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Using Technology and Media-Rich Platforms to help teach the Pythagorean Theorem

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Abstract

The history of mathematics is quite extensive, dating back to ancient times and elapsing several centuries. Many great mathematicians, philosophers, scientists, and scholars have contributed to mathematics on the subject we see today. For the purpose of this research project, we have chosen to highlight the great mathematician Pythagoras and his famous Pythagorean Theorem. We investigate the history of the Pythagorean Theorem, relationships among Pythagorean numbers, and unusual proofs of the Theorem. This paper provides a suggestive lesson plan, activity, and presentation for classroom use.

History of the Pythagorean Theorem

One of the most well-known theorems in our culture that many may remember from their earlier high school days is the Pythagorean theorem ($a^2 + b^2 = c^2$). Over the centuries, there have been many names for the Pythagorean theorem, including the Theorem of Pythagoras, the Hypotenuse theorem, and Euclid I 47 (since it is listed as Proposition 47 in Book I of Euclid's *Elements*). The Pythagorean theorem has had quite an impact, and there are several stories on its advancement (Maor, 2007).

In *The Ascent of Men*, Jacob Bronowski stated: "To this day, the theorem of Pythagoras remains the most important single theorem in the world of mathematics" (p.60). The Pythagorean theorem has influenced mathematics by virtually permeating every branch of science and playing a central role in various mathematical applications, such as geometry. Moreover, the classical theorem has over four hundred existence proofs, and it continues to

grow. In fact, on the original list of proofs, there is one from Albert Einstein from age twelve (Maor, 2007).

Upon further research on the history of the Pythagorean theorem, one can conclude that the Pythagorean formula was discovered independently, in some sense, by almost every ancient culture (Mackenzie, 2012). The Pythagorean theorem evolution goes back to over 2,500 years, to when Pythagoras theoretically first proved it. Notably, Pythagoras was not the first to discover the theorem, as recognized as the Babylonians, and probably the Egyptians and Chinese, at least, a thousand years ahead of him (Maor, 2007).

In 2000 BCE, Ancient clay tablets denote that the Babylonians created rules for the Pythagorean triplets. In ~1700 BCE, a famous Babylonian tablet called the Plimpton 322 includes a list of Pythagorean triplets (ex. 3-4-5). Discovering the tablet around 1945, which is currently on display at Yale University (Allen, 2003). Research proves that Pythagoras studied in Babylonia and might have learned the theorem there (Mackenzie, 2012). In 1700 BCE, Pierre de Fermat studied the Pythagorean theorem and conjectured that there were no solutions when n was greater than two. Wittingly, he left no proof and only wrote in the hand left margin that he knew this was not possible, but did not have enough room to write it. This conjecture was named the “Fermat’s Last Theorem” and was not proved until 1993 by Andrew Wiles, from Princeton University (Allen, 2003).

Modern algebra was on the rise in 1600 BCE, and the Pythagorean theorem assumed its now well-known algebraic form (Maor, 2007). In 1500 BCE, the Pythagorean theorem states: “The square of the lengths of the hypotenuse of the right triangle is equal to the sum of the squares of the legs” (Historical Timeline). Between 800 BCE- 600 BCE, an ancient Indian Mathematician discussed the Pythagorean theorem in Sulbasutras. In 569 BCE,

Pythagoras was born on the island of Samos in Greece. The Pythagorean theorem derived from the ancient Greek mathematician, Pythagoras. Many believe Pythagoras presented the first proof of the theorem, although it was discovered long before him (Allen 2003).

Furthermore, theories state that Pythagoras may have even traveled to India around 500 BCE, visiting eastern regions such as Egypt and Phoenicia. Supposedly, Pythagoras learned from the Phoenicians and Egyptians and their priests and acquired some early notions about mathematics (Aczel). “The Egyptians must have use this formula [$a^2 + b^2 = c^2$] or they couldn’t have built their pyramids, but they have never expressed it as... as useful theory,” from Joy Hakim, *The Story of Science* (page 78). Unfortunately, our knowledge of the Egyptians is not extensive due to the fragile mediums of evidence. The Babylonians wrote on clay tablets, which are virtually indestructible, whereas Egyptians wrote on delicate papyrus. The Egyptian monument, the “Great Pyramid of Cheops” is 756 feet on each side with a height of 481 feet. This example will require scientific information, which could include the Pythagorean theorem. However, did it really? The Rhind Papyrus consists of eighty-four problems and also contains Pyramid problems, but fails to reference them to the Pythagorean theorem (nor directly or indirectly). Another hypothesis states that Egyptians used a rope tied at equal intervals to measure distances, which could have led them to discover the 3-4-5 triangle. There is no evidence to support this theory (Maor, 2007).

Around 500 BCE- 200 BCE, the Chinese treatise, Chou Pei Suan Ching, offers a statement and demonstration of the Pythagorean theorem. Hippocrates of Chios (470-410 BCE) lived well before Euclid’s period, and researchers say he generally knew of the Pythagorean theorem, as he stated in Proposition 31 of Book VI: “In right-angled triangles, the figure on the side opposite the right angle equals the sum of the similar and similarly described figures on the

sides containing the right angle” (Alsina, 2011). In 300 BCE, Euclid’s *Elements* is published and includes a proof of the Pythagorean theorem (Mackenzie, 2012).

The Pythagorean theorem is one of the most well-known theorems in geometry, the most popular property of right angles, and contains more proofs than any other in mathematics. In 1876, a Pythagorean theorem proof was published in the *New England Journal of Education* (Volume 3, page 161), written by James Abram Garfield (1831-1881). During the time, Garfield was a member of the Ohio United States House of Representatives, until 1880, when he was elected the twentieth President of the United States. Sadly, he was assassinated only four months after being elected (Alsina, 2011).

In 1895, Lewis Carrol commented on the beauty of the Pythagorean theorem: “It is as dazzlingly beautiful now as it was in the day when Pythagoras first discovered it.” In 1927, Elisha S. Loomis (1852-1940), a mathematics teacher from Ohio, collected and wrote up three hundred seventy-one proofs in *The Pythagorean Preposition*. In 2004, the journal *Physics World*, awarded the Pythagorean theorem 4th place (out of 20), as voted by readers, for the twenty most beautiful equations in science (Maor, 2007).

The Pythagorean theorem has evolved and impacted numerous fields of math and science over the past decades. A wide variety of diverse cultures has contributed to its history. The Pythagorean theorem has more proofs than any other in mathematics, and each one offers something different. Today, we can only assume, Pythagoras, in all probability, supplied the first proof, and gets the name of this universal theorem.

Unusual relationships that exist among Pythagorean numbers

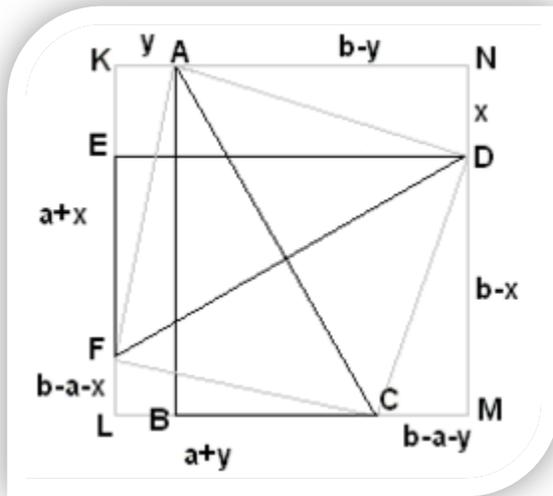
Did you know there is more to Pythagoras than the Pythagorean Theorem?

Pythagoras and his followers believed that all relationships to numbers could relate to and account for geometrical properties. Pythagoras and his supporters studied the properties of numbers that are familiar to us today. They studied even and odd numbers, prime numbers, and square numbers. Also, Pythagoras and his supporters believed that numerical properties were masculine or feminine, perfect or incomplete, and beautiful or ugly. Odd numbers were known to be masculine while even numbers were reflected feminine because they were weaker than the odd numbers. Furthermore, the odd numbers were considered the master numbers, because an odd plus an even always produces an odd. Also, two evens can never produce an odd number, while two odds always produce an even number.

Initially, Pythagoreans believed in the existence of whole numbers only; however, as time passed, they proved the existences of incommensurables. Incommensurables are what we know today as irrational numbers. Pythagoras and his followers also used patterns of dots to represent numbers. The resulting figures, created by the patterns of dots, are known as figure numbers. For example, nine dots can be arranged in three rows with three dots per row, creating the picture of a square. Similarly, ten dots can be organized into four rows, containing one, two, three, and four dots per row so as to form a triangle. Develop from the various figure numbers, Pythagoras, and his followers further derived relationships between numbers. They discovered that a square number could be subdivided by a diagonal line, into two triangular numbers; this is why we can say that square numbers are always the sum of two triangular numbers. For example, the square number of twenty-five is the sum of the triangular number ten and the triangular number fifteen.

Unusual Proofs of the Pythagorean Theorem

According to the writer A. Bogomolny, there are so many unusual ways to prove the Pythagorean Theorem. By incorporating those unique strategies to teach it in high school, teachers will be able to engage and lead students to learn the Pythagorean Theorem in a remarkable way. For instance, one of the cited proofs makes use of the area of the big square KLMN is b^2 . The square splits into four triangles and one quadrilateral:



The area of the big square KLMN is b^2 . The square divisions into one quadrilateral and four triangles:

$$b^2 = \text{Area}(KLMN)$$

$$= \text{Area}(AKF) + \text{Area}(FLC) + \text{Area}(CMD) + \text{Area}(DNA) + \text{Area}(AFCD)$$

$$= \frac{y(a+x)}{2} + \frac{(b-a-x)(a+y)}{2} + \frac{(b-a-y)(b-x)}{2} + \frac{x(b-y)}{2} + \frac{c^2}{2}$$

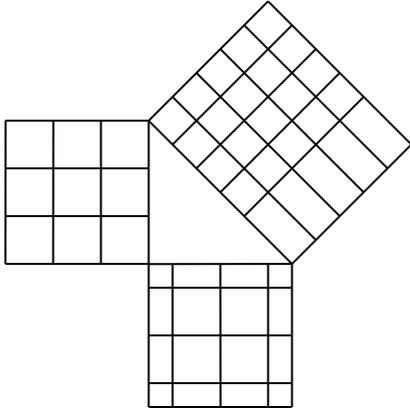
$$= \frac{[y(a+x) + b(a+y) - y(a+x) - x(b-y) - a \cdot a + (b-a-y)b + x(b-y) + c^2]}{2}$$

$$= \frac{[b(a+y) - a \cdot a + b \cdot b - (a+y)b + c^2]}{2}$$

$$= b^2/2 - a^2/2 + c^2/2. \text{ ((Bogomolny, A., 2012))}$$

Visual model of Pythagorean Theorem

The following model is a visualization of the Pythagorean Theorem:



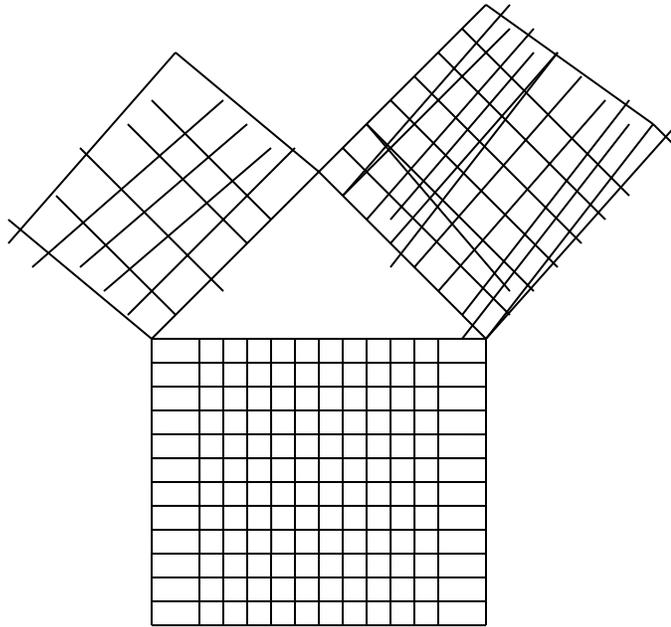
$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25$$

If you put three and four together, it will not equal five, so you can't put the original two squares together, to get the big square. The best way to get the two smaller squares to fit into the big square when they are combined is to square the number of tiles in each. So, three squared equals nine, and four squared equals sixteen, and nine plus sixteen equals twenty-five which will fit on the large square when combined.

Or this one:



$$\begin{aligned}5^2+12^2&=13^2 \\25+144&=169 \\169&=169\end{aligned}$$

If you put five and twelve together, it will not equal thirteen, so you can't put the original two squares together, to get the big square. The best way to get the two smaller squares to fit into the big square when they are combined is to square the number of tiles in each. So, five squared equals twenty-five and twelve squared equals one hundred and forty-four, and twenty-five plus one hundred and forty-four equals one hundred and sixty-nine, which will fit on the large square when combined. You can use any four sided figure you want to put together and form a right triangle in the middle.

Concluding Remarks

This record of the Pythagorean Theorem is far from a full documentation of the brilliant work of Pythagoras. It does, however, highlight some of the most important discoveries

regarding this Theorem. It is imperative to note that very few mathematicians have come close to, or would be able to discover the ideas they have, without it. The attached lesson plan, activity, and presentation aim to deliver this information in a student-friendly format. Through this lesson plan, it is our goal that students develop an understanding not only of the Theorem itself but of its place in history and importance in the development of modern day mathematics.

Lesson Plan

Lesson Plan: Applying the Pythagorean theorem

Grade Level: 6th - 9th Grade

Lesson Duration: 90 Minutes

Overview:

This lesson allows students to experience the Pythagorean theorem through the use of a media-rich interactive learning platform. Through an hands-on approach, students will deduce that the angle classification of a triangle is determined by its side measurements. Students will find the perimeter of polygons on the coordinate plane by triangulation. They will also integrate their reading, writing, and mathematical skills during this learning experience.

Objectives: Student will be able to:

- Find the square of a number using the area of a square.
- Find a square root of a number by using the length of the side of a square of the same area.
- Discover the Pythagorean theorem using the hands-on approach.
- Find the sides of a right triangle using the Pythagorean theorem.
- Conclude that the Pythagorean theorem is a relationship between areas.
- Use the Pythagorean theorem to find the right triangle and the length of an unknown side.

Materials:

- PowerPoint
- Copies of a Pythagorean assignment sheet (one per student)
- Set of calculators
- Graphic paper or dot paper
- Set of rulers
- Construction Paper Triangle

Technology Component:

- IPAD Cart
1. Introduction to the lesson starts with a brief lecture on the history of Pythagorean Theorem and right-triangle. The concepts of area and length are discussed, stressing the fact that the Pythagorean Theorem is fundamentally a relationship between areas.
 2. Students will watch a two minutes demo video on the Pythagorean Theorem, and students will then share with their partners, two facts about Pythagorean Theorem and two facts about two mathematicians' proofs of the Pythagorean Theorem. They also had to discuss and ask each other questions about their evidence by the think-pair-share method.
 3. Write Pythagorean Theorem equation down on the Promethean board and have a meaningful discussion about the different parts of the theorem. Comprehensively explain what a, b and c means in the equation. Explain that a and b are the legs of the triangle while c is the hypotenuse of the triangle.
 4. The classroom formation is arranged into groups. Each group is given four identical right triangles labeled with hypotenuse and legs. The students are asked to organize their triangles into a square with the four hypotenuses forming the perimeter of the square.
 5. Next, students are guided through the proof by being asked a series of questions about the arrangement of triangles.
 6. Allow 20-25 minutes for engaging discussion. Regroup the class together and call pairs of students to clarify how they will solve the problems. Feel free to ask open-ended questions (make sure you are not judging or communicating to students they are wrong or right.) If time is not permitted for all the problems discussion, the lesson will be extended to the next day for three minutes opening activity. The lesson could divide over two days. There are many ideas to discuss.

Accommodation:

- Student with specific learning disability in reading, assist the student by reading the directions and make sure that he or she understands.
- For a gifted and talented student in math and reading, challenge the student with story problems involving right triangles and using the Pythagorean Theorem.
- Student with attention deficit hyperactivity disorder (ADHD) and emotional disabilities, allow the student to work in a group setting so that the student can have a positive role model; in fact, teacher believes that students will stay on task better if his/her peers are on task. The teacher would allow the students to walk around during homework time for a short break.
- At the beginning of the intermediate levels of proficiency, English Language Learners should be exposed to samples of completed working assignments to model the correct format.

Assessments:

- During the activity, the teacher will implement an informal evaluation by circulating through the classroom, asking questions, and providing feedback.
- Exit Ticket: Students will take Quiz over Pythagorean Theorem. Only If the time permits.

Reflection

This project used a 9th grade Math class of twenty-six students, two of which are English language learners. It is a high achiever/general class setting, which is a double-coded class with varying ability levels. The lesson used for this project is entitled “Applying the Pythagorean Theorem.” In this lesson, the students examined the Pythagorean theorem to realize that in order to solve problems, the side measurement is critical in determining the angle measure classification of a triangle. When squaring sides, they can predict whether triangles will be right, obtuse, or acute. Using the Internet, the students will prove the Pythagorean Theorem, and that will enable them to use it to solve problems. Lastly, the students will demonstrate their knowledge of the Pythagorean Theorem by determining the perimeter of polygons on the coordinate plane by triangulation. This activity utilizes reading, writing, and mathematical skills in an interdisciplinary format.

At first, the activity begins with a brief lecture introducing right-triangles and the history of Pythagorean Theorem. The concepts of length and area are examined, stressing the fact that the Pythagorean Theorem is fundamentally a relationship between areas. Then, the students watched a demo instruction video on the Pythagorean theorem. Students then shared with their partners, two facts about Pythagorean Theorem and two facts about two mathematicians’ proofs of the Pythagorean Theorem. They also had to discuss and ask each other questions about their evidence using the think-pair-share method. The teacher walked around and listened to various responses and at the same time poses questions to elicit conversation.

The teacher changed questions around to make them open-ended questions. For example, “How do you think we are going to solve for the short side of the triangle?” One student responded, “the Pythagorean theorem is going to be our life jacket to save us.” The teacher then

asked, “What makes it a life jacket to saving us?” The student’s responses made the teacher cleared up any misconceptions. Closed questions deny students the opportunity of meaningful conversation. In looking back at the video, the teacher realized that her students will also benefit from a lesson on open and closed responses. If they are made more aware of this, they would be able to stimulate discussions that would provide opportunities for meaningful practice of the language of math. Some of the questions asked by students were closed questions. They were asking questions that elicited yes or no responses. The teacher modeled open-ended questions to the groups she visited; however, they need more practice as students were not able to grasp the concept fully.

The teacher made sure to visit her English language learner’s (ELL) student group. She placed them in the same small group for easy access. During one of her visits to their group, she noticed they were struggling with some words. So, she read the directions and the problem to assist with vocabulary comprehension. Also, she posed simple questions about the arrangement of triangles to guide them to answering the question. It is imperative that teacher’s repeat phrases and sentences so that ELLs can fully comprehend what is being said. Looking back at the video, she could see the “light bulb” come on for them as she repeated the sentences.

Discourse analysis has helped the teacher to look for recurring patterns in her questioning style. By analyzing how particular questions impacted her students’ responses, she made necessary changes, and she is now asking more challenging and thought-provoking questions to promote active and meaningful dialogue. For this reason, the final activity was designed to be independent group practice that will enable her to observe how well the students understood the main ideas covered in the lesson. It had them complete some procedural problems using the Pythagorean Theorem. Students will be giving problems that they need to apply relevant

properties that they learned in the lesson. There was a representation of practice problem that appropriately dealt with everything that was covered, which give the teacher a reasonably accurate assessment of their knowledge. Along with the discussions during class allowed the teacher to assess the levels of understanding for each student.

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