. He identified *nine* specific flaws and incorrect assumptions, each of which distorts the pricing model. A single flawed assumption is concerning enough, but may be accepted as part of an analytical process. However, nine flaws are exponentially more serious and bring into question the entire process of developing a pricing model. This led to an equally important question: Why do options traders need a pricing model at all?

There are no pricing models for other forms of investing or trading. A trader, speculator, or investor focusing on stocks, for example, relies on the market and its forces of supply and demand to set prices as a floating and ever-changing aspect of that auction market, and it works quite well. The market, with its informational efficiency at play, sets up a universe in which astute traders recognize when securities are overpriced or underpriced, all without a pricing model or formulation of what the price per share *should* be.

Since options are derivatives of their underlying securities, the pricing of an options contract is directly derived from movement in the underlying price, which varies and changes based on both price movement and historical volatility. The conclusion: A pricing model is a comfortable concept in theory, as Fischer Black himself observed as a certainty that could provide comfort to traders. However, Black also said, the assumptions that go into this desire for perfection rarely can be applied so that the pricing model works.

With these controversial realizations and the natural conflict between a comforting theory and a stark reality, it becomes clear that no one's answer addresses all of the questions or satisfies all of the beliefs within the options universe. By presenting alternatives, individuals on both sides may enter into a debate and may even learn from one another. Unfortunately, disagreement too often leads to shutting out of the opportunity to expand knowledge. It may only be hoped that the information in this book, especially surrounding these controversial topics, will lead to an advanced appreciation of a complex topic, whether through confirmation of what is believed today, or by discovery of alternative possibilities. That process—learning how to view a problem from a different point of view—is how everyone learns.

Spring Hill, USA Michael C. Thomsett

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Introduction—The Variability of Derivatives Trading

Abstract

Professional traders need practical solutions combining essential mathematical principles used to quantify options value. This should be practical and actionable, so that options traders are able to make clear and informed decisions. Every options trader deals with an array of calculations. This applies to pricing, of contracts, payoff expectations of specific strategies (including maximum profit and loss), probability of outcomes, and identification of risks. Traders want a convenient and practical reference guide to hedging portfolio risk; for evaluating pricing, payoff, probability and risk issues; and to convert complexity into sensible decisions. Among the challenges to articulating mathematical quantification of options is the problem of inaccuracies and estimates as part of so many formulas. Traders need and deserve a straightforward and accurate way to measure the essential ingredients of trading.

Every options trader deals with an array of calculations. This applies to pricing, of contracts, payoff expectations of specific strategies (including maximum profit and loss), probability of outcomes, and identification of risks.

The beginner is concerned with identifying risk and opportunity and focusing on a short list of strategies. However, the experienced options trader is more focused on a broad range of issues. Among these is the need for hedging to reduce portfolio risk as one aspect of options trading. The experienced options trader requires reference for evaluating these pricing, payoff, probability, and risk issues. These convenient and common sense

calculations are not easily found in free online sources, which tend to fall into one of two categories: First is the basic type of calculation aimed at the novice or general trader. Second is the esoteric level of advanced probability analysis and the Black-Scholes pricing model, aimed at the theoretical rather than at practical applications.

For the practical use of options math, notably for experienced individual traders or investors and for portfolio managers, the literature of the industry has lacked a practical reference. This is why this book has been written. Among the many challenges to articulating mathematical quantification of options trading is the problem of inaccuracies and estimates as part of so many formulas. One inaccuracy in a calculation is troubling, but can be adjusted in order to approximate a reliable outcome. However, multiple inaccuracies create exponential levels of unreliability.

A second problem with many mathematical applications is the lack of clarity in the methods used to calculate outcomes, such as probability, implied volatility, and pricing estimates. If traders are to rely on spreadsheets or calculators that include variables, how much reliance should be placed in the outcome? If the brokerage firm of options service is not able to willing to disclose its assumptions, the reliability of a simplified calculator cannot be known. This presents a deep and chronic problem for anyone interested in determining a reliable set of prices, risks, or trading opportunities.

The book begins with a review of some of the basics, as a means for setting the tone of the book and for defining a starting point for the mathematical principles addressed in the book. Chapter 1 is a discussion of trading goals and objectives, meant to reiterate the differences between investing and speculating and how options fit within that broad spectrum of risk tolerance. It also matches the specific risk tolerance attributes with risk identification specific to a range of options strategies. In order to identify the means for calculating expected outcomes based on well-defined goals and objectives, the chapter proposes development and use of a "probability matrix" designed to create a visual summary of the mathematical risk parameters associated with stock price behavior and, by association, of ever evolving options values and risks.

Chapter 2 further examines the rudimentary aspects of any trading program by exploring the role of fundamental and technical analysis for options trading. The concept of using both forms is not often addressed, neither is it common to suggest that both are of great value. However, as a starting point in determining which companies to include in an options-based program, the fundamental strength or weakness is an essential starting point to develop a sound program, not only for the obvious equity selection process but also to determine risk profiles for options strategies.

In Chap. 3, the pricing of options is the topic. This might seem an obvious and basic attribute of options trading, but in fact the methods by which premium levels are judged may involve a range of considerations including put/call parity and upper/lower price bounds.

Chapter 4 describes how dividends affect options returns and how a various number of dividend-related calculations have to be considered in the mathematical analysis of options trading.

It might appear that calculating returns on options trading is exceptionally basic. In practice, however, this may be one of the more complex aspects to options and often involves inconsistencies and confusion, even among experienced traders. Chapter 5 describes the problems and offers solutions in developing a consistent and reliable program for accurate comparisons and analysis of likely outcomes.

Chapter 6 describes the various forms of single-options trades, including meaning the most basic long calls and puts as well as a variety of short option trades. The chapter examines the critical importance of proximity as a timing issue for covered calls. Beyond the single-options trade is the popular spread in its many forms. Chapter 7 demonstrates the strategic payoff calculations of spread strategies, including proximity and its risk factors that affect pricing for the underlying security.

In Chap. 8, the same attributes are examined for straddles as alternative strategies—both high-risk and conservative—and further demonstrates how the math in quantifying risk is essential to understanding these devices. Chapter 9 describes the probability and risk of options trading, expanding to development of a probability matrix, use of VaR, expected and unexpected loss, and an overview of the mathematics of risk tolerance.

In Chap. 10, pricing models are examined and dissected. This chapter includes an analysis of the popular Black-Scholes pricing model and explores its benefits as well as its flaws. Moving beyond a discussion of pricing models, Chap. 11 demonstrates the many alternatives available to the options professional. Those who recognize the problems associated with reliance on pricing models, the Greeks, and calculations of implied volatility, will find practical value in reliance on technical timing, proximity analysis, and signals found in the forms of price, volume, momentum, and moving average analysis.

The intention and purpose of this book is to offer practical ideas for professional traders combining essential mathematical principles to quantify options value, with a practical and actionable approach to the broader struggle of identifying how to make informed decisions. Trade entry and exit timings are at the core of this effort, and options traders, such as eve-

ryone in the market, seek solutions to the timing challenges in an uncertain environment. The usual statistical analyses used elsewhere are rarely applicable. In most forms of statistics, a fixed field of known variables is involved. In the markets, however, the field is constantly in flux, changing not only daily but also by the minute. As a consequence, the random variables of the market necessitate an approach that moves beyond the comforting statistical certainty and expand to present a fresh and informed field of vision that acknowledges the dimensions of variation every trader faces.