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Research on Construction Engineering Project Risk Assessment with Some 2-Tuple Linguistic Neutrosophic Hamy Mean Operators

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Abstract: In this paper, we expand the Hamy mean (HM) operator, weighted Hamy mean (WHM), dual Hamy mean (DHM) operator, and weighted dual Hamy mean (WDHM) operator with 2-tuple linguistic neutrosophic numbers (2TLNNs) to propose a 2-tuple linguistic neutrosophic Hamy mean (2TLNHM) operator, 2-tuple linguistic neutrosophic weighted Hamy mean (2TLNWHM) operator, 2-tuple linguistic neutrosophic dual Hamy mean (2TLNDHM) operator, and 2-tuple linguistic neutrosophic weighted dual Hamy mean (2TLNWDHM) operator. Then, the multiple attribute decision-making (MADM) methods are proposed with these operators. Finally, we utilize an applicable example in risk assessment for construction engineering projects to prove the proposed methods.

Keywords: multiple attribute decision-making (MADM); neutrosophic numbers; 2-tuple linguistic neutrosophic numbers set (2TLNNSs); 2TLNHM operator; 2TLNWHM operator; 2TLNDHM operator; 2TLNWDHM operator; construction engineering projects; risk assessment

1. Introduction

Neutrosophic sets (NSs), which were proposed originally by Smarandache [1,2], have attracted the attention of many scholars, and NSs have acted as a workspace in depicting indeterminate and inconsistent information. A NS has more potential power than other modeling mathematical tools, such as fuzzy set [3], intuitionistic fuzzy set (IFS) [4] and interval-valued intuitionistic fuzzy set (IVIFS) [5]. But, it is difficult to apply NSs to solve real life problems. Therefore, Wang et al. [6,7] defined single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INS), which are characterized by a truth membership, an indeterminacy membership and a falsity membership. Hence, SVNSs and INSs can express much more information than fuzzy sets, IFSs and IVIFSs. Ye [8] proposed a multiple attribute decision-making (MADM) method with correlation coefficients of SVNSs. Broumi and Smarandache [9] defined the correlation coefficients of INSs. Biswas et al. [10] proposed the Technique for Order Preference by Similarity to an Ideal Solution(TOPSIS) method with SVNNs. Liu et al. [11] defined the generalized neutrosophic number Hamacher aggregation for SVNSs. Sahin and Liu [12] defined the maximizing deviation model under a neutrosophic environment. Ye [13] developed some similarity measures of INS. Zhang et al. [14] defined some aggregating operators with INNs. Ye [15] defined a simplified neutrosophic set (SNS). Peng et al. [16] developed aggregation operators under SNS. Peng et al. [17] investigated the outranking approach with SNS, and then Zhang et al. [18] extended Peng's approach. Liu and Liu [19] proposed a power averaging operator with SVNNs. Deli and Subas [20] discussed a novel method to rank SVNNs. Peng et al. [21] proposed multi-valued neutrosophic sets. Zhang et al. [22] gave the improved weighted correlation coefficient for interval neutrosophic sets. Chen and Ye [23] proposed Dombi operations for SVNSs. Liu and Wang [24] proposed the MADM method based on a SVN-normalized weighted Bonferroni mean. Wu et al. [25] proposed a cross-entropy and prioritized an aggregation operator with SNSs in MADM problems. Li et al. [26] developed SVNN Heronian mean operators in MADM problems. Zavadskas et al. [27] proposed a model for residential house elements and material selection using the neutrosophic MULTIMOORA method. Zavadskas et al. [28] studied the sustainable market valuation of buildings using the SVN MAMVA method. Bausys and Juodagalviene [29] investigated the garage location selection for residential houses using the WASPAS-SVNS method. Wu et al. [30] proposed some Hamacher aggregation operators under an SVN 2-tuple linguistic environment for MAGDM.

Although SVNS theory has been successfully applied in some areas, the SVNS is also characterized by truth membership degree, indeterminacy membership degree, and falsity membership degree information. However, all the above approaches are unsuitable for describing the truth membership degree, indeterminacy membership degree, and falsity membership degree information of an element of a set by linguistic variables on the basis of the given linguistic term sets, which can reflect a decision maker's confidence level when they are making an evaluation. In order to overcome this limit, we propose the concept of a 2-tuple linguistic neutrosophic numbers set (2TLNNSs) to solve this problem based on SVNS [6,7] and a 2-tuple linguistic information processing model [31]. Thus, how to aggregate these 2-tuple linguistic neutrosophic numbers is an interesting topic. To solve this issue, in this paper, we develop aggregation operators with 2TLNNs based on the traditional operator [32]. In order to do so, the remainder of this paper is set out as follows. In the next section, we propose the concept of 2TLNNSs. In Section 3, we propose Hamy mean (HM) operators with 2TLNNs. In Section 4, we give a numerical example for risk assessment of a construction engineering projects. Section 5 concludes the paper with some remarks.

2. Preliminaries

In this section, we propose the concept of using 2-tuple linguistic neutrosophic sets (2TLNSs) based on SVNSs [6,7] and 2-tuple linguistic sets (2TLSs) [31].

2.1. 2TLSs

Definition 1. *Let $S = \{s_i | i = 0, 1, \ldots, t\}$ be a linguistic term set with an odd cardinality. Any label, s_i, represents a possible value for a linguistic variable, and S can be defined as:*

$$S = \left\{ \begin{array}{l} s_0 = extremely\ poor,\ s_1 = very\ poor,\ s_2 = poor,\ s_3 = medium, \\ s_4 = good,\ s_5 = very\ good,\ s_6 = extremely\ good. \end{array} \right\} \quad (1)$$

Herrera and Martinez [27,28] developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, ρ_i), where s_i is a linguistic label for predefined linguistic term set S and ρ_i is the value of symbolic translation, and $\rho_i \in [-0.5, 0.5)$.

2.2. SVNSs

Let X be a space of points (objects) with a generic element in a fixed set, X, denoted by x. An SVNS, A, in X is characterized as the following [6,7]:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\} \quad (2)$$

where the truth membership function, $T_A(x)$, indeterminacy-membership, $I_A(x)$, and falsity membership function, $F_A(x)$, are single subintervals/subsets in the real standard $[0,1]$, that is, $T_A(x) : X \to [0,1], I_A(x) : X \to [0,1]$ and $F_A(x) : X \to [0,1]$. In addition, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Then, a simplification of A is denoted by $A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}$, which is a SVNS.

For a SVNS $\{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}$, the ordered triple components, $(T_A(x), I_A(x), F_A(x))$, are described as a single-valued neutrosophic number (SVNN), and each SVNN can be expressed as $A = (T_A, I_A, F_A)$, where $T_A \in [0,1], I_A \in [0,1], F_A \in [0,1]$ and $0 \leq T_A + I_A + F_A \leq 3$.

2.3. 2TLNSs

Definition 2. *Assume that $\varphi = \{\varphi_0, \varphi_1, \ldots, \varphi_t\}$ is a 2TLSs with an odd cardinality, $t+1$. If $\varphi = \langle (s_T, \alpha), (s_I, \beta), (s_F, \gamma) \rangle$ is defined for $(s_T, \alpha), (s_I, \beta), (s_F, \gamma) \in \varphi$ and $\alpha, \beta, \gamma \in [0, t]$, where $(s_T, \alpha), (s_I, \beta)$ and (s_F, γ) express independently the truth degree, indeterminacy degree, and falsity degree by 2TLSs, then 2TLNSs is defined as follows:*

$$\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle \tag{3}$$

where $0 \leq \Delta^{-1}\left(s_{T_j}, \alpha_j\right) \leq t, 0 \leq \Delta^{-1}\left(s_{I_j}, \beta_j\right) \leq t, 0 \leq \Delta^{-1}\left(s_{F_j}, \gamma_j\right) \leq t$, *and* $0 \leq \Delta^{-1}\left(s_{T_j}, \alpha_j\right) + \Delta^{-1}\left(s_{I_j}, \beta_j\right) + \Delta^{-1}\left(s_{F_j}, \gamma_j\right) \leq 3t$.

Definition 3. *Let $\varphi_1 = \langle (s_{T_1}, \alpha_1), (s_{I_1}, \beta_1), (s_{F_1}, \gamma_1) \rangle$ be a 2TLNN in φ. Then, the score and accuracy functions of φ_1 are defined as follows:*

$$S(\varphi_1) = \Delta \left\{ \frac{(2t + \Delta^{-1}(s_{T_1}, \alpha_1) - \Delta^{-1}(s_{I_1}, \beta_1) - \Delta^{-1}(s_{F_1}, \gamma_1))}{3} \right\}, S(\varphi_1) \in [0, t] \tag{4}$$

$$H(\varphi_1) = \Delta \left\{ \Delta^{-1}(s_{T_1}, \alpha_1) - \Delta^{-1}(s_{F_1}, \gamma_1) \right\}, H(\varphi_1) \in [-t, t]. \tag{5}$$

Definition 4. *Let $\varphi_1 = \langle (s_{T_1}, \alpha_1), (s_{I_1}, \beta_1), (s_{F_1}, \gamma_1) \rangle$ and $\varphi_2 = \langle (s_{T_2}, \alpha_2), (s_{I_2}, \beta_2), (s_{F_2}, \gamma_2) \rangle$ be two 2TLNNs, then*

(1) if $S(\varphi_1) < S(\varphi_2)$, then $\varphi_1 < \varphi_2$;
(2) if $S(\varphi_1) > S(\varphi_2)$, then $\varphi_1 > \varphi_2$;
(3) if $S(\varphi_1) = S(\varphi_2), H(\varphi_1) < H(\varphi_2)$, then $\varphi_1 < \varphi_2$;
(4) if $S(\varphi_1) = S(\varphi_2), H(\varphi_1) > H(\varphi_2)$, then $\varphi_1 > \varphi_2$;
(5) if $S(\varphi_1) = S(\varphi_2), H(\varphi_1) = H(\varphi_2)$, then $\varphi_1 = \varphi_2$.

Definition 5. *Let $\varphi_1 = \langle (s_{T_1}, \alpha_1), (s_{I_1}, \beta_1), (s_{F_1}, \gamma_1) \rangle$ and $\varphi_2 = \langle (s_{T_2}, \alpha_2), (s_{I_2}, \beta_2), (s_{F_2}, \gamma_2) \rangle$ be two 2TLNNs, $\zeta > 0$, then*

(1) $$\varphi_1 \oplus \varphi_2 = \left\{ \begin{array}{l} \Delta\left(t\left(\frac{\Delta^{-1}(s_{T_1}, \alpha_1)}{t} + \frac{\Delta^{-1}(s_{T_2}, \alpha_2)}{t} - \frac{\Delta^{-1}(s_{T_1}, \alpha_1)}{t} \cdot \frac{\Delta^{-1}(s_{T_2}, \alpha_2)}{t}\right)\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(s_{I_1}, \beta_1)}{t} \cdot \frac{\Delta^{-1}(s_{I_2}, \beta_2)}{t}\right)\right), \Delta\left(t\left(\frac{\Delta^{-1}(s_{F_1}, \gamma_1)}{t} \cdot \frac{\Delta^{-1}(s_{F_2}, \gamma_2)}{t}\right)\right) \end{array} \right\};$$

(2) $$\varphi_1 \otimes \varphi_2 = \left\{ \begin{array}{l} \Delta\left(t\left(\frac{\Delta^{-1}(s_{T_1},\alpha_1)}{t} \cdot \frac{\Delta^{-1}(s_{T_2},\alpha_2)}{t}\right)\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(s_{T_1},\beta_1)}{t} + \frac{\Delta^{-1}(s_{T_2},\beta_2)}{t} - \frac{\Delta^{-1}(s_{T_1},\beta_1)}{t} \cdot \frac{\Delta^{-1}(s_{T_2},\beta_2)}{t}\right)\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(s_{F_1},\gamma_1)}{t} + \frac{\Delta^{-1}(s_{F_2},\gamma_2)}{t} - \frac{\Delta^{-1}(s_{F_1},\gamma_1)}{t} \cdot \frac{\Delta^{-1}(s_{F_2},\gamma_2)}{t}\right)\right) \end{array} \right\};$$

(3) $$\zeta\varphi_1 = \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}(s_{T_1},\alpha_1)}{t}\right)^{\zeta}\right)\right), \Delta\left(t\left(\frac{\Delta^{-1}(s_{I_1},\beta_1)}{t}\right)^{\zeta}\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(s_{F_1},\gamma_1)}{t}\right)^{\zeta}\right) \end{array} \right\}, \zeta > 0;$$

(4) $$(\varphi_1)^{\zeta} = \left\{ \begin{array}{l} \Delta\left(t\left(\frac{\Delta^{-1}(s_{T_1},\alpha_1)}{t}\right)^{\zeta}\right), \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}(s_{I_1},\beta_1)}{t}\right)^{\zeta}\right)\right), \\ \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}(s_{F_1},\gamma_1)}{t}\right)^{\zeta}\right)\right) \end{array} \right\}, \zeta > 0.$$

2.4. HM Operator

Definition 6 [32]. *The Hamy mean (HM) operator is defined as follows:*

$$\mathrm{HM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \frac{\sum\limits_{1 \le i_1 < \ldots < i_x \le n} \left(\prod\limits_{j=1}^{x} \varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}, \tag{6}$$

where x is a parameter and $x = 1, 2, \ldots, n$, $i_1, i_2, \ldots, i_x$ are x integer values taken from the set $\{1, 2, \ldots, n\}$ of k integer values, C_n^x denotes the binomial coefficient and $C_n^x = \frac{n!}{x!(n-x)!}$.

3. Some 2TLNHM Operators

3.1. 2TLNHM Operator

In this section, we will combine HM and 2TLNNs and propose the 2-tuple linguistic neutrosophic Hamy mean (2TLNHM) operator.

Definition 7. *Let $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of 2TLNNs. The 2TLNHM operator is*

$$\mathrm{2TLNHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \frac{\bigoplus\limits_{1 \le i_1 < \ldots < i_x \le n} \left(\bigotimes\limits_{j=1}^{x} \varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}. \tag{7}$$

Theorem 1. *Let $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of 2TLNNs. The aggregated value from the 2TLNHM operators is also a 2TLNN where*

$$\begin{aligned}&\mathrm{2TLNHM}^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n)=\frac{\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}\\&=\left\{\begin{array}{l}\Delta\left(t\left(1-\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right),\\\Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right),\\\Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\end{array}\right\}.\end{aligned}\tag{8}$$

Proof:

$$\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}=\left\{\begin{array}{l}\Delta\left(t\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right),\Delta\left(t\left(1-\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)\right),\\\Delta\left(t\left(1-\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)\right)\end{array}\right\}.\tag{9}$$

Thus,

$$\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}=\left\{\begin{array}{l}\Delta\left(t\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right),\Delta\left(t\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right),\\\Delta\left(t\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\end{array}\right\}\tag{10}$$

Thereafter,

$$\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}=\left\{\begin{array}{l}\Delta\left(t\left(1-\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right),\\\Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right),\\\Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right)\end{array}\right\}.\tag{11}$$

Therefore,

$$\mathrm{2TLNHM}^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n)=\frac{\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}$$
$$=\left\{\begin{array}{l}\Delta\left(t\left(1-\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\end{array}\right\} \tag{12}$$

Hence, (7) is kept.

Then we need to prove that (7) is a 2TLNN. We need to prove two conditions, as follows:

① $0\le\Delta^{-1}(s_T,\alpha)\le t, 0\le\Delta^{-1}(s_I,\beta)\le t, 0\le\Delta^{-1}(s_F,\gamma)\le t.$

② $0\le\Delta^{-1}(s_T,\alpha)+\Delta^{-1}(s_I,\beta)+\Delta^{-1}(s_F,\gamma)\le 3t.$

Let

$$\frac{\Delta^{-1}(s_T,\alpha)}{t}=1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$
$$\frac{\Delta^{-1}(s_I,\beta)}{t}=\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}.$$
$$\frac{\Delta^{-1}(s_F,\gamma)}{t}=\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

□

Proof: ① Since $0\le\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\le 1$, we get

$$0\le\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\le 1 \text{ and } 0\le 1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\le 1 \tag{13}$$

Then,

$$0\le\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\le 1, \tag{14}$$

$$0\le 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\le 1. \tag{15}$$

That means $0\le\Delta^{-1}(s_T,\alpha)\le t$. Similarly, we can get $0\le\Delta^{-1}(s_I,\beta)\le t, 0\le\Delta^{-1}(s_F,\gamma)\le t$ so ① is maintained. ② Since $0\le\Delta^{-1}(s_T,\alpha)\le t, 0\le\Delta^{-1}(s_I,\beta)\le t, 0\le\Delta^{-1}(s_F,\gamma)\le t.$, $0\le\Delta^{-1}(s_T,\alpha)+\Delta^{-1}(s_I,\beta)+\Delta^{-1}(s_F,\gamma)\le 3t$. □

Example 1. *Let* $\langle(s_5,0),(s_2,0),(s_1,0)\rangle, \langle(s_4,0),(s_3,0),(s_4,0)\rangle, \langle(s_2,0),(s_5,0),(s_1,0)\rangle$ *and* $\langle(s_5,0),(s_1,0),(s_3,0)\rangle$ *be four 2TLNNs, and suppose* $x = 2$*, then according to (4), we have*

$$\text{2TLNHM}^{(2)}(\varphi_1,\varphi_2,\cdots,\varphi_n) = \frac{\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}$$

$$= \left\{ \begin{array}{l} \Delta\left(6\times\left(1-\left(\begin{array}{c}\left(1-\left(\frac{5}{6}\times\frac{4}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{5}{6}\times\frac{2}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{5}{6}\times\frac{5}{6}\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\frac{4}{6}\times\frac{2}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{4}{6}\times\frac{5}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{2}{6}\times\frac{5}{6}\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right)\right), \\ \Delta\left(6\times\left(\begin{array}{c}\left(1-\left(\left(1-\frac{2}{6}\right)\times\left(1-\frac{3}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{2}{6}\right)\times\left(1-\frac{5}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{2}{6}\right)\times\left(1-\frac{1}{6}\right)\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\left(1-\frac{3}{6}\right)\times\left(1-\frac{5}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{3}{6}\right)\times\left(1-\frac{1}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{5}{6}\right)\times\left(1-\frac{1}{6}\right)\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right), \\ \Delta\left(6\times\left(\begin{array}{c}\left(1-\left(\left(1-\frac{1}{6}\right)\times\left(1-\frac{4}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{1}{6}\right)\times\left(1-\frac{1}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{1}{6}\right)\times\left(1-\frac{3}{6}\right)\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\left(1-\frac{4}{6}\right)\times\left(1-\frac{1}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{4}{6}\right)\times\left(1-\frac{3}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{1}{6}\right)\times\left(1-\frac{3}{6}\right)\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right) \end{array} \right\}$$

$$= \langle(s_4,0.0235),(s_3,-0.1556),(s_2,0.2489)\rangle$$

Now, we will give some properties of a 2TLNHM operator.

Property 1. *(Idempotency) If* $\varphi_j = \left\langle\left(s_{T_j},\alpha_j\right),\left(s_{I_j},\beta_j\right),\left(s_{F_j},\gamma_j\right)\right\rangle (j=1,2,\ldots,n)$ *are equal, then*

$$2TLNHM^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n) = \varphi. \tag{16}$$

Proof: Since $\varphi_j = \varphi = \langle(s_T,\alpha),(s_I,\beta),(s_F,\gamma)\rangle$, then

$$\text{2TLNHM}^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n) = \frac{\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\varphi_{i_j}\right)^{\frac{1}{x}}}{C_n^x}$$

$$= \left\{ \begin{array}{l} \Delta\left(t\left(1-\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}(s_T,\alpha)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}(s_I,\beta)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}(s_F,\gamma)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \Delta\left(t\left(1-\left(\left(1-\left(\left(\frac{\Delta^{-1}(s_T,\alpha)}{t}\right)^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{C_n^x}}\right)\right), \Delta\left(t\left(\left(1-\left(\left(1-\frac{\Delta^{-1}(s_I,\beta)}{t}\right)^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(\left(1-\left(\left(1-\frac{\Delta^{-1}(s_F,\gamma)}{t}\right)^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{C_n^x}}\right) \end{array} \right\}$$

$$= \langle(s_T,\alpha),(s_I,\beta),(s_F,\gamma)\rangle = \varphi$$

□

Property 2. *(Monotonicity) Let* $\varphi_{a_j} = \left\langle\left(s_{T_{a_j}},\alpha_{a_j}\right),\left(s_{I_{a_j}},\beta_{a_j}\right),\left(s_{F_{a_j}},\gamma_{a_j}\right)\right\rangle (j=1,2,\ldots,n)$ *and* $\varphi_{b_j} = \left\langle\left(s_{T_{b_j}},\alpha_{b_j}\right),\left(s_{I_{b_j}},\beta_{b_j}\right),\left(s_{F_{b_j}},\gamma_{b_j}\right)\right\rangle (j=1,2,\ldots,n)$ *be two sets of 2TLNNs. If* $\Delta^{-1}\left(s_{T_{a_j}},\alpha_{a_j}\right) \le \Delta^{-1}\left(s_{T_{b_j}},\alpha_{b_j}\right)$, $\Delta^{-1}\left(s_{I_{a_j}},\beta_{a_j}\right) \ge \Delta^{-1}\left(s_{I_{b_j}},\beta_{b_j}\right)$ and $\Delta^{-1}\left(s_{F_{a_j}},\gamma_{a_j}\right) \ge \Delta^{-1}\left(s_{F_{b_j}},\gamma_{b_j}\right)$ *hold for all j, then*

$$2TLNHM^{(x)}(\varphi_{a_1},\varphi_{a_2},\cdots,\varphi_{a_n}) \le 2TLNHM^{(x)}(\varphi_{b_1},\varphi_{b_2},\cdots,\varphi_{b_n}) \tag{17}$$

Proof: Let $\varphi_{a_j} = \left\langle \left(s_{T_{a_j}}, \alpha_{a_j}\right), \left(s_{I_{a_j}}, \beta_{a_j}\right), \left(s_{F_{a_j}}, \gamma_{a_j}\right) \right\rangle$ and $\varphi_{b_j} = \left\langle \left(s_{T_{b_j}}, \alpha_{b_j}\right), \left(s_{I_{b_j}}, \beta_{b_j}\right), \left(s_{F_{b_j}}, \gamma_{b_j}\right) \right\rangle$, given that $\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right) \leq \Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)$, we can obtain

$$\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right)}{t} \right) \leq \prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)}{t} \right), \tag{18}$$

$$1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right)}{t} \right) \right)^{\frac{1}{x}} \geq 1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)}{t} \right) \right)^{\frac{1}{x}}. \tag{19}$$

Thereafter,

$$\left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \geq \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \tag{20}$$

Furthermore,

$$1 - \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \leq 1 - \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}}. \tag{21}$$

That means $\Delta^{-1}(s_{T_a}, \alpha_a) \leq \Delta^{-1}(s_{T_b}, \alpha_b)$. Similarly, we can obtain $\Delta^{-1}(s_{I_a}, \beta_a) \geq \Delta^{-1}(s_{I_b}, \beta_b)$ and $\Delta^{-1}(s_{F_a}, \gamma_a) \geq \Delta^{-1}(s_{F_b}, \gamma_b)$.

If $\Delta^{-1}(s_{T_a}, \alpha_a) < \Delta^{-1}(s_{T_b}, \alpha_b), \Delta^{-1}(s_{I_a}, \beta_a) \geq \Delta^{-1}(s_{I_b}, \beta_b)$ and $\Delta^{-1}(s_{F_a}, \gamma_a) \geq \Delta^{-1}(s_{F_b}, \gamma_b)$,
$\text{2TLNHM}^{(x)}(\varphi_a, \varphi_a, \cdots, \varphi_a) < \text{2TLNHM}^{(x)}(\varphi_b, \varphi_b, \cdots, \varphi_b)$

If $\Delta^{-1}(s_{T_a}, \alpha_a) = \Delta^{-1}(s_{T_b}, \alpha_b), \Delta^{-1}(s_{I_a}, \beta_a) > \Delta^{-1}(s_{I_b}, \beta_b)$ and $\Delta^{-1}(s_{F_a}, \gamma_a) > \Delta^{-1}(s_{F_b}, \gamma_b)$,
$\text{2TLNHM}^{(x)}(\varphi_a, \varphi_a, \cdots, \varphi_a) < \text{2TLNHM}^{(x)}(\varphi_b, \varphi_b, \cdots, \varphi_b)$

If $\Delta^{-1}(s_{T_a}, \alpha_a) = \Delta^{-1}(s_{T_b}, \alpha_b), \Delta^{-1}(s_{I_a}, \beta_a) = \Delta^{-1}(s_{I_b}, \beta_b)$ and $\Delta^{-1}(s_{F_a}, \gamma_a) = \Delta^{-1}(s_{F_b}, \gamma_b)$,
$\text{2TLNHM}^{(x)}(\varphi_a, \varphi_a, \cdots, \varphi_a) = \text{2TLNHM}^{(x)}(\varphi_b, \varphi_b, \cdots, \varphi_b)$

So, Property 2 is right. □

Property 3. *(Boundedness) Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. If* $\varphi_i^+ = \left(max_i\left(s_{T_j}, \alpha_j\right), min_i\left(s_{I_j}, \beta_j\right), min_i\left(s_{F_j}, \gamma_j\right)\right)$ *and* $\varphi_i^+ = \left(max_i\left(s_{T_j}, \alpha_j\right), min_i\left(s_{I_j}, \beta_j\right), min_i\left(s_{F_j}, \gamma_j\right)\right)$*, then*

$$\varphi^- \leq 2TLNHM^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^+. \tag{22}$$

From Property 1,

$$\begin{array}{l} \text{2TLNHM}^{(x)}\left(\varphi_1^-, \varphi_2^-, \cdots, \varphi_n^-\right) = \varphi^- \\ \text{2TLNHM}^{(x)}\left(\varphi_1^+, \varphi_2^+, \cdots, \varphi_n^+\right) = \varphi^+ \end{array}.$$

From Property 2,

$$\varphi^- \leq \text{2TLNHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^+.$$

3.2. The 2TLNWHM Operator

In an actual MADM, it is important to consider attribute weights. This section proposes a 2-tuple linguistic neutrosophic weighted Hamy mean (2TLNWHM) operator as follows.

Definition 8. *Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs with a weight vector,* $w_i = (w_1, w_2, \ldots, w_n)^T$*, thereby satisfying* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$*. Then, we can define the 2TLNWHM operator as follows:*

$$\mathrm{2TLNWHM}_w^{(x)}(\varphi_1, \varphi_2, \ldots, \varphi_n) = \frac{\underset{1 \le i_1 < \ldots < i_x \le n}{\oplus} \left(\overset{x}{\underset{j=1}{\otimes}} \left(\varphi_{i_j}\right)^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x}. \tag{23}$$

Theorem 2. *Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. The aggregated value determined using a 2TLNWHM operator is also a 2TLNN, where*

$$\begin{aligned} &\mathrm{2TLNWHM}_w^{(x)}(\varphi_1, \varphi_2, \ldots, \varphi_n) = \frac{\underset{1 \le i_1 < \ldots < i_x \le n}{\oplus} \left(\overset{x}{\underset{j=1}{\otimes}} \left(\varphi_{i_j}\right)^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x} \\ &= \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \right), \\ \Delta\left(t\left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right), \\ \Delta\left(t\left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \end{array} \right\}. \end{aligned} \tag{24}$$

Proof: From Definition 5, we can obtain,

$$\left(\varphi_{i_j}\right)^{w_{i_j}} = \left\{ \begin{array}{l} \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right), \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right) \right), \\ \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right) \right) \end{array} \right\}. \tag{25}$$

Thus,

$$\overset{x}{\underset{j=1}{\otimes}} \left(\varphi_{i_j}\right)^{w_{i_j}} = \left\{ \begin{array}{l} \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right) \right), \Delta\left(t\left(1 - \prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right) \right), \\ \Delta\left(t\left(1 - \prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right) \right) \end{array} \right\} \tag{26}$$

Therefore,

$$\left(\overset{x}{\underset{j=1}{\otimes}} \left(\varphi_{i_j}\right)^{w_{i_j}} \right)^{\frac{1}{x}} = \left\{ \begin{array}{l} \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right), \Delta\left(t\left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right) \end{array} \right\}. \tag{27}$$

Thereafter,

$$\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\left(\varphi_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{x}}=\left\{\begin{array}{l}\Delta\left(t\left(1-\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right)\end{array}\right\} \tag{28}$$

Furthermore,

$$\begin{array}{l}2\mathrm{TLNWHM}_w^{(x)}(\varphi_1,\varphi_2,\ldots,\varphi_n)=\frac{\underset{1\le i_1<\ldots<i_x\le n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\left(\varphi_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{x}}}{C_n^x}.\\ =\left\{\begin{array}{l}\Delta\left(t\left(1-\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right),\\ \Delta\left(t\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\end{array}\right\}.\end{array} \tag{29}$$

Hence, (23) is kept.

Then we need to prove that (23) is a 2TLNN. We need to prove two conditions as follows:

① $0\le\Delta^{-1}(s_T,\alpha)\le t, 0\le\Delta^{-1}(s_I,\beta)\le t, 0\le\Delta^{-1}(s_F,\gamma)\le t.$

② $0\le\Delta^{-1}(s_T,\alpha)+\Delta^{-1}(s_I,\beta)+\Delta^{-1}(s_F,\gamma)\le 3t.$

Let

$$\frac{\Delta^{-1}(s_T,\alpha)}{t}=1-\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_I,\beta)}{t}=\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_F,\gamma)}{t}=\left(\prod\limits_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod\limits_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}.$$

□

Proof. ① Since $0 \leq \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \leq 1$, we get

$$0 \leq \prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \leq 1 \text{ and } 0 \leq 1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \leq 1 \tag{30}$$

Then,

$$0 \leq \prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \leq 1 \tag{31}$$

$$0 \leq 1 - \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \leq 1 \tag{32}$$

That means $0 \leq \Delta^{-1}(s_T, \alpha) \leq t$. Similarly, we can get $0 \leq \Delta^{-1}(s_I, \beta) \leq t, 0 \leq \Delta^{-1}(s_F, \gamma) \leq t$. So, ① is maintained; ② Since $0 \leq \Delta^{-1}(s_T, \alpha) \leq t, 0 \leq \Delta^{-1}(s_I, \beta) \leq t, 0 \leq \Delta^{-1}(s_F, \gamma) \leq t$. $0 \leq \Delta^{-1}(s_T, \alpha) + \Delta^{-1}(s_I, \beta) + \Delta^{-1}(s_F, \gamma) \leq 3t$. □

Example 2. *Let* $\langle (s_5, 0), (s_2, 0), (s_1, 0) \rangle, \langle (s_4, 0), (s_3, 0), (s_4, 0) \rangle, \langle (s_2, 0), (s_5, 0), (s_1, 0) \rangle$ *and* $\langle (s_5, 0), (s_1, 0), (s_3, 0) \rangle$ *be four 2TLNNs,* $w = (0.2, 0.3, 0.4, 0.1)$ *and suppose* $x = 2$*, then according to (23), we have*

$$\text{2TLNHM}^{(2)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \frac{\underset{1 \leq i_1 < \ldots < i_x \leq n}{\oplus} \left(\overset{x}{\underset{j=1}{\otimes}} \left(\varphi_{i_j}\right)^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x}$$

$$= \left\{ \begin{array}{l}
\Delta \left\{ 6 \times \left\{ 1 - \left\{ \begin{array}{l} \left(1 - \left(\left(\frac{5}{6}\right)^{0.2} \times \left(\frac{4}{6}\right)^{0.3}\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(\frac{5}{6}\right)^{0.2} \times \left(\frac{2}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(\frac{5}{6}\right)^{0.2} \times \left(\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \\ \times \left(1 - \left(\left(\frac{4}{6}\right)^{0.3} \times \left(\frac{2}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(\frac{4}{6}\right)^{0.3} \times \left(\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(\frac{2}{6}\right)^{0.4} \times \left(\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \end{array} \right\}^{\frac{1}{C_4^2}} \right\} \right\}, \\
\Delta \left\{ 6 \times \left\{ \begin{array}{l} \left(1 - \left(\left(1 - \left(\frac{2}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{3}{6}\right)^{0.3}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{2}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{5}{6}\right)^{0.4}\right)\right)^{\frac{1}{2}}\right) \\ \times \left(1 - \left(\left(1 - \left(\frac{2}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{1}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{3}{6}\right)^{0.3}\right) \times \left(1 - \left(\frac{5}{6}\right)^{0.4}\right)\right)^{\frac{1}{2}}\right) \\ \times \left(1 - \left(\left(1 - \left(\frac{3}{6}\right)^{0.3}\right) \times \left(1 - \left(\frac{1}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{5}{6}\right)^{0.4}\right) \times \left(1 - \left(\frac{1}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \end{array} \right\}^{\frac{1}{C_4^2}} \right\}, \\
\Delta \left\{ 6 \times \left\{ \begin{array}{l} \left(1 - \left(\left(1 - \left(\frac{1}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{4}{6}\right)^{0.3}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{1}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{1}{6}\right)^{0.4}\right)\right)^{\frac{1}{2}}\right) \\ \times \left(1 - \left(\left(1 - \left(\frac{1}{6}\right)^{0.2}\right) \times \left(1 - \left(\frac{3}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{4}{6}\right)^{0.3}\right) \times \left(1 - \left(\frac{1}{6}\right)^{0.4}\right)\right)^{\frac{1}{2}}\right) \\ \times \left(1 - \left(\left(1 - \left(\frac{4}{6}\right)^{0.3}\right) \times \left(1 - \left(\frac{3}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \times \left(1 - \left(\left(1 - \left(\frac{1}{6}\right)^{0.4}\right) \times \left(1 - \left(\frac{3}{6}\right)^{0.1}\right)\right)^{\frac{1}{2}}\right) \end{array} \right\}^{\frac{1}{C_4^2}} \right\}
\end{array} \right\}.$$

$$= \langle (s_5, 0.3604), (s_1, -0.0344), (s_1, -0.3963) \rangle$$

Now, we will discuss some properties of the 2TLNWHM operator.

Property 4. *(Monotonicity) Let* $\varphi_{a_j} = \left\langle \left(s_{T_{a_j}}, \alpha_{a_j}\right), \left(s_{I_{a_j}}, \beta_{a_j}\right), \left(s_{F_{a_j}}, \gamma_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\varphi_{b_j} = \left\langle \left(s_{T_{b_j}}, \alpha_{b_j}\right), \left(s_{I_{b_j}}, \beta_{b_j}\right), \left(s_{F_{b_j}}, \gamma_{b_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *be two sets of 2TLNNs. If* $\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right) \leq \Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)$, $\Delta^{-1}\left(s_{I_{a_j}}, \beta_{a_j}\right) \geq \Delta^{-1}\left(s_{I_{b_j}}, \beta_{b_j}\right)$ *and* $\Delta^{-1}\left(s_{F_{a_j}}, \gamma_{a_j}\right) \geq \Delta^{-1}\left(s_{F_{b_j}}, \gamma_{b_j}\right)$ *hold for all j, then*

$$\text{2TLNWHM}^{(x)}(\varphi_{a_1}, \varphi_{a_2}, \cdots, \varphi_{a_n}) \leq \text{2TLNWHM}^{(x)}(\varphi_{b_1}, \varphi_{b_2}, \cdots, \varphi_{b_n}) \quad (33)$$

The proof is similar to 2TLNWHM; it is omitted here.

Property 5. *(Boundedness) Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. If* $\varphi_i^{+} = \left(\max_i\left(s_{T_j}, \alpha_j\right), \min_i\left(s_{I_j}, \beta_j\right), \min_i\left(s_{F_j}, \gamma_j\right)\right)$ *and* $\varphi_i^{+} = \left(\max_i\left(s_{T_j}, \alpha_j\right), \min_i\left(s_{I_j}, \beta_j\right), \min_i\left(s_{F_j}, \gamma_j\right)\right)$, *then*

$$\varphi^{-} \leq \text{2TLNWHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^{+}. \quad (34)$$

From theorem 2, we get

$$\text{2TLNWHM}_w^{(x)}((\varphi_1^{-}, \varphi_2^{-}, \cdots, \varphi_n^{-})) = \frac{\underset{1 \leq i_1 < \ldots < i_x \leq n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\left(\min\varphi_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{x}}}{C_n^x}$$
$$= \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\min\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \\ \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\max\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\max\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right) \end{array} \right\}, \quad (35)$$

$$\text{2TLNWHM}_w^{(x)}((\varphi_1^{+}, \varphi_2^{+}, \cdots, \varphi_n^{+})) = \frac{\underset{1 \leq i_1 < \ldots < i_x \leq n}{\oplus}\left(\overset{x}{\underset{j=1}{\otimes}}\left(\max\varphi_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{x}}}{C_n^x}$$
$$= \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\max\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \\ \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\min\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\min\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right) \end{array} \right\}. \quad (36)$$

From property 4, we get

$$\varphi^{-} \leq 2\text{TLNWHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^{+} \tag{37}$$

It is obvious that the 2TLNWHM operator lacks the property of idempotency.

3.3. The 2TLNDHM Operator

Based on the Hamy mean (HM) operator [32], we propose the dual Hamy mean (DHM) operator.

Definition 9. *The DHM operator is defined as follows:*

$$\text{DHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(\frac{\sum_{j=1}^{x} \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}. \tag{38}$$

where x is a parameter and $x = 1, 2, \ldots, n$, $i_1, i_2, \ldots, i_x$ are x integer values taken from the set $\{1, 2, \ldots, n\}$ of k integer values, C_n^x denotes the binomial coefficient and $C_n^x = \frac{n!}{x!(n-x)!}$.

In this section, we propose the 2-tuple linguistic neutrosophic DHM (2TLNDHM) operator.

Definition 10. *Let $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of 2TLNNs. The 2TLNDHM operator is:*

$$2\text{TLNDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\underset{1 \leq i_1 < \ldots < i_x \leq n}{\otimes} \left(\frac{\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}. \tag{39}$$

Theorem 3. *Let $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of 2TLNNs. The aggregated value determined using 2TLNDHM operators is also a 2TLNN where*

$$\begin{aligned} & 2\text{TLNDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\underset{1 \leq i_1 < \ldots < i_x \leq n}{\otimes} \left(\frac{\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \\ & = \left\{ \begin{array}{l} \Delta\left(t \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right), \\ \Delta\left(t \left(1 - \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \right), \\ \Delta\left(t \left(1 - \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n} \left(1 - \left(\prod_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right) \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \right) \end{array} \right\} \end{aligned} \tag{40}$$

Proof:

$$\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j} = \left\{ \begin{array}{l} \Delta\left(t\left(1-\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)\right), \Delta\left(t\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)\right), \\ \Delta\left(t\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)\right) \end{array} \right\} \tag{41}$$

Thus,

$$\frac{\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j}}{x} = \left\{ \begin{array}{l} \Delta\left(t\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right), \Delta\left(t\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right), \\ \Delta\left(t\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right) \end{array} \right\} \tag{42}$$

Thereafter,

$$\underset{1\le i_1<\ldots<i_x\le n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j}}{x}\right) = \left\{ \begin{array}{l} \Delta\left(t\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right), \\ \Delta\left(t\left(1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right), \\ \Delta\left(t\left(1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)\right) \end{array} \right\} \tag{43}$$

Therefore,

$$\begin{array}{l} \text{2TLNDHM}^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n) = \left(\underset{1\le i_1<\ldots<i_x\le n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}} \varphi_{i_j}}{x}\right)\right)^{\frac{1}{C_n^x}} \\ = \left\{ \begin{array}{l} \Delta\left(t\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \\ \Delta\left(t\left(1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right) \end{array} \right\}. \end{array} \tag{44}$$

Hence, (39) is kept.

Then, we need to prove that (39) is a 2TLNN. We need to prove two conditions as follows:

① $0 \le \Delta^{-1}(s_T,\alpha) \le t, 0 \le \Delta^{-1}(s_I,\beta) \le t, 0 \le \Delta^{-1}(s_F,\gamma) \le t.$

② $0 \le \Delta^{-1}(s_T,\alpha) + \Delta^{-1}(s_I,\beta) + \Delta^{-1}(s_F,\gamma) \le 3t.$

Let

$$\frac{\Delta^{-1}(s_T,\alpha)}{t} = \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_I,\beta)}{t} = 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_F,\gamma)}{t} = 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

□

Proof. ① Since $0 \le \frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t} \le 1$, we get

$$0 \le \prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right) \le 1 \text{ and } 0 \le 1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}} \le 1. \tag{45}$$

Then,

$$0 \le \prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right) \le 1 \tag{46}$$

$$0 \le \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}} \le 1 \tag{47}$$

That means $0 \le \Delta^{-1}(s_T,\alpha) \le t$. Similarly, we can get $0 \le \Delta^{-1}(s_I,\beta) \le t, 0 \le \Delta^{-1}(s_F,\gamma) \le t$. So, ① is maintained. ② Since $0 \le \Delta^{-1}(s_T,\alpha) \le t, 0 \le \Delta^{-1}(s_I,\beta) \le t, 0 \le \Delta^{-1}(s_F,\gamma) \le t$. $0 \le \Delta^{-1}(s_T,\alpha)+\Delta^{-1}(s_I,\beta)+\Delta^{-1}(s_F,\gamma) \le 3t$. □

Example 3. *Let* $\langle(s_5,0),(s_2,0),(s_1,0)\rangle, \langle(s_4,0),(s_3,0),(s_4,0)\rangle, \langle(s_2,0),(s_5,0),(s_1,0)\rangle$ *and* $\langle(s_5,0),(s_1,0),(s_3,0)\rangle$ *be four 2TLNNs, and suppose* $x=2$*, then according to (39), we have*

$$\text{2TLNDHM}^{(2)}(\varphi_1,\varphi_2,\cdots,\varphi_n) = \left(\underset{1\le i_1<\ldots<i_x\le n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}}\varphi_{i_j}}{x}\right)\right)^{\frac{1}{C_n^x}}$$

$$= \left\{\begin{array}{l}
\Delta\left(6\times\left(\begin{array}{l}\left(1-\left(\left(1-\frac{5}{6}\right)\times\left(1-\frac{4}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{5}{6}\right)\times\left(1-\frac{2}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{5}{6}\right)\times\left(1-\frac{5}{6}\right)\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\left(1-\frac{4}{6}\right)\times\left(1-\frac{2}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{4}{6}\right)\times\left(1-\frac{5}{6}\right)\right)^{\frac{1}{2}}\right)\times\left(1-\left(\left(1-\frac{2}{6}\right)\times\left(1-\frac{5}{6}\right)\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right),\\
\Delta\left(6\times\left(1-\left(\begin{array}{l}\left(1-\left(\frac{2}{6}\times\frac{3}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{2}{6}\times\frac{5}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{2}{6}\times\frac{1}{6}\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\frac{3}{6}\times\frac{5}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{3}{6}\times\frac{1}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{5}{6}\times\frac{1}{6}\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right)\right),\\
\Delta\left(6\times\left(1-\left(\begin{array}{l}\left(1-\left(\frac{1}{6}\times\frac{4}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{1}{6}\times\frac{1}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{1}{6}\times\frac{3}{6}\right)^{\frac{1}{2}}\right)\\ \times\left(1-\left(\frac{4}{6}\times\frac{1}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{4}{6}\times\frac{3}{6}\right)^{\frac{1}{2}}\right)\times\left(1-\left(\frac{1}{6}\times\frac{3}{6}\right)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\right)\right)
\end{array}\right\}$$

$$= \langle(s_4,0.1802),(s_3,-0.4123),(s_2,0.0680)\rangle$$

Similar to the 2TLNHM operator, we can get the properties, as follows.

Property 6. *(Idempotency) If* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *are equal, then*

$$2\text{TLNDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \varphi. \tag{48}$$

Property 7. *(Monotonicity) Let* $\varphi_{a_j} = \left\langle \left(s_{T_{a_j}}, \alpha_{a_j}\right), \left(s_{I_{a_j}}, \beta_{a_j}\right), \left(s_{F_{a_j}}, \gamma_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\varphi_{b_j} = \left\langle \left(s_{T_{b_j}}, \alpha_{b_j}\right), \left(s_{I_{b_j}}, \beta_{b_j}\right), \left(s_{F_{b_j}}, \gamma_{b_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *be two sets of 2TLNNs. If* $\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right) \leq \Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right), \Delta^{-1}\left(s_{I_{a_j}}, \beta_{a_j}\right) \geq \Delta^{-1}\left(s_{I_{b_j}}, \beta_{b_j}\right)$ *and* $\Delta^{-1}\left(s_{F_{a_j}}, \gamma_{a_j}\right) \geq \Delta^{-1}\left(s_{F_{b_j}}, \gamma_{b_j}\right)$ *hold for all j, then*

$$2\text{TLNDHM}^{(x)}(\varphi_{a_1}, \varphi_{a_2}, \cdots, \varphi_{a_n}) \leq 2\text{TLNDHM}^{(x)}(\varphi_{b_1}, \varphi_{b_2}, \cdots, \varphi_{b_n}) \tag{49}$$

Property 8. *(Boundedness) Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. If* $\varphi_i^+ = \left(\max_i\left(s_{T_j}, \alpha_j\right), \min_i\left(s_{I_j}, \beta_j\right), \min_i\left(s_{F_j}, \gamma_j\right)\right)$ *and* $\varphi_i^+ = \left(\max_i\left(s_{T_j}, \alpha_j\right), \min_i\left(s_{I_j}, \beta_j\right), \min_i\left(s_{F_j}, \gamma_j\right)\right)$ *then*

$$\varphi^- \leq 2\text{TLNDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^+. \tag{50}$$

3.4. The 2TLNWDHM Operator

In an actual MADM, it is important to consider attribute weights; in this section we shall propose the 2-tuple linguistic neutrosophic weighted DHM (2TLNWDHM) operator.

Definition 11. *Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs with the weight vector,* $w_i = (w_1, w_2, \ldots, w_n)^T$*, thereby satisfying* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$*. If*

$$2\text{TLNWDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\underset{1 \leq i_1 < \ldots < i_x \leq n}{\otimes} \left(\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j} \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}. \tag{51}$$

Theorem 4. *Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. The aggregated value by using 2TLNWDHM operators is also a 2TLNN where*

$$\mathrm{2TLNWDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\underset{1 \le i_1 < \ldots < i_x \le n}{\otimes} \left(\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j} \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}$$

$$= \left\{ \begin{array}{l} \Delta\left(t\left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \le i_1 < \ldots < i_x \le n} \left(1 - \left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right)^{\frac{1}{C_n^x}} \right) \right) \end{array} \right\} \quad (52)$$

Proof. From Definition 5, we can obtain that

$$w_{i_j} \varphi_{i_j} = \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right) \right), \\ \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right), \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right) \end{array} \right\} \quad (53)$$

Then,

$$\overset{x}{\underset{j=1}{\oplus}} w_{i_j} \varphi_{i_j} = \left\{ \begin{array}{l} \Delta\left(t\left(1 - \prod\limits_{j=1}^{x} \left(\left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right) \right) \right), \\ \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right) \right), \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right) \right) \end{array} \right\} \quad (54)$$

Thus,

$$\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j} \varphi_{i_j}}{x} = \left\{ \begin{array}{l} \Delta\left(t\left(1 - \left(\prod\limits_{j=1}^{x} \left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \right), \\ \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right), \Delta\left(t\left(\prod\limits_{j=1}^{x} \left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t} \right)^{w_{i_j}} \right)^{\frac{1}{x}} \right) \end{array} \right\} \quad (55)$$

Therefore,

$$\underset{1\le i_1<\ldots<i_x\le n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j}\varphi_{i_j}}{x}\right)=\left\{\begin{array}{l}\Delta\left(t\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right),\\ \Delta\left(t\left(1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right),\\ \Delta\left(t\left(1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)\right)\end{array}\right\} \quad (56)$$

Therefore,

$$2\text{TLNWDHM}^{(x)}(\varphi_1,\varphi_2,\cdots,\varphi_n)=\left(\underset{1\le i_1<\ldots<i_x\le n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j}\varphi_{i_j}}{x}\right)\right)^{\frac{1}{C_n^x}}$$

$$=\left\{\begin{array}{l}\Delta\left(t\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right),\\ \Delta\left(t\left(1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right),\\ \Delta\left(t\left(1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right)\end{array}\right\} \quad (57)$$

Hence, (51) is kept.

Then, we need to prove that (51) is a 2TLNN. We need to prove two conditions as follows:

① $0\le\Delta^{-1}(s_T,\alpha)\le t, 0\le\Delta^{-1}(s_I,\beta)\le t, 0\le\Delta^{-1}(s_F,\gamma)\le t.$

② $0\le\Delta^{-1}(s_T,\alpha)+\Delta^{-1}(s_I,\beta)+\Delta^{-1}(s_F,\gamma)\le 3t.$

Let

$$\frac{\Delta^{-1}(s_T,\alpha)}{t}=\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j},\alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_I,\beta)}{t}=1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{I_j},\beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

$$\frac{\Delta^{-1}(s_F,\gamma)}{t}=1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{x}\left(\frac{\Delta^{-1}\left(s_{F_j},\gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}$$

□

Proof. ① Since $0 \leq \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t} \leq 1$, we get

$$0 \leq \prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right) \leq 1 \text{ and } 0 \leq 1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}} \leq 1 \tag{58}$$

Then,

$$0 \leq \prod_{1 \leq i_1 < \ldots < i_x \leq n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right) \leq 1 \tag{59}$$

$$0 \leq \left(\prod_{1 \leq i_1 < \ldots < i_x \leq n}\left(1-\left(\prod_{j=1}^{x}\left(1-\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}} \leq 1. \tag{60}$$

That means $0 \leq \Delta^{-1}(s_T, \alpha) \leq t$. Similarly, we can get $0 \leq \Delta^{-1}(s_I, \beta) \leq t, 0 \leq \Delta^{-1}(s_F, \gamma) \leq t$ so ① is maintained. ② Since $0 \leq \Delta^{-1}(s_T, \alpha) \leq t, 0 \leq \Delta^{-1}(s_I, \beta) \leq t, 0 \leq \Delta^{-1}(s_F, \gamma) \leq t$. $0 \leq \Delta^{-1}(s_T, \alpha) + \Delta^{-1}(s_I, \beta) + \Delta^{-1}(s_F, \gamma) \leq 3t$. □

Example 4. *Let $\langle (s_5, 0), (s_2, 0), (s_1, 0) \rangle, \langle (s_4, 0), (s_3, 0), (s_4, 0) \rangle, \langle (s_2, 0), (s_5, 0), (s_1, 0) \rangle$ and $\langle (s_5, 0), (s_1, 0), (s_3, 0) \rangle$ be four 2TLNNs, $w = (0.2, 0.3, 0.4, 0.1)$ and suppose $x = 2$, then according to (51), we have*

$$\mathrm{2TLNWDHM}^{(2)}(\varphi_1, \varphi_2, \cdots, \varphi_n) = \left(\underset{1 \leq i_1 < \ldots < i_x \leq n}{\otimes}\left(\frac{\overset{x}{\underset{j=1}{\oplus}} w_{i_j}\varphi_{i_j}}{x}\right)\right)^{\frac{1}{C_n^x}}$$

$$= \left\{\begin{array}{l}
\Delta\left\{6 \times \left\{\begin{array}{l}
\left(1-\left(\left(1-\frac{5}{6}\right)^{0.2} \times \left(1-\frac{4}{6}\right)^{0.3}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(1-\frac{5}{6}\right)^{0.2} \times \left(1-\frac{2}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \\
\times\left(1-\left(\left(1-\frac{5}{6}\right)^{0.2} \times \left(1-\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(1-\frac{4}{6}\right)^{0.3} \times \left(1-\frac{2}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \\
\times\left(1-\left(\left(1-\frac{4}{6}\right)^{0.3} \times \left(1-\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(1-\frac{2}{6}\right)^{0.4} \times \left(1-\frac{5}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right)
\end{array}\right\}^{\frac{1}{C_4^2}}\right\}, \\
\Delta\left\{6 \times \left\{1-\left\{\begin{array}{l}
\left(1-\left(\left(\frac{2}{6}\right)^{0.2} \times \left(\frac{3}{6}\right)^{0.3}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{2}{6}\right)^{0.2} \times \left(\frac{5}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{2}{6}\right)^{0.2} \times \left(\frac{1}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \\
\left(1-\left(\left(\frac{3}{6}\right)^{0.3} \times \left(\frac{5}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{3}{6}\right)^{0.3} \times \left(\frac{1}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{5}{6}\right)^{0.4} \times \left(\frac{1}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right)
\end{array}\right\}^{\frac{1}{C_4^2}}\right\}\right\}, \\
\Delta\left\{6 \times \left\{1-\left\{\begin{array}{l}
\left(1-\left(\left(\frac{1}{6}\right)^{0.2} \times \left(\frac{4}{6}\right)^{0.3}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{1}{6}\right)^{0.2} \times \left(\frac{1}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{1}{6}\right)^{0.2} \times \left(\frac{3}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \\
\times\left(1-\left(\left(\frac{4}{6}\right)^{0.3} \times \left(\frac{1}{6}\right)^{0.4}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{4}{6}\right)^{0.3} \times \left(\frac{3}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right) \times \left(1-\left(\left(\frac{1}{6}\right)^{0.4} \times \left(\frac{3}{6}\right)^{0.1}\right)^{\frac{1}{2}}\right)
\end{array}\right\}^{\frac{1}{C_4^2}}\right\}\right\}
\end{array}\right\}$$

$$= \langle (s_1, 0.3339), (s_5, 0.0807), (s_5, -0.4164) \rangle$$

Now, we will discuss some properties of the 2TLNWDHM operator.

Property 9. *(Monotonicity) Let* $\varphi_{a_j} = \left\langle \left(s_{T_{a_j}}, \alpha_{a_j}\right), \left(s_{I_{a_j}}, \beta_{a_j}\right), \left(s_{F_{a_j}}, \gamma_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\varphi_{b_j} = \left\langle \left(s_{T_{b_j}}, \alpha_{b_j}\right), \left(s_{I_{b_j}}, \beta_{b_j}\right), \left(s_{F_{b_j}}, \gamma_{b_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *be two sets of 2TLNNs. If* $\Delta^{-1}\left(s_{T_{a_j}}, \alpha_{a_j}\right) \leq \Delta^{-1}\left(s_{T_{b_j}}, \alpha_{b_j}\right)$, $\Delta^{-1}\left(s_{I_{a_j}}, \beta_{a_j}\right) \geq \Delta^{-1}\left(s_{I_{b_j}}, \beta_{b_j}\right)$ *and* $\Delta^{-1}\left(s_{F_{a_j}}, \gamma_{a_j}\right) \geq \Delta^{-1}\left(s_{F_{b_j}}, \gamma_{b_j}\right)$ *hold for all j, then*

$$\text{2TLNWDHM}^{(x)}(\varphi_{a_1}, \varphi_{a_2}, \cdots, \varphi_{a_n}) \leq \text{2TLNWDHM}^{(x)}(\varphi_{b_1}, \varphi_{b_2}, \cdots, \varphi_{b_n}) \tag{61}$$

Property 10. *(Boundedness) Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs. If* $\varphi_i^+ = \left(\max_i \left(s_{T_j}, \alpha_j\right), \min_i \left(s_{I_j}, \beta_j\right), \min_i \left(s_{F_j}, \gamma_j\right)\right)$ *and* $\varphi_i^+ = \left(\max_i \left(s_{T_j}, \alpha_j\right), \min_i \left(s_{I_j}, \beta_j\right), \min_i \left(s_{F_j}, \gamma_j\right)\right)$*, then*

$$\varphi^- \leq \text{2TLNWDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^+ \tag{62}$$

From Theorem 4,

$$\begin{aligned} &\text{2TLNWDHM}^{(x)}(\varphi_1^+, \varphi_2^+, \cdots, \varphi_n^+) \\ &= \left\{ \begin{array}{l} \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\max\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\min\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\min\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right) \end{array} \right\} \end{aligned} \tag{63}$$

$$\begin{aligned} &\text{2TLNWDHM}^{(x)}(\varphi_1^-, \varphi_2^-, \cdots, \varphi_n^-) \\ &= \left\{ \begin{array}{l} \Delta\left(t\left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(1 - \frac{\min\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\max\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right), \\ \Delta\left(t\left(1 - \left(\prod\limits_{1 \leq i_1 < \ldots < i_x \leq n}\left(1 - \left(\prod\limits_{j=1}^{x}\left(\frac{\max\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_{i_j}}\right)^{\frac{1}{x}}\right)\right)^{\frac{1}{C_n^x}}\right)\right) \end{array} \right\} \end{aligned} \tag{64}$$

From Property 9,

$$\varphi^- \leq \text{2TLNWDHM}^{(x)}(\varphi_1, \varphi_2, \cdots, \varphi_n) \leq \varphi^+ \tag{65}$$

It is obvious that 2TLNWDHM operator lacks the property of idempotency.

4. Numerical Example and Comparative Analysis

4.1. Numerical Example

The constructional engineering projects have the following characteristics: large investment, many participants, complex project environment, and a wide range of risk factors on the basis of the Engineering Procurement Construction (EPC) mode. Therefore, it is necessary to analyze and assess the constructional engineering project's risks during the life cycle; a risk assessment is good for implementing projects and completing project goals. During the process of implementation, constructional engineering projects face aspects of risk—political, economic, social and natural and other aspects of risks. These risks have a great influence on our construction company, and produce many factors with high probability that are difficult to estimate and quantify. Thus, we shall give a numerical example for construction engineering project risk assessment with 2TLNNs in order to illustrate the method proposed in this paper. There are five possible construction engineering projects $A_i (i = 1, 2, 3, 4, 5)$ to evaluate. The expert selects four attributes to evaluate the five possible construction engineering projects: ① G_1 is the construction work environment; ② G_2 is the construction site safety protection measures; ③ G_3 is the safety management ability of the engineering project management; and ④ G_4 is the safety production responsibility system. The five possible construction engineering projects, $A_i (i = 1, 2, 3, 4, 5)$, will be evaluated using the 2TLNNs by the decision maker using the above four attributes (whose weighting vector is $\omega = (0.2, 0.1, 0.5, 0.2)$ and expert weighting vector is $\omega = (0.2, 0.4, 0.4)$, which are listed in Tables 1–3.

Table 1. 2-tuple linguistic neutrosophic numbers (2TLNN) decision matrix (R_1).

	G_1	G_2	G_3	G_4
A_1	$<(s_4, 0), (s_3, 0)\ (s_2, 0)>$	$<(s_5, 0), (s_3, 0)\ (s_1, 0)>$	$<(s_4, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_2, 0), (s_3, 0)\ (s_2, 0)>$
A_2	$<(s_3, 0), (s_2, 0)\ (s_4, 0)>$	$<(s_4, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_3, 0)\ (s_3, 0)>$
A_3	$<(s_5, 0), (s_4, 0)\ (s_3, 0)>$	$<(s_4, 0), (s_4, 0)\ (s_3, 0)>$	$<(s_2, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_3, 0)\ (s_2, 0)>$
A_4	$<(s_2, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_5, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_3, 0)\ (s_5, 0)>$	$<(s_3, 0), (s_1, 0)\ (s_1, 0)>$
A_5	$<(s_4, 0), (s_3, 0)\ (s_1, 0)>$	$<(s_5, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_1, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$

Table 2. 2TLNN decision matrix (R_2).

	G_1	G_2	G_3	G_4
A_1	$<(s_3, 0), (s_2, 0)\ (s_3, 0)>$	$<(s_3, 0), (s_3, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_1, 0)\ (s_3, 0)>$
A_2	$<(s_2, 0), (s_3, 0)\ (s_3, 0)>$	$<(s_3, 0), (s_3, 0)\ (s_3, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_4, 0)\ (s_3, 0)>$
A_3	$<(s_2, 0), (s_3, 0)\ (s_3, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_2, 0), (s_3, 0)\ (s_1, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_4, 0)>$
A_4	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_2, 0), (s_2, 0)\ (s_3, 0)>$	$<(s_3, 0), (s_4, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_1, 0)\ (s_2, 0)>$
A_5	$<(s_3, 0), (s_2, 0)\ (s_1, 0)>$	$<(s_3, 0), (s_4, 0)\ (s_3, 0)>$	$<(s_4, 0), (s_1, 0)\ (s_1, 0)>$	$<(s_2, 0), (s_3, 0)\ (s_2, 0)>$

Table 3. 2TLNN decision matrix (R_3).

	G_1	G_2	G_3	G_4
A_1	$<(s_3, 0), (s_3, 0)\ (s_1, 0)>$	$<(s_4, 0), (s_2, 0)\ (s_1, 0)>$	$<(s_4, 0), (s_4, 0)\ (s_3, 0)>$	$<(s_4, 0), (s_1, 0)\ (s_3, 0)>$
A_2	$<(s_2, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_4, 0)\ (s_4, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_3, 0)>$	$<(s_2, 0), (s_1, 0)\ (s_3, 0)>$
A_3	$<(s_2, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_4, 0), (s_5, 0)\ (s_2, 0)>$	$<(s_2, 0), (s_4, 0)\ (s_4, 0)>$
A_4	$<(s_3, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_2, 0), (s_1, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_4, 0)\ (s_5, 0)>$	$<(s_5, 0), (s_3, 0)\ (s_1, 0)>$
A_5	$<(s_3, 0), (s_3, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_2, 0)>$	$<(s_3, 0), (s_2, 0)\ (s_3, 0)>$	$<(s_5, 0), (s_3, 0)\ (s_4, 0)>$

Then, we utilize the approach developed to select the best construction engineering projects.

Definition 12. *Let* $\varphi_j = \left\langle \left(s_{T_j}, \alpha_j\right), \left(s_{I_j}, \beta_j\right), \left(s_{F_j}, \gamma_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of 2TLNNs with the weight vector,* $w_i = (w_1, w_2, \ldots, w_n)^T$*, thereby satisfying* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$*, then we can obtain*

$$\text{2TLNNWAA}(\varphi_1, \varphi_2, \ldots, \varphi_n) = \sum_{j=1}^{n} w_j \varphi_j$$
$$= \left\{ \begin{array}{l} \Delta\left(t\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_j}\right)\right), \\ \Delta\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_j}\right)\right), \Delta\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_j}\right)\right) \end{array} \right\} \quad (66)$$

$$\text{2TLNNWGA}(\varphi_1, \varphi_2, \ldots, \varphi_n) = \sum_{j=1}^{n} \left(\varphi_j\right)^{w_j}$$
$$= \left\{ \begin{array}{l} \Delta\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta^{-1}\left(s_{T_j}, \alpha_j\right)}{t}\right)^{w_j}\right)\right), \Delta\left(t\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}\left(s_{I_j}, \beta_j\right)}{t}\right)^{w_j}\right)\right), \\ \Delta\left(t\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}\left(s_{F_j}, \gamma_j\right)}{t}\right)^{w_j}\right)\right) \end{array} \right\} \quad (67)$$

Step 1. According to the 2TLNNs, $r_{ij} (i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can calculate all 2TLNNs r_{ij} by using the 2-tuple linguistic neutrosophic numbers weighted average (2TLNNWA) operator and 2-tuple linguistic neutrosophic numbers weighted geometric (2TLNNWG) operator to get the overall 2TLNNs $A_i (i = 1, 2, 3, 4, 5)$ of the construction engineering projects. Then, the results are shown in Table 4.

Table 4. The aggregating results by the 2TLNNWAA operator.

	G_1	G_2
A_1	<(s_3, 0.2337), (s_3, −0.4492), (s_2, −0.2174)>	<(s_4, −0.0477), (s_3, −0.4492), (s_1, 0.3195)>
A_2	<(s_2, 0.2236), (s_2, 0.3522), (s_3, −0.2981)>	<(s_4, −0.3522), (s_3, 0.1037),(s_3, 0.1037)>
A_3	<(s_3, −0.0314), (s_2, 0.0477), (s_3, −0.4492)>	<(s_3, 0.2337), (s_2, 0.2974), (s_2, 0.1689)>
A_4	<(s_3, −0.1777), (s_1, 0.3195), (s_2, 0.0000)>	<(s_3, −0.0314), (s_1, 0.3195), (s_2, 0.3522)>
A_5	<(s_3, 0.2337), (s_3, −0.4492), (s_1, 0.3195)>	<(s_4, −0.4082), (s_3, −0.3610), (s_2, 0.3522)>
	G_3	G_4
A_1	<(s_4, −0.3522), (s_2, −0.2589), (s_2, 0.3522)>	<(s_4, −0.2974), (s_1, 0.2457), (s_3, −0.2337)>
A_2	<(s_3, 0.000), (s_2, 0.0000), (s_2, 0.3522)>	<(s_3, −0.1037), (s_2,0.1689), (s_3, 0.0000)>
A_3	<(s_3, −0.0314), (s_3, −0.0458), (s_2, −0.4843)>	<(s_3, -−0.1037), (s_3, −0.1381), (s_3,0.4822)>
A_4	<(s_3, 0.2337), (s_4, −0.2236), (s_3, 0.4657)>	<(s_4, 0.0668), (s_2, −0.4482), (s_1,0.3195)>
A_5	<(s_3, 0.4492), (s_2, −0.4843), (s_2, −0.4482)>	<(s_4, −0.1689), (s_3, −0.2337),(s_3, −0.3610)>

Step 2. According to Table 4, we can calculate the r_{ij} of all 2TLNNs by using the 2TLNWHM (2TLNWDHM) operator to get the overall 2TLNNs $A_i (i = 1, 2, 3, 4, 5)$ of the construction engineering projects, A_i. Suppose that $x = 2$, then the aggregating results are as shown in Table 5.

Table 5. The aggregating results of the construction engineering projects by the 2TLNWHM (2TLNWDHM) operator.

	2TLNWHM	**2TLNWDHM**
A_1	<(s_5, 0.3257), (s_1, −0.4776), (s_1, −0.4012)>	<(s_1, 0.1564), (s_5, −0.4119), (s_5, −0.2877)>
A_2	<(s_5, 0.0468), (s_1, −0.3632), (s_1, −0.2177)>	<(s_1, −0.1456), (s_5, −0.2124), (s_5, −0.0171)>
A_3	<(s_5, 0.1002), (s_1, −0.2491), (s_1, −0.3474)>	<(s_1, −0.1092), (s_5, −0.0603), (s_5, −0.1946)>
A_4	<(s_5, 0.2143), (s_1, −0.3927), (s_1, −0.3263)>	<(s_1, 0.0275), (s_5, −0.2639), (s_5, −0.1713)>
A_5	<(s_5, 0.2941), (s_1, −0.3870), (s_1, −0.4981)>	<(s_1, −0.1127), (s_5, −0.2367), (s_5, −0.4415)>

Step 3. In accordance with the aggregating results shown in Table 5, the score functions of the construction engineering projects are shown in Table 6.

Table 6. The score functions of the construction engineering projects.

	2TLNWHM	2TLNWDHM
A_1	$(s_5, 0.4015)$	$(s_1, 0.2853)$
A_2	$(s_5, 0.2092)$	$(s_1, 0.0280)$
A_3	$(s_5, 0.2323)$	$(s_1, 0.0486)$
A_4	$(s_5, 0.3111)$	$(s_1, 0.1542)$
A_5	$(s_5, 0.3930)$	$(s_1, 0.2636)$

Step 4. In accordance with the scores shown in Table 6 and the comparison formulas of the score functions, the ordering of the construction engineering projects are shown in Table 7. The best construction engineering project is A_1.

Table 7. Ordering of the construction engineering projects.

	Ordering
2TLNWHM	$A_1 > A_5 > A_4 > A_3 > A_2$
2TLNWDHM	$A_1 > A_5 > A_4 > A_3 > A_2$

4.2. Influence of the Parameter on the Final Result

In order to show the effects on the ranking results by changing parameters of x in the 2TLNWHM (2TLNWDHM) operators, all the results are shown in Tables 8 and 9.

Table 8. Ranking results for different operational parameters of the 2TLNWHM operator.

	$s(A_1)$	$s(A_2)$	$s(A_3)$	$s(A_4)$	$s(A_5)$	Ordering
$x = 1$	0.9134	0.8799	0.8865	0.9070	0.9065	$A_1 > A_4 > A_5 > A_3 > A_2$
$x = 2$	0.9003	0.8682	0.8720	0.8852	0.8988	$A_1 > A_5 > A_4 > A_3 > A_2$
$x = 3$	0.8953	0.8642	0.8661	0.8696	0.8958	$A_5 > A_1 > A_4 > A_3 > A_2$
$x = 4$	0.8927	0.8621	0.8627	0.8571	0.8942	$A_5 > A_1 > A_2 > A_3 > A_4$

Table 9. Ranking results for different operational parameters of the 2TLNWDHM operator.

	$s(A_1)$	$s(A_2)$	$s(A_3)$	$s(A_4)$	$s(A_5)$	Ordering
$x = 1$	0.1922	0.1489	0.1557	0.1796	0.1817	$A_1 > A_5 > A_4 > A_3 > A_2$
$x = 2$	0.2142	0.1713	0.1748	0.1924	0.2106	$A_1 > A_5 > A_4 > A_3 > A_2$
$x = 3$	0.2239	0.1815	0.1852	0.1965	0.2252	$A_5 > A_1 > A_4 > A_3 > A_2$
$x = 4$	0.2293	0.1878	0.1922	0.1985	0.2342	$A_5 > A_1 > A_4 > A_3 > A_2$

4.3. Comparative Analysis

Then, we compare our proposed method with the LNNWAA operator and LNNWGA operator [33] and cosine measures of linguistic neutrosophic numbers [34]. The comparative results are shown in Table 10.

Table 10. Ordering of the construction engineering projects.

	Ordering
LNNWAA [33]	$A_5 > A_1 > A_4 > A_3 > A_2$
LNNWGA [33]	$A_5 > A_1 > A_3 > A_2 > A_4$
${C^{w_1}}_{LNNs}$ [34]	$A_5 > A_1 > A_4 > A_2 > A_3$
${C^{w_2}}_{LNNs}$ [34]	$A_5 > A_1 > A_4 > A_2 > A_3$

From above, we determine that the optimal construction engineering projects to show the practicality and effectiveness of the proposed approaches. However, the LNNWAA operator and LNNWGA operator do not consider the information about the relationship between the arguments being aggregated and thus, cannot eliminate the influence of unfair arguments on the decision result. Our proposed 2TLNWHM and 2TLNWDHM operators consider the information about the relationship among arguments being aggregated.

5. Conclusions

In this paper, we investigated the MADM problems with 2TLNNs. Then, we utilized the Hamy mean (HM) operator, weighted Hamy mean (WHM) operator, dual Hamy mean (DHM) operator and weighted dual Hamy mean (WDHM) operator to develop some Hamy mean aggregation operators with 2TLNNs: 2-tuple linguistic neutrosophic Hamy mean (2TLNHM) operator, 2-tuple linguistic neutrosophic weighted Hamy mean (2TLNWHM) operator, 2-tuple linguistic neutrosophic dual Hamy mean (2TLNDHM) operator, and 2-tuple linguistic neutrosophic weighted dual Hamy mean (2TLNWDHM) operator. The prominent characteristics of these proposed operators were studied. Then, we utilized these operators to develop some approaches to solve MADM problems with 2TLNNs. Finally, a practical example for construction engineering project risk assessment was given to show the developed approach. In the future, the application of the 2TLNNs needs to be investigated under uncertain [35–46] and fuzzy environments [47–54].

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