

complete, up-to-date textbook on an increasingly important subject

Robust Systems Theory and Applications covers both the techniques used in linear robust control analysis/synthesis and in robust (control-oriented) identification. The main analysis and design methods are complemented by elaborated examples and a group of worked-out applications that stress specific practical issues: nonlinearities, robustness against changes in operating conditions, uncertain infinite dimensional plants, and actuator and sensor limitations. Designed expressly as a textbook for master's and first-year PhD students, this volume:

• introduces basic robustness concepts in the context of SISO systems described by Laplace transforms, establishing connections with well-known classical control techniques

• presents the internal stabilization problem from two different points of view: algebraic and state-space

• discusses the four basic problems in robust control and the Loop shaping design method

• presents the optimal \mathcal{H}_2 control problem from a different viewpoint, including an analysis of the robustness properties of \mathcal{H}_2 controllers and a treatment of the generalized problem

• presents the \mathcal{H}_∞ control problem using both the state-space approach developed in the 1980s and a Linear Matrix Inequality approach (developed in the mid 1990s) that encompasses more general problems

• discusses more general types of uncertainties (parametric and mixed type) and synthesis as a design tool

• presents an overview of optimal ℓ^1 control theory and covers the fundamentals of its norm approximation

• presents the basic tools of model order reduction

• includes a tutorial on robust identification

• contains numerous end-of-chapter problems and worked-out examples of robust control

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ROBUST SYSTEMS

ROBUST SYSTEMS THEORY AND APPLICATIONS

RICARDO S. SÁNCHEZ-PEÑA
MARIO SZNAIER



**ROBUST SYSTEMS
THEORY AND APPLICATIONS**

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Ricardo S. Sánchez-Peña

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To our parents
Lolita and Miguel, Rebeca and Marcos

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PREFACE

Robustness against disturbances and model uncertainty is at the heart of control practice. Indeed, in the (completely unrealistic) case where both all external disturbances and a model of the system to be controlled are exactly known, there is no need for feedback: Optimal performance can be achieved with an open loop controller.

The main ingredients of present day robust control theory were already present in the classical work of Bode and in many popular frequency domain-based design techniques. With the advent of state-space methods in the mid 1960s the issue of robustness took a backseat to other topics, but was never completely abandoned by the control community, especially control practitioners. Interest in robust control rose again in the late 1970s where it was shown that many popular control methods (including optimal LQR control and controller design based on the cascade of an observer and state feedback) led to closed-loop systems very sensitive to model perturbations. Moreover, widely accepted “ad-hoc” recipes for “improving robustness,” such as using artificially large noise levels in the design, had precisely the opposite effect.

Robust control has undergone extensive developments in the past two decades, leading to powerful formalisms, such as \mathcal{H}_∞ , μ -synthesis/analysis and, more recently, ℓ^1 optimal control, that, coupled with newly developed control-oriented identification techniques, have been successfully applied to challenging practical problems. A salient feature of the framework is that it is oriented towards applications and thus is based on “practical,” realistic assumptions.

There are many excellent books that cover specialized topics (\mathcal{H}_2 , \mathcal{H}_∞ , ℓ^1 , parametric uncertainty, linear matrix inequalities) with others scattered in the technical journals. Our intention in writing this book is to provide a self-contained overview of robust control that illustrates all the issues involved, ranging from the transformation of experimental signals from the physical plant to a set of models (robust identification), to the synthesis of a controller for that set of models (robust control). The purpose of the book is twofold: to serve as a textbook for courses at the Master’s/beginning Ph.D. level and as a reference for control practitioners. It assumes that the reader has a background in classical and state-space control methods. In order to keep the

text size at a manageable level, in some cases only basic results are covered, and the reader is referred to more specialized literature for further coverage.

In all cases we have strived to link the theory with practical applications. To this end, in addition to the examples covered throughout the book, the last chapter contains several worked out application problems that stress specific practical issues: nonlinearities, unknown time delays, infinite dimensional plants, actuator and sensor limitations. They are all extracted from our practical experience in different engineering fields. Furthermore, due to the fact that most of the problems to be solved by the theory presented here are computer-intensive, we stress the algorithmic and computational aspect of the solution along with the mathematical theory.

We (and our graduate students) have tried to eliminate obvious mistakes. However, as anyone who has tried knows, it is virtually impossible to make a “perfect” book. We encourage readers to send corrections, comments and general feedback to either one of the authors. Finally, as attested by the list of more than 300 references, we have tried to give credit where it is due. However, owing to the sheer volume of literature published on robust control, we may have inadvertently failed to do so on occasion. We apologize in advance to readers or authors who may feel that this is the case, and we encourage them to send us comments.

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Preliminary versions of the book were tested in courses offered by the authors at Penn State University and the University of Buenos Aires. Feedback from students enrolled in these courses was fundamental to its fine tuning. We are also indebted to our graduate and post-doctoral students, Takeshi Amishima, Pablo Anigstein, Juanyu Bu, Tamer Inanc, Cecilia Mazzaro, Pablo Parrilo and Zi-Qin Wang, for reviewing the book in great detail and providing some of the material. In particular, Cecilia developed very efficiently the

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