Lesson 17: The Area of a Circle

Use reference sheet

$$C = \pi d$$
or
$$C = 2\pi r$$

Classwork

Exercises 1–3

Solve the problem below individually. Explain your solution.

1. Find the radius a circle if its circumference is 37.68 inches. ~~Use π as 3.14~~

r?

Why did I choose this formula?

$$C = 2\pi r$$
$$37.68 = 2\pi r$$
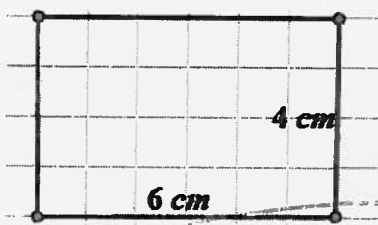$$\left(\frac{1}{2\pi}\right) 37.68 = \frac{1}{2\pi}(2\pi r)$$
$$6 \approx r$$

*The radius is **about** 6 inches.*

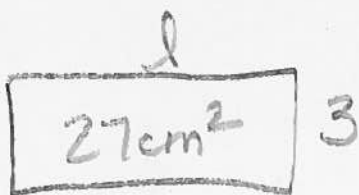2. Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.

① Count unit squares

② L · W

4 cm

6 cm

area = 24 cm²

3. Find the length of a rectangle if the area is 27 cm² and the width is 3 cm.

ℓ

27 cm²   3

$$3\ell = 27$$
$$\frac{1}{3}(3\ell) = 27\left(\frac{1}{3}\right)$$
$$\ell = 9 \text{ cm}$$

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Lesson 17:   The Area of a Circle

Exploratory Challenge

To find the formula for the area of a circle, cut a circle into 16 equal pieces.
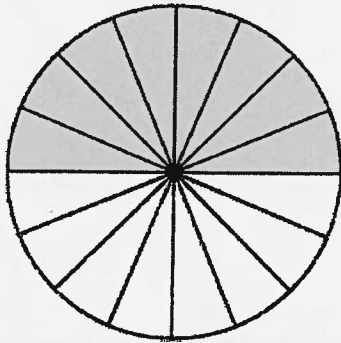
Arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate "rectangle."

"radius"

"half of the circumference"

Move half to the other end.
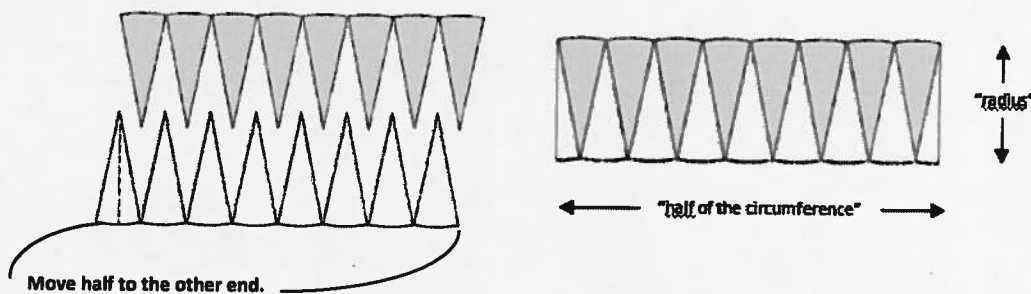
The circumference is $2\pi r$, where the radius is r. Therefore, half of the circumference is πr.

r

πr
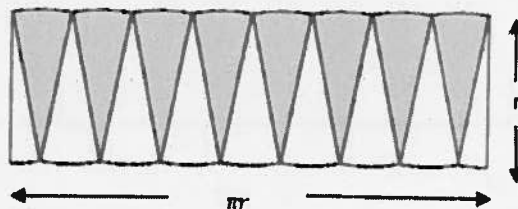
What is the area of the "rectangle" using the side lengths above?

$A = bh$

$A = \pi r \cdot r$

Are the areas of the "rectangle" and the circle the same?

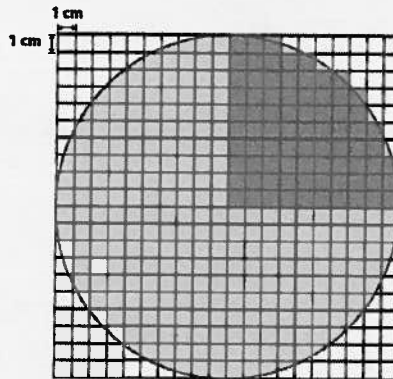yes → the pieces were just rearranged

reference sheet! →

If the area of the rectangular shape and the circle are the same, what is the area of the circle?

$$A = \pi \cdot r \cdot r \quad \text{or} \quad A = \pi r^2$$

Example 1

Use the shaded square centimeter units to approximate the area of the circle.

What is the radius of the circle?

$$10 \text{ cm}$$

What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

Count how many squares are in the "¼ section" then × 4.

Using the diagram, how many squares were used to cover one-fourth of the circle?

$$\approx 79 \text{ cm}^2$$

What is the area of the entire circle?

$$79 \times 4$$
$$\approx 316 \text{ cm}^2$$

* Try the formula *
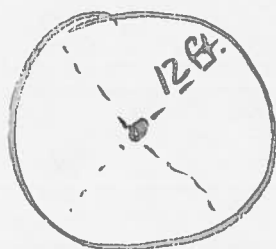
$$A = \pi r r$$
$$A = \pi \cdot 10 \cdot 10$$
$$A = 100 \pi$$
$$A \approx 314 \text{ cm}^2$$

Example 2

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?

$r = 12$ ft.

$A = ?$

$A = \pi r^2$

$A = \pi \cdot r \cdot r$

$A = \pi \cdot 12 \cdot 12$ ✓ exact

$A = 144\pi$ ft^2

$A \approx 452$ ft^2 ← approximate

What information is needed to solve the problem?

The formula!

Example 3

Suzanne is making a circular table out of a square piece of wood. The radius of the circle that she is cutting is 3 feet. How much waste will she have for this project? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?

$r = 3$ ft

waste

$(6 \cdot 6) - (\pi \cdot 3 \cdot 3)$
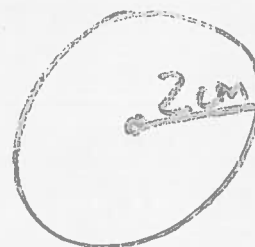
$36 - (9\pi)$

≈ 8 ft^2 of waste

What information is needed to solve the problem?

The area of the square & the circle to determine the waste.

What information do we need to determine the area of the square and the circle?

How will we determine the waste?

Does your solution answer the problem as stated?

Exercises 4–6

4. A circle has a radius of 2 cm.

    a. Find the exact area of the circular region.

$$A = \pi r^2$$
$$A = \pi r r$$
$$A = \pi \cdot 2 \cdot 2$$
$$A = 4\pi \ cm^2$$

    b. Find the approximate area using ~~3.14 to approximate π~~. π key.

$4 \cdot \pi$ on calc → $\approx 12.57 \ cm^2$

5. A circle has a radius of 7 cm.

    a. Find the exact area of the circular region.

$$A = \pi r^2$$
$$A = \pi \cdot r \cdot r$$
$$A = \pi \cdot 7 \cdot 7$$
$$A = 49\pi \ cm^2$$

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b. Find the approximate area using $\frac{22}{7}$ to approximate π. ~~π Key.~~

$$49\pi \approx 153.9 \text{ cm}^2$$

c. What is the circumference of the circle?

$r = 7$
$d = 14$

$C = \pi d$
$C = \pi \cdot 14$
$C = 14\pi \text{ cm}$

6. Joan determined that the area of the circle below is 400π cm². Melinda says that Joan's solution is incorrect; she believes that the area is 100π cm². Who is correct and why?
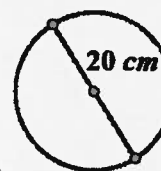
Melinda is correct.
Joan used the diameter instead of the radius.

$A = \pi r^2$
$A = \pi r r$
$A = \pi \cdot 10 \cdot 10$
$A = 100\pi \text{ cm}^2$

20 cm

$r = 10$
$d = 20$

Relevant Vocabulary

CIRCULAR REGION (OR DISK): Given a point C in the plane and a number $r > 0$, the *circular region (or disk)* with center C and radius r is the set of all points in the plane whose distance from the point C is less than or equal to r.

The boundary of a disk is a circle. The *area of a circle* refers to the area of the disk defined by the circle.

(handwritten: Scroll down ⇊)

Exit Ticket: Sample Solutions

Complete each statement using the words or algebraic expressions listed in the word bank below.

1. The length of the *height* of the rectangular region approximates the length of the *radius* of the circle.
2. The *base* of the rectangle approximates the length of one-half of the circumference of the circle.
3. The circumference of the circle is $2\pi r$.
4. The *diameter* of the *circle* is 2r.
5. The ratio of the circumference to the diameter is π.
6. Area (circle) = Area of *(rectangle)* $= \frac{1}{2} \cdot$ circumference $\cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r = \pi r^2$

Problem Set Sample Solutions

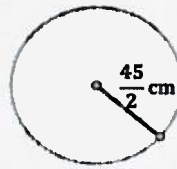1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)

   21 cm   —   81 ft   — $\frac{45}{2}$ cm
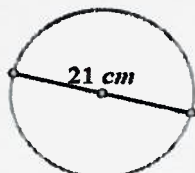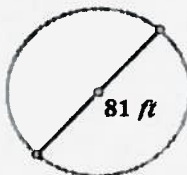
   346.5 cm²   —   5,155.1 ft²   —   1,591.1 cm²

2. A circle has a diameter of 20 inches.

   a. Find the exact area, and find an approximate area using $\pi \approx 3.14$.

   If the diameter is 20 in., then the radius is 10 in. If $A = \pi r^2$, then $A = \pi \cdot (10 \text{ in.})^2$ or 100π in². $A \approx (100 \cdot 3.14) \text{ in}^2 \approx 314 \text{ in}^2$.

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 20 in., then the circumference is $C = \pi d$ or $C \approx 3.14 \cdot 20$ in. ≈ 62.8 in.

3. A circle has a diameter of 11 inches.

   a. Find the exact area and an approximate area using $\pi \approx 3.14$.

   If the diameter is 11 in., then the radius is $\frac{11}{2}$ in. If $A = \pi r^2$, then $A = \pi \cdot \left(\frac{11}{2} \text{ in.}\right)^2$ or $\frac{121}{4}\pi$ in².

   $$A \approx \left(\frac{121}{4} \cdot 3.14\right) \text{ in}^2 \approx 94.985 \text{ in}^2$$

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 11 inches, then the circumference is $C = \pi d$ or $C \approx 3.14 \cdot 11$ in. ≈ 34.54 in.

4. Using the figure below, find the area of the circle.
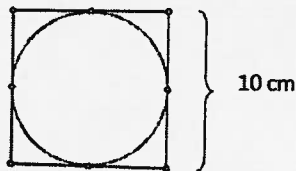
10 cm

In this circle, the diameter is the same as the length of the side of the square. The diameter is 10 cm; so, the radius is 5 cm. $A = \pi r^2$, so $A = \pi(5 \text{ cm})^2 = 25\pi \text{ cm}^2$.

5. A path bounds a circular lawn at a park. If the inner edge of the path is 132 ft. around, approximate the amount of area of the lawn inside the circular path. Use $\pi \approx \frac{22}{7}$.

    The length of the path is the same as the circumference. Find the radius from the circumference; then, find the area.

$$C = 2\pi r$$
$$132 \text{ ft.} \approx 2 \cdot \frac{22}{7} \cdot r$$
$$132 \text{ ft.} \approx \frac{44}{7} r$$
$$\frac{7}{44} \cdot 132 \text{ ft.} \approx \frac{7}{44} \cdot \frac{44}{7} r$$
$$21 \text{ ft.} \approx r$$

$$A \approx \frac{22}{7} \cdot (21 \text{ ft.})^2$$
$$A \approx 1386 \text{ ft}^2$$

6. The area of a circle is $36\pi \text{ cm}^2$. Find its circumference.

    Find the radius from the area of the circle; then, use it to find the circumference.

$$A = \pi r^2$$
$$36\pi \text{ cm}^2 = \pi r^2$$
$$\frac{1}{\pi} \cdot 36\pi \text{ cm}^2 = \frac{1}{\pi} \cdot \pi r^2$$
$$36 \text{ cm}^2 = r^2$$
$$6 \text{ cm} = r$$

$$C = 2\pi r$$
$$C = 2\pi \cdot 6 \text{ cm}$$
$$C = 12\pi \text{ cm}$$

7. Find the ratio of the area of two circles with radii 3 cm and 4 cm.

    The area of the circle with radius 3 cm is $9\pi \text{ cm}^2$. The area of the circle with the radius 4 cm is $16\pi \text{ cm}^2$. The ratio of the area of the two circles is $9\pi : 16\pi$ or $9 : 16$.

8. If one circle has a diameter of 10 cm and a second circle has a diameter of 20 cm, what is the ratio of the area of the larger circle to the area of the smaller circle?

   The area of the circle with the diameter of 10 cm has a radius of 5 cm. The area of the circle with the diameter of 10 cm is $\pi \cdot (5 \text{ cm})^2$, or $25\pi \text{ cm}^2$. The area of the circle with the diameter of 20 cm has a radius of 10 cm. The area of the circle with the diameter of 20 cm is $\pi \cdot (10 \text{ cm})^2$ or $100\pi \text{ cm}^2$. The ratio of the diameters is 20 to 10 or 2: 1, while the ratio of the areas is 100π to 25π or 4: 1.

9. Describe a rectangle whose perimeter is 132 ft. and whose area is less than 1 ft². Is it possible to find a circle whose circumference is 132 ft. and whose area is less than 1 ft²? If not, provide an example or write a sentence explaining why no such circle exists.

   A rectangle that has a perimeter of 132 ft. can have a length of 65.995 ft. and a width of 0.005 ft. The area of such a rectangle is 0.329975 ft², which is less than 1 ft². No, because a circle that has a circumference of 132 ft. has a radius of approximately 21 ft.

   $$A = \pi r^2 = \pi(21)^2 = 1387.96 \neq 1$$

10. If the diameter of a circle is double the diameter of a second circle, what is the ratio of the area of the first circle to the area of the second?

    If I choose a diameter of 24 cm for the first circle, then the diameter of the second circle is 12 cm. The first circle has a radius of 12 cm and an area of $144\pi \text{ cm}^2$. The second circle has a radius of 6 cm and an area of $36\pi \text{ cm}^2$. The ratio of the area of the first circle to the second is 144π to 36π, which is a 4 to 1 ratio. The ratio of the diameters is 2, while the ratio of the areas is the square of 2, or 4.

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