

# Design of Parallel Algorithms

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# Thank you



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This presentation borrows heavily from information freely available on the web by

Ian Foster and Blaise Barney

(see references)

# Outline

- Basics of Parallel Algorithm Design
  - Partitioning
  - Communication
  - Agglomeration
  - Mapping
- Final Thoughts
- References

# Design of Parallel Algorithms

- Ensure that you understand fully the problem and/or the serial code that you wish to make parallel
- Identify the program **hotspots**
  - These are places where most of the computational work is being done
  - Making these sections parallel will lead to the most improvement
  - **Profiling** can help to determine the hotspots (more on this tomorrow)
- Identify **bottlenecks** in the program
  - Some sections of the code are disproportionately slow
  - It is often possible to restructure a code to minimize the bottlenecks
- Sometimes it is possible to identify a different computational algorithm that has much better scaling properties

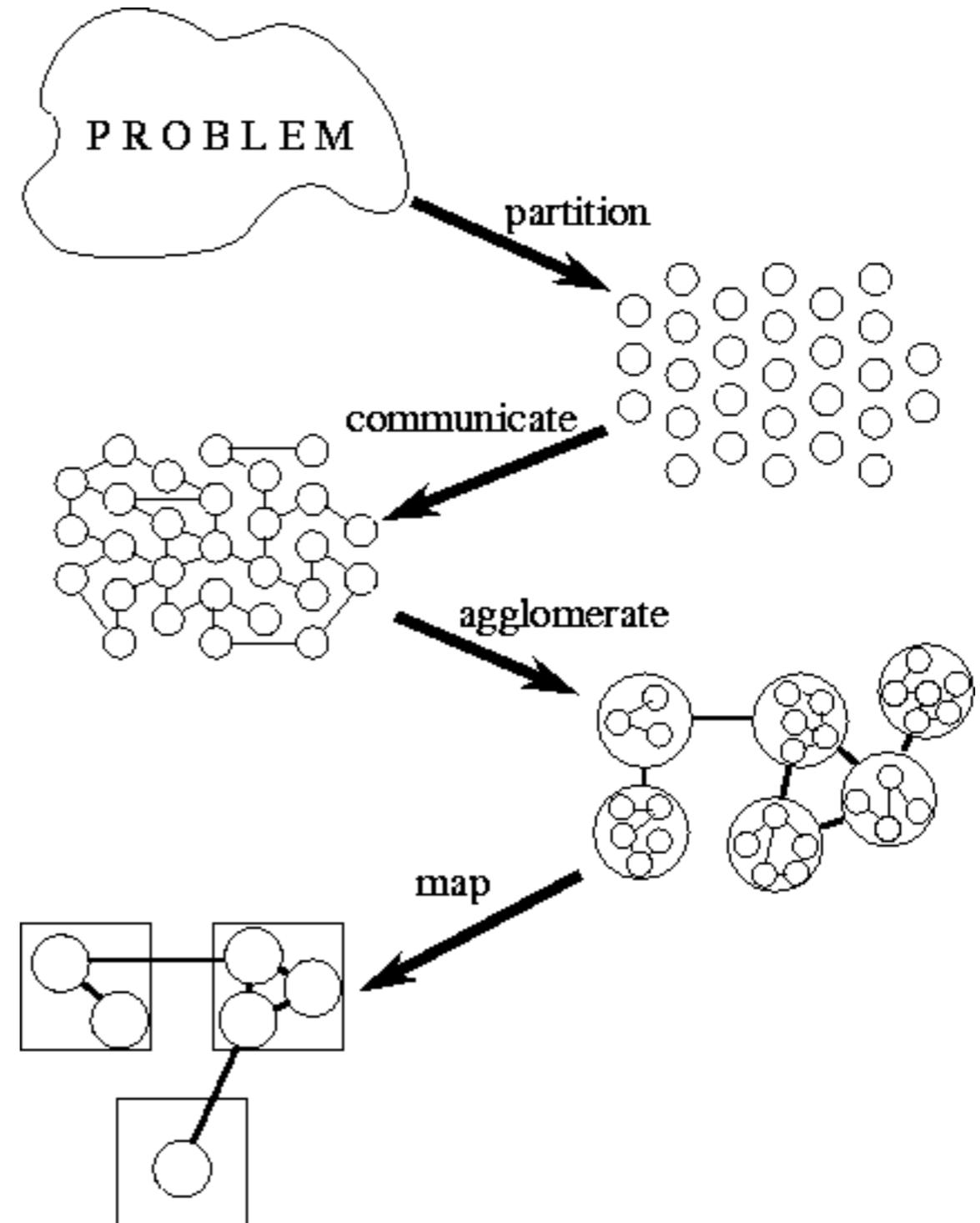
## Methodological Approach to Parallel Algorithm Design:

1) Partitioning

2) Communication

3) Agglomeration

4) Mapping

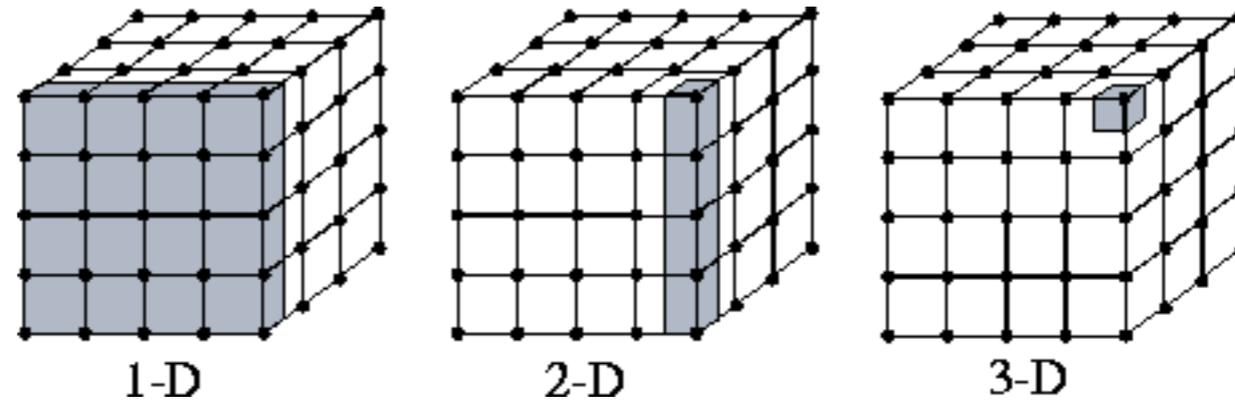


# Partitioning

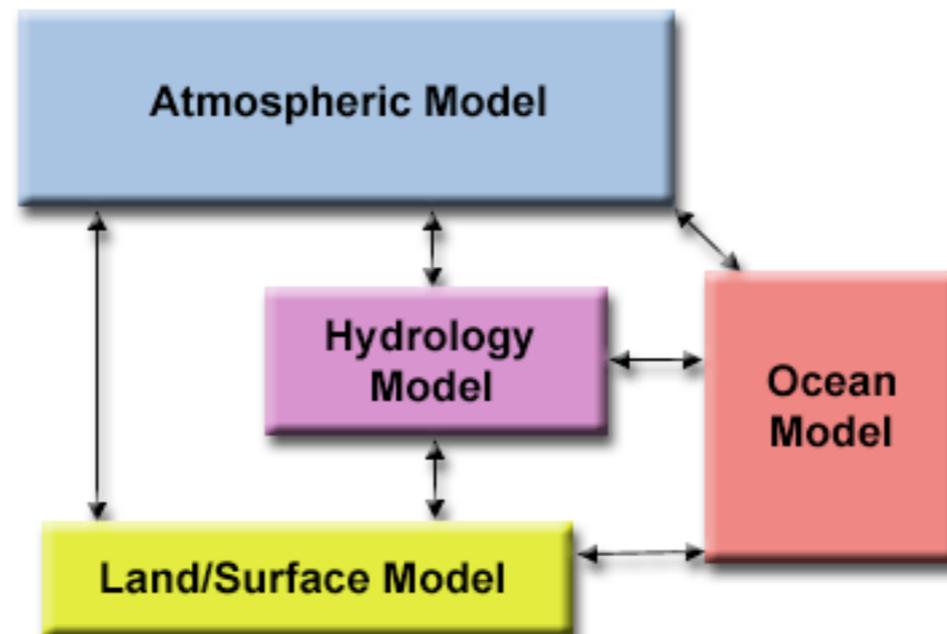
- Split both the computation to be performed and the data into a large number of small tasks (**fine-grained**)

Two primary ways of decomposing the problem:

- Domain Decomposition



- Functional Decomposition



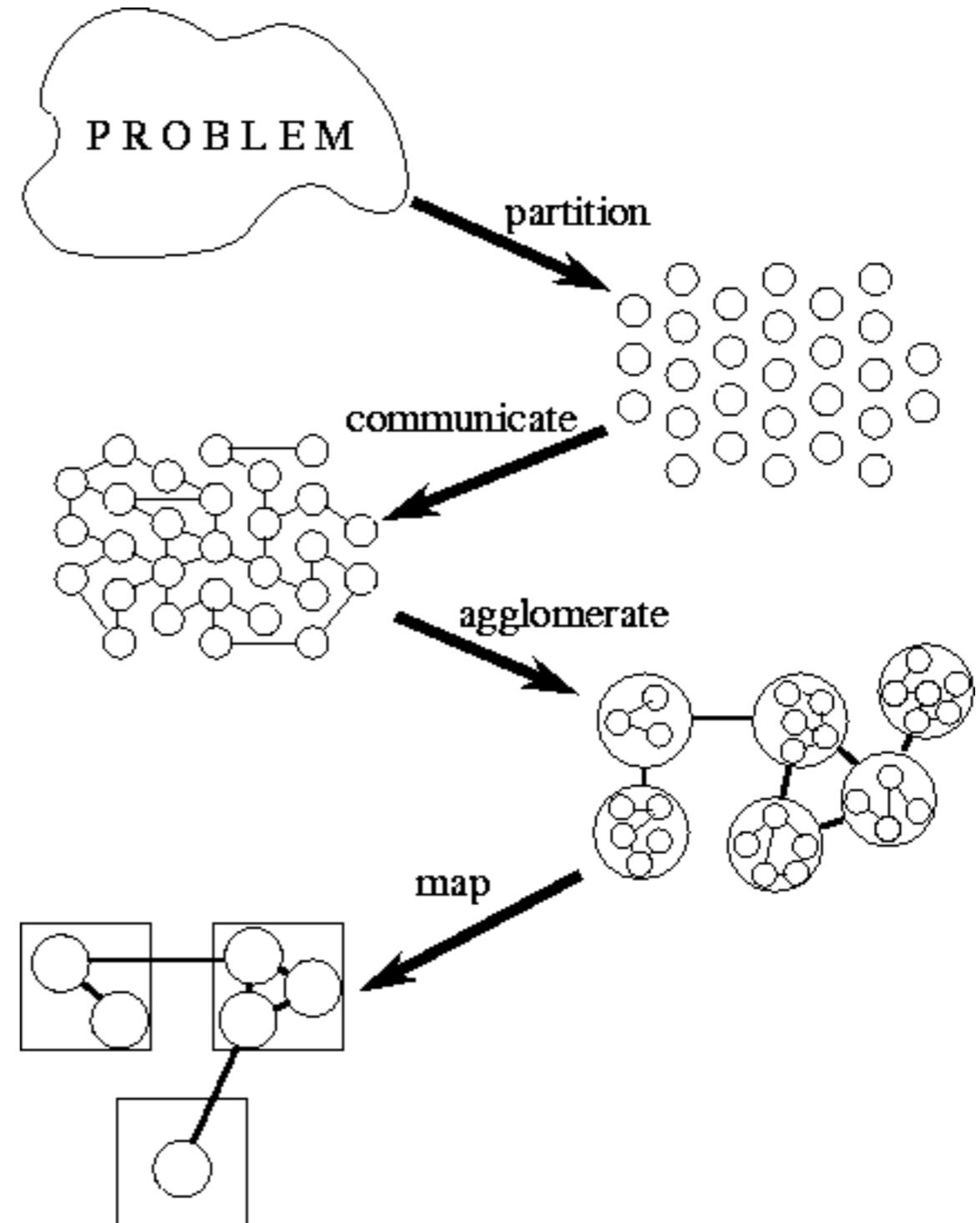
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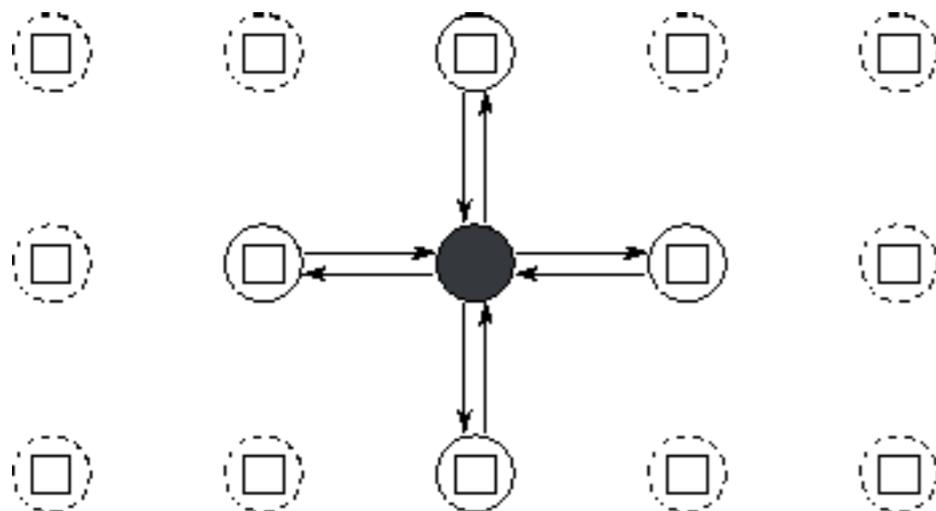


# Communication

- Identify the necessary communication between the fine-grained tasks to perform the necessary computation
- For functional decomposition, this task is often relatively straightforward
- For domain decomposition, this can be a challenging task.  
We'll consider some examples:

## Finite Difference Relaxation:

$$f_{i,j}^{t+1} = \frac{4f_{i,j}^t + f_{i-1,j}^t + f_{i+1,j}^t + f_{i,j-1}^t + f_{i,j+1}^t}{8}$$

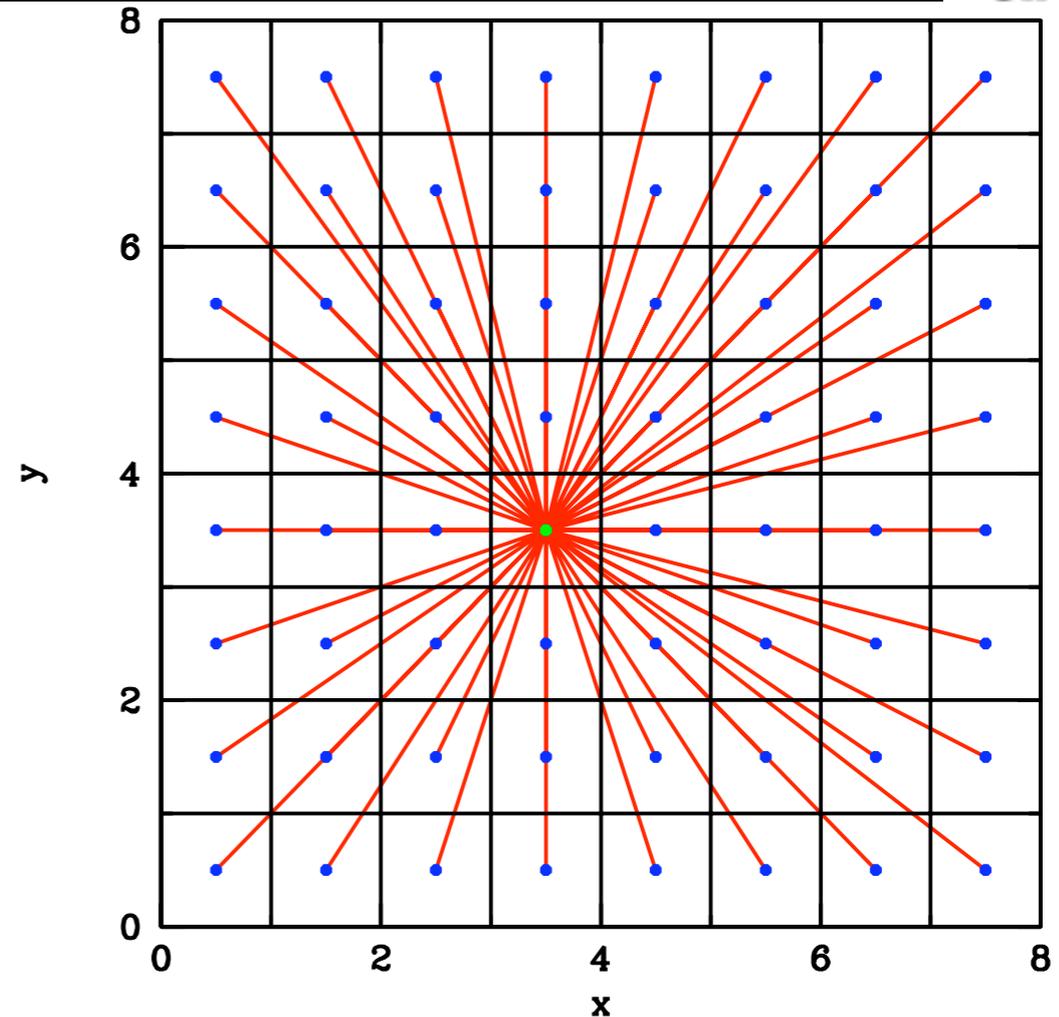


- This is a **local** communication, involving only neighboring tasks

# Communication

## Gravitational N-Body Problems:

- This is a **global** communication, requiring information from all tasks



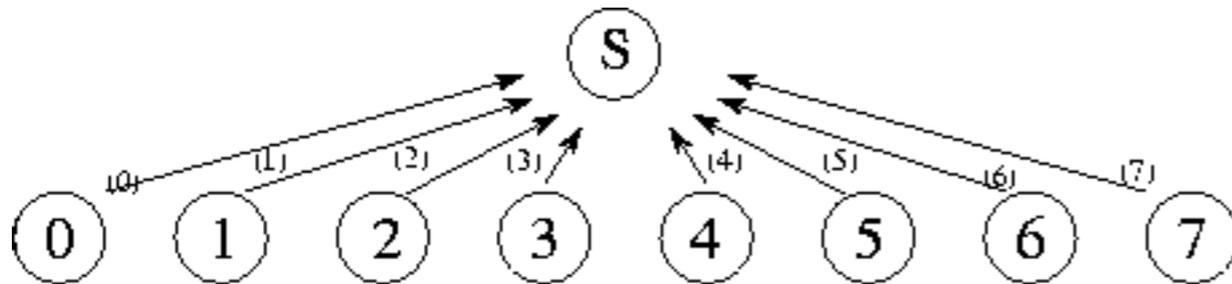
When communication is necessary, it is important to employ a scheme that executes the communications between different tasks **concurrently**.

# Schemes for Global Communication

Consider the problem of summing the values on  $N=8$  different processors

- This is an example of a parallel process generically called **reduction**.

Method 1: Summing by a **Manager task, S**



- Requires  $N=8$  communications
- If all processors require the sum, it will require  $2N=16$  communications

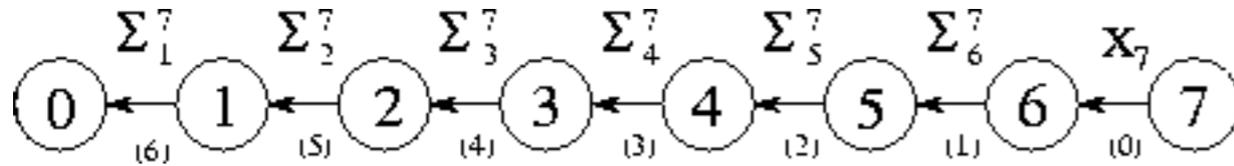
**This is a poor parallel algorithm!**

- Two properties of this method hinder parallel execution:
  - The algorithm is **centralized**, the manager participation in all interactions
  - The algorithm is **sequential**, without communications occurring concurrently

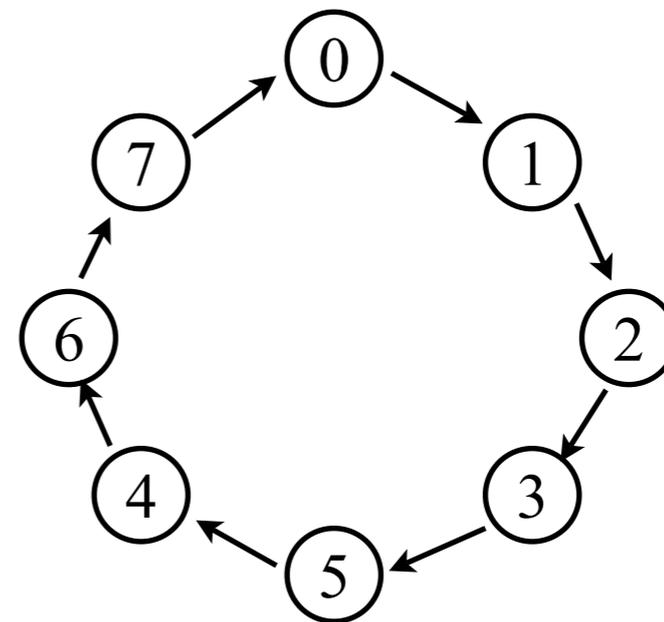
# Schemes for Global Communication

## Method II: Line or Ring Communications

- By **decentralizing**, one can achieve some savings



- Requires  $N-1=7$  communications, but it is still **sequential**
- If all processors require the sum, we can achieve this result with the same number of **concurrent** communications
  - By arranging the communications in a ring, we can distribute the sum at all processors in  $N-1=7$  communication steps.



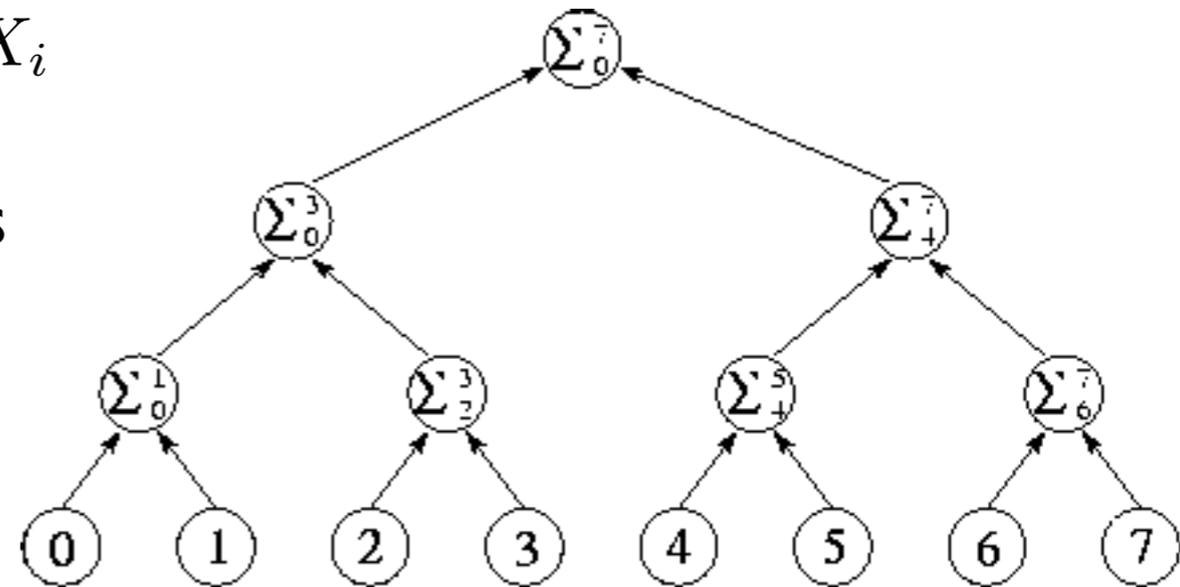
# Schemes for Global Communication

## Method III: Tree Communications

- But we can do better by using a **divide and conquer** approach to the problem
  - Split problem into two of equivalent size, to be performed concurrently

$$\sum_{i=0}^{N-1} X_i = \sum_{i=0}^{N/2-1} X_i + \sum_{i=N/2}^{N-1} X_i$$

- Recursive application of this principle leads to a tree approach
- Requires  $\log_2 N=3$  communication steps
- Distribution of the sum to all processors can be accomplished with the same  $\log_2 N=3$  communication steps.

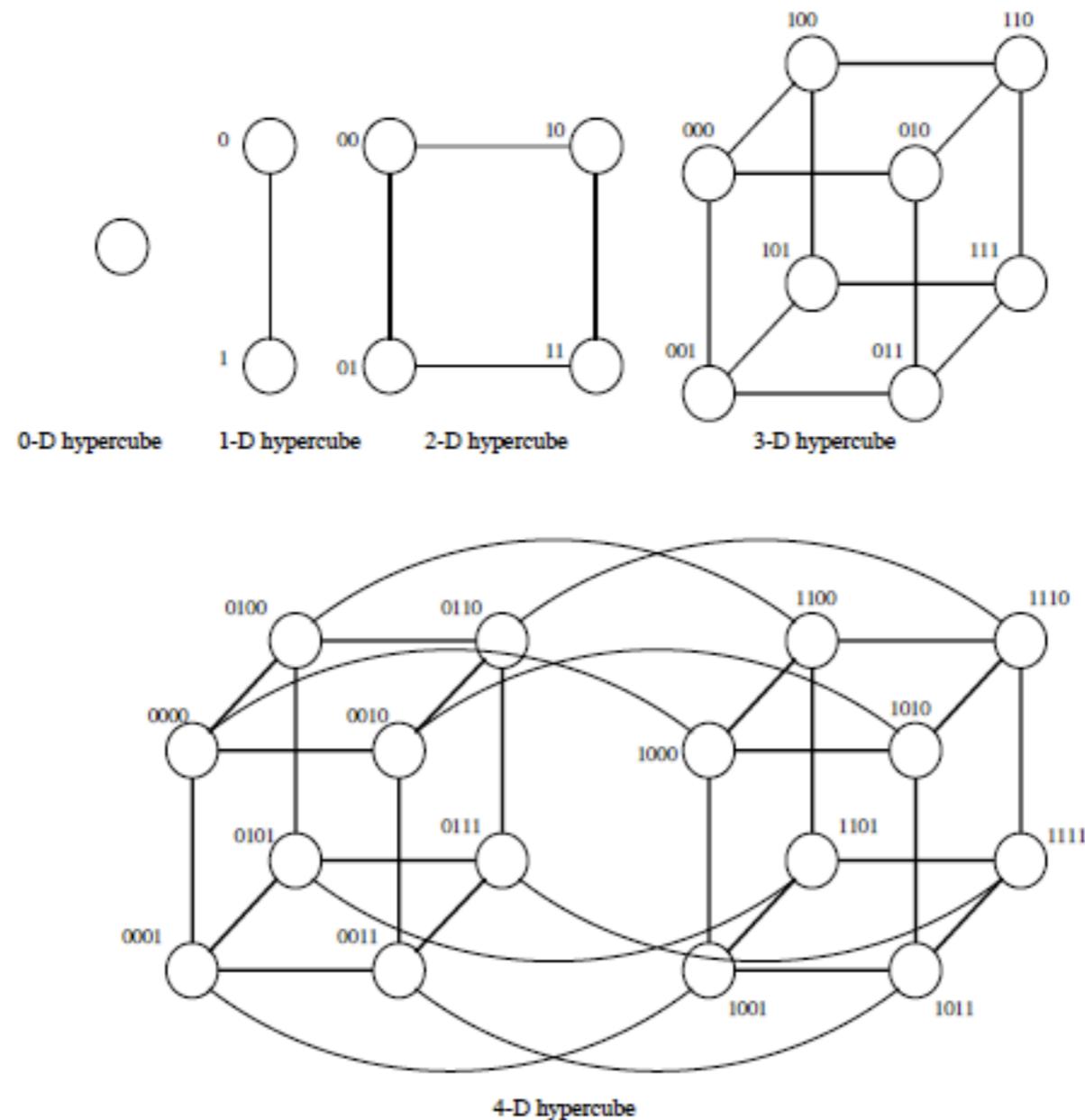


This is called a **hypercube** communication scheme

# Hypercube Communication

In Hypercube Communications,

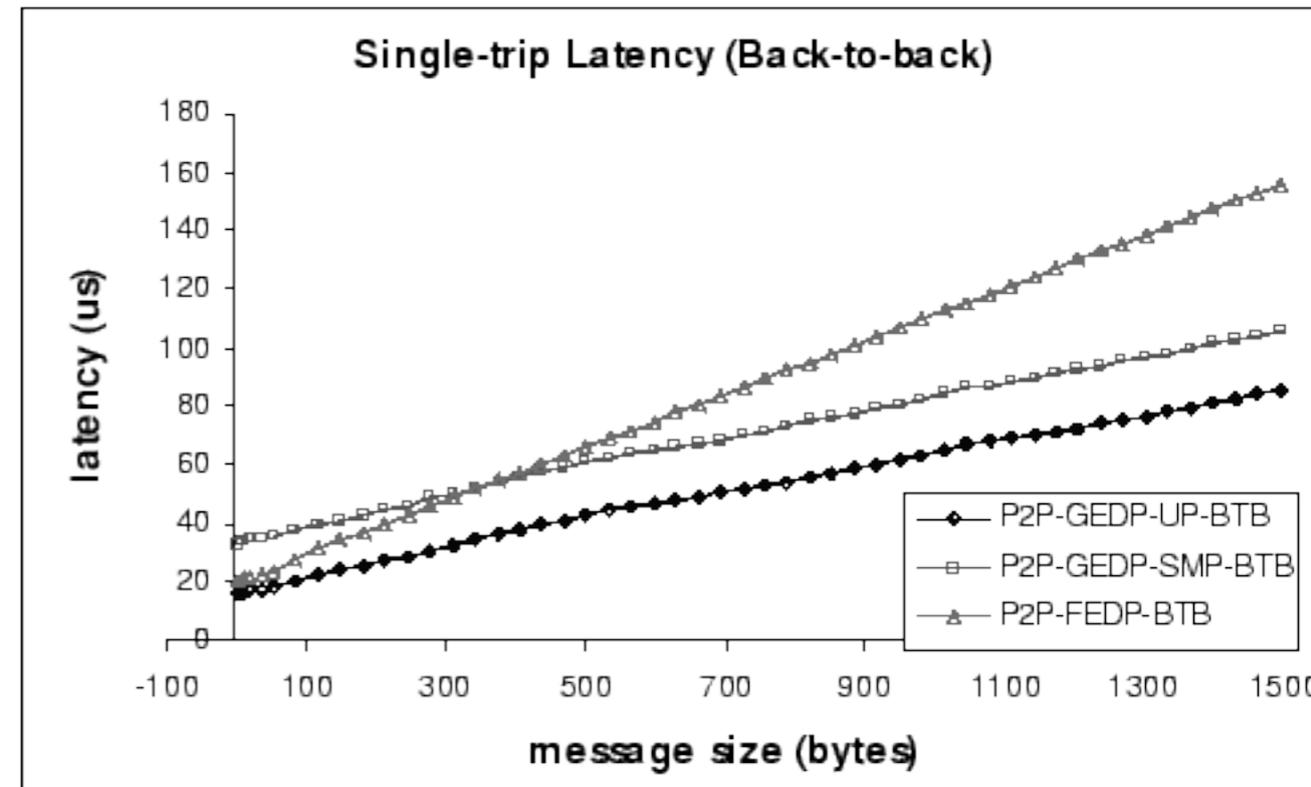
- All tasks communicate with one other tasks at each step,
- At each step, the task passes along all of the information it has gathered up to that point



# Communication: Latency vs. Bandwidth

## Cost of Communications (Overhead):

- **Latency:** The time it takes to send a minimal message (1 bit) from A to B
- **Bandwidth:** The amount of data that can be communicated per unit of time



## Factors to consider:

- Sending many small messages will cause latency to dominate the communications overhead
  - It is better to package many small messages into one large message
- The less information that needs to be transmitted, the less time the communications will require.
- It is often best to have all necessary communication occur at the same time

# Synchronous vs. Asynchronous Communication

Consider a communication involving a message sent from task A to task B

## Synchronous Communication:

- Task A sends the message, and must wait until task B receives message to move on
- Also known as **blocking** communication

## Asynchronous Communication:

- After task A has sent the message, it can move on to do other work. When task B receives the message doesn't matter to task A.
- Also known as **non-blocking** communication
- Requires care to insure that different tasks don't get wildly out of step, possibly leading to race conditions or deadlocks.

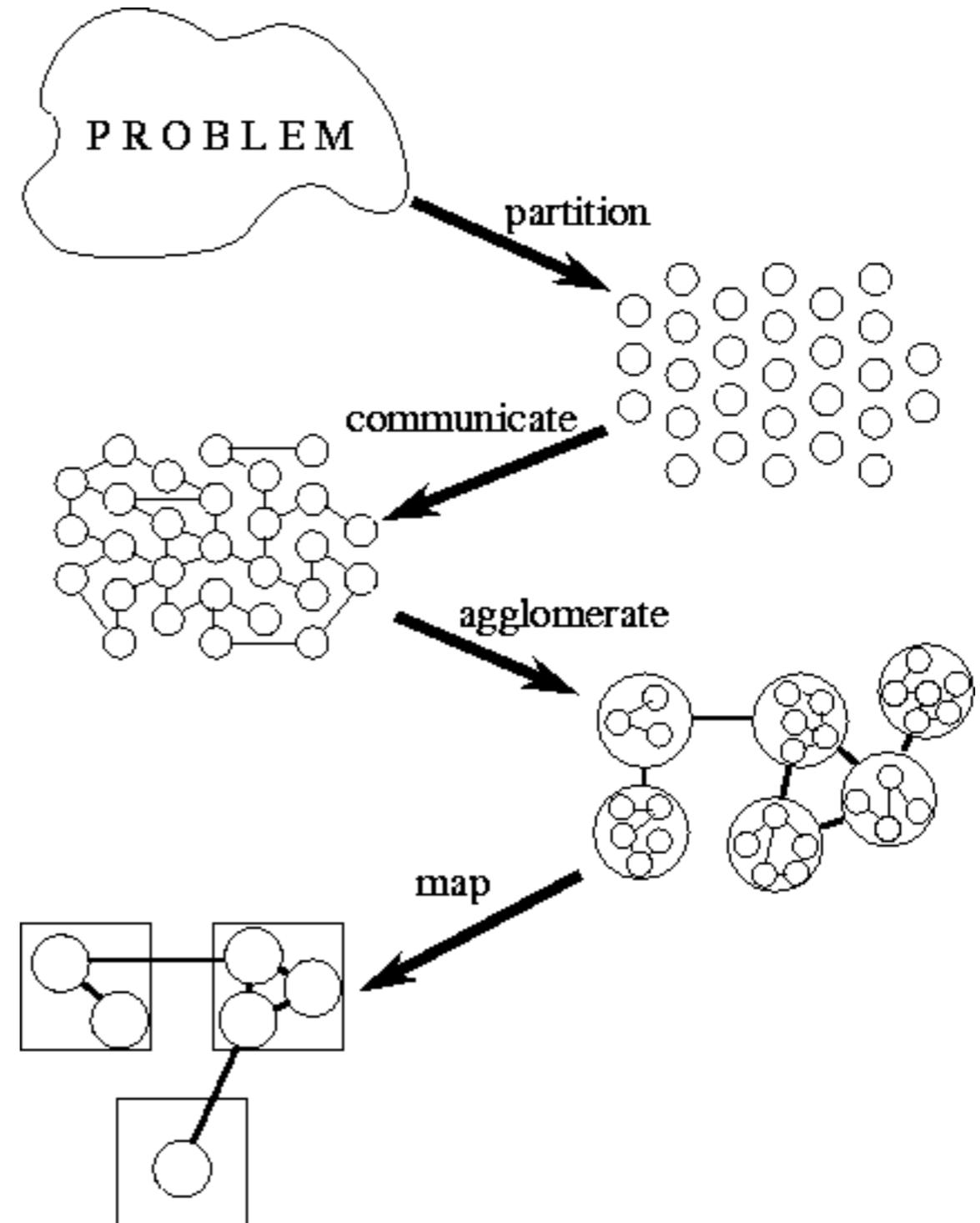
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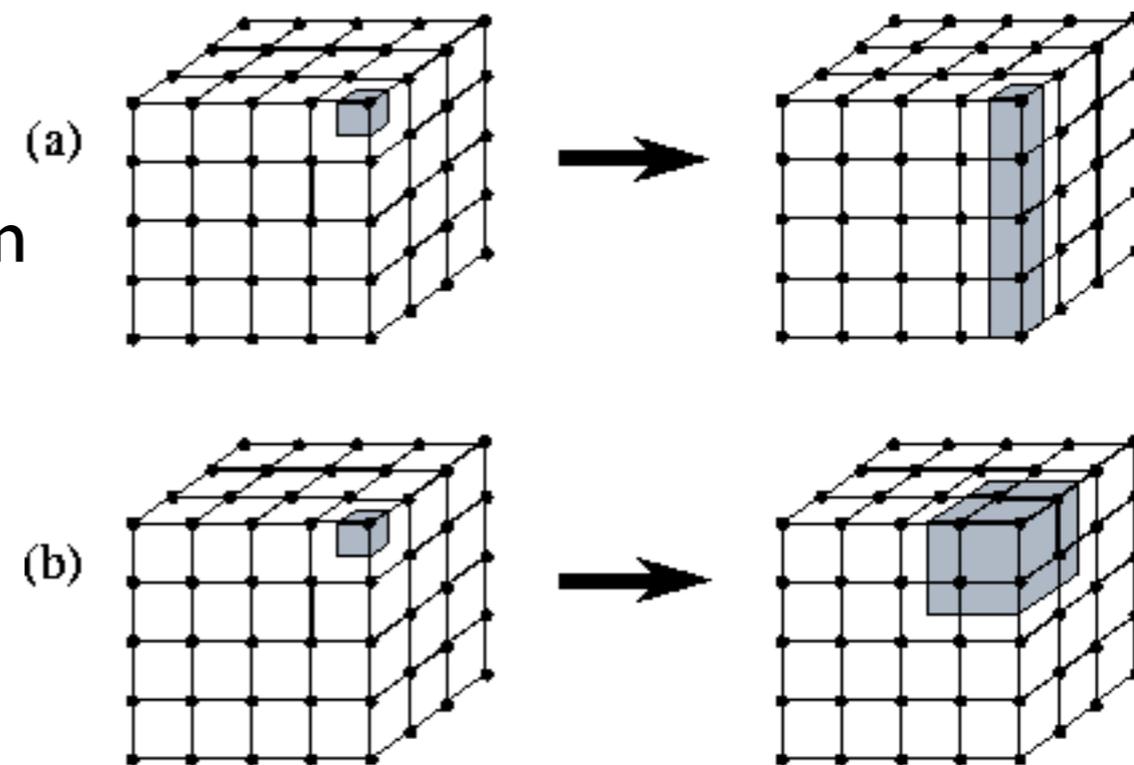


# Agglomeration

- Fine-grained partitioning of a problem is generally not an efficient parallel design
  - Requires too much communication of data to be efficient
- Agglomeration is required to achieve data **locality** and good **performance**

## Agglomeration:

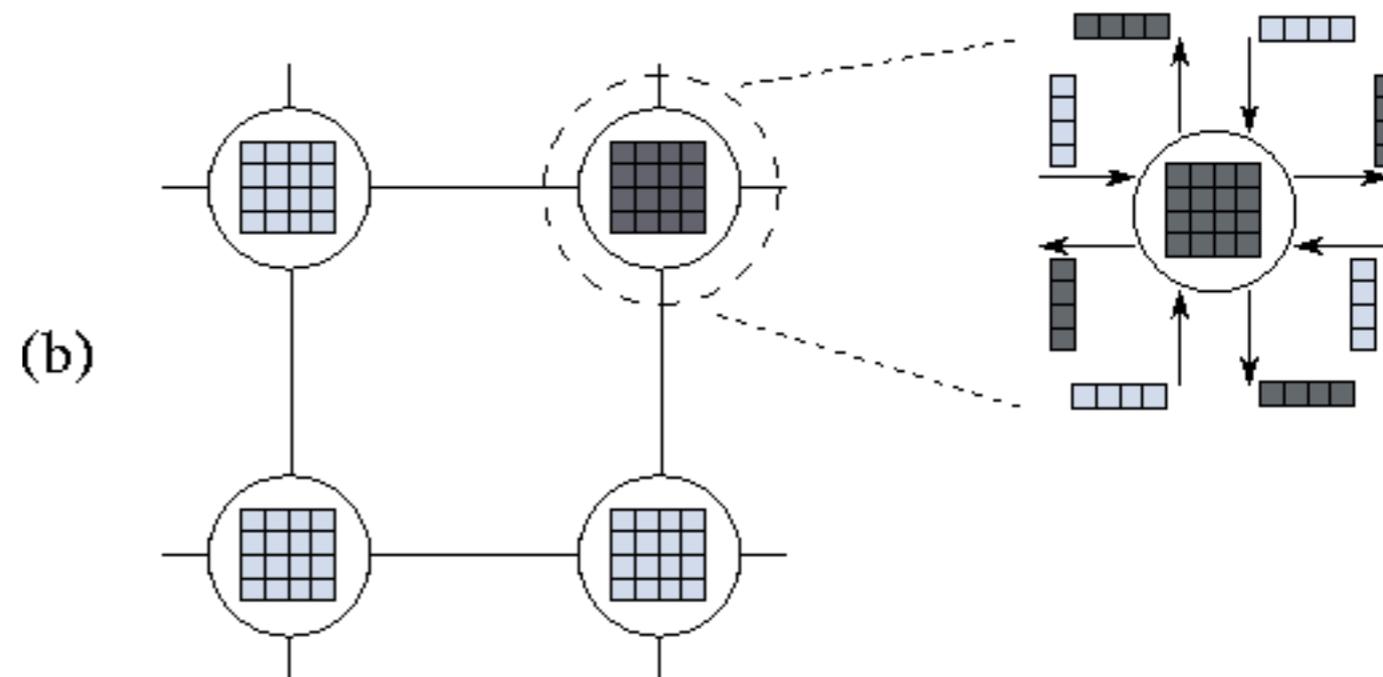
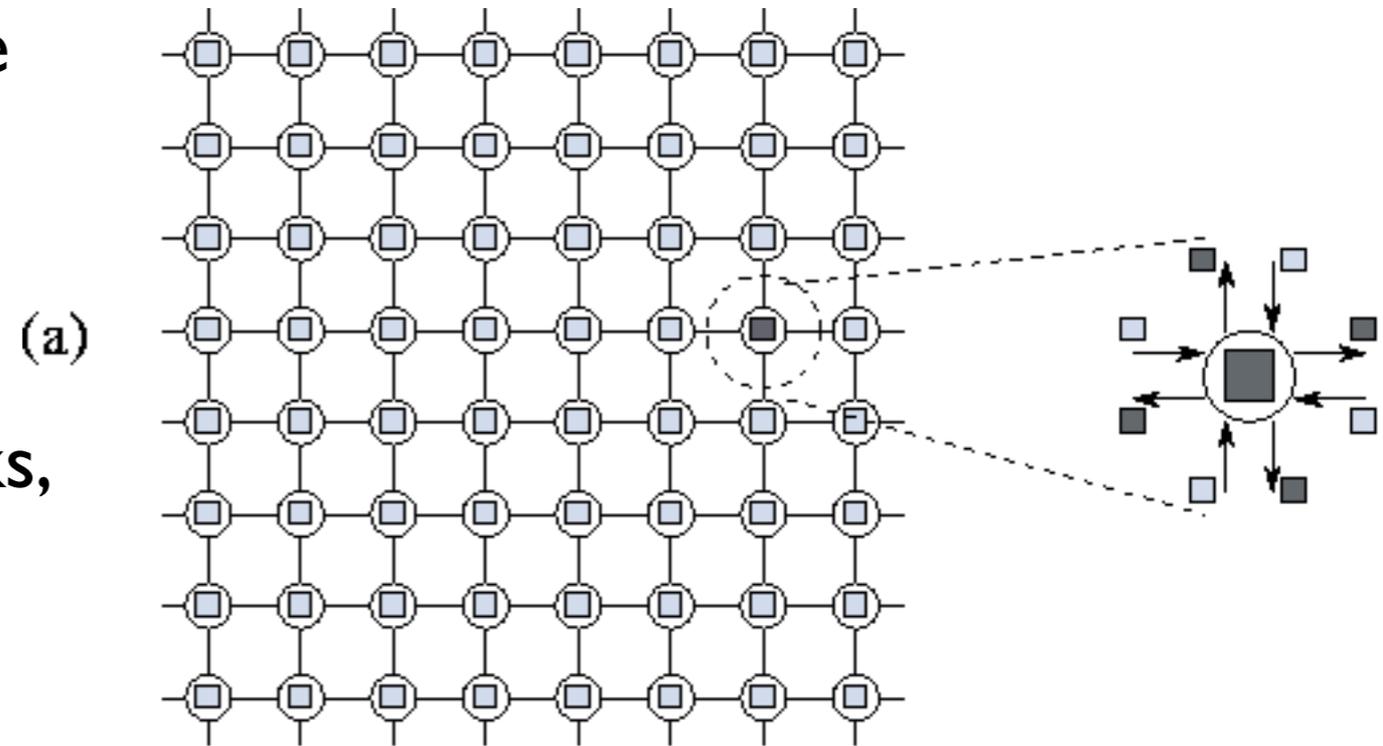
- Combine the many **fine-grained** tasks from partitioning into fewer **coarse-grained** tasks of larger size
- This task must take into account the details of the problem in order to achieve an algorithm with good scaling properties and good efficiency



# Granularity

**Granularity** is the ratio of local computation to communication.

- Agglomeration is used to increase the granularity, improving performance since communication is slow compared to computation.
- By combining many finely grained tasks, we reduce both:
  - (i) **number of communications**
  - (ii) **size of communications**
- In (a), updating 16 points requires
  - (i)  $16 \times 4 = 64$  communications
  - (ii) passing 64 “bits”
- In (b), updating 16 points requires
  - (i) 4 communications
  - (ii) passing 16 “bits”



# Surface-to-Volume in Domain Decomposition

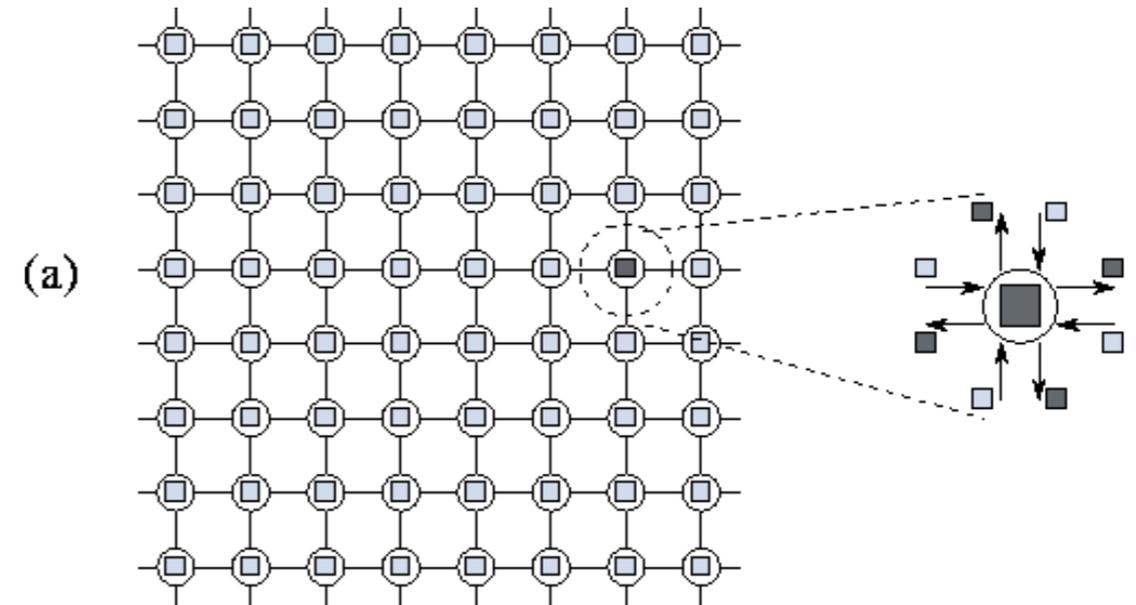
For domain decomposition in problems with local data dependency,  
(ex. finite difference):

- Communication is proportional to subdomain surface area
- Computation is proportional to volume of the subdomain

For this 2-D problem:

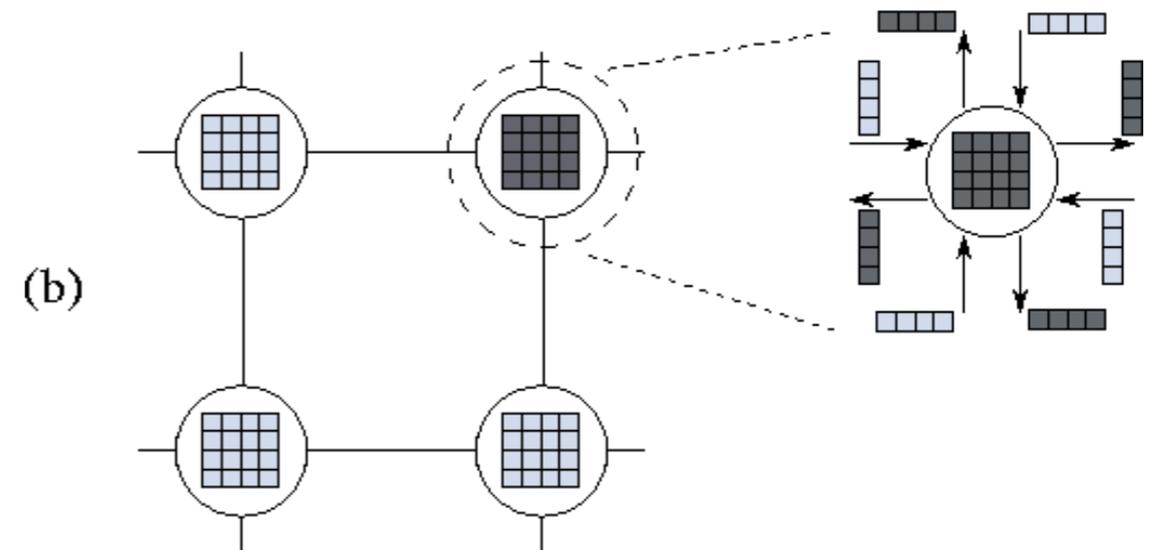
(a) Surface  $S = 4d$  & Area  $A = d^2$

Thus,  $\frac{S}{A} = \frac{4}{d}$



(b) Surface  $S = 16d$  & Area  $A = 16d^2$

Thus,  $\frac{S}{A} = \frac{1}{4d}$

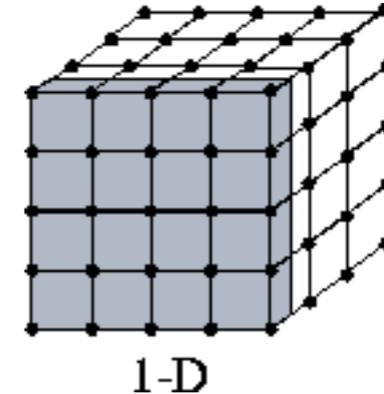


**Decrease of surface-to-volume ratio is  
equivalent to increased granularity**

# Other Factors in Agglomeration

## Maintaining flexibility:

- It is possible to make choices in designing a parallel algorithm that limit flexibility
- For example, if 3-D data is decomposed in only 1-D, it will limit the scalability of the application



We'll see this later in the weak scaling example of HYDRO

## Replication of Data and/or Computation:

- Sometimes significant savings in communication can be made by replicating either data or computation
- Although from a serial point of view this seems inefficient and wasteful, because communication is much slower than computation, it can often lead to significant improvements in performance.

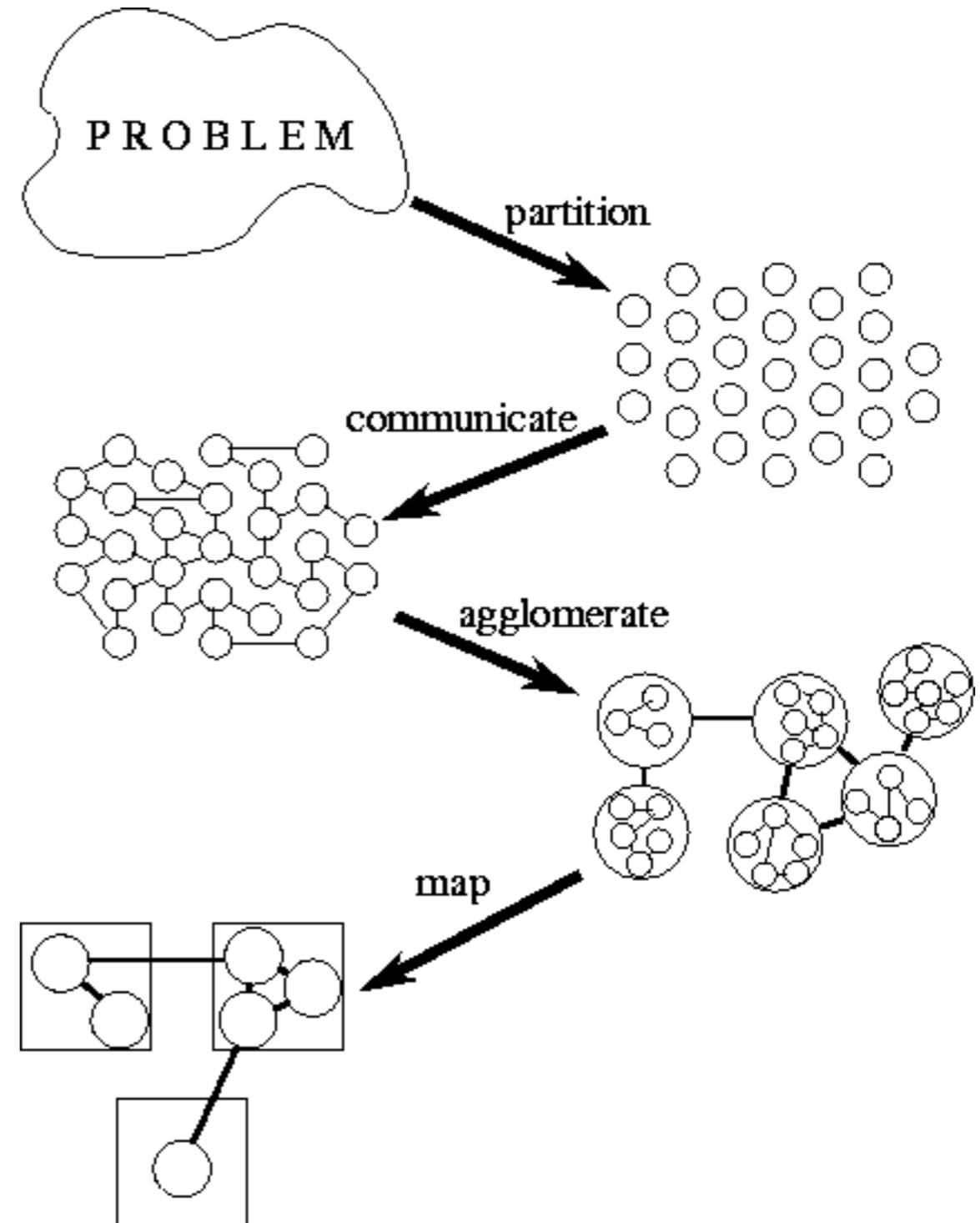
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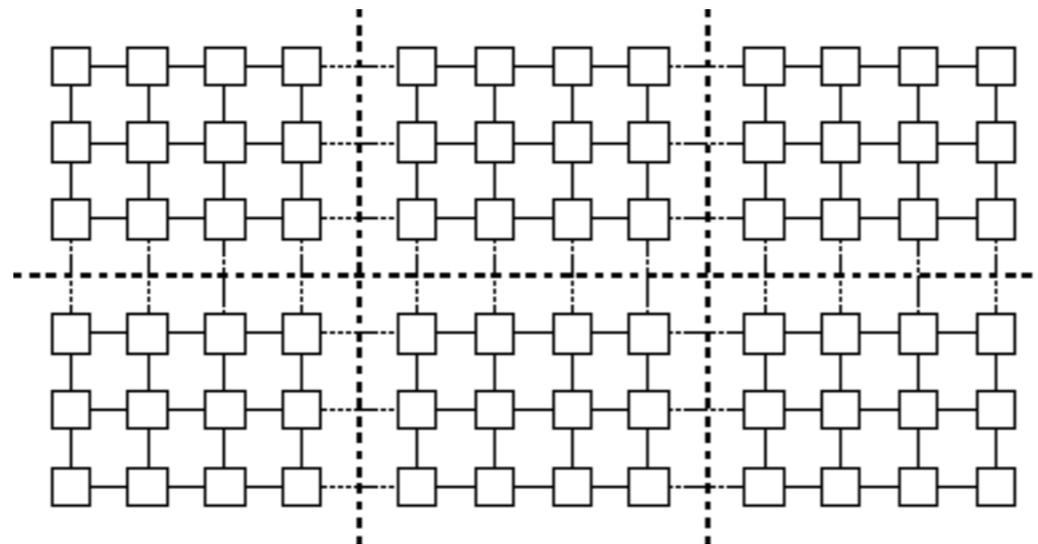
4) Mapping



# Mapping

## Mapping Coarse-grained Tasks to Processors:

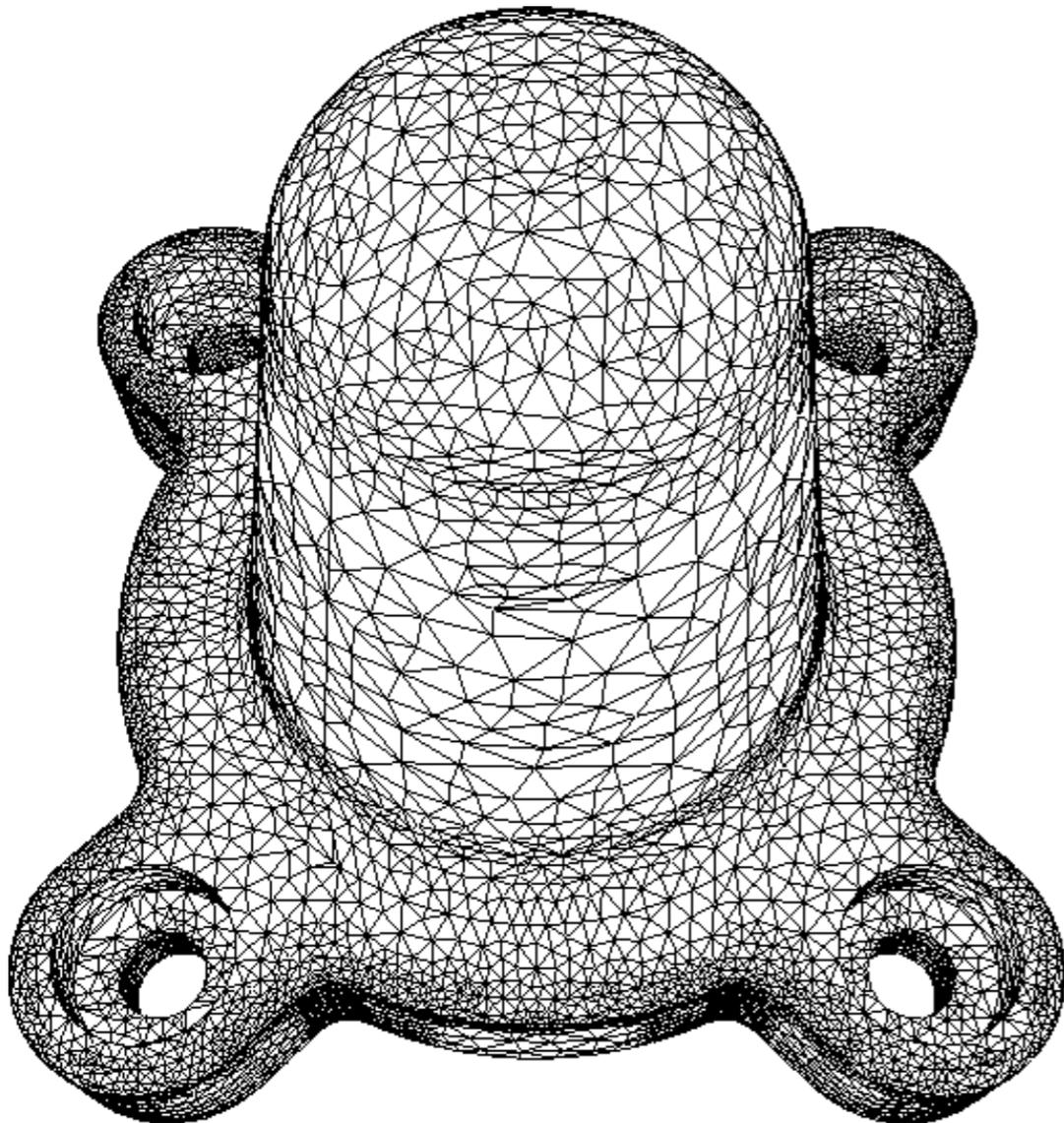
- **Goal:** To minimize total execution time
- **Guidelines:**
  - Tasks that can execute concurrently map to **different** processors
  - Tasks that communicate frequently map to the **same** processor



- For many domain decomposition approaches, the agglomeration stage decreases the number of coarse-grained tasks to exactly the number of processors, and the job is done
- In general, however, one wants to map tasks to achieve good **load balancing**

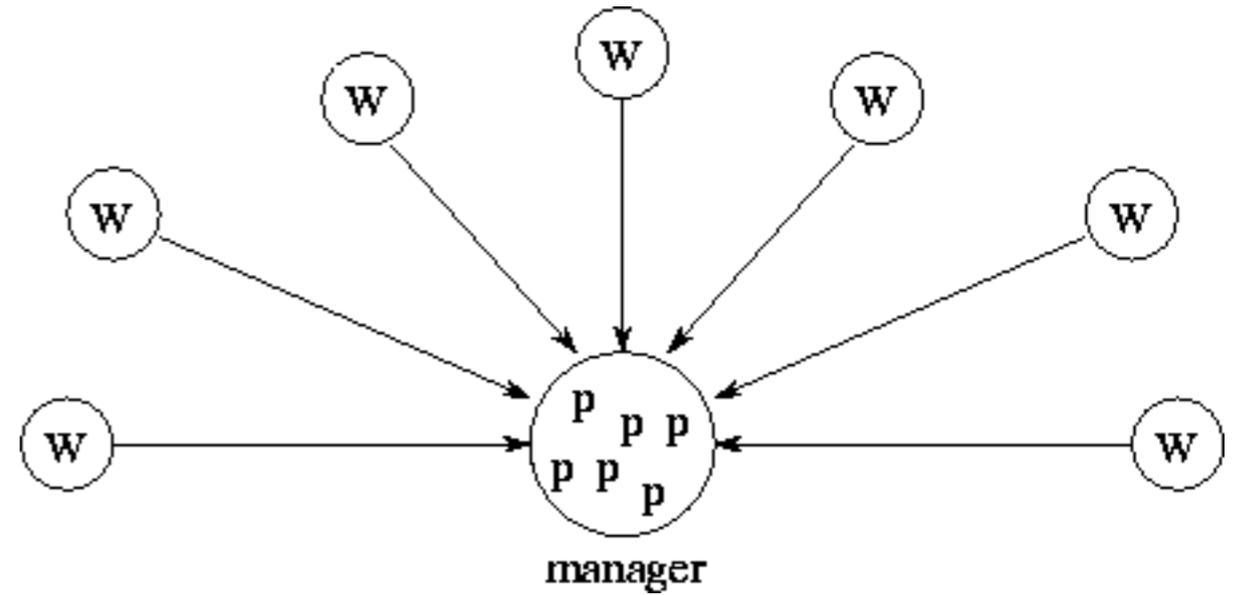
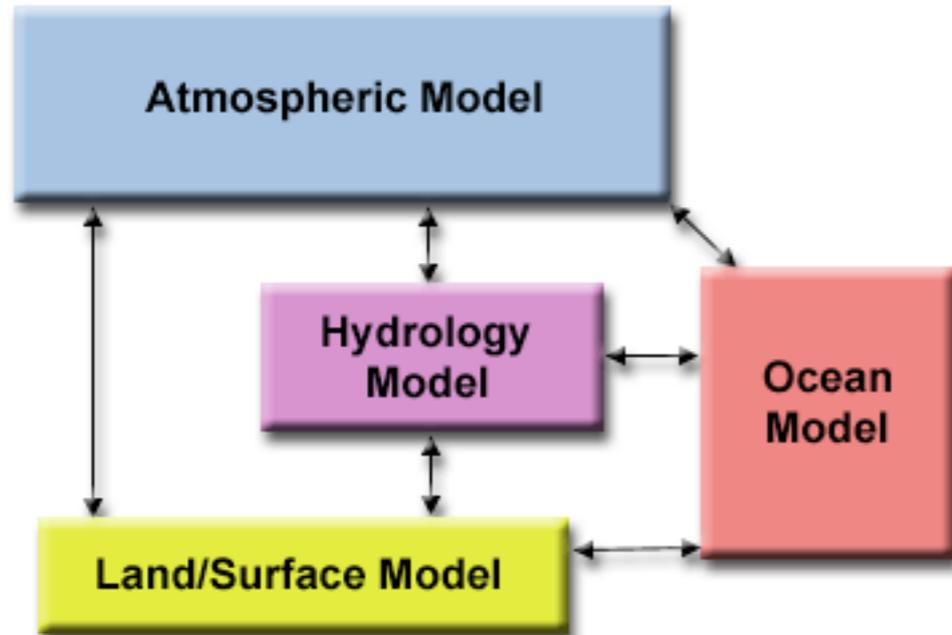
# Load Balancing

- Good parallel scaling and efficiency requires that all processors have an **equal amount of work**
- Otherwise, some processors will sit around **idle**, while others are completing their work, leading to a less efficient computation
- Complicated Load Balancing algorithms often must be employed.



# Load Balancing

- For problems involving functional decomposition or a master/slave design, load balancing can be a very significant challenge



# Parting Thoughts

- Part of the challenge of parallel computing is that the most efficient parallelization strategy for each problem generally requires a unique solution.
- It is generally worthwhile spending significant time considering alternative algorithms to find an optimal one, rather than just implementing the first thing that comes to mind
- But, consider the time required to code a given parallel implementation
  - You can use a less efficient method if the implementation is much easier.
  - You can always improve the parallelization scheme later. Just focus on making the code parallel first.

**TIME is the ultimate factor in choosing a parallelization strategy---Your Time!**

# References

## Introductory Information on Parallel Computing

- **Designing and Building Parallel Programs**, Ian Foster  
<http://www.mcs.anl.gov/~itf/dbpp/>  
-Somewhat dated (1995), but an excellent online textbook with detailed discussion about many aspects of HPC. This presentation borrowed heavily from this reference
- **Introduction to Parallel Computing**, Blaise Barney  
[https://computing.llnl.gov/tutorials/parallel\\_comp/](https://computing.llnl.gov/tutorials/parallel_comp/)  
-Up to date introduction to parallel computing with excellent links to further information
- **MPICH2: Message Passage Interface (MPI) Implementation**  
<http://www.mcs.anl.gov/research/projects/mpich2/>  
-The most widely used Message Passage Interface (MPI) Implementation
- **OpenMP**  
<http://openmp.org/wp/>  
-Application Program Interface (API) supports multi-platform shared-memory parallel programming in C/C++ and Fortran
- **Numerical Recipes**  
<http://www.nr.com/>  
-Incredibly useful reference for a wide range of numerical methods, though not focused on parallel algorithms.
- **The Top 500 Computers in the World**  
<http://www.top500.org/>  
-Updated semi-annually list of the Top 500 Supercomputers

# References

## Introductory Information on Parallel Computing

- **Message Passing Interface (MPI)**, Blaise Barney  
<https://computing.llnl.gov/tutorials/mpi/>  
-Excellent tutorial on the use of MPI, with both Fortran and C example code
- **OpenMP**, Blaise Barney  
<https://computing.llnl.gov/tutorials/openMP/>  
-Excellent tutorial on the use of OpenMP, with both Fortran and C example code
- **High Performance Computing Training Materials, Lawrence Livermore National Lab**  
<https://computing.llnl.gov/?set=training&page=index>  
-An excellent online set of webpages with detailed tutorials on many aspects of high performance computing.