The Grammar According to West

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I have been accumulating these observations for a number of years. Writing textbooks has led me to think about choices I make in writing mathematics. I have also noted the writing errors commonly made by my thesis students. Here I have collected my conclusions from both situations.

The first aim of this document is to educate my own students and to save time in editing their theses. Since it exists, I am making it available publicly in the hope that others may find it useful; if you don't (or if you object on principle), then please ignore it. Perhaps it will sensitize some writers of mathematics (especially students) to issues they may not have thought about, where small changes will produce writing that is easier to read by wider audiences.

I consider points of English grammar and aspects of mathematical usage and notation. My intent is not to make writing rigid, but rather to make it transparent, so that the reader is not distracted by ambiguities or awkwardness in the flow of the narrative. A reader should not need to pause or backtrack to understand the organization of a sentence.

In mathematical conversations, one takes many shortcuts that would not be acceptable in precise mathematical writing. The context is known by all participants, and shortcuts evolve to save time. Also, the speaker can immediately clarify anything ambiguous. Lacking these protections, written mathematics must use language more carefully.

Many mathematicians will object to some of my recommendations. Many time-honored practices in the writing of mathematics are grammatically incorrect. These mistakes in writing cause no difficulty for readers with sufficient mathematical sophistication or familiarity with the subject. I argue that it is unnecessary to restrict the audience to such readers. A modicum of care leads to better writing habits that make mathematics more easily accessible to a wider and less specialized audience.

I apologize in advance for my own grammatical errors. Habits die hard, and it is easy to err in applying principles of writing.

Issues of English grammar

1. *"Which" vs "that".* The following two sentences have different meanings: 1) "She will attend our meetings that concern calculus." 2) "She will attend our meetings, which concern calculus." Sentence (1) states that among our meetings, she will attend those concerning calculus and perhaps no others. Sentence (2) states that all the meetings concern calculus, and she will attend them all.

When the phrase after the relative pronoun specifies a further restriction of the class that has just been introduced, the correct pronoun is "that", and the subsequent phrase tells which of the items in the class are those being discussed. If the subsequent phrase speaks about the totality of the class, then the proper pronoun is "which". When "that" and "which" both seem usable, use "that" when the sense is "having the property that", and use "which" when the sense is "all of which" or "the only one of which". Often a comma is appropriate before "which". Usually "that" is correct when an indefinite article ("a" or "an") has been used on the word being modified.

2. *Immediacy of antecedents.* When using “which”, “that”, “where”, or other words to introduce explanatory or descriptive phrases, the subsequent phrase modifies the most recent item. For example, “an embedding of G on a surface which has no crossings” indicates that the surface has no crossings, not that the embedding has no crossings. Making the comment on crossings apply to the embedding requires rewriting: “On a specified surface, consider an embedding of G, which has no crossings”. Here “which” is proper because every embedding has no crossings; an embedding is a drawing *that* has no crossings.

3. *Definite articles vs. possessives.* The definite article “the” specifies uniqueness. Possessives also play this role. It is incorrect to use both together, because the possessive already provides definite specification. For example, we write “Greene’s Theorem” but not “the Greene’s Theorem”; this is a theorem proved by Greene, not by “the Greene”.

When discussing a result by two authors, we cannot put possessives on both names, and making only the second name possessive would be wrong. Hence we write “the Greene–Kleitman Theorem”. Here “the” serves as a definite article for the unique object “Greene–Kleiman Theorem”. When the result is less celebrated, one can indicate the possessive by “of”, as in “the theorem of Greene and Kleitman”.

4. *Titles of results.* In the examples above, “Theorem” is capitalized. When there is only one instance of an object, and the name of it involves a person, it plays the role of a proper noun and its name is a title. Another example is “the Cauchy–Schwarz Inequality”.

5. *Indefinite nouns and articles.* Indefinite nouns need no articles. We speak of “a theorem” or “a result”, since these are definite nouns, but “work” is an indefinite noun and does not take the indefinite article “a”. We say simply “This is work of mine”, not “This is a work of mine”, and “This is joint work with my colleague,” not “This is a joint work”.

As another example, “graph colorability” is an indefinite concept, so we do not say “Next we discuss the graph colorability”. In contrast, “chromatic number” may be indefinite or specific. We may say “Next we discuss chromatic number”, referring to a general concept, or “Next we discuss the chromatic number of this graph”, since this graph has only one value as its chromatic number.

Articles may also be inappropriate when referring to one instance of a conceptual noun. We say “The variable has expected value 4” rather than “The variable has the expected value 4” and “The vertex has degree 4” rather than “The vertex has the degree 4”. Never say “the vertex has a degree 4” (the vertex has only one degree).

6. *“Best possible”.* “Best possible” is an adjective; it indicates sharpness. We write “This result is best possible”, just as we would write “This result is sharp”. Writing “This result is the best possible” says that this result is better than all other results. The article should not be used here.

7. *“Every”, “distinct”, and “unique”.* The word “distinct” has the same meaning as “different”. Two things can be distinct, but one thing cannot be distinct. The word “every” is *singular*; it means “each one”. Because of this, we write “all values” or “every value”; not “all value” or “every values”. The consequence of these two observations is that the sentence “Every value is distinct” is incorrect; it has no meaning. Some students think it means that each value is different from every other value, but it does not.

The word "unique" indicates that there is only one of the items being described. It does not mean that this item is different from other items. Some students think that "The function f maps the points in A to unique points in B" is a statement that f is injective, but it is not. Every function from A to B maps each point in A to a unique point in B.

8. *Commas, semicolons, and conjunctions.* Punctuation is used to shape sentences to encourage the reader to pause at the places that will make understanding easier. Missing commas may cause the reader to stop and re-read in order to parse the words correctly even though the sentence otherwise would be clear. The same applies to excessive commas.

An *appositive* is a noun or noun phrase that renames or substitutes for another noun or noun phrase immediately preceding or following it. It can be recognized by the fact that omitting it would yield a clear and complete sentence and that the additional information in it is not grammatically essential to the statement being made. It should be set off by commas: "His book, the best book on the subject, took years to write. An appositive in the middle of a sentence cannot have a comma on only one side.

When an appositive is short enough or contains essential information, the commas are omitted: "My friend Bob is a student." In mathematical writing, a similar situation applies when notation is introduced: "The *degree* $d(v)$ of a vertex v is the number of neighbors of v." Here "$d(v)$" is a brief appositive. One could argue that the notation for "degree" is not essential to the sense of the sentence, but putting commas around very short appositives of this nature produces very choppy sentences. A speaker need not pause for such appositives, and hence it makes sense to omit the commas.

Conjunctions (such as "and" or "but") require a preceding comma when used to separate clauses. Since a conjunction joins two things, no sentence should begin with "And" or "But". An important sentence form is "If . . ., then . . ."; here each clause has an introductory word, and there should be a comma before "then" (an exception can be made for a brief implication contained within a clause already set off by a comma.) The nature of conjuctions is one reason no sentence should begin with an implicative "Then".

Compound sentences consist of two complete sentences with no conjunction separating them. They need a semicolon (not a comma!) to separate the two parts; this sentence is an example. This form is used especially when the second sentence clarifies the first.

A clause requires a subject and a verb. When "and" joins two parts of a sentence that do not both stand on their own with a subject and a verb, there should be no comma before it. The comma in "We will prove the lemma, and then the theorem" is incorrect.

Many commas within clauses are wrong. Journalists often introduce spurious commas, as in the following published example: "In February, the graduate student in Electrical and Computer Engineering, was awarded the A–B–C Prize." Spurious commas sometimes appear in definitions written by non-native speakers (see later item on *Definitions*).

A *serial comma* is a comma appearing after the penultimate element of a list. For a discussion of its use, see http://en.wikipedia.org/wiki/Serial_comma . It is generally safest to use a comma in this situation. (As one example, compare "Under the conditions $1 \le i, k \le r$ and m even" with "Under the conditions $1 \le i$, $k \le r$, and m even"; the two sentences have different meanings.) For a list of lists, clarity can be achieved by using semicolons or by changing "and" to "&" within list items.

9. *Quotations and ends of sentences.* It is traditional correct style in English grammar that all terminal punctuation comes inside quotation marks. This convention arose from the technical aspects of printing presses. Its purpose was to lessen the danger of breakage of fixed metal type in printing presses. In the era of electronic publishing of mathematics, this justification is obsolete, and we can replace the convention with logical punctuation. When the material being quoted is treated as an item within the sentence and is not itself a sentence, the terminal punctuation logically comes outside the quotation marks.

10. *The naked "This".* When "This" is used as the subject of a sentence, its antecedent is the most recent noun. If the desired antecedent is the preceding paragraph or some other object, then a noun should be inserted, as in "This discussion implies" or "This inequality implies" instead of merely "This implies". One way to treat this issue is to view "this" only as an adjective, not as a pronoun.

11. *Passive voice.* Good writers of English minimize the use of passive voice. This accepted principle applies also in writing mathematics. Active verbs make the exposition more engaging. On the other hand, judicious use of the passive voice can be appropriate.

12. *"If" and "when".* For ease of understanding, a sentence that begins with "If" should at some point later have ", then" to start the conclusion. The word "then" should not be omitted. When the statement is simple and readability would be improved by omitting "then", the sentence should instead start with "When" or "For", as in this sentence itself. Note that a comma still follows the condition introduced by "When" or "For".

13. *"Then."* Used at the beginning of a sentence, the English word "Then" is temporal. The implicative conjunction "then" begins a clause and cannot begin a sentence. The common two-sentence mathematical construction

"Let <hypothesis>. Then <conclusion>."

is grammatically incorrect and can always be avoided.

Since the implicative sense of "then" is so common in mathematics, use of the temporal sense should be avoided. Usually the temporal "then" at the beginning of a sentence can be changed to "Now" or "Next" with less confusion and essentially the same meaning.

14. *Words of hypothesis.* When what follows is a clause, "suppose" or "assume" should be followed by "that". On the other hand, "Assume the hypothesis." is a simple imperative sentence, with verb "Assume" and object "the hypothesis". There is no subsequent verb and hence no later clause. The principle is the same in "Assume $x + y \leq 10$" if one treats the notation as a noun unit. When the relation is viewed as a verb, "that" should be used.

Exception: If the instruction is used in an informal way without precise mathematical notation, then "that" is usually dropped to avoid ponderous language. For example, "I suppose we should go now" and "Suppose the hypothesis is true" would be awkward with "that". If the notation is very short and the verb is a simple "is", then one might again seek a simple expression, as in "Suppose G is a graph", but in this case it is better to write "Let G be a graph".

"Let" and "Suppose". Compare "Suppose $x = 1$" and "Let $x = 1$"; the second sentence is better. The first assumes the truth of an equality; the equation is a unit. The second is more active. Because we never say "Let that . . .", it is understood that the equality sign is the verb. This usage of "Let" is an exception to the treatment of expressions as nouns (see "Expressions as units" below).

15. *Words of conclusion.* A long proof does not fit in a single sentence; often one needs a word to start a sentence that states a conclusion. Avoiding "Then" leaves several other choices, such as "Therefore", "Hence", "Thus", and "So". Purists require a comma after every such introductory word or phrase (as they do after "Finally", "On the other hand", "In 1965", etc.). This can make language overly formal.

Among these choices, "Therefore" is the most formal, likely to introduce a major conclusion and take a comma. Because "Hence" and "Thus" are single syllables, they can be used without commas to indicate the flow of argument without making the writing choppy. Because of its usage in English, "So" is too informal to introduce a sentence of conclusion (with or without comma). Reserve "so" for use as a conjuction: "The graph is connected, so each vertex is reachable from every other vertex."

16. *"Such that" vs. "so that".* "So that" means "in such a way that". Use "such that" when imposing a condition and "so that" when producing a construction in a certain way. In particular, "so that" requires a verb or action and specifies the way in which the action is done. "Such that" generally imposes a condition on a noun structure. Compare "Consider a graph such that no vertex is isolated" and "Color the graph so that no two adjacent vertices have the same color; what follows "such that" modifies "graph", but what follows "so that" is a condition on the coloring.

17. *"Pairwise" and "mutually".* Old-fashioned mathematics took the old-fashioned word "mutually" to describe a binary relation satisfied by all pairs in a set, as in "a set of mutually orthogonal Latin squares". In English usage, "mutual" indicates symmetry. Hence modern mathematics should avoid using "mutually" in this way. Instead, the word "pairwise" states exactly what is meant. The substitution becomes even more important due to modern terms like "mutual independence" in which "mutual" explicitly does not mean pairwise.

18. *Adjectival forms of names.* Some graph theorists use "Hamilton cycle" to mean a spanning cycle in a graph, but this is grammatically incorrect, and they use it only out of habit. They would never say "Abel group". When describing a type of cycle, the modifier must be an adjective, so we use an adjectival form of the name: "Hamiltonian cycle". The same applies to "Euler circuit" and "Eulerian circuit".

19. *Numerals and spelled-out numbers.* In standard English writing, numbers less than 10 usually are spelled in full, while numbers more than 10 are written in numerals. In mathematical writing, the basis for the distinction is different. Numbers less than 10 are spelled out only when used as adjectives expressing the quantity of objects in a set. They remain as numerals when designating the numerical value of an operator. For example, "The two vertices both have degree 3" or "A cycle of length 4 has four edges".

20. *Contractions.* Because mathematical writing is formal, contractions ("can't", "won't", etc.) should usually be avoided. They introduce a sudden informality that is inconsistent with the tone of proof.

Issues of mathematical style

21. *Definitions.* Words being defined should be distinguished by italics (or perhaps boldface in a textbook context). When italics are used to indicate a word being defined, it is unnecessary to use "called" or "said to be"; the use of italics announces that this is the term being defined and replaces these words.

Definitions written by non-native speakers sometimes contain errant commas. In each sentence below, the comma should be deleted.

"A *bipartite graph*, is a graph that is 2-colorable".

"A graph is *bipartite*, if it is 2-colorable".

The examples above illustrate the italicization used in two forms for the definition of a property of a structure. When using the word for the property as a adjective directly modifying the noun, the term being defined is the name for structures that have the property; hence the full term *bipartite graph* is italicized. When the property itself is being defined and is placed as a predicate adjective, only the adjective is italicized.

22. *Double-Duty Definitions.* In general, one cannot make a statement about an object before the object has been defined. Similarly, one cannot use notation in a formula unless the notation has previously been defined. In particular, these tasks cannot correctly be accomplished at the same time with one instance of the notation. For example, "The *neighborhood* of a vertex v is $N(v) = \{u\colon uv \in E(G)\}$" is incorrect; with a subject and a verb before the equation, the equation is a single unit. We have defined the neighborhood of v to be a particular equation, and we have not defined the notation $N(v)$.

Of course, readers with sufficient sophistication or familiarity have no trouble understanding what is meant, but why disenfranchise other readers? One can as easily write "The *neighborhood* of a vertex v, denoted $N(v)$, is $\{u\colon uv \in E(G)\}$". Alternatively, one can introduce the notation as an appositive in a conventional position immediate after the term defined: "The *neighborhood* $N(v)$ of a vertex v is $\{u\colon uv \in E(G)\}$".

A common egregious Double-Duty definition is "Let $G = (V, E)$ be a graph". The sentence defines the equation $G = (V, E)$ to be a graph. Of course, the author intends simultaneously to introduce notation for a particular graph and its vertex set and edge set, but this is not what the sentence says. For this reason, a better practice is to write "Let G be a graph" and use the general operators V and E to refer to the vertex and edge sets of G as $V(G)$ and $E(G)$ (see also later item on "*Operators vs. constants*".)

A more subtle example is "For each $1 \le i \le n$,". The introduction of the notation i has been lost because the inequalities impose conditions on it before it is defined. Since the expression is a unit, grammatically the phrase is referring to each inequality written in this way. Correct alternatives that express the intended meaning include "For all i such that $1 \le i \le n$", "For $i \in [n]$", and "For $1 \le i \le n$". The third option is slightly different from the others; it means "whenever i is such that the conditions hold", implicitly introducing i in a specified range but avoiding the grammatical problem.

23. *Parenthetic or wordless restrictions.* Many writers impose restrictions parenthetically or via commas, thereby omitting words in sentences; this is common in mathematics but imposes unnecessary potential difficulty for the reader. For example, "Suppose there is an edge xy ($\neq e$) in G such that" should be "Suppose that G has an edge xy other than e such that". Similarly, "For $k \leq m$ with k even" is easier to read than "For $k \leq m$ (k even)" or "For $k \leq m$, k even". One can also put words within parentheses: "For $k \leq m$ (where k is even)" or "Suppose that G has an edge xy (with $xy \neq e$) such that". Note that "Suppose that there is an edge $xy \neq e$ in G such that" is a Double-Duty Definition.

24. *Expressions as units.* Notational expressions should be viewed as single objects (nouns, essentially), without assuming that the reader will replace relational symbols by appropriate verbs. For example, "there exists $i < j$ with $x_i = x_j$" ascribes a property to the inequality $i < j$ (and is a Doubly-Duty Definition of i). To introduce i with a constrained value, one should write "there exists i such that $i < j$ and $x_i = x_j$". Consider also "The number of nonneighbors is $n - 1 - d(u) \geq i$." The number of nonneighbors is not an inequality, it is a number. The inequality should be observed after stating that the number of nonneighbors is $n - 1 - d(u)$.

Exceptions. Applying this principle with the simplest expressions leads to ponderous writing. Hence it is reasonable to allow limited exceptions; here we list two.

1) We write "Choose $x \in V(G)$ such that x has minimum degree." We are choosing x, not the expression "$x \in V(G)$". The convention is almost universal that a membership or containment symbol can be read as "in" when introducing an object. (One can treat nonmembership in the same way, though this is less common.)

2) We write "Let $G' = G - x$". When introducing notation for an object or expression by a single imperative verb ("let", "set", "put", "choose", etc.), we read the equality symbol as the verb "equal". Note, however, that the sentence should not simultaneously attempt to say something else about the expression (see "Double Duty Definitions").

If the introductory part of the sentence is longer, then we may already have a noun and a verb, and hence the expression again becomes a unit. For example, "Include each vertex independently with probability $p = (\ln n)/n$" should be "Include each vertex independently with probability p, where $p = (\ln n)/n$".

25. *Mixing words and notation:* We cannot compare words with notation via a relational symbol. Do not write "Consider a graph G with maximum degree $\leq k$". Grammatically, the sentence does not indicate where the inequality starts. If one side is written in words, then the relation must also be written in words. This restriction is a logical consequence of treating the notation as a unit; in that viewpoint, the sentence above says that the maximum degree of G equals the expression $\leq k$.

Similarly, one should not use quantification symbols ($\forall$, $\exists$) to substitute for words in sentences.

These restrictions do not apply to shorthand notation used to save space on slides for a lecture, since the slides summarize the lecture and are accompanied orally by sentences.

Avoiding the mixing of words and notation also explains why an expression involving addition should be enclosed in parentheses when used to modify a noun. For example, we write "k-connected graph" or "$(k+1)$-connected graph", but not "$k+1$-connected graph" or "$k - 1$-connected graph.

26. *Sentence beginnings.* Never begin a sentence with notation. Always one can prepend a specifier (such as "The graph G is" instead of "G is") or rewrite the sentence in another way to avoid starting with notation. Following this rule makes mathematics easier to read.

27. *Contiguous formulas.* Again for ease of reading, avoid placing two formulas consecutively with only a comma separating them. For example, "For $x < 0$, $x^2 > 0$" looks more like a specification of two conditions than like one condition and a conclusion. Similarly, "For some k with $k < n$, $n - k + f(n) < n/2$" requires the reader to stop and go back to insert the missing words. The mathematics will be easier to read if the formulas are separated by the comma plus "it follows that", "we have", etc.

When the second formula is just notation for an object, the intervening English can specify the type of object, as in "When $k = 2$, the graph G is Eulerian."

One can always rewrite to avoid separating notational expressions by only a comma. Sometimes it is very easy, as in changing "For every bipartite graph G, $\chi(G) \leq 2$" to "If G is bipartite, then $\chi(G) \leq 2$".

28. *Specifying two instances.* It is common but ungrammatical to write "Let x, y be vertices in G". Indeed, this is an instance of two formulas separated by a comma. To see what can go wrong, consider the following clause: "Since $a \| b$ and a, b are maximal and minimal". What was meant was: "Since $a \| b$, and a is maximal, and b is minimal". In general, the comma within a list of two elements should be replaced with "and". This is imperative when discussing the two elements as individual items. For example, "If x, y are adjacent" should be "If x and y are adjacent" or "If $\{x, y\}$ is a pair of adjacent vertices".

Exceptions. With a list of size at least three, omission of "and" does not cause as much confusion, and including it introduces ponderousness and awkwardness. Hence there is not much objection to "Let x, y, z be the vertices of T," although one could rewrite this as "Let $\{x, y, z\}$ be the vertex set of T" to be more precise.

Another sensible exception is "Choose $x, y \in V(G)$". Here the relation is between each variable and the set, and we accept this as a single formula. Similarly, many mathematicians write, "For $n, m \geq 2$" to mean the conjunction of $n \geq 2$ and $m \geq 2$. The exception for the membership symbol is consistent with other exceptions for the membership symbol; doing it with inequalities is less defensible. Avoid doing it with equalities (see "Equality between a variable and a list" below).

29. *Words indicating universal quantification.* The word "any" can mean "some" or "all" in different contexts, so it can be imprecise. It is clearer to use "each" or "every" as a universal quantifier when referring to a singular object.

Numbered plural variables cause difficulty. In English, "for every two elements" is awkward because "every" is singular. Thus here it is better to say "for any two elements". The presence of "for" is suggestive of the universal quantification and helps avoid ambiguity. Nevertheless, there may still be confusion: consider the sentence "Form G' from G by adding an edge joining any two vertices with distance 2 in G." Some readers will think that only one edge is added, so this exception must be used with care.

Avoiding "any" is not imperative. Evaluate its use in context, making sure to prevent misinterpretation. Note also that "any" can be a good substitute for "an arbitrary".

Using an indefinite article ("a" or "an") as a universal quantifier can be dangerous, as in "Prove that a bipartite graph has no odd cycle." Putting "must" before the conclusion suggests universality; using "every" is clearer. Although the indefinite quantifier is correct, some may interpret "a" as "one" or "some", turning universality into existence.

30. *Position of universal quantifiers.* Although there is a preference for specifying the universe over which a formula or statement holds before stating it, when there is a single universal quantification the sentence may read better with the quantifier at the end. This order has the added benefit of emphasizing the conclusion when the context is easily understood. For example, one might prefer to write "For every graph G that is bipartite, $\chi(G) \le 2$" as "Always $\chi(G) \le 2$ when G is bipartite".

31. *"Maximal" vs. "maximum".* Many mathematicians use these words interchangeably. One can make a useful distinction by using "maximum" to compare numbers or sizes and "maximal" to compare sets or other objects. For example, a *maximal* object of type A is an object of type A that is not contained in any other object of type A. A *maximum* object of type A is a largest object of type A; here "maximum" is an abbreviation for "maximum-sized". For example, in a graph we may speak of "maximal independent sets" and "maximum independent sets"; these are convenient terms for distinct concepts that are both important.

32. *"Less" vs. "fewer".* Use "less" when comparing numbers, and use "fewer" when referring to a set of objects. For example, "the number of edges is less than k" is correct, as is "the graph has fewer than k edges".

33. *Apples and oranges.* Comparing incomparable quantities is often called "comparing apples and oranges". One cannot compare a set with an integer; it is incorrect to write "Sperner proved that no antichain of subsets of an n-set is larger than $\binom{n}{\lfloor n/2 \rfloor}$". One must distinguish between a set and its size. Here one can write "no antichain has size greater than $\binom{n}{\lfloor n/2 \rfloor}$" or "no antichain has more than $\binom{n}{\lfloor n/2 \rfloor}$ elements".

34. *Bounds and inequalities.* Many non-native speakers use "bound of" when they mean "bound on". If $x \le k$, then we have an upper bound of k on x. Using "bound of x" for "bound on x" can become confusing when comparing parameters. We do not want to say that the maximum degree $\Delta(G)$ is a bound of the chromatic number $\chi(G)$; when $\Delta(G) = k$ we want to say that $\Delta(G)$ establishes a bound *of* k *on* the chromatic number of the graph.

Many mathematicians, particularly analysts, use the English word "estimate" to have the meaning of the English word "bound" (both for nouns and verbs). They write "now we estimate this quantity" when they mean "now we prove an upper bound on this quantity". In English, "estimate" means "approximate"; both upper and lower bounds are needed to give an estimate. The common usage of analysts is incorrect English, even if they are assuming an unstated implicit lower bound of 0.

35. *Partial case.* In English, we do not say that one result is a "partial case" of another. We say that it is a special case. This is another issue of translation from one language to another.

Terminology and notation (especially in discrete mathematics)

36. *Definition symbol ":=".* Some mathematicians use this symbol to indicate that the preceding symbol is being defined to mean the subsequent object. If this occurs in a sentence like "Let $[n] := \{1, \dots, n\}$", then the verb states that the notation is being defined, and the special notation is unnecessary. If it occurs in a sentence about the object being defined, such as "Consider a coloring of $[n] := \{1, \dots, n\}$", then it is an improper Double-Duty Definition and should be rewritten: "Consider a coloring of $[n]$, where $[n] = \{1, \dots, n\}$." Reading ":=" requires thinking "be/is defined to be"; this awkward notation is never needed and encourages grammatical problems.

37. *Equality between a variable and a list.* Many mathematicians write "for $m = 1, 2, \dots, n$" to mean "for $m \in \{1, 2, \dots, n\}$" or "for $1 \le m \le n$". The first expression is mathematically incorrect; it sets the value of m to be a list of numbers. The same principle applies to writing "$i = 1, 2$" to name two cases; this should be $i \in \{1, 2\}$.

Common usage of "Big Oh" notation also suffers from this problem. The expression "$f(n) = O(n^2)$" does not mean that the value $f(n)$ equals the set represented by the notation $O(n^2)$. To be precise, what is meant is "$f(n) \in O(n^2)$"; Knuth has written at length on this subject. Unfortunately, it is convenient to do arithmetic with these classes of functions, so this problem will not go away. It stems from the difficulty of distinguishing between an equivalence class and its elements, along with the fact that no good notation exists for this equivalence relation (unlike "$\equiv$" for congruence). An unsatisfying compromise is to use the membership symbol where the grammar of computation permits, in order to ensure that the meaning of the concept is understood.

38. *Sequences, series, and lists.* In mathematics, a sequence is a function whose domain is the set of natural numbers (perhaps with a shift of the initial element). Discrete mathematicians abuse this term in using it for a finite set in a linear order. A good name for such an object is *list*. An n-tuple is a list of length n. It is an abuse of terminology to say "a sequence of length n".

The usage of "series" in English is contrary to its usage in mathematics. In English a "series" usually consists of finitely many occurrences in order, as in the "World Series" or the title "A Series of Unfortunate Events". In mathematics a series is an infinite sum.

Writing "$v_1, v_2, \dots, v_n$" for an indexed n-tuple is a style used to suggest that the elements are indexed by the first n positive integers with no skips. However, the most natural interpretation of the expression "$v_1, \dots, v_n$" is exactly the same. The appropriate convention is that indices in a list are consecutive unless the author explicitly indicates otherwise. Another reason to eliminate v_2 from the expression is that "$v_1, v_2, \dots, v_n$" forbids the possibility $n = 1$.

Note also that the sentence "Let $x_1 < \cdots < x_n$ be a list of integers" is a Double-Duty Definition; the writer attempts simultaneously to introduce notation for the elements of a list and to impose inequalities on them. The expression $x_1 < \cdots < x_n$ denotes a set of relations, not a list; what is meant is "Let $x_1, \dots, x_n$ be integers such that $x_1 < \cdots < x_n$". Similarly, a chain under inclusion is a list of sets $A_1, \dots, A_k$ such that $A_1 \subseteq \cdots \subseteq A_k$; the expression "$A_1 \subseteq \cdots \subseteq A_k$" is not itself a chain.

39. *Multicharacter operators.* A string of letters in notation denotes the product of individual quantities. Therefore, any operator whose notation is more than one character should be in a different font, generally roman. This convention is well understood for trigonometric, exponential, and logarithmic functions, and it applies equally well to such operators as dimension (dim), crossing number (cr), etc.

40. *Operators vs. constants.* We never use f to denote the value of a function f at a point x. The same principle applies to graph parameters and other operators. For example, the maximum degree of a graph G is denoted $\Delta(G)$. Here Δ is a function, not a number, and hence Δ should not be used to denote the value of the function Δ on a particular graph. In particular, it is poor writing to refer to the class of graphs with maximum degree Δ, while it is equally easy to refer to the class of graphs with maximum degree D.

It is true that "We write $V = V(G)$ and $\Delta = \Delta(G)$" is a convenient and common mnemonic device. It generally does not cause problems when discussing only one graph at a time. The difference between this situation and that of a function $f\colon \mathbb{R} \to \mathbb{R}$ is that we rarely focus only on the value of f at one point. Nevertheless, the same notation should not be used for distinct concepts in a single discussion, and this principle applies here. (A certain textbook author violated this principle in the use of $n(G)$ and $e(G)$ for the numbers of vertices and edges in a graph G; the error will be corrected in the next edition.)

41. *Notation with graphs.* A graph is not a set. When G is a graph and h is a vertex in it, it makes no sense to write $h \in G$, since h could just as easily be an edge. A graph consists of a vertex set and an edge set; one should write $v \in V(G)$ and $e \in E(G)$. This is also the reason why the convenient notations $|G|$ and $||G||$ are mathematically inconsistent for the order and size of a graph.

42. *Cliques and complete subgraphs.* These terms traditionally were used interchangeably in graph theory. There is a difference between referring to a set of pairwise adjacent vertices in a graph (the opposite of an independent set of vertices) and referring to a subgraph isomorphic to a complete graph. We need terminology for such a vertex set, and "clique" seems to be the best choice.

43. *Between.* An object that is between two other objects separates them; this is the common mathematical sense of "between". Hence referring to an edge with endpoints u and v as an edge "between" u and v is inconsistent mathematical usage. One can always say "an edge *joining* u and v" instead. In a planar embedding of a graph, we could say that an edge shared by the boundaries of two faces is an an edge between them.

44. *\setminus.* The operator $\setminus$ most often denotes difference of sets. Hence it is somewhat misleading or old-fashioned (and looks rather pompous) to use it for deletion of elements, as in "$G \setminus e$". Use "$G - e$" instead. Also, the notation $G \setminus H$ is easily confused with G/H (especially by students). Of course, there are some contexts (matroids and various algebraic topics), where these notations have special meanings and are quite important, but I think that for simple set difference $A - B$ is preferable.