

The Academic Perspective

Mathletics Diagnostic Tests

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Introduction

The Mathletics suite of stand-alone or LAN-delivered objective tests now comprises some 4600 questions spanning 175 different skills areas from GCSE to level 1 university mathematics. Much use is made of multi-choice questions where sensible, but the incorrect use of rules leads to a wrong choice being made. This means that highly-focused feedback can often tell a student not just that they are wrong, but why, see [1]. Different question libraries can be used alone (e.g. a sine rule “track and field” test); or in combinations, such as an algebra pentathlon, calculus marathon, or in diagnostic tests spanning a wider range of skills at an assumed Higher Education (HE) entry level (post-GCSE or post A-Level mathematics).

The Mathletics resource can be used in both formative and summative modes – for detailed results for various student cohorts and comparison with unseen written examination marks see [2]. Many students are assessment driven, and this has led to (easily created) bespoke tests for mathematics modules taken by maths students and those of other departments. Formative and group learning is encouraged by including each student's best ever mark for a test in their overall module mark (but the student may still be required to pass the unseen written exam at the end of the module). More recently, the resource has been used to provide remedial courses for engineering and computer science students lacking fluent GCSE maths skills.

Content

- **NUMBERS:** General Arithmetic, Arithmetic Models, Terminology, Decimals & Scientific Notation, Fractions, Power of Numbers, Percentages, Numerical Sequences, Units and Dimensions, Surds, General Complex Numbers, Add Complex, Multiply and Divide Complex, Argand diagram, Complex Polynomials, DeMoivre, Definitions and Notation, Divisibility and Prime Numbers, Euclidean Algorithm, Modular Arithmetic, $U(n)$ Groups 1 & 2.
- **PROBABILITY & STATISTICS:** Basic Probability, Combinations, Permutations, Probability Trees, Data Types, Data Display, Analysis of Data, Shapes of Data, Definitions of Measure, Measures of Location, Measures of Dispersion, Correlation, Binomial Distribution, Cumulative Binomial Distribution, Poisson Distribution, Cumulative Poisson Distribution, Normal Distribution 1 & 2, Central Limit Theorem, Confidence Intervals, Hypothesis Testing 1, 2 & 3, Basic Queuing, Little's Laws
- **ALGEBRA:** Proportionality, Linear Equations, Modelling, Co-ordinates, Sequences 1 & 2, Expanding Brackets, Factorisation, Flow Charts, Rearranging Equations, Indices 1 & 2, Simplification, Solving Equations, Using Formulae, APs, GPs, Sigma Notation, Completing the Square, Inequalities, Simultaneous Equations, Growth/Decay, Pascal Triangle, Binomial Theorem, Polynomial Multiplication, Polynomial Division, Min/Max of Quadratics, Partial Fractions 1 & 2.

- **FUNCTIONS:** Recognising Graphs, Limits of Functions, Domains and Ranges 1 & 2, Inverse Functions, Function Manipulation, Symmetry of Functions, Degrees \leftrightarrow Radians, Trigonometry Definitions, Special Trigonometry Values, Trigonometry Equations 1 & 2, Trigonometry Graphs 1 & 2, Reciprocal Trigonometry Functions, Sine Rule, Cosine Rule, Combining Signals, General Trigonometry Solutions, Trigonometry Identities, Logarithms and Exponentials.
- **DIFFERENTIATION:** General Differentiation, Differentiation of Powers, Differentiation of Products, Differentiation of Quotients, Chain Rule, Differentiation of Logarithms and Exponentials, Differentiation of Trigonometry, Differentiation of Hyperbolics, Differentiation of Inverse Trig., Differentiation of Inverse Hyperbolics, Logarithmic differentiation, Parametric Differentiation, Implicit Differentiation Series & Expansions, Differentiation of Functions of two variables.
- **INTEGRATION:** Integration of Polynomials, Integration of Algebraic Functions, Integration of Rational Functions, Integration of Trigonometry Functions, Integration of Hyperbolic Functions, Integration by Substitution, Integration by Partial Fractions 1 & 2, Integration by Parts 1 & 2, Double Integration, ODEs: First Order Integrating Factors 1, 2 & 3, First Order Solution by Integrating Factors with bc's, First order Separable 1, 2, 3, First Order Solution by Separation with bc's, Second Order Complementary Function, Second Order Particular Integral, Second BVP, Second Order Classification
- **LAPLACE TRANSFORMS:** Basic Laplace Transforms 1 & 2, First Shift Theorem, Inverse Laplace Transforms and Applications to ODEs.
- **NUMERICAL METHODS:** Initial Conditions and Notation, Taylor Series for ODEs, Second, Third and Fourth Order Runge Kutta, Pivoted Systems.
- **STATICS:** Loaded Beams, Resolving Forces, Pivoted Systems
- **VECTORS & MATRICES:** General Vector, Vector Addition, Dot and Cross Products, Triple Products, Lines and Planes, 2x2 and 3x3 Numeric and Algebraic Determinants, Applications of Determinants, Matrix Addition and Scalar Multiplication, Matrix Multiplication, Inverse Matrix, Non-Square Systems, Translation, Rotation and Shear, 2x2 and 3x3 Eigenvalues and Eigenvectors
- **GRAPH THEORY:** Basics, Digraphs, Colourings, Networks, Paths and Walks 1 & 2.

Participants

Increasingly, Mathletics has become a workhorse of our teaching at Brunel University; in the 1999/2000 academic year some 600 foundation, mathematics, engineering and biological sciences students took over 23,000 diagnostic and continual assessment tests, whilst for 2000/01 the tally was 25,000. This year (2002/03) about 300 students took diagnostic tests in freshers' week.

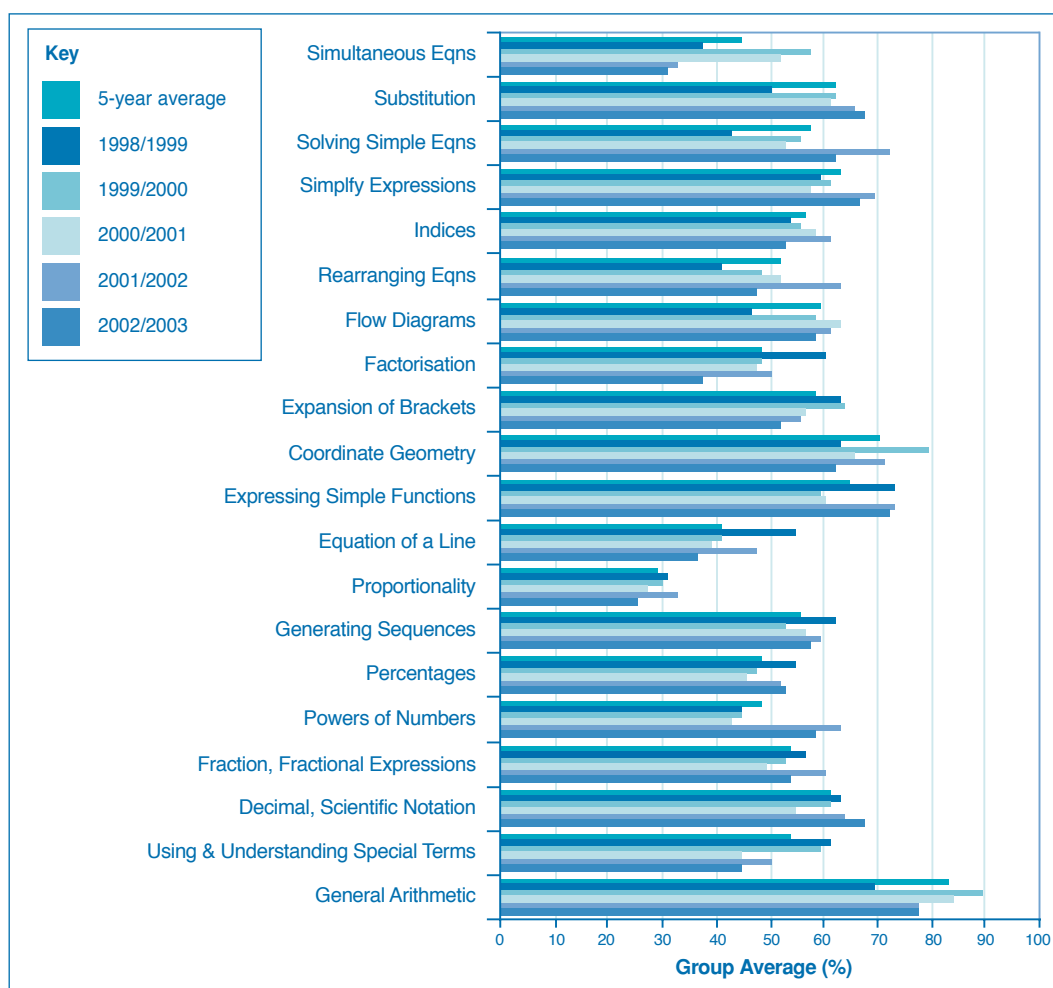


Figure 1: Zero Level Diagnostic Tests 1998/2003 – Year-on-year group averages for Foundation Students. Updated from [2].

The current snapshot (taken November 2002) is typical; level 1 undergraduates in mathematics have just completed a calculus test and are about to start one on linear algebra; systems engineers are mid-way through a set of tests, whilst Biologists and Foundations of Science, IT and Engineering have completed an extensive set of algebra tests and are starting a similar set on functions and (polynomial) calculus. About six maths staff are involved, two of whom manage all the answer files.

Much of this activity takes place during students' free time, but some happens in staffed workshops. The teaching dynamic is interesting here; staff are able to jump in when a student has questions and teach alongside the student. This results in the lecturer being on the same side as the student, physically and metaphorically, so it's "us against the PC" rather than "staff versus student". Since the student is also focused on the problem, this makes for extremely effective learning.

Feedback to Students/Staff

Diagnostic tests are followed-up with emails to each student/student tutor with results and advice. Discussions often result in students revising or learning material and retaking the test. Equally important is the whole class view of first years skills, which informs level 1 lecturers and admissions tutors. An advantage of CAL is the speed and comparative ease with which large amounts of data can be interpreted in a useful (actionable) way, see Figure 1.

Dissemination/Developments

Mathletics is available free on request and about 120 universities and colleges in the UK have already been sent a copy. Given the blurred division between school and university mathematics, much of the above content has been explicitly linked to GCSE, A/S-Level and A-Level topics and repackaged as "Mathletics for schools". Again this (free) resource is proving popular.

[3] describes recent technical developments to a web-based version of Mathletics, such as dynamic questions and diagrams that change according to random parameters chosen at runtime.

References

- [1] *Setting Objective Tests in Mathematics with QM Designer*, Maths, Stats & OR, February, Greenhow, M. (2000).
- [2] *Answer Files – what more do they reveal?* LTSN Maths, Stats & OR Network, CAA Series <http://ltsn.mathstore.ac.uk/articles/maths-caa-series/jan2002/index.shtml>, Greenhow, M. (2002).
- [3] *Using Question Mark Perception v3 for Testing Mathematics*, MSOR Connections 2(3): August, Greenhow, M. and Nichols, D. (2002).