

**HONORS PRECALCULUS**

**Prove the following identities-**

1.)  $(\cos x - \sin x)^2 = 1 - 2 \sin x \cos x$

2.)  $\tan x + \sec x = \frac{\cos x}{1 - \sin x}$

3.)  $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2 \csc^2 x$

4.)  $\frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$

5.)  $\cot^2 x - \cos^2 x = \cos^2 x \cot^2 x$

6.)  $\frac{\tan x}{\sec x - 1} = \frac{\sec x + 1}{\tan x}$

Solve for  $0 \leq x < 2\pi$

1.  $\sin x = \frac{1}{2}$

2.  $\tan x = 1$

3.  $2\sin x + \sqrt{3} = 0$

4.  $2\cos x + \sqrt{3} = 0$

5.  $2\cos x - \sqrt{3} = 0$

6.  $\tan x + 1 = 0$

7.  $\sqrt{3} \cot x + 1 = 0$

8.  $\csc^2 x = 1$

9.  $\sec^2 x - 1 = 0$

10.  $4\sec^2 x = 8$

11.  $2\cos^2 x = 1$

12.  $4\cos^2 x - 3 = 0$

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**Without a calculator compute the following:**

13.  $\sin 105^\circ$

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**Lets  $\sin x = -5/13$  and  $\cos y = 4/5$  not in quadrant four find each of the following:**

14.  $\sin 2x$

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15.  $\tan 2x$

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16.  $\cos (x+y)$

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**verify the following identities**

17.  $\sin^2 x(1 + \cot^2 x) = 1$

18.  $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

Prove each identity:

$$1. \quad \sec x - \tan x \sin x = \frac{1}{\sec x}$$

$$2. \quad \frac{1 + \cos x}{\sin x} = \csc x + \cot x$$

$$3. \quad \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$4. \quad \frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

$$5. \quad \cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

$$6. \quad \csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$$

$$7. \quad \frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

$$8. \quad \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$9. (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$$

$$10. (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$$

$$11. \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1 - \cot\theta}{1 + \cot\theta}$$

$$12. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$13. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$14. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$15. \frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$$

$$16. \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

## 5.1 and 5.2 Review: Trigonometric Equations and Trigonometric identities

Prove the following trigonometric identities.

$$1. \quad \frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$$

$$2. \quad \frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$$

$$3. \quad \frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2 \csc^2 x$$

$$4. \quad (\cot^2 x + 1)(\sin^2 x - 1) = -\cot^2 x$$

$$5. \quad \csc x + \cot x = \frac{\sin x}{1 - \cos x}$$

$$6. \quad \sqrt{\frac{1 + \sin x}{1 - \sin x}} = \frac{1 + \sin x}{|\cos x|}$$

Find all solutions to the following trigonometric equations

7.  $2\cos x - 1 = 0$

8.  $\tan x \sin^2 x = 2 \tan x$

9.  $3\sec^2 x - 4 = 0$

10.  $2\cos^2 x = 1$

11.  $4\cos^2 x - 3 = 0$





Use identities to find all solutions in the interval  $[0, 2\pi)$ .

7.  $2\sin 3x \cdot \cos x = 0$

8.  $\cos 3x + \cos x = 0$

Use identities to find the general solution, i.e., ALL solutions, to the following.

9.  $\cos 3x = \sin 2x$

10.  $\cos^2 x = \sin^2\left(\frac{x}{2}\right)$

Prove the following identities:

11.  $\sin 3x = 3\cos^2 x \sin x - \sin^3 x$

12.  $\sec 2x = \frac{\csc^2 x}{\csc^2 x - 2}$

### Law of Sines

1. In triangle  $EFG$ ,  $e = 4.56$ ,  $E = 43^\circ$ , and  $G = 57^\circ$ . Solve the triangle.
2. In triangle  $ABC$ ,  $b = 24$ ,  $B = 38^\circ$ , and  $C = 21^\circ$ . Solve the triangle.
3. In triangle  $ABC$ ,  $a = 15.6$  in.,  $A = 89^\circ$ , and  $b = 18.4$  in. Solve the triangle.
4. In triangle  $ABC$ ,  $A = 36.5^\circ$ ,  $a = 24$ , and  $b = 34$ . Solve the triangle.
5. In triangle  $ABC$ ,  $a = 20.01$  cm,  $b = 10.07$  cm, and  $A = 30.3^\circ$ . Solve the triangle.

### Law of Cosines

Solve each  $ABC$  triangle with the given information.

1.  $a = 6.8, c = 2.4, B = 10.5^\circ$

2.  $a = 30.4, b = 28.9, c = 31.6$

3.  $a = 3, b = 4, c = 7$

4.  $a = 3.1, b = 2.9, C = 121.3^\circ$

### Laws of Sines and Cosines

1. Points A and B are on opposite sides of the Grand Canyon. Point C is 200 yards from A. Angle B measures  $87^{\circ} 37'$  and angle C measures  $67^{\circ} 12'$ . What is the distance between A and B?
2. Two observers are standing on shore  $\frac{1}{2}$  mile apart at points A and B and measure the angle to a sailboat at a point C at the same time. Angle A is  $63^{\circ} 24'$  and angle B is  $56^{\circ} 36'$ . Find the distance from each observer to the sailboat.
3. A person at A looks due east and sees a UFO with an angle of elevation of  $40^{\circ}$ . At the same instant, another person, 1.0 miles due west of A looks due east and sights the same UFO with an angle of elevation of  $25^{\circ}$ . Find the distance between A and the UFO. How far is the UFO above the ground?
4. A vertical flagpole is attached to the top edge of a building. A man stands 400 feet from the base of the building. From his viewpoint, the angle of elevation to the bottom of the flagpole is  $60^{\circ}$ ; to the top is  $62.5^{\circ}$ . Determine the height of the flagpole.
5. When a boy stands on the bank of a river and looks across to the other bank, the angle of depression is  $12^{\circ}$ . If he climbs to the top of a 10-ft tree and looks across to the other bank, the angle of depression is  $15^{\circ}$ . What is the distance from the first position of the boy to the other bank of the river? How wide is the river?
6. In a recreation park a children's slide is 27 feet long and makes an angle of  $39^{\circ}$  with the ground. Its top is reached by a ladder 18 feet long. What is the angle of inclination of the ladder?
7. A small town is separated from the local power plant by mountainous terrain and several lakes. Until now, electrical power has been routed through a nearby city. The recent development of a stronger wire permits a direct line to be constructed. Sighting from the town, the angle between the city and the power plant is  $77.17^{\circ}$ . The distance between the city and the town is 123 km. The distance from the power plant to the city is 156 km. What is the distance "as the crow flies" between the town and the power plant?
8. Two pedestrians walk from opposite ends of a city block to a point on the other side of the street. The angle formed by their paths is  $25^{\circ}$ . One pedestrian

walks 300 feet, the other walks 320 feet. How long is the city block?

9. Points A and B are sighted from point C. If  $C = 98^\circ$ ,  $AC = 128$  m and  $BC = 96$  m, how far apart are points A and B?

10. Two sides and the included angle of a parallelogram have measures 3.2, 4.8, and  $54^\circ 24'$  respectively. Find the lengths of the diagonals.

11. The lengths of two sides of a parallelogram are 24.6 inches and 38.2 inches. The angle at one vertex has measure  $108^\circ 42'$ . Find the lengths of the diagonals.

12. A bridge is supported by triangular braces. If the sides of each brace have lengths 63 feet, 46 feet and 40 feet, find the measure of the angle opposite the 46 ft side.

13. The measures of two sides of a parallelogram are 28 in and 42 in. If the longer diagonal has measure 58 in, find the measures of the angles at the vertices.

14. On a baseball diamond with 90-foot sides, the pitcher's mound is 60.5 feet from home plate. How far is the pitcher's mound from third base?

15. On a map, Orlando is 178 mm due south of Niagara Falls, Denver is 273 mm from Orlando, and Denver is 235 mm from Niagara Falls. Find the bearing of Denver from Orlando. Find the bearing of Denver from Niagara Falls.

From <http://www.gadsdenst.cc.al.us/math/index.htm> Gadsden State Community College in Alabama