Create new identities...using the ones we already have.

Use the angle sum identity to express sin(2A).

$\sin(A+A)$ $\qquad \sin(A+B) = \sin A\cos B + \cos A \sin B$

$= \sin A\cos A + \cos A \sin A$

$= 2\sin A\cos A$

$\sin 2A = 2\sin A\cos A$

Use the angle sum identity to express cos(2A).

$\cos(A+A)$ $\qquad \cos(A+B) = \cos A\cos B - \sin A \sin B$

$\cos A\cos A - \sin A \sin A$

$\cos^2 A - \sin^2 A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$ $\qquad \cos^2 A = 1 - \sin^2 A$

$\cos 2A = \cos^2 A - (1 - \cos^2 A)$

$\boxed{\cos 2A = 2\cos^2 A - 1}$

$\cos 2A = (1 - \sin^2 A) - \sin^2 A$

$\boxed{\cos 2A = 1 - 2\sin^2 A}$

Section 5.3: Double Angle Identites

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

Example: Let's practice using this identity to find sin90°

$\sin(90^\circ)$

$\sin(2A) = \sin(2 \cdot \overset{A}{45^\circ})$

$2\sin A\cos A$ $\qquad 2\sin 45^\circ \cos 45^\circ$

$\dfrac{2}{1} \cdot \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$\dfrac{4}{4}$

1

Find the exact value for cos 600º

$$\cos 2A = \cos 600^\circ$$

match

$2A = 600^\circ$

$A = 300^\circ$

$$\cos 600^\circ = \cos(2\cdot 300^\circ)$$
$$= \cos^2 A - \sin^2 A$$
$$= \cos^2 300^\circ - \sin^2 300^\circ$$
$$= (\cos 300^\circ)^2 - (\sin 300^\circ)^2$$
$$= \left(\frac{1}{2}\right)^2 - \left(-\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= -\frac{2}{4}$$
$$\cos 600^\circ = -\frac{1}{2}$$

Example: Use a double angle formula to find the exact value of tan 2*A* if cos *A* = -3/4 and *A* is in quadrant 3.

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Lets get $\tan A = \frac{\sin A}{\cos A}$

$$\tan A = \frac{-\frac{\sqrt{7}}{4}}{-\frac{3}{4}} = \frac{\sqrt{7}}{3}$$

A in Q3

$\cos A = -\frac{3}{4}$

Use an identity

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A + \left(-\frac{3}{4}\right)^2 = 1$

$\sin^2 A + \frac{9}{16} = 1$

$\sin^2 A = \frac{16}{16} - \frac{9}{16}$

$\sin^2 A = \frac{7}{16}$

$\sin A = \pm\sqrt{\frac{7}{16}}$

$\sin A = \pm\frac{\sqrt{7}}{4}$

$\sin A = \frac{-\sqrt{7}}{4}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$
$$= \frac{2\left(\frac{\sqrt{7}}{3}\right)}{1-\left(\frac{\sqrt{7}}{3}\right)^2}$$
$$= \frac{\frac{2\sqrt{7}}{3}}{\frac{9}{9} - \frac{7}{9}}$$
$$= \frac{\frac{2\sqrt{7}}{3}}{\frac{2}{9}} = \frac{\cancel{2}\sqrt{7}}{\cancel{3}}\cdot\frac{\cancel{9}\,3}{\cancel{2}} = 3\sqrt{7}$$

$$\tan 2A = 3\sqrt{7}$$

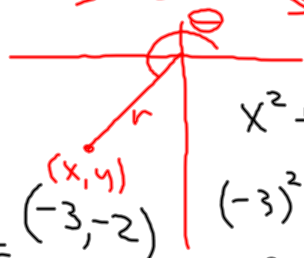
Example: If $\tan\theta = 2/3$ and θ is in quadrant 3, then find $\cos 2\theta$.

θ Q3

Use $\tan\theta = \frac{2}{3} = \frac{y}{x}$

$\cancel{\frac{\sin\theta}{\cos\theta} = \frac{2}{3}}$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(\frac{-3}{\sqrt{13}}\right)^2 - 1$$
$$= 2\left(\frac{9}{13}\right) - 1$$
$$= \frac{18}{13} - \frac{13}{13}$$
$$\cos 2A = \frac{5}{13}$$



$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}}$

$x^2 + y^2 = r^2$

$(-3)^2 + (-2)^2 = r^2$

$9 + 4 = r^2$

$13 = r^2$

$\sqrt{13} = r$

Simplify each of the following by recognizing the form of an identity.

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$\sin 2A = 2\sin A\cos A$

$2\sin 10^{\circ}\cos 10^{\circ}$

$2\sin A\cos A = \sin 2(10^{\circ})$

$= \sin 20^{\circ}$

$\cos^2 \overset{A}{\pi/8} - \sin^2 \overset{A}{\pi/8}$ $= \cos 2\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos^2 A - \sin^2 A = \cos 2A$