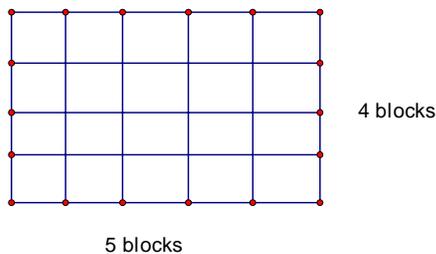


Section 7.2 Area

The Area of Rectangles and Triangles

We encounter two dimensional objects all the time. We see objects that take on the shapes similar to squares, rectangle, trapezoids, triangles, and many more. Did you every think about the properties of these geometric shapes? The properties of these geometric shapes include perimeter, area, similarity, as well as other properties. One of the properties we will first examine is area. The **area** of an object is the amount of surface that the object occupies. The area of object depends on its shape. Different shapes use different formulas to compute the area. For example, the area of a rectangle is length of the rectangle multiplied by the width of the rectangle. Let's examine rectangles further to see why the formula of a rectangle is $Area = length \times width$. If we had a rectangle that was 5 blocks by 4 blocks, then we can take a short cut by multiply 5 blocks by 4 blocks to find the area.



$$A = l \cdot w = (5)(4) = 20 \text{ square units}$$

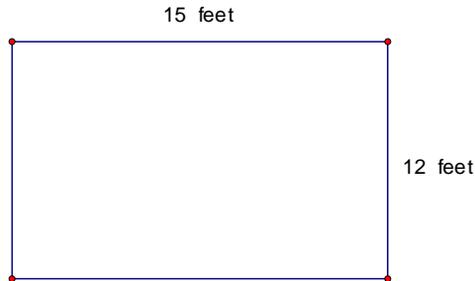
Similarity the area of other objects can be used to find the area.

Key Formulas

Object	Shape	Formula
Square	<p>s = side</p>	$A = s^2$
Rectangle	<p>l = length w = width</p>	$A = l \cdot w$
Triangle	<p>h = height b = base</p>	$A = \frac{1}{2}bh$

Example 1

Suppose you wanted to install new carpet in one room in your house that is 15 feet by 12 feet, how many square feet of carpet would you need?



To find the answer, simply find the area of the room.

$$\text{Area} = (15 \text{ feet})(12 \text{ feet}) = 180 \text{ square feet}$$

Now, suppose that price of carpet at the store you are buying carpet from is \$10.50 per square yard. Roughly, how much is it going to cost to put carpet down in this room?

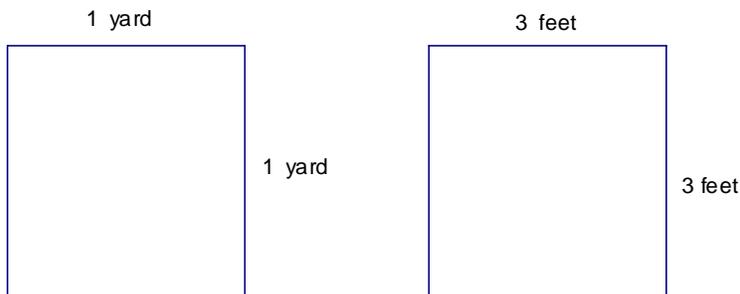
First, find the area of the room in square yards.

$$180 \text{ square feet} \times \frac{1 \text{ square yards}}{9 \text{ square feet}} = 20 \text{ square yards}$$

Now, you can find the cost. $\text{Cost} = (\$10.50/\text{square yards})(20 \text{ square yards}) = \$ 210.00$

Note: Why is 1 square yard equal to 9 square feet?

If you were to look at a square that is 1 yard by 1 yard, then that square would also be 3 feet by 3 feet. Remember 1 yard is 3 feet.



Therefore, the area of the squares in square yards and square feet would be as follows:

$$A = (1 \text{ yard})(1 \text{ yard}) = 1 \text{ sq.yd.} \quad \text{and} \quad A = (3 \text{ feet})(3 \text{ feet}) = 9 \text{ sq. feet}$$

Since the two rectangles are the same, $1 \text{ square yard} = 9 \text{ square feet}$

Example 2

Suppose you wanted to put down hardwood floors in your living room which is roughly 12 feet by 10 feet. If the cost of the hardwood flooring is \$5.00 per square foot, find the approximate cost not including labor to put down hardwood flooring in your living room?

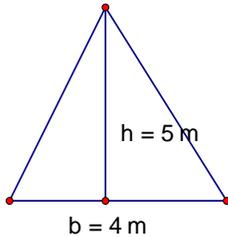
First, the area of the room, then multiply by rate of \$5.00 per square foot.

$$A = (12 \text{ feet})(10 \text{ feet}) = 120 \text{ square feet}$$

$$\text{Cost} = (120 \text{ square feet})(\$ 5.00 / \text{square foot}) = \$600.00$$

Example 3

Find area of triangle with a base of 4 meters and a height of 6 meters.



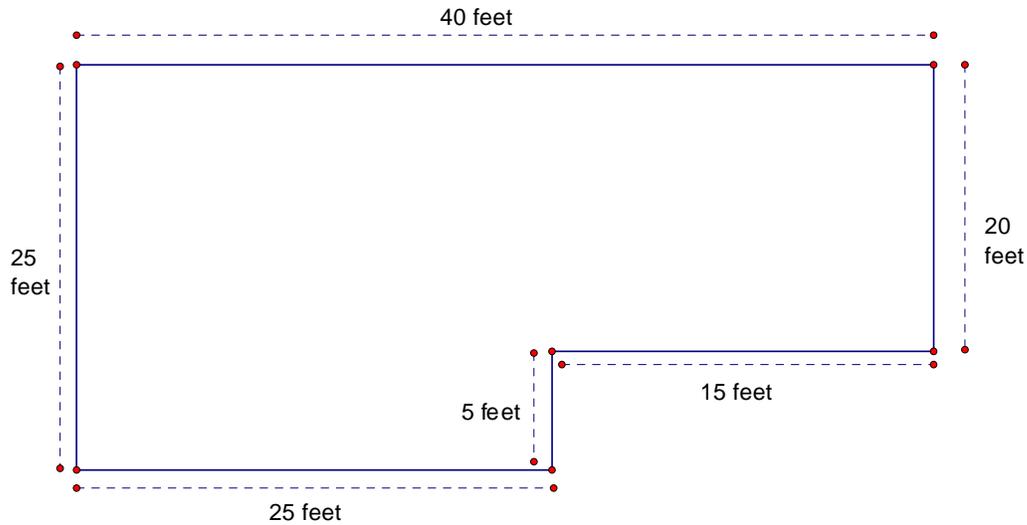
$$A = \frac{1}{2}(4m)(5m)$$

$$A = (2m)(5m)$$

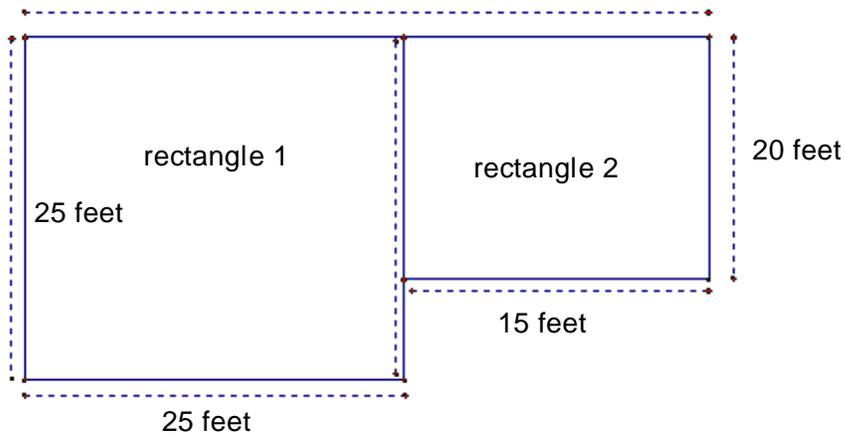
$$A = 10m^2$$

Example 4

Given the floor plans for the 1st floor of a house below, find the area or square footage of the 1st floor of this house.



Solution: Simply divide the rectangle up into two rectangles and find the area of each rectangle



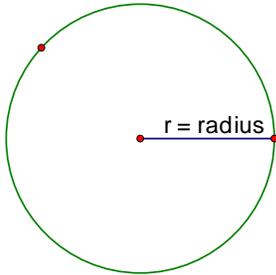
$$\text{Area of rectangle 1: } A = (25 \text{ ft})(25 \text{ ft}) = 625 \text{ square feet}$$

$$\text{Area of rectangle 2: } A = (20 \text{ ft})(15 \text{ ft}) = 300 \text{ square feet}$$

$$\text{Total Area} = 625 \text{ square feet} + 300 \text{ square feet} = 925 \text{ square feet}$$

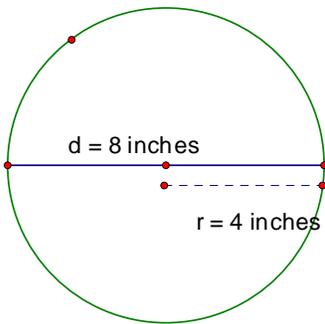
The Area of a Circle

The area of a circle is given by the formula $A = \pi r^2$ where r is the radius of circle.



Example 5

Using 3.14 for pi, find the area and circumference of a circle with a diameter of 8 inches.



First find the radius of the circle by dividing the diameter by 2.

$$r = 8 \text{ in} / 2 = 4 \text{ in}$$

Next, find the area

$$A = \pi r^2 = 3.14(4 \text{ in})^2 = 3.14(16 \text{ in}^2) = 50.24 \text{ sq inches}$$

Now, find the circumference.

$$C = \pi d = 3.14(8 \text{ in}) = 25.12 \text{ inches}$$

Math History Excursion: Egyptian Geometry

A question that one might ask is what was the first civilization to try to understand or explain the concept of area? The answer to this question is mostly likely the Egyptians. The Egyptian as mentioned earlier in the chapter surveyed and divided up their land. The Egyptians were also successful at measuring out distances using their own system of measurements. In the Egyptian system, they had three units of measure for distance which were cubits, palms and fingers. Here is a diagram that shows the relationship between these units of measure.

Egyptians Units of Measure

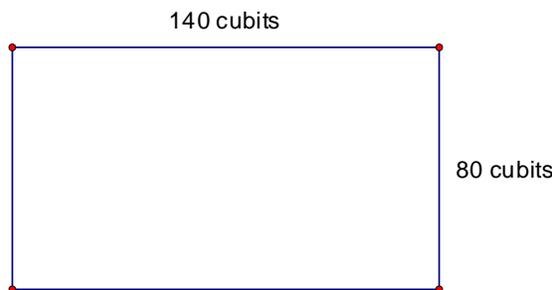
$$1 \text{ cubit} = 7 \text{ palms}$$

$$1 \text{ palm} = 4 \text{ fingers}$$

$$1 \text{ setat} = 10,000 \text{ square cubits}$$

Note: To give us an idea of the magnitude of these measurements, a finger would roughly be the length of your finger, and a palm would be the length of your hand.

Once they were able to measure the dimension of a rectangular shaped lot they were able to find the area of the rectangular lot by multiplying the length of the lot by the width of the lot. For example, suppose there is a rectangular shaped Egyptian lot that is 140 cubits by 80 cubits, what would be the area of the lot?



Since the area of a rectangle is length times width ($A = l \cdot w$), you would multiply 140 cubits times 80 cubits which would give us an area of 11,200 square cubits.

$$A = l \cdot w = (140 \text{ cubits})(80 \text{ cubits}) = 11,200 \text{ square cubits}$$

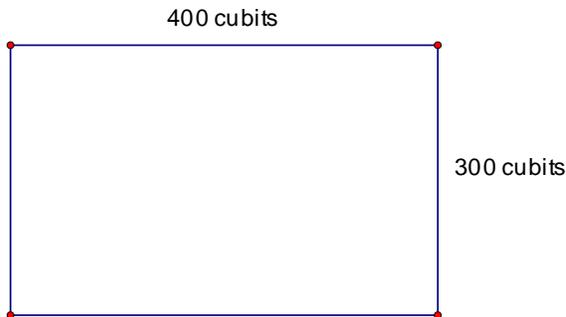
Notice that in the above table of Egyptian units, 1 setat is equal to 10,000 square cubits, so the answer to this problem can be expressed in setats as well as square cubits.

$$11,200 \text{ square cubit} \times \frac{1 \text{ setat}}{10,000 \text{ square cubits}} = \frac{11,200}{10,000} \text{ setats} = 1.12 \text{ setats}$$

The setat was the Egyptian unit of measure that was used specifically to measure area much like an acre measures area in the English system. Now, let's try an example.

Example 6

Find the area of a rectangular shaped Egyptian shaped lot that measures 300 cubits by 400 cubits in square cubits and setats.



$$\text{Area} = 300 \text{ cubits} \times 400 \text{ cubits} = 120,000 \text{ square cubits}$$

$$120,000 \text{ square cubit} \times \frac{1 \text{ setat}}{10,000 \text{ square cubits}} = \frac{120,000}{10,000} \text{ setats} = 12 \text{ setats}$$

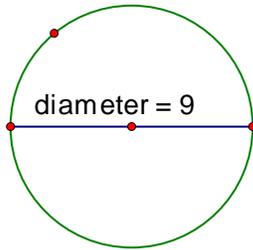
Problem 48 of the Rhind Papyrus and Circles

One the most important discoveries about Egyptian geometry was in regard to circles. Problem 48 of the Rhind Papyrus¹ states that “The area of a circle of diameter 9 is the same as a square of side 8”. If we investigate this further we can find an approximation for pi.

¹ The Rhind Papyrus is one of two famous papyri which accounts for most of our history about Egyptian geometry. This ancient papyrus was not found until 1858 when a Scottish archeologist name Henry Rhind discovered them in accident in Luxor, Egypt. These papyri that were written by the scribe Ahmes around 1650 BC contained 84 problems with the solutions.

Problem 48 of the Rhind Papyrus

The area of a circle of diameter 9 is the same as a square of side 8



First, find the diameter of the circle

$$\text{diameter} = 9 \Rightarrow \text{radius} = \frac{9}{2}$$

Find the area of each region.

$$\text{Area of circle} = \pi r^2 = \pi \left(\frac{9}{2}\right)^2 = \frac{81}{4} \pi$$

$$\text{Area of square} = (8)^2 = 64$$

If we set the areas equal, we get the following equation.

$$64 = \frac{81}{4} \pi$$

$$\pi = 64 \left(\frac{4}{81}\right)$$

$$\pi = \frac{256}{81}$$

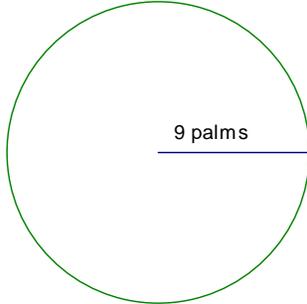
Using this simple calculation you will get an approximate value for pi which is the ratio $\frac{256}{81}$. If divide the value of 256 by 81, you will get a value that is approximately equal to

3.16. Earlier in the chapter we used the modern approximation for pi which is about 3.1416..., so as we can see this value of $\frac{256}{81}$ is fairly close to the modern approximation.

Recall that the Egyptians did not use formulas, so they had no way to calculate the area of a circle. The Egyptian knowledge of circles was same as their knowledge of right triangles in that they only understood these objects through specific examples. However, we can still use the current formula for the area of the circle along with the Egyptian approximation for pi to compute the area of a circle. Given that the formula for the area of circle is $A = \pi r^2$ and the circumference is $C = \pi d$, use the Egyptian value for Pi to answer the following questions.

Example 7

Use the Egyptian value for pi to find the area of a circle with a radius of 9 palms. Then, use the value of 3.14 for pi to find the area of the circle. Compare the two values.



The area of the circle can be found by substituting $\frac{256}{81}$ in for π and 9 palms in for radius into the area of a circle formula.

Solution:

Find the area using the Egyptian $\frac{256}{81}$ value for Pi.

$$A = \pi r^2 = \frac{256}{81}(9 \text{ palms})^2 = \frac{256}{81}(81 \text{ square palms}) = 256 \text{ square palms}$$

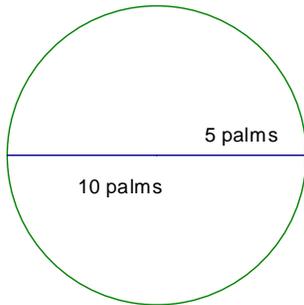
Find the area using 3.14 as Pi.

$$A = \pi r^2 = 3.14(9 \text{ palms})^2 = 3.14(81 \text{ square palms}) = 254.34 \text{ square palms}$$

Notice that the area using the Egyptian value for pi is fairly close to the area found using the exact value for pi.

Example 8

Find the circumference of a circle with a radius of 5 palms using the Egyptian value for pi.



First find the diameter of the circle.

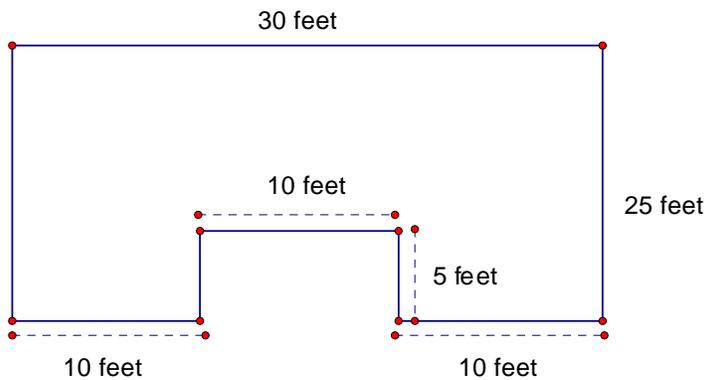
$$d = 2(5 \text{ palms}) = 10 \text{ palms}$$

Next, the circumference of the circle can be found by substituting $\frac{256}{81}$ in for π and 10 palms in for the diameter into the circumference formula.

$$C = \pi d = \frac{256}{81}(10 \text{ palms}) = \frac{2560}{81} \text{ palms} \approx 31.6 \text{ palms}$$

Exercises

- 1) Find the area of a rectangular shaped Egyptian shaped lot that measures 200 cubits by 300 cubits in square cubits and setats.
- 2) You want to put down hardwood floors in your living room that is roughly 18 feet by 16 feet. If the cost of the hardwood flooring is \$5.00 per square foot, find the approximate cost not including labor to put down hardwood flooring in your living room?
- 3) You want to put down carpet in your living room that is roughly 6 yards by 2 and half yards. If the cost of the carpet is \$4.00 per square yard, find the approximate cost not including labor to put down hardwood flooring in your living room?
- 4) Given the floor plans for the 1st floor of a house below, find the area or square footage on the 1st floor of this house.



- 5) Use the Egyptian value for Pi to find the area of a circle with a radius of 4 palms. Then, use the value of 3.14 for Pi to find the area of the circle. Compare the two values.
- 6) Using 3.14 for Pi, find the area and circumference of a circle with a diameter of 8 inches.
- 7) $6 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$
- 8) $20 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ yd}^2$
- 9) $100 \text{ square cubits} = \underline{\hspace{2cm}} \text{ setats}$
- 10) Using the internet or other mathematics textbook, find some other Egyptian units of measurement other than cubits, setats, palms, and fingers.