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Preface

The basic topic of this book is solving problems from system and control theory using convex optimization. We show that a wide variety of problems arising in system and control theory can be reduced to a handful of standard convex and quasiconvex optimization problems that involve matrix inequalities. For a few special cases there are "analytic solutions" to these problems, but our main point is that they can be solved numerically in all cases. These standard problems can be solved in polynomial-time (by, e.g., the ellipsoid algorithm of Shor, Nemirovskii, and Yudin), and so are tractable, at least in a theoretical sense. Recently developed interior-point methods for these standard problems have been found to be extremely efficient in practice. Therefore, we consider the original problems from system and control theory as solved.

This book is primarily intended for the researcher in system and control theory, but can also serve as a source of application problems for researchers in convex optimization. Although we believe that the methods described in this book have great practical value, we should warn the reader whose primary interest is applied control engineering. This is a research monograph: We present no specific examples or numerical results, and we make only brief comments about the implications of the results for practical control engineering. To put it in a more positive light, we hope that this book will later be considered as the *first* book on the topic, not the most readable or accessible.

The background required of the reader is knowledge of basic system and control theory and an exposure to optimization. Sontag's book *Mathematical Control Theory* [Son90] is an excellent survey. Further background material is covered in the texts *Linear Systems* [Kai80] by Kailath, *Nonlinear Systems Analysis* [Vid92] by Vidyasagar, *Optimal Control: Linear Quadratic Methods* [AM90] by Anderson and Moore, and *Convex Analysis and Minimization Algorithms I* [HUL93] by Hiriart–Urruty and Lemaréchal.

We also highly recommend the book *Interior-point Polynomial Algorithms in Convex Programming* [NN94] by Nesterov and Nemirovskii as a companion to this book. The reader will soon see that their ideas and methods play a critical role in the basic idea presented in this book.