Section 5.3: Double Angle, Power Reduction and Half-Angle Formulas

Each formula used in this section can be derived using the 'grandparent' formulas from the previous section.

I. Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

(only has one version)

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ (has THREE versions)

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

II. Power Reduction Formulas

(from cosine's last two double angle formulas)

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

III. Half-Angle Formulas

(from the power reduction formulas ... replace θ with $\alpha/2$)

$$\sin(\alpha/2) = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos(\alpha/2) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

Now let's put them to some use ...

I. Using the Double Angle Formulas

ex) Given that $\sin\theta = \frac{2}{5}$ and θ acute, evaluate the following

a) $\sin 2\theta$

b) $\cos 2\theta$

ex) Verify the following identities:

a) $(\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\theta$

b) $\dfrac{\sin^2 2x}{1 - \cos 2x} = 2\cos^2 x$

ex) Verify that $\sin 4\theta = 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$

II. Using the Power Reduction Formulas

Their main purpose is to reduce the power of SQUARED sine and cosine expressions.

ex) Rewrite the expression $16\sin^2\theta$ as a single powered cosine expression.

ex) Rewrite the expression $12\sin^2\beta\cos^2\beta$ as a single powered cosine expression.

III. Using the Half-Angle Formulas*

***The " + " or " – " is determined by the half-angle's quadrant location**

ex) Use the half-angle formula to determine the exact value of :

a) $\sin(22.5^\circ)$

b) $\cos(105^\circ)$

ex) Given that $\cos\theta = -\frac{1}{4}$ and θ lies in quadrant III, evaluate the following:

a) $\sin\frac{\theta}{2}$

b) $\cos\frac{\theta}{2}$