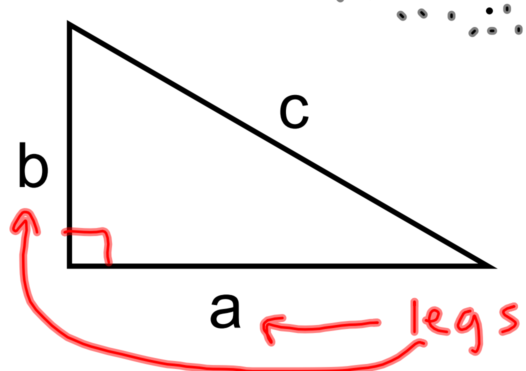
8.1 Geometry - The Pythagorean Theorem

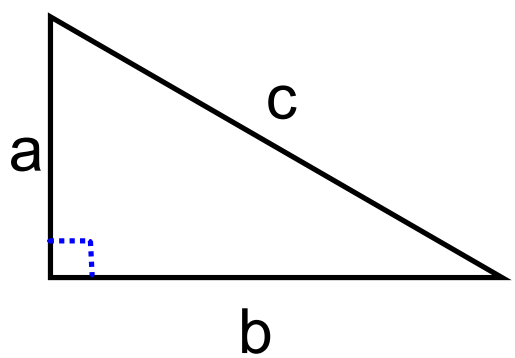
$$a^2 + b^2 = c^2$$



Theorem 8-1 The Pythagorean Theorem

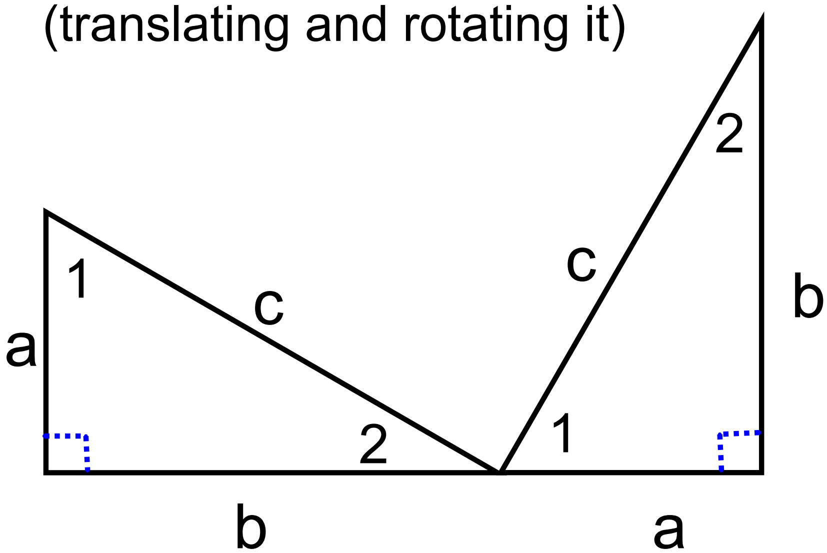
In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse

Start with a right triangle with legs of length a and b and hypotenuse length c.

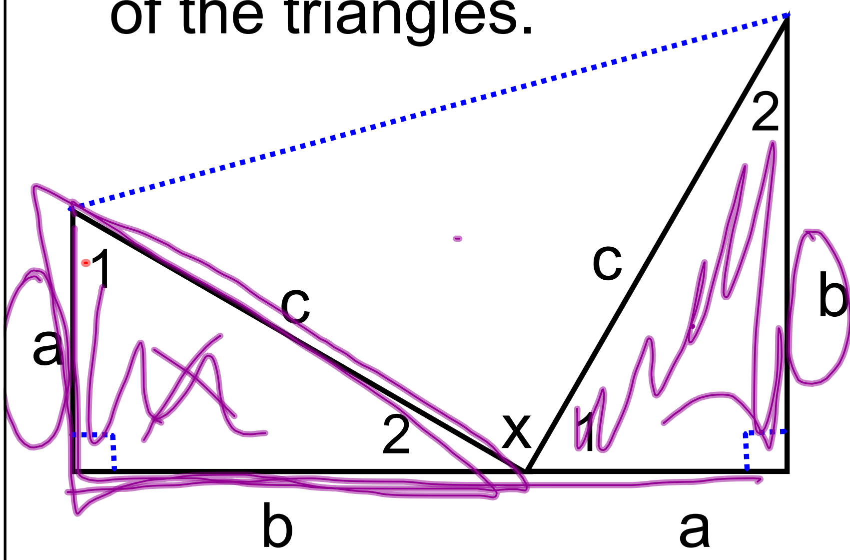


Duplicate this triangle in the following way

(translating and rotating it)



Create a trapezoid by connecting the vertices of the triangles.



$A = \frac{1}{2}h(b_1 + b_2)$

How do we know x is 90 degrees?

Area 1 = Area 2

$\left(\frac{1}{2}(b+a)(b+a) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2\right)2$

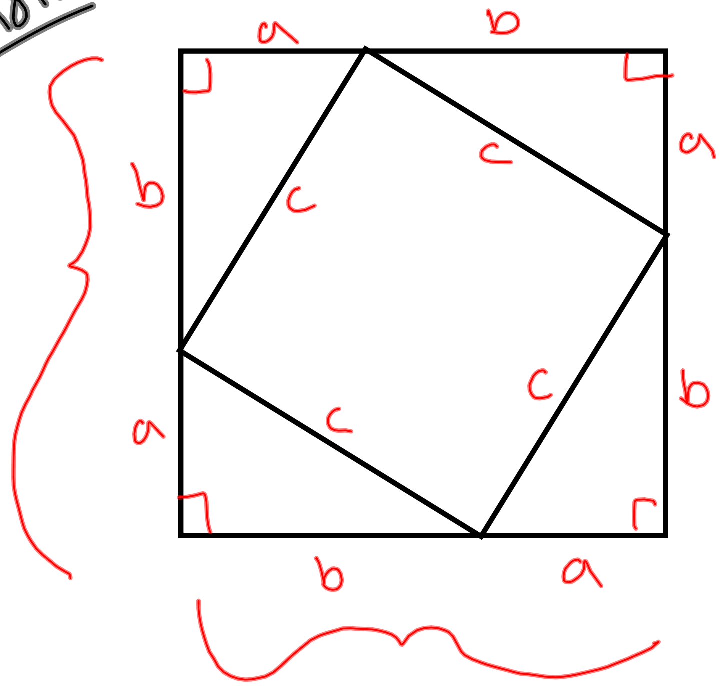
$(b+a)(b+a) = ab + ab + c^2$

$b^2 + 2ab + a^2 = 2ab + c^2$

$-2ab \qquad -2ab$

$\boxed{b^2 + a^2 = c^2}$

A-HA!
Behold



$$(a+b)^2 = c^2 + 4\left(\tfrac{1}{2}ab\right)$$

$$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$$

$$a^2 + b^2 = c^2$$

Pythagorean Triples

A set of 3 whole numbers a, b, c that satisfy the equation $a^2+b^2=c^2$

Examples:

3,4,5 → 6, 8, 10

5,12,13 → 10, 24, 26

8,15,17 → 16, 30, 34

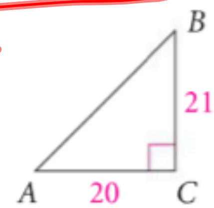
9, 12, 15

when any pythagorean triple is multiplied by a whole # the result is a pyth. triple also

$\boxed{c = \text{hypotenuse}}$

Examples:

Find the length of the hypotenuse of $\triangle ABC$. Do the lengths of the sides of $\triangle ABC$ form a Pythagorean triple?



$20^2 + 21^2 = c^2$

$400 + 441 = c^2$

$\sqrt{841} = \sqrt{c^2}$

$c = 29$

A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. ~~Do the lengths of the sides form a Pythagorean triple?~~

$a^2 + b^2 = c^2$

$10^2 + b^2 = 25^2$

$100 + b^2 = 625$

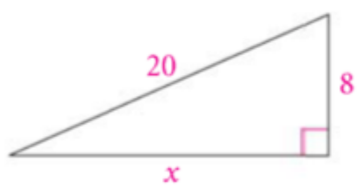
$b^2 = 525$

$b = \sqrt{525} \approx 22.9$

$22.9 < 25$

Example

Find the value of x. Leave your answer in simplest radical form



$$8^2 + x^2 = 20^2$$
$$-8^2 \qquad -8^2$$

$$20^2 - 8^2 = x^2$$
$$400 - 64 = x^2$$
$$336 = x^2$$

$$x = \sqrt{336}$$
$$x = \sqrt{4}\sqrt{84}$$
$$= 2\sqrt{84}$$
$$= 2\sqrt{21 \cdot 4} = 2\sqrt{4} \cdot \sqrt{21}$$
$$= 4\sqrt{21}$$

Theorem 8-2 The Converse of the Pythagorean Theorem

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle

If $a^2 + b^2 = c^2$

then

a, b, c are sides

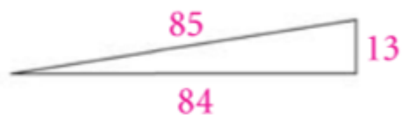
of a rt. triangle.

$$84^2 + 13^2 \stackrel{?}{=} 85^2$$

$$7225 = 7225$$

Example:

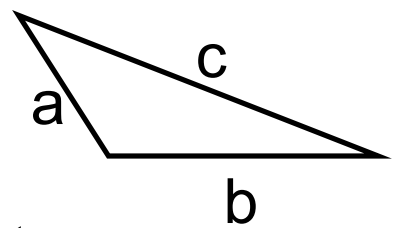
Is this a right triangle?



It is a right $\triangle$

<u>Theorem 8-3</u>

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, the triangle is obtuse.

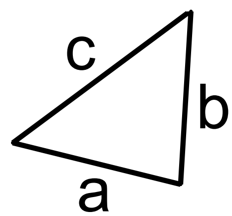


$c^2 > a^2 + b^2$ the triangle is obtuse

C= the longest side

Theorem 8-4

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, the triangle is acute.



$c^2 < a^2 + b^2$ the triangle is acute

Ex. side lengths

6, 11, 14

determine what kind of $\triangle$.

$6^2 + 11^2 = 157$

$14^2 = 196$ $6^2+11^2 < 14^2$

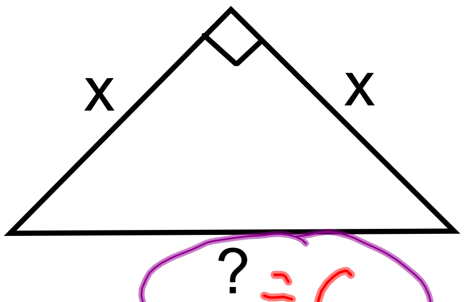
Obtuse

Examples

A triangle has sides of lengths 7, 8, and 9. Classify the triangle by its angles.

Classify the triangle whose side lengths are 6, 11, and 14 as acute, obtuse, or right.

8.2 Special Right Triangles



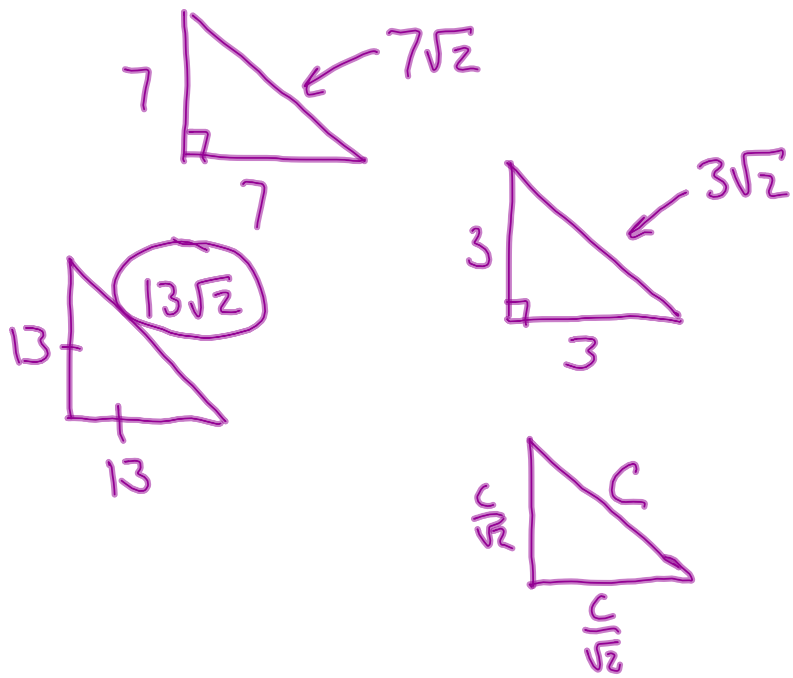
Find the length of the hypotenuse, in terms of x.

$$x^2 + x^2 = c^2$$

$$\sqrt{2x^2} = \sqrt{c^2}$$

$$\sqrt{2}\,x = c$$

$$x\sqrt{2}$$



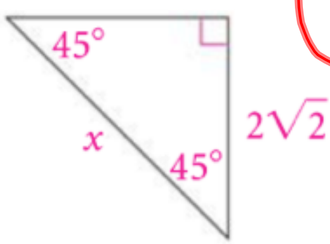
7
7√2
7
13√2
13
13
3√2
3
3
c
c/√2
c/√2

8-5 45-45-90 Triangle Theorem

In a 45-45-90 triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

hypotenuse = $\sqrt{2}$ * leg

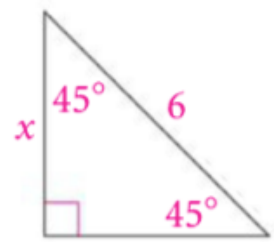
Examples:



$$(\sqrt{2})^2 = 2$$

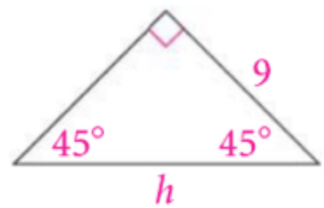
$$(2\sqrt{2})\sqrt{2}$$

$$2\cdot\sqrt{2}\cdot\sqrt{2} = 2\cdot\sqrt{2\cdot 2} = 2\sqrt{4}$$
$$= 2\cdot 2 = 4$$



$$\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2}$$

Example:



Find the length of a leg of a 45°-45°-90° triangle with a hypotenuse of length 10.

Example:

A square garden has sides 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot.

Find the length of the hypotenuse of a $45°$-$45°$-$90°$ triangle with legs of length $5\sqrt{3}$.

A square garden has sides 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot.

Theorem 8-6 30-60-90 Triangle Theorem

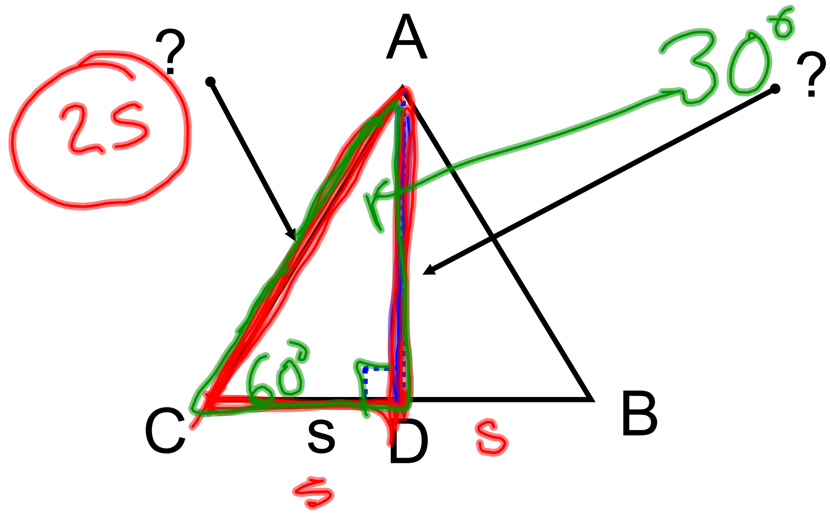
In a 30-60-90 triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

hypotenuse = 2* shorter leg

longer leg = $\sqrt{3}$ shorter leg

Given: ΔABC is a an equilateral triangle with perpendicular bisector AD

Prove: AC=2s and AD=$\sqrt{3}$ s



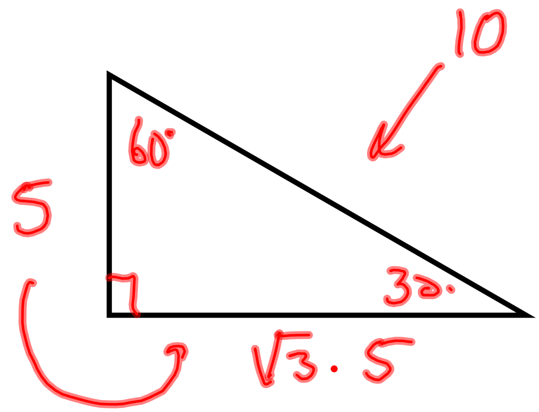
$$s^2+b^2=(2s)^2$$

$$s^2+b^2=4s^2$$

$$-s^2 \qquad -s^2$$

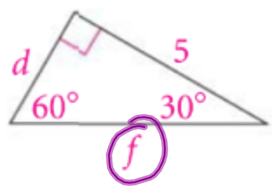
$$\sqrt{b^2}=\sqrt{3s^2}$$

$$\boxed{b=\sqrt{3}\cdot s}$$


10
60°
5
30°
√3 · 5

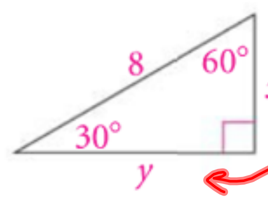
Examples:

Find the value of each variable.



$d = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

$f = \frac{10\sqrt{3}}{3}$



$x = 4$

$4\sqrt{3}$

Road Signs The moose warning sign at the left is an equilateral triangle. The height of the sign is 1 m. Find the length s of each side of the sign to the nearest tenth of a meter.