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Short Communication

Improving connectivity in vehicular ad hoc networks: An analytical study ☆

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Abstract

Connectivity in vehicular ad hoc networks may degrade dramatically in sparse traffic and also high speed highways. In this paper we study a way to improve the connectivity by adding some extra nodes with higher transmission range which we call mobile base-stations. These nodes can also offer commercial services (e.g. advertisement, video, audio, etc.) to the vehicles on roads. Besides, the financial profit of those services also depends on a satisfactory connectivity. We use an equivalent $M/G/\infty$ queuing model in order to investigate the connectivity. We further take into account the case when some vehicles do not participate in the network either because they are not equipped with wireless transceivers or some other reasons like security concerns. Moreover, connectivity in presence of fixed Road Side Units (RSUs) is also studied. Our proposed analytical model can be used to find the optimum values of the number of base-stations as well as their transmission range in order to achieve a desired degree of connectivity.

Keywords: VANETs; Connectivity; Infinite server queue; Mobile base-station; Road Side Unit

1. Introduction

Vehicular ad hoc networks (VANETs) are formed spontaneously between moving vehicles on roads. The performance of this type of networks is affected by characteristics of road's traffic. From theory of traffic [1] we know that the traffic state on a road can be studied in two different phases. First when the density of vehicles is low, vehicles can drive as fast as they want or wish. This state which is called free-flow state, holds until the density reaches a threshold called critical value. Beyond this density, some breakdown locations appear on the highway which lead to formation of some queues of vehicles. This phase is called forced-flow state. From the communication point of view which we persue in VANETs, different challenges should be addressed in each traffic state. Obviously, connectivity is satisfactory in the forced-flow state while it deteriorates at light load corresponding to the free-flow state in which it might not be possible to transfer messages to other vehicles because of disconnections. In this paper we address the connectivity in the free-flow state.

Connectivity in mobile ad hoc networks (MANETs) has a mature body of research, but it has been little-studied in VANETs. Most of the existing works are simulation studies (e.g.[2]). Recently in [3] an analytical model is proposed which studies the effects of traffic flow and vehicles' speed as well as the transmission range of vehicles on connectivity. The obtained results show that when the road's traffic is sparse and vehicles drive with high speed, the connectivity can be quite poor. In real-life implementations connectivity of VANETs also deteriorates because of

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non-participant vehicles. Since the traffic state and vehicles' speed as well as participation rate are not under the control of network and application designer, one possible way to improve the connectivity is to add some nodes with higher transmission range, called base-stations in this paper, to the network. These extra nodes (vehicles) can also offer commercial services like advertisement, video, audio, etc., to the vehicles on roads. On the other side, the commercial benefit of those services also depends on a good connectivity.

To the best of our knowledge, there is no work in the literature addressing improving connectivity in VANETs. However in the context of conventional ad hoc networks the idea is investigated in a few works. In [4,5] a static network is modeled as a geometric graph. Then the problem of finding optimum positions for extra nodes is reduced to the problem of Euclidean Steiner minimal tree which is NP-hard. The authors therefore proposed some heuristic approaches including an algorithm based on the Minimum Spanning Tree (MST) [4] problem and a Delaunary Triangulation-based algorithm [5]. Clearly these methods are not applicable in highly mobile networks like VANETs. In [6] the idea of improving connectivity by inserting fixed and wired base stations is studied. The authors showed that for the one dimensional case, adding fixed infrastructure improves connectivity significantly. However, similar to [4,5] they assume that the transmission range of the base-stations is the same as the transmission range of ordinary nodes. Moreover, in case of mobile networks, the authors in [7], studied the problem by adding some intelligent nodes called Mobile Infrastructure Nodes (MINEs) which dynamically move to suitable positions in order to re-create disconnected links. For doing that, MINEs need to collect information about the position of network nodes by using a specific protocol called Mobile Infrastructure Location Exchange (MILE). The idea of having some nodes move freely in VANETs seems to us not applicable on a large scale. Our work is different from the above-mentioned work in that in our proposal the base-station vehicles travel along the road in the same way as ordinary vehicles, thus they are mobile and wireless. Furthermore, we assume a more realistic assumption in which the transmission range of the base-stations is larger than that of the ordinary vehicles.

In order to investigate the connectivity in VANETs, we use the results of the work of Miorandi and Altman [8] that identified the equivalence between (i) the busy period of an infinite server queue and the connectivity distance in an ad hoc network, and (ii) that between the number of customers served during the busy period and the number of mobiles in a connected cluster in the ad hoc network. This is obtained when the inter-arrival times in the infinite server queue have the same distribution as the distance between successive nodes and when the service times have the same distribution as the transmission range of the mobiles. Therefore, we need to obtain the distribution function of inter-vehicle distances as well as the distribution of the vehicles' transmission range. From now on, we shall use VANET's terms instead of the queuing terms. In the sequel we study connectivity while we are interested in the following metrics:

- Connectivity distance, defined as the length of a connected path from any given vehicle.
- The number of vehicles in a connected spatial cluster (platoon) or a connected path from any given vehicle.

The rest of this paper is organized as follows. In Section 2 we first obtain the distribution function of inter-vehicle distances as well as the distribution function of vehicles' transmission range. Then in Section 3 we study the connectivity using the equivalent $M/G/\infty$ queuing model as well as the connectivity in presence of fixed Road Side Units (RSUs). In Section 5 we investigate the model numerically along with common traffic statistical data. Finally the paper will be concluded in Section 6.

2. The distribution of inter-vehicle distances and vehicles' transmission range

Assume an observer stands at an arbitrary point of an uninterrupted highway (i.e., without traffic lights, etc.). The vehicles pass the observer with exponentially distributed inter-arrival times with parameter λ_{veh}. In reality it is possible that some vehicles do not participate in the network mainly because of the following two reasons: (1) incomplete market penetration as it is predicted to take rather long time until all vehicles appearing on roads are equipped with wireless transceivers. (2) Unwillingness of drivers because of some reasons like security concerns, etc. From connectivity point of view, a non-participant vehicle in VANETs can be seen as an analogue of an unreliable sensor in the context of large scale sensor networks; the latter case is studied in [9]. Denote by P_{pa}, the probability that a given vehicle participates in the network, then the distribution of the number of participant vehicles can be seen as a thinned Poisson process with parameter $P_{pa}\lambda_{veh}$. Also assume the base-stations are issued with the inter-departure time which is distributed exponentially with rate λ_{BS}. Therefore, we can describe the distribution of number of vehicles (including ordinary participant vehicles and the base-stations), passing the observer, as a Poisson process with parameter $\lambda = P_{pa}\lambda_{veh} + \lambda_{BS}$. Furthermore, assume that there are N discrete levels of constant speed $v_i (i = 1, \ldots, N)$ in the highway where the speeds are *i.i.d.* and independent of the inter-arrival times. This assumption practically holds in the free-flow traffic state in the traffic theory terminology [1]. Denote the rate of arrivals of cars at each level of speed by $\lambda_i (i = 1, \ldots, N)$ where $\sum_{i=1}^{N} \lambda_i = \lambda$, thus, the occurrence probability of each speed level is λ_i/λ. Then it can be shown that inter-vehicle distances are *i.i.d* and exponentially distributed with parameter $\sum_{i=1}^{N} \frac{\lambda_i}{v_i} = \lambda \mathbb{E}(1/V)$ as follows (see the proof in [3]).

$$P(L > x) = 1 - F_L(x) = \mathrm{e}^{-\sum_{i=1}^{N} \frac{\lambda_i}{v_i} x} = \mathrm{e}^{-\lambda \mathbb{E}(1/V) x}, \tag{1}$$

where V is a random variable representing the vehicles' speed. Moreover, Let R_1 and R_2, respectively, be the transmission range of the ordinary vehicles and that of the base-stations. We assume the receiver range of both are the same while $R_2 > R_1$. Furthermore, the ratio of the number of base-stations to all vehicles is denoted by $q = \frac{\lambda_{BS}}{\lambda_{BS} + P_{pa}\lambda_{veh}}$. Thus $p = 1 - q$ is the probability that a random node in the highway is ordinary vehicle. Then the distribution function of the random variable R which represents the transmission range of vehicles can be described as follows:

$$H_R(\alpha) = \begin{cases} 0 & \text{if} \quad \alpha < R_1 \\ p & \text{if} \quad R_1 \leqslant \alpha < R_2 \\ 1 & \text{if} \quad \alpha \geqslant R_2 \end{cases}. \tag{2}$$

3. Connectivity analysis

Since the inter-vehicle distances are exponentially distributed as in (1) and vehicles' transmission ranges are distributed as in (2), we use the equivalent $M/G/\infty$ for investigating the connectivity. From [10] we know the Laplace transform of the probability density function (p.d.f) of the random variable d which represents the connectivity distance is given as:

$$f_d(s) = 1 + \frac{s}{\lambda \mathbb{E}(1/V)} - \frac{1}{\lambda \mathbb{E}(1/V) p^*(s)}, \tag{3}$$

where $p^*(s)$ is the Laplace transform of $p_0(t)$ defined as: $p_0(t) = \mathrm{e}^{-\lambda \mathbb{E}(1/V) \int_0^t (1 - H_R(\alpha)) d\alpha}$. Hence considering (2) after some algebra we obtain the following:

$$p^*(s) = \frac{(1 - \mathrm{e}^{-(s+\xi)R_1})}{s + \xi} - \frac{\mathrm{e}^{-\xi p R_1}(\mathrm{e}^{-(s+\xi(1-p))R_2})}{s + \xi(1-p)} + \frac{\mathrm{e}^{-\xi p R_1}(\mathrm{e}^{-(s+\xi(1-p))R_1})}{s + \xi(1-p)} + \frac{\mathrm{e}^{-sR_2}\mathrm{e}^{-\xi((1-p)R_2 + pR_1)}}{s}, \tag{4}$$

where $\xi = \lambda \mathbb{E}(1/V)$. Then by substituting (4) in (3) we will reach the p.d.f of the connectivity distance. Consequently, the tail probability of connectivity distance: $P_d(\alpha) = P(d > \alpha) = 1 - F_d(\alpha)$ can be found by inverting its complementary cumulative distribution function (c.c.d.f) defined as:

$$P_d^*(s) = \frac{1 - f_d(s)}{s} = -\frac{1}{\lambda \mathbb{E}(1/V)} + \frac{1}{\lambda \mathbb{E}(1/V) s p^*(s)}, \tag{5}$$

where $p^*(s)$ is given in Eq. (4). Since the resulted expression may not be inverted explicitly, we resort to the Gaver–Stehfest method for the numerical inversion [11] in the continuing. However, from [12] we know that the expectation of connectivity distance is given as: $\frac{1}{\lambda \mathbb{E}(1/V) P_0} - \frac{1}{\lambda \mathbb{E}(1/V)}$ where

$$P_0 = \lim_{t \to \infty} P_0(t) = \mathrm{e}^{-\lambda \mathbb{E}(1/V)((1-p)R_2 + pR_1)}. \tag{6}$$

As a result the average connectivity distance is obtained as below:

$$\mathbb{E}(d) = \frac{1 - \mathrm{e}^{-\lambda \mathbb{E}(1/V)\{(1-p)R_2 + pR_1\}}}{\lambda \mathbb{E}(1/V) \mathrm{e}^{-\lambda \mathbb{E}(1/V)\{(1-p)R_2 + pR_1\}}}. \tag{7}$$

Furthermore, we are able to find the average number of vehicles in a platoon which is given by $\lambda \mathbb{E}(1/V)\bar{c}$, where $\bar{c} = \frac{1}{\lambda \mathbb{E}(1/V) P_0}$, is the average distance between the beginning of two consecutive platoons [12]. Thus, expected value of the random variable N representing the platoon size is obtained as below:

$$\mathbb{E}(N) = \frac{1}{P_0} = \mathrm{e}^{\lambda \mathbb{E}(1/V)\{(1-p)R_2 + pR_1\}}. \tag{8}$$

The proposed analytical model provides a tool to design a system with a desired degree of connectivity. For a given scenario (i.e. the speeds' distribution as well as the values of R_1 and λ_{veh}) one can find the best values for λ_{BS} and/or R_2 in order to achieve a desired degree of connectivity while taking into account some design constraints. Note that in practice the number of base-stations and/or their transmission range is constrained from technical and/or financial points of view.

4. Discussion

In the case of a network with different transmission ranges, studying the connectivity using the equivalent infinite server queuing model addresses one-way connectivity. As shown in Fig. 1, when there is a gap in the network, X and Y may not be able to communicate with each other. In presence of a base-station, say A, Y can reach X because A due to its higher transmission range can reach the ordinary vehicle B. However, still X cannot reach Y because B cannot reach A. It should be stressed that most of the safety applications just need one-way data transmission (e.g. when a vehicle informs the approaching vehicles about the occurrence of an accident). However, for the comfort applications (e.g. internet access on roads) the connectivity should be two-way, because data communication protocols need sending and receiving packets simultaneously. Our model can also cover this case considering the following remarks: Let P_{1-way} be the tail probability of one-way connectivity distance (either right-to-left or left-to-right). Then define $\overline{P}_{1-way} = 1 - P_{1-way}$. Now if P_{2-way} stands for the tail probability of 2-way connectivity distance, the following expression always holds:

$$1 - 2\overline{P}_{1-way} \leqslant P_{2-way} \leqslant P_{1-way}. \tag{9}$$

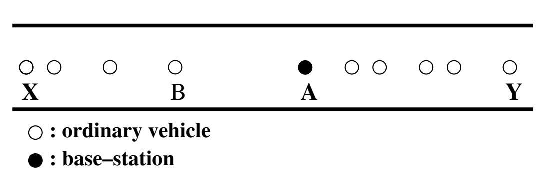

Fig. 1. One-way and two-way connectivity.

From Eq. (9) one can find the lower and the upper bounds on the tail probability of 2-way connectivity distance. If the tail probability of one-way connectivity is sufficiently large, then the margins for the tail probability of two-way connectivity distance will be tight and the results of our model will be more accurate.

4.1. Connectivity in presence of fixed Road Side Units

One of the promising applications of VANETs is the commercial applications in which Road Side Unit (RSU) offer services to the vehicles on roads. Thus studying the connectivity distance from a RSU is practically important. Let R_{RSU} and R_1 be, respectively, the transmission range of the RSU and that of the all vehicles. Denote by d_{RSU} the random variable representing the distance with which the RSU can communicate. This distance can be obtained by taking into account two independent distances: (1) the distance covered by the RSU with its own transmission range, and (2) the distance covered by the ad hoc network formed between vehicles. Consider the car whose location is the smallest among all those who are larger than $R_{RSU} - R_1$, and let X be its distance from that point. If $X > R_1$ then this car is not connected to the RSU and the connectivity distance is R_{RSU}. If $X \leqslant R_1$ then at point $R_{RSU} - R_1 + X$ a new connectivity distance d is started.

It follows from Fig. 2 that the connectivity distance from the location of the RSU is given as follows:

$$d_{RSU} = R_{RSU} + 1\{X \leqslant R_1\}(X + d - R_1), \tag{10}$$

where d is the random variable representing connectivity distance as defined in Section 1. Then the average connectivity distance is obtained as below:

$$\bar{d}_{RSU} = \mathbb{E}(d_{RSU}) = R_{RSU} + (\hat{d} - R_1)P(X \leqslant R_1) + \mathbb{E}(X \cdot 1\{X \leqslant R_1\}), \tag{11}$$

where $\hat{d}$ is the average connectivity distance obtained by considering $p = 1$ in Eq. (7). Note that

$$\mathbb{E}[X \cdot 1\{X \leqslant R_1\}] = \mathbb{E}[X] - \mathbb{E}[X|X > R_1]P(X > R_1). \tag{12}$$

Since X is exponentially distributed with parameter $\xi = \lambda\mathbb{E}(1/V)$, we have

$$\begin{aligned}\mathbb{E}[X| > XR_1] &= \int_0^\infty P(X > t|X > R_1)dt \\ &= R_1 + \int_{R_1}^\infty P(X > t|X > R_1)dt \\ &= R_1 + \int_{R_1}^\infty P(X > t - R_1)dt = R_1 + 1/\xi.\end{aligned} \tag{13}$$

Then as a result

$$\begin{aligned}\mathbb{E}[X \cdot 1\{X \leqslant R_1\}] &= \frac{1}{\xi} - \left(\frac{1}{\xi} + R_1\right)e^{-\xi R_1} \\ &= \left(\frac{1}{\xi} + R_1\right)(1 - \mathrm{e}^{-\xi R_1}) - R_1.\end{aligned} \tag{14}$$

Finally we obtain the following explicit expression for the average connectivity distance from a RSU:

$$\bar{d}_{RSU} = R_{RSU} - R_1 + \left(\frac{1}{\xi} + \hat{d}\right)(1 - e^{-\xi R_1}). \tag{15}$$

Furthermore, N_{RSU}, the random variable representing the number of vehicles with which a RSU can communicate, can also be studied. Indeed,

$$N_{RSU} = \mathcal{A}^1(R_{RSU}) + 1\{X \leqslant R_1\}[N - \mathcal{A}^2(R_1 - X)], \tag{16}$$

where N is the random variable which represents platoon size as defined in Section 1, and $\mathcal{A}^i(x), i = 1, 2$ are two Poisson distributed random variables, each with mean ξx. Then the average number of vehicles with which the RSU can communicate (in each direction) is given as:

$$\overline{N}_{RSU} = R_{RSU}\xi + \mathbb{E}(N)P(X \leqslant R_1) - \mathbb{E}(\mathcal{A}^2(R_1 - X) \cdot 1\{X \leqslant R_1\}). \tag{17}$$

Since we have

$$\begin{aligned}\mathbb{E}(\mathcal{A}^2(R_1 - X) \cdot 1\{X \leqslant R_1\}) &= \int_0^{R_1} \xi \mathrm{e}^{-\xi x}\mathbb{E}[\mathcal{A}^2(R_1 - x)]dx \\ &= \int_0^{R_1} \xi^2(R_1 - x)\mathrm{e}^{-\xi x}dx \\ &= \xi R_1 - (1 - \mathrm{e}^{-\xi R_1}),\end{aligned} \tag{18}$$

then we obtain the following explicit expression for the average number of vehicles with which the RSU can communicate (in each direction)

$$\overline{N}_{RSU} = R_{RSU}\xi + \widehat{N}(1 - \mathrm{e}^{-\xi R_1}) - \mathrm{e}^{-\xi R_1} - \xi R_1 + 1, \tag{19}$$

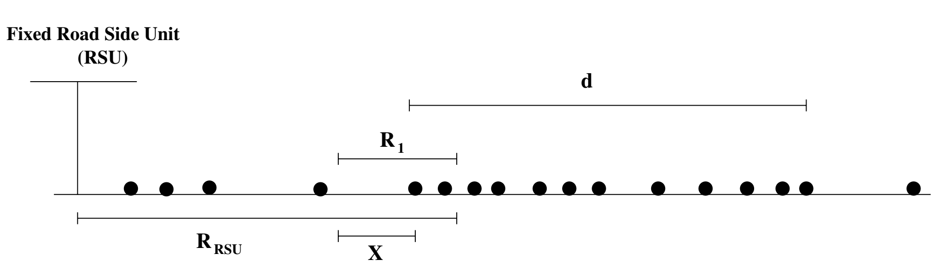

Fig. 2. Connectivity distance from a RSU.

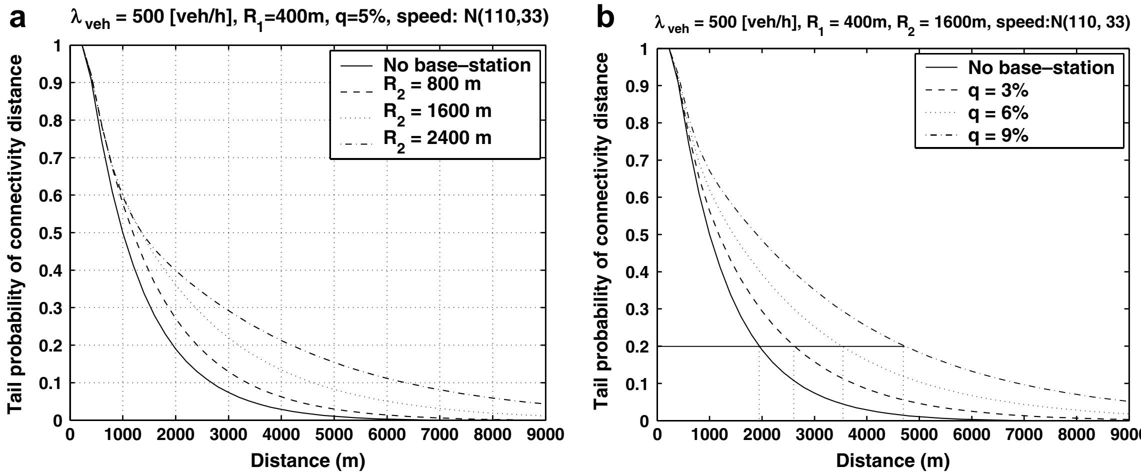

Fig. 3. The improvement of the tail probability of connectivity distance (a) for different base-station's transmission ranges, (b) for different number of base-stations, when $P_{pa} = 1$.

where $\widehat{N}$ is the average platoon size obtained by taking $p = 1$ in Eq. (8).

5. Numerical study

In this section we evaluate the model numerically along with common traffic statistics while we mainly focus on the base-stations and their effect on connectivity. According to [1] in the free-flow traffic state λ_{veh} is below 1000 $[veh/\text{h}]$ and speeds are found out to be normally distributed as $N(\mu, \sigma)$ where μ and σ are average and standard deviation values. We shall use a truncated version of this distribution to avoid dealing with negative speeds or even to avoid getting close to zero speed [3]. The latter would otherwise cause problems in (1) and elsewhere; in fact it can be seen that a speed of zero does not make sense since a car cannot cross the observer if it has speed zero. In the sequel we use $\mu = 110$ km/h, $\sigma = 33$ km/h and the left and right truncation points corresponding to the minimum and the maximum speeds are 11 km/h and 209 km/h, respectively. The exact values of all parameters are reported in each plot.

Fig. 3 shows the results of numerically inversion [11] of (5). It illustrates that by increasing the number of base-stations or their transmission range, longer connectivity distances are more probable. For example Fig. 3b shows that in case of no-base-station, for the tail probability equal to 0.2, the achievable connectivity distance is about 1900 m. While it is increased to 2600 m, 3500 m and 4700 m when, respectively, $q = 3\%, q = 6\%$ and $q = 9\%$. In other words when 9% of vehicles are base-station, the connectivity distance at which its tail probability is 0.2, increases by 2800 m in comparison to the case when there is no-base-station. Thus it shows more than 140% increase which is quite considerable. Besides, as it follows from Fig. 3a and b when the connectivity distance increases the relative improvement of its tail probability, due to increase of the number of base-stations or their transmission range, also increases. Furthermore, as the tail probability of connectivity distance decreases the amount of improvement of the achievable connectivity distance also increases.

In order to observe the improvement discussed above more clearly, we depict in Fig. 4, the average connectivity distance and the average platoon size in terms of q, for different values of R_2. As the figure shows, when 10% of all vehicles are base-stations and the transmission range of base-stations is 2400 m, the average platoon size and the average connectivity distance are increased by about 400%.

Moreover, as it follows from (7) and (8), the average connectivity distance and the average platoon size depend on only the average value of transmission ranges. Thus in different scenarios with the same average transmission range, the average connectivity distance and the average platoon size remain fixed. We investigate the question of whether the tail probabilities of connectivity distance are also the same, in Fig. 5. The figure shows the comparison of the tail probability of connectivity distance for two different scenarios. As it can be seen, in spite of the same average transmission range, the tail probabilities are not the same. Indeed there is a range of distances at which the tail probability of connectivity distance of one scenario is lager than the other. For instance, in Fig. 5, the scenario with more heterogenous transmission ranges shows higher tail probability for distances larger than 6000 m. While the scenario which is more homogeneous shows higher tail probability for distances smaller than 6000 m. This fact can be described in that the long-ranged base-stations fill large gaps successfully such that longer distances are more likely to be observed.

Finally the effect of non-participant vehicles on the tail probability of connectivity distance is illustrated in Fig. 6. It follows from the figure that connectivity deteriorates markedly in low-participant scenarios. For instance, Fig. 6a shows that for a tail probability equal to 0.2, the achievable connectivity distance decreases by about 1500 m when the participation probability decreases from

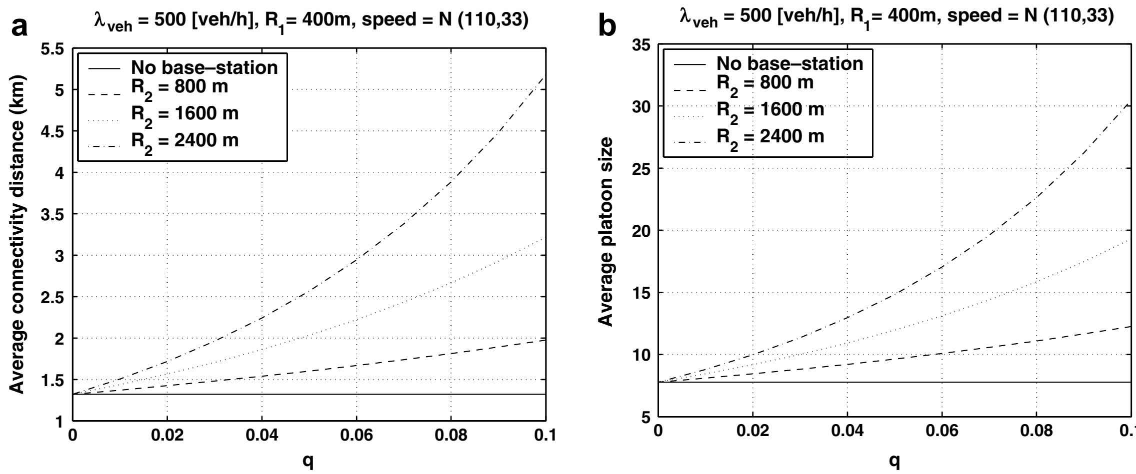

Fig. 4. The improvement of (a) the average connectivity distance, (b) the average platoon size for different base-stations' transmission range, when $P_{pa} = 1$.

$P_{pa} = 1$ to $P_{pa} = 0.75$. Thus it shows about 37% decrease and also the figure shows that the same relative decrease is observed for a decrease of P_{pa} by 25% from any initial participation rate. Furthermore, the amount of decrease of the achievable connectivity distance increases as the tail probability decreases. Note that in practice the base-stations may be added to the network partially for mitigating the connectivity impairment due to incomplete participation. However, for a given scenario of base-stations, the amount of amendment is not uniform for all values of connectivity distance. This issue is illustrated in Fig. 6b where the improvement achieved by base-stations is more considerable when the connectivity distances are larger than 1500 m.

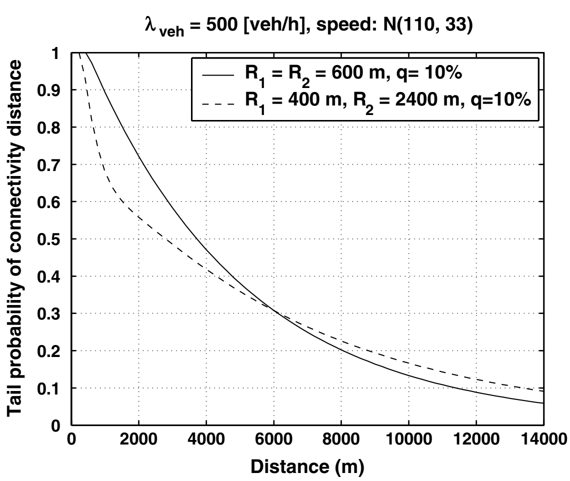

Fig. 5. The comparison of the tail probability of connectivity distance for two scenarios with same average transmission range.

6. Conclusions

Using the equivalent $M/G/\infty$ queuing model we studied the connectivity in VANETs in presence of some added nodes, which we called mobile base-stations. We obtained expressions for the expectation of the connectivity distance and the tail probability of connectivity distance as well as the expected value of the number of vehicles in a spatial cluster. We then present explicit expressions for the

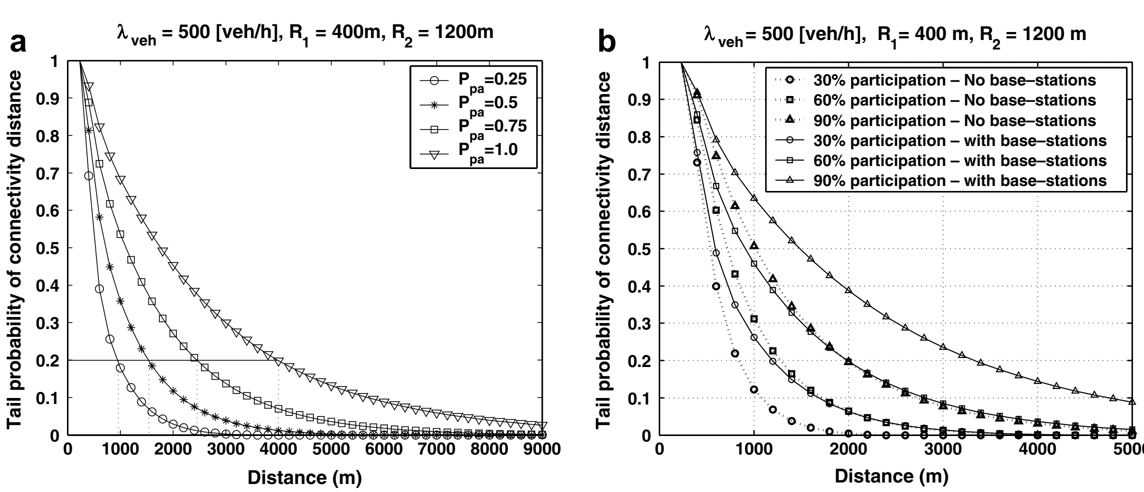

Fig. 6. (a) The effect of incomplete participation on the tail probability of connectivity distance. (b) Mitigating the connectivity impairment by adding base-stations, $q = 10\%$.

expected value of connectivity distance from a RSU as well as the average number of vehicles with which a RSU can communicate. We also studied the effect of incomplete node participation on connectivity. Our analytical model can be used to obtain the number of base-stations and also their transmission range in order to achieve a desired level of connectivity.

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