LESSON 8.4 **Rhombuses, Rectangles, and Squares.**

Vocabulary

A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.

A **square** is a parallelogram with four congruent sides and four right angles.

Rhombus Corollary: A quadrilateral is a rhombus if and only if it has four congruent sides.

Rectangle Corollary: A quadrilateral is a rectangle if and only if it has four right angles.

Square Corollary: A quadrilateral is a square if and only if it is a rhombus and a rectangle.

Theorem 8.11: A parallelogram is a rhombus if and only if its diagonals are perpendicular.

Theorem 8.12: A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

Theorem 8.13: A parallelogram is a rectangle if and only if its diagonals are congruent.

EXAMPLE 1

Use properties of special quadrilaterals

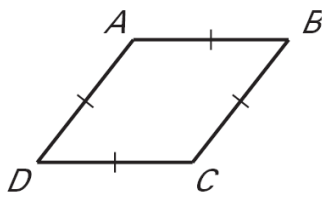
For any rhombus *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your reasoning.

a. $AB > BC$

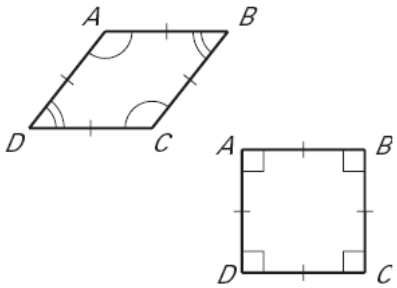
b. $m\angle A > m\angle B$

Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. So, $AB = BC$. The statement is *never* true.



b. By definition, a rhombus is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent. Because $\angle A$ and $\angle B$ are not opposite angles, they are not necessarily congruent, and $m \notin A$ could be greater than $m \notin B$. If rhombus *ABCD* is a square, then $m \notin A = m \notin B = 90°$. So, the statement is *sometimes* true.



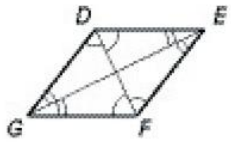
Exercises for Example 1

For any rhombus *DEFG*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your reasoning.

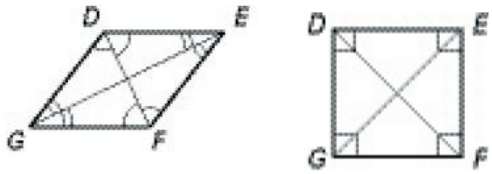
1. $\angle DEG \cong \angle FEG$
2. $\angle DEG \cong \angle EFD$
3. $\overline{DG} \cong \overline{GF}$

SOLUTIONS:

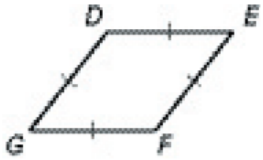
1. always



2. sometimes

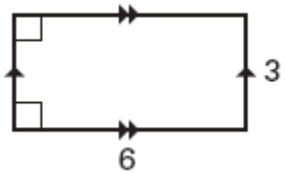


3. always



EXAMPLE 2 **Classify special quadrilaterals**

Classify the special quadrilateral. *Explain* your reasoning.

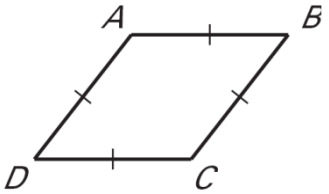


Solution
The quadrilateral is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent, so all four angles of the quadrilateral are right angles. By the Rectangle Corollary, the quadrilateral is a rectangle. Because the four sides are not congruent, the rectangle is not a square.

EXAMPLE 3 **List properties of special parallelograms**

Sketch rhombus *ABCD*. List everything you know about it.
Solution
By definition, you need to draw a figure with the following properties:

- The figure is a parallelogram.
- The figure has four congruent sides.



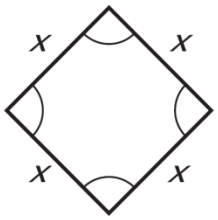
Because *ABCD* is a parallelogram, it also has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

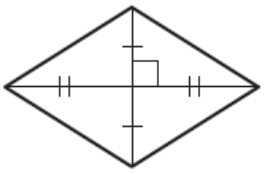
By Theorem 8.11, the diagonals of *ABCD* are also perpendicular.

Exercises for Examples 2 and 3 **Classify the special quadrilateral. *Explain* your reasoning.**

4.



5.

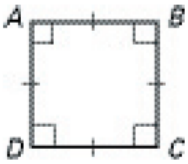


6. Sketch square *ABCD*. List everything you know about it.

SOLUTIONS TO #4, 5, AND 6

4. The quadrilateral has four congruent sides, so it is a rhombus. Because all four angles are congruent, by the Corollary to Theorem 8.1, the measure of each angle is $360° \div 4 = 90°$, and the quadrilateral is a rectangle. So, by the Square Corollary, the quadrilateral is a square.

5. The diagonals bisect each other, so by Theorem 8.10 the quadrilateral is a parallelogram. The diagonals are perpendicular, so by Theorem 8.11 the parallelogram is a rhombus.

6. 

By definition, square *ABCD* is a parallelogram with four congruent sides and four right angles. Because *ABCD* is a parallelogram, it also has these properties: Opposite sides are parallel and congruent; opposite angles are congruent; consecutive angles are supplementary; and diagonals bisect each other. Because squares are also rectangles, by Theorem 8.12, the diagonals of *ABCD* are congruent.