String theories at strong coupling and string duality

In this lecture we mainly follow section 14 of [1].

1 Introduction

The perturbative picture of the different superstring theories is shown in figure 1. There are five different theories, some ofwhich are related by perturbative string dualities (T-duality) upon compactification.

Our purpose in this lecture is to study the strong coupling limit of these theories. We will find out that this limit is surprisingly quite simple, and is usually described in terms of a weakly coupled dual theory. In this description further, non-perturbative, dualities relate all the different string theories. This implies that the different perturbative string theories all arise in different limits of a unique underlying theory, as some moduli are tuned. The situation is shown in figure 2. This is analogous to how 10d type IIA and IIB are recovered starting from a unique theory (type II on $\mathbf{S^1}$) in the two limits of large radius and small radius (large T-dual radius).

The main tool used in the exploration of the strong coupling regime is to follow the properties of BPS states as the coupling becomes strong. This can be done because such properties are protected by the supersymmetry of these states. Some of these states, which are non-perturbative and very heavy in the weakly coupled regime, become light in the strong coupling regime, and correspond to the states that dominate this regime, and provide the elementary, perturbative, degrees of freedom of the dual theory, which is weakly coupled in that regime.

An intuitive argument indicating which BPS states dominate the dy-

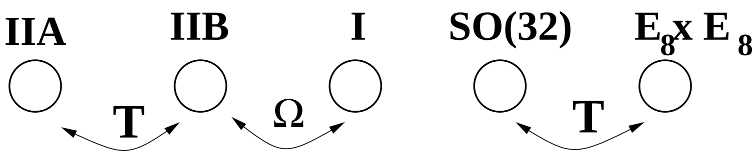

Figure 1: The different 10d supersymmetric superstring theories in perturbation theory.

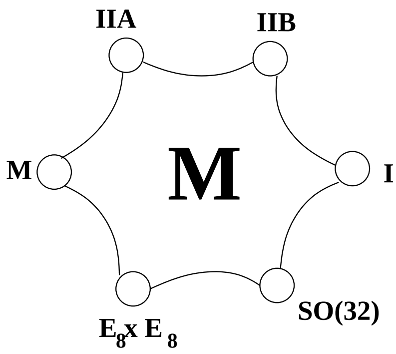

Figure 2: Map of the moduli space of the underlying theory and its different known limits.

namics at strong coupling is to associate an scale to them. For instance, the tension of a Dp-brane $T_p \simeq \alpha'^{-(p+1)/2}/g_s$ defines a mass scale $M \simeq \alpha'^{-1/2} g_s^{-1/(p+1)}$. This implies that in the strong coupling limit the lightest mass scale corresponds to the lowest p Dp-brane, suggesting these are the states dominating the low-enery dynamics in that regime.

2 The type IIB $SL(2, \mathbf{Z})$ self-duality

The basic reference is [3], see also [2].

2.1 Type IIB S-duality

At strong coupling, the lightest mass scale is set by the D1-brane states, $M \simeq \alpha'^{-1/2} g_s^{-1/2}$. The fact that these objects dominate the dynamics at strong coupling suggests that the strong coupling limit is described by a string theory. It is reasonable to imagine that it moreover corresponds to a weakly interacting string theory, hence it should correspond to one of the string theories we have studied. And the only string theories with the correct amount of supersymmetry are the type IIA and type IIB theories.

The natural proposal is that in fact the strong coupling limit of type IIB theory is described by a weakly coupled dual type IIB theory. In fact, in the low-enery limit the theory is described by type IIB supergravity, which is known to have a symmetry relating weak and strong coupling. The action of this symmetry, known as S-duality, relates the massless fields of the theory at coupling g_s (denoted as unprimed) with those of the theory at coupling $g_s' = 1/g_s$ (denoted as primed), as follows

$$\begin{gathered} a' = a \quad ; \quad \phi' = -\phi \quad ; \quad B_2' = \tilde{B}_2 \quad ; \quad \tilde{B}_2' = b_2 \\ C_4' = C_4 \quad ; \quad G' = G \end{gathered} \tag{1}$$

where G is the metric in the Einstein frame. The reason why we can trust the form of the type IIB supergravity action is that its form is fixed by supersymmetry (up to higher derivative terms, which are not relevant at low energies).

The proposal is that this symmetry of the supergravity limit is an exact symmetry of the full string theory! As a consequence, the theory at $g_s \to \infty$ is described by a perturbative type IIB theory, the transformed under S-duality, which is weakly coupled $g_s' \to 0$.

2.2 Additional support

We would like to mention additional evidence supporting this proposal.

• The D1-branes in the original theory are the fundamental strings of the dual one. Therefore the D1-brane 2d world-volume theory should be of the same kind as that of a fundamental type IIB string. In fact, D-brane worldvolume spectra were computed in previous lecture. For a D1-brane we have a 2d $U(1)$ gauge boson (which is non-dynamical in 2d), 8 2d real scalars $X^i(\sigma^1, \sigma^2)$ in the 8_V of the transverse $SO(8)$ Lorentz group, and 8 2d fermions $\Theta^\alpha(\sigma^1, \sigma^2)$, transforming in the 8_C of $SO(8)$. This is precisely the 2d field content of a type IIB fundamental string in the Green-Schwarz formalism (see comment in page 19 in lecture on type II superstrings).

• The BPS states of both theories agree. For instance

IIB at g_s		IIB at $g_s' = 1/g_s$
F1	$\longleftrightarrow$	D1
D1	$\longleftrightarrow$	F1
D3	$\longleftrightarrow$	D3
D5	$\longleftrightarrow$	NS5
NS5	$\longleftrightarrow$	D5

The tensions and charges of the objects match. Also, they have equivalent world-volume field theories, as we have seen for the D1/F1 and as follows from the discussion of world-volume modes for the D5/NS5-branes in the lecture on non-perturbative states in string theory.

2.3 $SL(2,\mathbf{Z})$ duality

In fact, type IIB supergravity has a larger symmetry group, $SL(2,\mathbf{R})$, the group of unit determinant 2×2 real matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Introducing the type IIB complex coupling $\tau = a + ie^{-\phi}$, one such transformation relates the theory at coupling τ to the theory at coupling τ', by the following action on the massless fields

$$\begin{aligned} \tau' &= \frac{a\tau + b}{c\tau + d} \\ \begin{pmatrix} B_2' \\ \tilde{B}_2' \end{pmatrix} &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ \tilde{B}_2 \end{pmatrix} \\ G' &= G \quad , \quad C_4' = C_4 \end{aligned} \tag{2}$$

As we argued two lectures ago, not the full $SL(2,\mathbf{R})$ can be an exact symmetry of the quantum theory, since it does not respect the discrete lattice of charges of brane states in the theory. However, an $SL(2,\mathbf{Z})$ subgroup of it does respect it, and we propose that it is an exact symmetry of the full quantum theory. In fact, this group is generated by the above S-duality transformation $\tau \to -1/\tau$, and a transformation $\tau \to \tau + 1$, which simply shift $a \to a + 1$ leaving all other fields invariant. The latter is knonw to be a symmetry to all orders in perturbation theory (it is a 0-form gauge field, so gauge invariance implies that it has no non-derivative couplings), so it is natural to propose that it is a symmetry at the non-perturbative level.

This proposal has several interesting implications.

• For instance, it implies the existence of an infinite set of points in the τ moduli space which are related by $SL(2,\mathbf{Z})$ to weak coupling; that is, whose dynamics is equivalent to a perturbative IIB string theory once described in suitable $SL(2,\mathbf{Z})$ dual variables.

• It implies that the spectrum of BPS states in type IIB string theory must arrange in $SL(2,\mathbf{Z})$ multiplets. In particular, it must contain an $SL(2,\mathbf{Z})$ orbit of string-like objects, denoted (p,q)-strings. The $(1,0)$- and $(0,1)$-strings correspond to the F1- and D1-strings. Indeed, at a point τ dual to weak coupling by an $SL(2,\mathbf{Z})$ duality, the object becoming the perturbative one is the

(p, q)-string related to the F1-string by the same $SL(2, \mathbf{Z})$-transformation. Simlarly we have (p, q) 5-branes; in these cases the p, q labels transform under $SL(2, \mathbf{Z})$ as a doublet, which means that a (p, q) object can be regarded as a bound state of p $(1, 0)$ objects and q $(0, 1)$ objects. There are also (p, q) 7-branes, but they have a more involved $SL(2, \mathbf{Z})$ transformation rule and cannot be properly regarded as bound states of the 'elementary' solitons. The existence of these (p, q)-branes as supergravity solitons is guaranteed from the fact that $SL(2, \mathbf{Z})$ is a subgroup of the supergravity symmetry group.

Toroidal compactification has been already discussed in the lecture on non-perturbative objects in string theory. So we refer the reader to the corresponding section.

3 Type IIA and M-theory on $\mathbf{S}^1$

The original paper discussing this is [3]

3.1 Strong coupling proposal

The type IIA theory strong coupling dynamics at low energies is dominated by the D0-branes, with a mass scale of $M \simeq \alpha'^{-1/2} g_s^{-1}$. There is no BPS string becoming light in the strong coupling regime, and this suggests that the strong coupling limit is *not* described by a string theory. Instead what one finds at strong coupling is that states with n D0-branes form an infinite tower of states, with massess $M_n \simeq \frac{n}{g_s} M_s$, which is becoming extremely light. This suggests that the strong coupling limit is a decompactification limit of some 11d theory.

Indeed, there exists and 11d supergravity theory with the correct amount of supersymmetry (32 supercharges), and which upon Kaluza-Klein compactification on a circle of radius R leads to an effective theory (neglecting KK replicas of massless modes) given by 10d type IIA supergravity, with

$$g_s = (M_{11}R)^{3/2} \tag{3}$$

where M_{11} is the 11d Planck scale, and R is measured in the 11d metric.

More explictly, 11d supergravity is described by a metric G, a 3-form $C_3$and and 11d gravitino. The matching of massless 11d fields and massless type IIA 10d fields is

$$\begin{array}{rcl} G_{MN} & \longrightarrow & G_{\mu\nu} \\ & & G_{\mu,10} \to A_\mu \\ & & G_{10,10} \to \phi \\ C_{MNP} & \longrightarrow & C_{\mu\nu\rho} \\ & & C_{\mu\nu,10} \to B_{\mu\nu} \\ \Psi_{M,\alpha} & \longrightarrow & \psi_{\mu\alpha}, \psi_{\mu\dot\alpha}, \psi_{10,\alpha}, \psi_{10,\dot\alpha} \end{array} \tag{4}$$

This suggests that type IIA at strong coupling is a new 11d quantum theory, whose low energy limit is 11d supergravity. The microscopic nature of this 11d theory is completely unknown (let us emphasize again that it it not a string theory), and it is simply called M-theory. The facts we know about M-theory are

- At low energies it reduces to 11d supergravity

- It contains 1/2 BPS states corresponding to a 2-brane and a 5-brane (denoted M2- and M5-brane). These can be constructed as BPS solutions of the 11d supergravity equations of motion, and argued to exist in the full microscopic theory (whatever it is) due to their BPS property.

- M-theory compactified on a circle of radius R is completely equivalent to full-fledged type IIA string theory at coupling $g_s = (M_{11}R)^{3/2}$.

It is interesting to point out that M-theory in 11d does not have any scalar field, and consequently does not have any dimensionless coupling constant. This means that there is no parameter which can be taken small to obtain a perturbative description, so the theory is intrinsecally non-perturbative. Once compactified on a circle, however, there is one dimensionless quantity $M_{11}R$, which can be taken small to lead to a perturbative theory: this is precisely perturbative type IIA string theory.

3.2 Further comments

Let us provide some additional support for the proposal. For instance there is a precise matching of BPS states in both theories, as follows [1]

IIA at g_s	$\longleftrightarrow$	M-theory on $\mathbf{S^1}$ of radius R
D0-branes	$\longleftrightarrow$	KK momenta of 11d supergravity multiplet
F1	$\longleftrightarrow$	wrapped M2
D2	$\longleftrightarrow$	unwrapped M2
D4	$\longleftrightarrow$	wrapped M5
NS5	$\longleftrightarrow$	unwrapped M5
D6	$\longleftrightarrow$	Kaluza-Klein monopole

The tensions of these objects agree completely, and it is possible to show that they have equivalent world-volume field theories. In particular one can show that the worldvolume theory of an M2-brane wrapped on $\mathbf{S^1}$ reduces to the world-sheet theory of a fundamental type IIA string.

We would like to mention that the M-theory proposal implies very interesting properties for the D0-branes, since they are, from an 11d viewpoint,

[1] The D8-brane is however a bit problematic, since it is a source of the type IIA mass parameter, and there is no 11d version of supergravity which reduces to massive IIA theory. This is in a sense an open issue.

simply 11d gravitons (and partners) with non-zero momentum along the circle. For instance, a 11d graviton with n units of momentum is not the same state as n 11d gravitons with 1 unit of momentum each, although the have the same mass and charge. This implies that there should exist a bound states of n D0-branes (with zero binding energy) in type IIA theory. Moreover, scattering of this kind of states should reproduce the supergravity interactions in 11d!

This line of though has led to a proposal to define microscopically M-theory, known as the M(atrix) theory proposal [4]. It is based on describing the complete dynamics of 11d M-theory from the world-line gauge theory on stacks of D0-branes. This is a 1d quantum mechanics of $U(n)$ gauge fields, and its partners under the 16 unbroken supersymmetries (9 scalars and fermions). In this description, spacetime arises as the moduli space of scalars in the 1d theory; 11d gravitons are bound states in this quantum mechanics system; scattering of supergravity modes in 11d is recovered by interactions of wavepackets of bound states.

M(atrix) theory has led to very interesting results in 11d and in compactifications preserving enough supersymmetry (toroidal compactifications, etc). However, difficulties have typically arisen in trying to study more involved situations with less supersymmetry.

4 M-theory on $\mathbf{T^2}$ vs type IIB on $\mathbf{S^1}$

The original discussion is in [5].

There must be a direct link between M-theory compactified on $\mathbf{T^2}$ and type IIB compactified on $\mathbf{S^1}$. This can be seen by regarding M-theory on $\mathbf{T^2}$ as type IIA on $\mathbf{S^1}$ and performing a T-duality to type IIB on (a T-dual) $\mathbf{S^1}$.

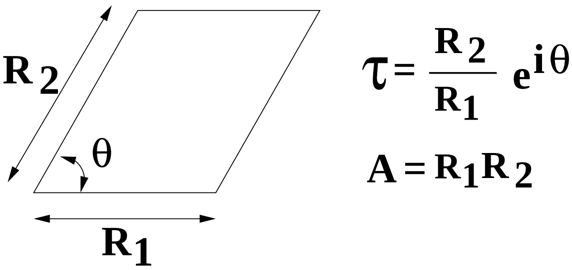

Figure 3: Complex structure parameter and area of a two-torus.

11d			M
			$\downarrow \mathbf{S^1}$
10d	IIB		IIA/M
	$\downarrow \mathbf{S^1}$		$\downarrow \mathbf{S^1}$
9d	IIB	$\stackrel{T}{\longleftrightarrow}$	IIA/M

We can perform the matching of both theories even when the circles are not small, and propose they are equivalent, with the following relations.

Moduli: The τ complex coupling of type IIB theory matches with the complex structure parameter of the M-theory $\mathbf{T^2}$, $\tau = \frac{R_1}{R_2} e^{i\theta}$ (see figure 3). The radius R of the IIB $\mathbf{S^1}$ is related to the area of the M-theory $\mathbf{T^2}$ $A = R_1 R_2$ by $M_{11}^3 \, A = 1/R$.

Duality groups: The $SL(2, \mathbf{Z})$ duality group of type IIB theory (already present in 10d) matches the $SL(2, \mathbf{Z})$ invariance group of the $\mathbf{T^2}$ geometry, corresponding to large diffeomorphisms of $\mathbf{T^2}$. This is hence a nice geometric interpretation for the IIB self-duality group.

BPS states: Let us give some examples on the matching of BPS states

IIB on $\mathbf{S^1}$	$\longleftrightarrow$	M-theory on $\mathbf{T^2}$
unwrapped (p,q) string	$\longleftrightarrow$	M2 wrapped on (p,q) cycle in $\mathbf{T^2}$
wrapped (p,q) string	$\longleftrightarrow$	KK momentum of 11d sugra multiplet along (p,q) direction
momentum in $\mathbf{S^1}$	$\longleftrightarrow$	M2 wrapped on $\mathbf{T^2}$
wrapped (p,q) 5-brane	$\longleftrightarrow$	M5-brane wrapped on (p,q) cycle
unwrapped (p,q) 5-brane	$\longleftrightarrow$	KK-monopole with isometry along (p,q)
unwrapped D3	$\longleftrightarrow$	M5 wrapped on $\mathbf{T^2}$
wrapped D3	$\longleftrightarrow$	unwrapped M2

The tensions of all objects agree, and they have equivalent world-volume theories.

Hence type IIB on $\mathbf{S^1}$ with radius R and coupling τ is equivalent to M-theory on a $\mathbf{T^2}$ with complex structure τ and area $A \simeq 1/R$. In particular notice that the decompactified 10d type IIB string theory can be obtained by taking M-theory on a $\mathbf{T^2}$ in the limit of vanishing area. In this limit, a tower of light states arises from M2-branes wrapped on $\mathbf{T^2}$, these are interpreted as the KK modes on the $\mathbf{S^1}$ of the dual IIB theory, which is in the decompactification limit.

5 Type I / $SO(32)$ heterotic duality

See [3] and [6].

5.1 Strong coupling of Type I theory

In 10d type I theory at strong coupling, the lightest mass scale is set by the D1-branes, with $M \simeq \alpha'^{-1/2} g_s^{-1/2}$. This suggests that the strong coupling behaviour is controlled by a string, and that the strong coupling limit may correspond to a dual string theory.

From the amount of supersymmetry, the dual string theory could be a dual type I theory, or a dual $SO(32)$ or $E_8 \times E_8$ heterotic theory. However, the D1-string of the original theory is BPS, so the dual string theory should have an F1 BPS state. This is not present in type I theory, so the strong coupling dynamics cannot correspond to a dual type I theory. Out of the two heterotic theories, the fact that the $SO(32)$ heterotic has the same gauge group as type I theory suggests that it is the correct candidate to describe the strong coupling limit of type I.

In fact, restricting to low energies, the low energy supergravity action for type I and $SO(32)$ heterotic theories is the same, up to redefinitions of the fields, as follows.

$$\begin{aligned} \phi_{\rm typeI} &= -\phi_{\rm het.} \quad \to \quad (g_s)_{\rm het} = 1/(g_s)_{\rm typeI} \\ G_{\rm typeI} &= e^{-\phi_{\rm het.}} G_{\rm het.} \quad , \quad (A_{SO(32)})_{\rm typeI} = (A_{SO(32)})_{\rm het.} \quad , \quad (H_{3,RR})_{\rm typeI} = (H_3)_{\rm het.} \end{aligned} \tag{5}$$

This suggest that the type I theory at coupling g_s is exactly equivalent to the $SO(32)$ heterotic at coupling $1/g_s$. And in particular that the strong coupling limit of type I theory is described by a weakly coupled $SO(32)$ heterotic string theory, and viceversa.

5.2 Further comments

5.3 Additional support

We would like to mention additional evidence supporting this proposal.

• The D1-branes in the original type Itheory are the fundamental strings of the dual heterotic theory. Therefore the type I D1-brane 2d world-volume theory should be of the same kind as that of a fundamental $SO(32)$ heterotic string. In fact, D-brane worldvolume spectra were computed in previous lecture. For a D1-brane, in the 11 sector we have an $O(1) = \mathbf{Z_2}$ gauge symmetry, 8 2d real scalars $X^i(\sigma^1, \sigma^2)$ in the 8_V of the transverse $SO(8)$ Lorentz group, and 8 2d rightmoving chiral fermions $\Theta^\alpha(\sigma^1 - \sigma^2)$, transforming in the 8_C of $SO(8)$. In addition in the 19 and 91 sectors we have 32 2d left-moving chiral fermions $\lambda^I(\sigma^1 + \sigma^2)$, singlets under the Lorentz $SO(8)$ and transforming in the fundamental of the $SO(32)$ spacetime group. This is precisely the 2d field content of a heterotic fundamental string in the Green-Schwarz formalism. The fact that it is the $SO(32)$ follows from the fact that the fermions λ^I are odd under the $\mathbf{Z_2}$ gauge symmetry, and so in building gauge invariant states of the 2d theory they suffer a GSO projection acting in the same way on the 32 2d internal fermions.

- The BPS states of both theories agree. For instance

type I at g_s		$SO(32)$ Heterotic at $g_s' = 1/g_s$
D1	$\longleftrightarrow$	F1
D5	$\longleftrightarrow$	NS5

We would like to conclude with a comment. The $SO(32)$ heterotic theory contains massive states in the spinor representation of $SO(32)$, of dimension 2^{15}. They correspond to states with internal 16d momentum

$$P = \frac{1}{2}(\pm, \ldots, \pm) \quad \text{with even number of minu signs.} \tag{6}$$

These states are non-BPS, but are stable due to charge conservation (there are no states lighter than them with the same charge). A prediction of heterotic/typeI duality is that states with those quantum numbers exist in type I theory. These do not appear in perturbative type I theory, or in the non-perturbative BPS states. In the lecture on stable non-BPS D-branes we will discuss the nature of these objects.

6 M-theory on $\mathbf{S^1/Z_2}$ / $E_8 \times E_8$ heterotic

The strong coupling limit of the $E_8 \times E_8$ heterotic is difficult to analyze directly, as we will understand later on. It is somewhat easier (although highly non-trivial) to derive it starting from the discussion of compactifications of M-theory in the unique compact 1d space which is not the circle: the interval. The discussion follows the original paper [7] (wee [8] for more advanced discussions).

6.1 Horava-Witten theory

Consider the compactification of M-theory on $\mathbf{S^1}$, modded out by a $\mathbf{Z_2}$ action, with generator acting by

$$\begin{aligned} \theta : \quad & x^{10} \to -x^{10} \\ & C_3 \to -C_3 \end{aligned} \tag{7}$$

which is a symmetry of the theory (at least at the supergravity level, so we are assuming implicitly this to be a symmetry of microscopic M-theory). The action on C_3 is required so that the term in the 11d supergravity action $\int_{11d} C_3 \wedge G_4 \wedge G_4$ (with $G_4 = dC_3$) is invariant.

The quotient space is an interval (see figure 4), so that spacetime has two 10d boundaries sitting at $x^{10} = 0, \pi R$.

It is important to understand the we do not have a microscopic description of M-theory, and such a description would be required to construct an orbifold of M-theory from first principles. This is because at the fixed points of the orbifold (the boundaries of spacetime) there may be additional states which are not obtained simply from the effective field analysis. They would be the analogues of twisted sectors in string theory constructions. We will not be able to obtaine these states from first principles, but happily the

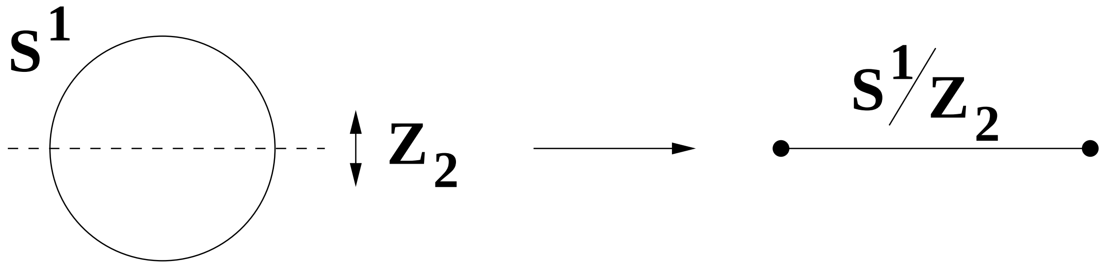

Figure 4: The quotient of a circle by a reflection under a diameter is an interval $I = \mathbf{S^1}/\mathbf{Z_2}$.

consistency condition of cancellation of anomalies will be enough to show that the existence of these states, and their precise spectrum.

Let us start constructing the orbifold. We expect that the 10d theory will contain a sector given by the $\mathbf{Z_2}$ invariant states in the compactification of M-theory on $\mathbf{S^1}$ (this is the analogue of the untwisted sector in string theory orbifolds). Ignoring KK replicas, we have

11 field		10 field	$\mathbf{Z_2}$ parity	Surviving field
G_{MN}	$\longrightarrow$	$G_{\mu\nu}$	$+$	$G_{\mu\nu}$
		$G_{\mu,10} \to A_\mu$	$-$	—
		$G_{10,10} \to \phi$	$+$	ϕ
C_{MNP}	$\longrightarrow$	$C_{\mu\nu\rho}$	$-$	—
		$C_{\mu\nu,10} \to B_{\mu\nu}$	$+$	$B_{\mu\nu}$
$\Psi_{M,\alpha}$	$\longrightarrow$	$\psi_{\mu\alpha}$, $\psi_{\mu\dot{\alpha}}$, $\psi_{10,\alpha}$, $\psi_{10,\dot{\alpha}}$	$+, -, +, -$	$\psi_{\mu\alpha}$, $\psi_{10,\dot{\alpha}}$

The content of massless 10d surviving fields is exactly that of 10d $\mathcal{N} = 1$ supergravity. This content is chiral, and leads to 10d anomalies, hence the theory as it stands is inconsistent.

If M-theory is consistent at the quantum level it should lead to an additional set of states. Moreover, one can check that from the 11d viewpoint the anomalies are localized on the 10d fixed locus of the orbifold. This is because in the bulk of the spacetime away from the boundaries the local dynamics is still described by 11d M-theory, which is non-chiral, while it is at the boundaries that the orbifold projection introduces chirality. The new fields cancelling the anomaly must be localized on the orbifold fixed points, as expected.

From our discussion of anomalies in heterotic theories, we know that there are two possible sets of fields that can cancel (in a very miraculous way) the anomaly of the 10d $\mathcal{N} = 1$ supergravity multiplet. One of them is a 10d $\mathcal{N} = 1$ $SO(32)$ vector multiplet, and the other is a 10d $\mathcal{N} = 1$ $E_8 \times E_8$ vector multiplet. Clearly only the later set of fields can be split into two fixed points and cancel the two sources of anomaly, so they provide the only candidate set of multiplets that M-theory must contain in order to lead to a consisten compactification.

That is, compactification of M-theory on the interval $\mathbf{S^1}/\mathbf{Z_2}$ contains one E_8 10d $\mathcal{N} = 1$ vector multiplet at each of the two 10d boundaries of spacetime, see figure 5. This is known as Horava-Witten theory or Horava-Witten compactification of M-theory.

Notice that the final theory has the same massless spectrum as the $E_8 \times E_8$ heterotic theory. Moreover, the effective action of both theories is determined by supersymmetry, and agrees if the heterotic string coupling constant and the M-theory radius are related by $g_s = (M_{11}R)^{3/2}$. It is then natural to propose that the $E_8 \times E_8$ heterotic string theory at coupling g_s is completely equivalent to M-theory on $\mathbf{S^1}/\mathbf{Z_2}$ with radius R, related to g_s as above.

The strong coupling regime of the $E_8 \times E_8$ heterotic string theory corresponds to the large radius limit of the M-theory compactification. We can now understand why it is difficult to determine directly the strong coupling regime directly. The sign of the opening up of the extra dimension is the

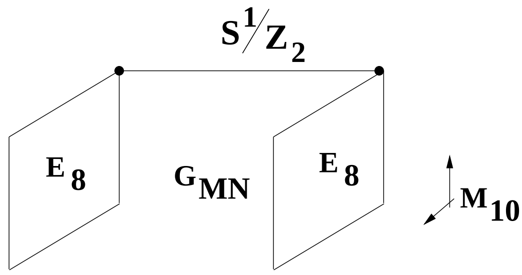

Figure 5: The strong coupling description of $E_8 \times E_8$ heterotic involves the compatification of M-theory on a space with two 10d boundaries. Gravity propagates in 11d, while gauge interactions are localized on the 10d subpaces at the boundaries.

appearance of KK momentum modes, but these are not BPS states, due to the $\mathbf{Z_2}$ projection in M-theory language: the gauge boson that would carry the charge of these states is projected out by $\mathbf{Z_2}$; equivalentely, momentum is not a conserved charge due to violation of translational invariance in the $\mathbf{S^1}$ due to the existence of preferred points (the orbifold fixed points).

6.2 Additional support

We can also match BPS states in the two theories, as follows

$E_8 \times E_8$ heterotic at g_s		M-theory on $\mathbf{S^1}/\mathbf{Z_2}$ at R
F1	$\longleftrightarrow$	wrapped M2 (see fig. 6)
NS5	$\longleftrightarrow$	unwrapped M5

Notice that other states in M-theory on $\mathbf{S_1}$, which are projected out by $\mathbf{Z_2}$ (like a warpped M5-branes, or an unwrapped M2-branes) are correctly absent in heterotic theory.

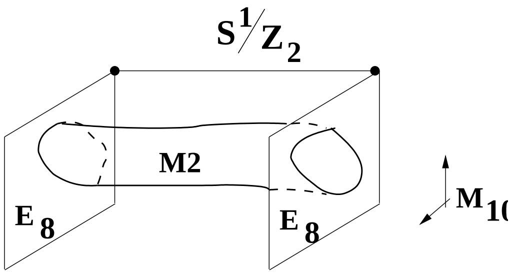

Figure 6: The fundamental heterotic string arising in the Horava-Witten viewpoint from a M-theory M2-brane stretched along the interval. Note that it knows about the existence of the two E_8's, thus explaining why the heterotic string has (in the fermionic formulation) 2d fermions charged under the cartans of both group factors.

7 $SO(32)$ het/typeI on $\mathbf{S^1}$ vs M-theory on $\mathbf{S^1} \times (\mathbf{S^1}/\mathbf{Z_2})$

In this section we describe a relation between Horava-Witten theory compactified to 9d on a circle with type I theory. We will find that the type I picture in terms of D-branes, in a T-dual version, provides further insight into the appearance of the E_8 gauge multiplets on the boundaries of the interval. See section 14.5 in [1].

We consider the following chain of dualities

11d							M
							$\downarrow \mathbf{S^1}/\mathbf{Z_2}$
10d			type I	$\stackrel{S}{\longleftrightarrow}$	$SO(32)$ het		$E_8 \times E_8$/HW
			$\downarrow \mathbf{S^1}$		$\downarrow \mathbf{S^1}$		$\downarrow \mathbf{S^1}$
9d	type I'	$\stackrel{T}{\longleftrightarrow}$	type I	$\longleftrightarrow$	$SO(32)$ het	$\stackrel{T}{\longleftrightarrow}$	$E_8 \times E_8$/HW

Following the duality carefully allows to derive the Horava-Witten picture from type I' theory on $\mathbf{S^1}$.

T-duality relates the 9d $SO(32)$ and $E_8 \times E_8$ heterotic theories if there are Wilson lines turned on, breaking the gauge group to $SO(16)$ (see lecture on toroidal compactification of heterotic strings). We can now use the S-dual version of $SO(32)$ heterotic theory, and relate type I on $\mathbf{S^1}$ with Wilson lines breaking to $SO(16)^2$ with $E_8 \times E_8$ heterotic theory.

In fact, it is more useful to use the T-dual of type I theory, namely type I' theory, where the Wilson lines correspond to D8-brane positions (see lecture on type I toroidal compactification). We are interested in locating 16 D8-branes on top of each of the two O8-planes in the ΩR quotient of type IIA on $\mathbf{S^1}$, a configuration which leads to $SO(16)^2$ gauge group.

Thus we have a relation between IIA modded out by ΩR, $R: s^9 \to -x^9$ (with $SO(16)$ gauge multiplets on top of each of the fixed points of R) and $E_8 \times E_8$ theory (on $\mathbf{S^1}$ with Wilson lines breaking to $SO(16)^2$. We now only need to identify in type I' language what is the limit that corresponds to taking large $\mathbf{S^1}$ radius and strong coupling in the heterotic side. It can be seen to correspond also to large radius and large coupling in type I' picture.

Recall now that in the bulk of the type I' theory, away form the O8-planes, the local dynamics is that of type IIA theory. Since we are taking a strong coupling limit, a new dimension will open up (D0's are beoming light), lifting our configuration to M-theory. We recover a picture of M-theory on $\mathbf{S^1}/\mathbf{Z_2}$ (and on large circle). At the same time, we should see our $SO(16)$ gauge groups enhancing to E_8's. Indeed this is the case: near the O8-planer there are stuck D0-branes (which cannot move off into the bulk), which lead to additional light particles (in vector multiplets) transforming in the chiral spinors representation 128 of $SO(16)$ and enhancing the group to E_8 [2]

[2]The open string sector of 08 and 80 strings leads to fermionic zero modes on the

The result is exactly the Horava-Witten picture. The advantage of the present approach is that it provides a more intuitive interpretation of the E_8 gauge multiplets living on the boundaries of spacetime. The type I' picture has managed to make part of these multiplets perturbative and familiar.

Another additional advantage of the present picture is that is clarifies a little bit the role of D8-branes in the lift to M-theory, at least in this particular context. Another important feature of this picture is that it allows to understand some subtle details in the matching with heterotic string theory (namely the appearance of exceptional gauge symmetries), but these are beyond the scopes of these notes, see [6] for details.

8 Final remarks

As promised in the introduction, the study of strong coupling behaviour of string theory has enriched our picture of these theories, and shown they are all related in an intricate web of dualities, see figure 2. The duality web get even more intricate as we compactify in more involveld geometries.

We have learnt the lesson that different string theories are simply different perturbative limits of a unique underlying theory. This underlying theory has moreover a limit described by an 11d theory, which reduces at low energies to 11d supergravity.

The theory underlying all string theories and the 11d theory is sometimes referred to as M-theory as well, in a broad sense (M-theory is often used in a restricted sense to refer to the 11d theory underlying 11d supergravity).

D0-brane worldline, transforming in the representation 16 of $SO(16)$. In the quantum mechanics of these particles, quantization of the fermion zero modes implies these particles transform in the spinor representation; there is a $\mathbf{Z_2}$ gauge symmetry on the D0-brane volume that forces us to project out on of the chiral representations.

There are several proposals to define M-theory (in a broad sense) microscopically, but for the moment a complete definition is lacking: We do not have a complete definition of string theory beyond the perturbative corners.

Altough the discoveries in this lecture may make us feel a bit uncomfortable, we should realize that the final picture is extremely beautiful. For instance, in perturbation theory it seemed that we had five different and seemingly disconnected solutions/proposals to provide a quantum consistent description of gravity and gauge interactions. Non-perturbatively we find that in fact there is a unique answer to this problem. The issue is to extract the fundamental physical principles underlying this theory in an intrinsic way (not tied to any particular perturbative limit).

References

[1] J. Polchinski, 'String theory' vol 2.

[2] C. M. Hull, P. K. Townsend, 'Unity of superstring dualities', Nucl. Phys. B438 (1995) 109, hep-th/9410167.

[3] E. Witten,'String theory dynamics in various dimensions', Nucl. Phys. B443 (1995) 85, hep-th/9503124.

[4] T. Banks, W. Fischler, S. H. Shenker, L. Susskind, 'M theory as a matrix model: A Conjecture', Phys. Rev. D55 (1997) 5112, hep-th/9610043.

[5] J. H. Schwarz, 'An SL(2,Z) multiplet of type IIB superstrings', Phys. Lett. B360 (1995) 13, ERRATUM-ibid. B364 (1995) 252, hep-th/9508143;
P. S. Aspinwall, 'Some relationships between dualities in string theory', Nucl. Phys. Proc. Suppl. 46 (1996) 30, hep-th/9508154.

[6] J. Polchinski, E. Witten, 'Evidence for heterotic - type I string duality', Nucl. Phys. B460 (1996) 525, hep-th/9510169.

[7] P. Horava, E, Witten, 'Heterotic and type I string dynamics from eleven-dimensions', Nucl. Phys. B460 (1996) 506, hep-th/9510209.

[8] P. Horava, E, Witten, 'Eleven-dimensional supergravity on a manifold with boundary', Nucl. Phys. B475 (1996) 94, hep-th/9603142.