

1. Double Angle Formulas

$$\sin(2u) = 2 \sin u \cos u$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\begin{aligned} \cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u \end{aligned}$$

2. EXAMPLE: Solving a Multiple Angle Equation

Find all solutions of  $2 \cos x + \sin 2x = 0$

3. EXAMPLE: Using double-angle formulas

(a) If  $x = 4 \sin \theta$ ,  $0 < \theta < \frac{\pi}{2}$ , express  $\sin 2\theta$  in terms of  $x$ .

(b) Use the following to find  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ .

$$\cos \theta = \frac{5}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi$$

4. EXAMPLE: Using double-angle formulas to prove an identity

Prove the identity  $\sin 3x = 3 \sin x - 4 \sin^3 x$

5. Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

6. EXAMPLE: Reducing a power

Rewrite  $\sin^4 x$  as a sum of first powers of the cosines of multiple angles.

7. Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

8. EXAMPLE: Using a Half-Angle Formula

Evaluate  $\cos 105^\circ$  using a half-angle formula.