On the Geometry of the General Solution for the Vacuum Field of the Point-Mass

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The black hole, which arises solely from an incorrect analysis of the Hilbert solution, is based upon a misunderstanding of the significance of the coordinate radius r. This quantity is neither a coordinate nor a radius in the gravitational field and cannot of itself be used directly to determine features of the field from its metric. The appropriate quantities on the metric for the gravitational field are the proper radius and the curvature radius, both of which are functions of r. The variable r is actually a Euclidean parameter which is mapped to non-Euclidean quantities describing the gravitational field, namely, the proper radius and the curvature radius.

1 Introduction

The variable r has given rise to much confusion. In the conventional analysis, based upon the Hilbert metric, which is almost invariably and incorrectly called the "Schwarzschild" solution, r is taken both as a coordinate and a radius in the spacetime manifold of the point-mass. In my previous papers [1, 2] on the general solution for the vacuum field, I proved that r is neither a radius nor a coordinate in the gravitational field (M_g, g_g), as Stavroulakis [3, 4, 5] has also noted. In the context of (M_g, g_g) r is a Euclidean parameter in the flat spacetime manifold (M_s, g_s) of Special Relativity. Insofar as the point-mass is concerned, r specifies positions on the real number line, the radial line in (M_s, g_s), not in the spacetime manifold of the gravitational field, (M_g, g_g). The gravitational field gives rise to a mapping of the *distance* $D = |r - r_0|$ between two *points* $r,\ r_0 \in \Re$ into (M_g, g_g). Thus, r becomes a *parameter* for the spacetime manifold associated with the gravitational field. If $R_p \in (M_g, g_g)$ is the proper radius, then the gravitational field gives rise to a mapping ψ,

$$\psi : |r - r_0| \in (\Re - \Re^-) \to R_p \in (M_g, g_g)\,, \qquad (A)$$

where $0 \leqslant R_p < \infty$ in the gravitational field, on account of R_p being a *distance* from the point-mass located at the point $R_p(r_0) \equiv 0$.

The mapping ψ must be obtained from the geometrical properties of the metric tensor of the solution to the vacuum field. The r-parameter location of the point-mass does not have to be at $r_0 = 0$. The point-mass can be located at any point $r_0 \in \Re$. A test particle can be located at any point $r \in \Re$. The point-mass and the test particle are located at the end points of an interval along the *real line* through r_0 and r. The distance between these points is $D = |r - r_0|$. In (M_s, g_s), r_0 and r may be thought of as describing 2-spheres about an origin $r_c = 0$, but only the distance between these 2-spheres enters into consideration. Therefore, if two test particles are located, one at any point on the 2-sphere $r_0 \neq 0$ and one at a point on the 2-sphere $r \neq r_0$ on the radial line through r_0 and r, the distance between them is the length of the radial interval between the 2-spheres, $D = |r - r_0|$. Consequently, the domain of both r_0 and r is the real number line. In this sense, (M_s, g_s) may be thought of as a parameter space for (M_g, g_g), because ψ maps the *Euclidean distance* $D = |r - r_0| \in (M_s, g_s)$ into the *non-Euclidean proper distance* $R_p \in (M_g, g_g)$: the radial line in (M_s, g_s) is precisely the real number line. Therefore, the required mapping is appropriately written as,

$$\psi : |r - r_0| \in (M_s, g_s) \to R_p \in (M_g, g_g)\,. \qquad (B)$$

In the pseudo-Euclidean (M_s, g_s) the polar coordinates are $r,\ \theta, \varphi$, but in the pseudo-Riemannian manifold (M_g, g_g) of the point-mass and point-charge, r is *not* the radial coordinate. Conventionally there is the persistent misconception that what are polar coordinates in Minkowski space must also be polar coordinates in Einstein space. This however, does not follow in any rigorous way. In (M_g, g_g) the variable r is nothing more than a real-valued parameter, of no physical significance, for the true radial quantities in (M_g, g_g). The parameter r *never* enters into (M_g, g_g) directly. Only in Minkowski space does r have a direct physical meaning, as mapping (B) indicates, where it is a radial coordinate. Henceforth, when I refer to the radial coordinate or r-parameter I always mean $r \in (M_s, g_s)$.

The solution for the gravitational field of the simple configurations of matter and charge requires the determination of the mapping ψ. The orthodox analysis has completely failed to understand this and has consequently failed to solve the problem.

The conventional analysis simply looks at the Hilbert metric and makes the following unjustified assumptions, tacitly or otherwise;

(a) *The variable r is a radius and/or coordinate of some kind in the gravitational field.*

(b) *The regions $0 < r < 2m$ and $2m < r < \infty$ are both valid.*

(c) *A singularity in the gravitational field must occur only where the Riemann tensor scalar curvature invariant (Kretschmann scalar) $f = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is unbounded.*

The orthodox analysis has never proved these assumptions, but nonetheless simply takes them as given, finds for itself a curvature singularity at $r = 0$ in terms of f, and with legerdemain reaches it by means of an *ad hoc* extension in the ludicrous Kruskal-Szekeres formulation. However, the standard assumptions are incorrect, which I shall demonstrate with the required mathematical rigour.

Contrary to the usual practise, one *cannot* talk about extensions into the region $0 < r < 2m$ or division into R and T regions until it has been rigorously established that the said regions are valid to begin with. One *cannot* treat the r-parameter as a radius or coordinate of any sort in the gravitational field without first demonstrating that it is such. Similarly, one *cannot* claim that the scalar curvature must be unbounded at a singularity in the gravitational field until it has been demonstrated that this is truly required by Einstein's theory. Mere *assumption* is *not* permissible.

2 The basic geometry of the simple point-mass

The usual metric g_s of the spacetime manifold (M_s, g_s) of Special Relativity is,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \qquad (1)$$

The foregoing metric can be statically generalised for the simple (i. e. non-rotating) point-mass as follows,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \quad (2a)$$

$$A, B, C > 0 ,$$

where A, B, C are analytic functions. I emphatically remark that *the geometric relations between the components of the metric tensor of (2a) are precisely the same as those of (1)*.

The standard analysis writes (2a) as,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \quad (2b)$$

and claims it the most general, which is incorrect. The form of $C(r)$ cannot be pre-empted, and must in fact be rigorously determined from the general solution to (2a). The physical features of (M_g, g_g) must be determined exclusively by means of the resulting $g_{\mu\mu} \in (M_g, g_g)$, *not* by foisting upon (M_g, g_g) the interpretation of elements of (M_s, g_s) in the misguided fashion of the orthodox relativists who, having written (2b), incorrectly treat r in (M_g, g_g) precisely as the r in (M_s, g_s).

With respect to (2a) I identify the coordinate radius, the r-parameter, the radius of curvature, and the proper radius as follows:

(a) The coordinate radius is $D = |r - r_0|$.

(b) The r-parameter is the variable r.

(c) The radius of curvature is $R_c = \sqrt{C(r)}$.

(d) The proper radius is $R_p = \int \sqrt{B(r)}dr$.

The orthodox motivation to equation (2b) is to evidently obtain the circumference χ of a great circle, $\chi \in (M_g, g_g)$ as,

$$\chi = 2\pi r ,$$

to satisfy its unproven assumptions about r. But this equation is only formally the same as the equation of a circle in the Euclidean plane, because in (M_g, g_g) it describes a non-Euclidean great circle and therefore does not have the same meaning as the equation for the ordinary circle in the Euclidean plane. The orthodox assumptions distort the fact that r is only a real parameter in the gravitational field and therefore that (2b) is not a general, but a particular expression, in which case the form of $C(r)$ has been fixed to $C(r) = r^2$. Thus, the solution to (2b) can only produce a particular solution, not a general solution in terms of $C(r)$, for the gravitational field. Coupled with its invalid assumptions, the orthodox relativists obtain the Hilbert solution, a correct *particular form* for the metric tensor of the gravitational field, but interpret it incorrectly with such a great thoroughness that it defies rational belief.

Obviously, the spatial component of (1) describes a sphere of radius r, centred at the point $r_0 = 0$. On this metric $r \geqslant r_0$ is usually assumed. Now in (1) the distance D between two points on a radial line is given by,

$$D = |r_2 - r_1| = r_2 - r_1 . \qquad (3)$$

Furthermore, owing to the "origin" being usually fixed at $r_1 = r_0 = 0$, there is no distinction between D and r. Hence r is both a coordinate *and* a radius (distance). However, the correct description of points by the spatial part of (1) must still be given in terms of *distance*. Any point in any direction is specified by its *distance* from the "origin". It is this distance which is the important quantity, not the coordinate. It is simply the case that on (1), in the usual sense, the distance and the coordinate are identical. Nonetheless, the distance from the designated "origin" is still the important quantity, not the coordinate. It is therefore clear that the designation of an origin is arbitrary and one can select *any* $r_0 \in \Re$ as the origin of coordinates. Thus, (1) is a special case of a general expression in which the origin of coordinates is arbitrary and the distance from the origin to another point does not take the same value as the coordinate designating it. The "origin" $r_0 = 0$ has *no intrinsic meaning*. The relativists and the mathematicians have evidently failed to understand this elementary geometrical fact. Consequently, they have managed to attribute to $r_0 = 0$ miraculous qualities of which it is not worthy, one of which is the formation of the black hole.

Equations (1) and (2a) are not sufficiently general and so their forms suppress their true geometrical characteristics. Consider two points P_1 and P_2 on a radial line in Euclidean 3-space. With the usual Cartesian coordinates let P_1 and P_2 have coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively. The distance between these points is,

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = \\ = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2 + |z_1 - z_2|^2} \geqslant 0 \,. \tag{4}$$

If $x_1 = y_1 = z_1 = 0$, D is usually called a radius and so written $D \equiv r$. However, one may take P_1 or P_2 as an origin for a sphere of radius D as given in (4). Clearly, a general description of 3-space must rightly take this feature into account. Therefore, the most general line-element for the gravitational field in quasi-Cartesian coordinates is,

$$ds^2 = F dt^2 - G\left(dx^2 + dy^2 + dz^2\right) - \\ - H\left(|x - x_0|dx + |y - y_0|dy + |z - z_0|dz\right)^2 , \tag{5}$$

where $F, G, H > 0$ are functions of

$$D = \sqrt{|x - x_0|^2 + |y - y_0|^2 + |z - z_0|^2} = |r - r_0| \,,$$

and $P_0\,(x_0, y_0, z_0)$ is an arbitrary origin of coordinates for a sphere of radius D centred on P_0.

Transforming to spherical-polar coordinates, equation (5) becomes,

$$ds^2 = -H|r - r_0|^2 dr^2 + F dt^2 - \\ - G\left(dr^2 + |r - r_0|^2 d\theta^2 + |r - r_0|^2 \sin^2\theta d\varphi^2\right) = \\ = A(D)dt^2 - B(D)dr^2 - C(D)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \tag{6}$$

where $A, B, C > 0$ are functions of $D = |r - r_0|$. Equation (6) is just equation (2a), but equation (2a) has suppressed the significance of distance and the arbitrary origin and is therefore invariably taken with $D \equiv r \geqslant 0$, $r_0 = 0$.

In view of (6) the most general expression for (1) for a sphere of radius $D = |r - r_0|$, centred at some $r_0 \in \Re$, is therefore,

$$ds^2 = dt^2 - dr^2 - \left(r - r_0\right)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) = \tag{7a}$$

$$= dt^2 - \frac{\left(r - r_0\right)^2}{|r - r_0|^2} dr^2 - \left|r - r_0\right|^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) = \tag{7b}$$

$$= dt^2 - \left(d|r - r_0|\right)^2 - \left|r - r_0\right|^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \tag{7c}$$

The spatial part of (7) describes a sphere of radius $D = |r - r_0|$, centred at the arbitrary point r_0 and reaching to some point $r \in \Re$. Indeed, the curvature radius R_c of (7) is,

$$R_c = \sqrt{\left(r - r_0\right)^2} = \left|r - r_0\right| , \tag{8}$$

and the circumference χ of a great circle centred at r_0 and reaching to r is,

$$\chi = 2\pi\left|r - r_0\right| . \tag{9}$$

The proper radius (distance) R_p from r_0 to r on (7) is,

$$R_p = \int_0^{|r - r_0|} d\left|r - r_0\right| = \int_{r_0}^{r} \left[\frac{r - r_0}{\left|r - r_0\right|}\right] dr = \left|r - r_0\right| . \tag{10}$$

Thus $R_p \equiv R_c \equiv D$ on (7), owing to its pseudo-Euclidean nature.

It is evident by similar calculation that $r \equiv R_c \equiv R_p$ in (1). Indeed, (1) is obtained from (7) when $r_0 = 0$ and $r \geqslant r_0$ (although the absolute value is suppressed in (1) and (7a)). The geometrical relations between the components of the metric tensor are *inviolable*. Therefore, in the case of (1), the following obtain,

$$D = |r| = r \,, \\ R_c = \sqrt{|r|^2} = \sqrt{r^2} = r \,, \\ \chi = 2\pi|r| = 2\pi r \,, \\ R_p = \int_0^{|r|} d\,|r| = \int_0^{r} dr = r \,. \tag{11}$$

However, equation (1) hides the true arbitrary nature of the origin r_0. Therefore, the correct geometrical relations have gone unrecognized by the orthodox analysis. I note, for instance, that G. Szekeres [6], in his well-known paper of 1960, considered the line-element,

$$ds^2 = dr^2 + r^2 d\omega^2 \,, \tag{12}$$

and proposed the transformation $\overline{r} = r - 2m$, to allegedly carry (12) into,

$$ds^2 = d\overline{r}^2 + \left(\overline{r} - 2m\right)^2 d\omega^2 \,. \tag{13}$$

The transformation to (13) by $\overline{r} = r - 2m$ is incorrect: by it Szekeres should have obtained,

$$ds^2 = d\overline{r}^2 + \left(\overline{r} + 2m\right)^2 d\omega^2 \,. \tag{14}$$

If one sets $r = \overline{r} - 2m$, then (13) obtains from (12). Szekeres then claims on (13),

> "*Here we have an apparent singularity on the sphere $\overline{r} = 2m$, due to a spreading out of the origin over a sphere of radius $2m$. Since the exterior region $\overline{r} > 2m$ represents the whole of Euclidean space (except the origin), the interior $\overline{r} < 2m$ is entirely disconnected from it and represents a distinct manifold.*"

His claims about (13) are completely false. He has made an incorrect assumption about the origin. His equation (12) describes a sphere of radius r centred at $r = 0$, being identical to the spatial component of (1). His equation (13) is precisely the spatial component of equation (7) with $r_0 = 2m$ and $r \geqslant r_0$, and therefore actually describes a sphere of radius $D = \overline{r} - 2m$ centred at $\overline{r}_0 = 2m$. His claim that $\overline{r} = 2m$ describes a sphere is due to his invalid assumption that $\overline{r} = 0$ has some intrinsic meaning. It did not come from his transformation. The claim is false. Consequently there is no interior region at all and no distinct manifold anywhere. All Szekeres did unwittingly was to move the origin for a sphere from the coordinate value $r_0 = 0$ to the coordinate value $r_0 = 2m$. In fact, he effectively repeated the same error committed by Hilbert [8] in 1916, an error, which in one guise or another, has been repeated relentlessly by the orthodox theorists.

It is now plain that r is neither a radius nor a coordinate in the metric (6), but instead gives rise to a *parameterization* of the relevant radii R_c and R_p on (6).

Consider (7) and introduce a test particle at each of the *points* r_0 and r. Let the particle located at r_0 acquire mass. The coordinates r_0 and r do not change, however in the gravitational field (M_g, g_g) the distance between the point-mass and the test particle, and the radius of curvature of a great circle, centred at r_0 and reaching to r in the parameter space (M_s, g_s), will no longer be given by (11).

The solution of (6) for the vacuum field of a point-mass will yield a mapping of the Euclidean *distance* $D = |r - r_0|$ into a non-Euclidean proper radius $R_P(r)$ in the pseudo-Riemannian manifold (M_g, g_g), locally generated by the presence of matter at the r-parameter $r_0 \in (M_s, g_s)$, i. e. at the invariant point $R_p(r_0) \equiv 0$ in (M_g, g_g).

Transform (6) by setting,

$$R_c = \sqrt{C(D(r))} = \frac{\chi}{2\pi}\,, \tag{15}$$

$$D = |r - r_0|\,.$$

Then (6) becomes,

$$ds^2 = A^*(R_c)dt^2 - B^*(R_c)dR_c^2 - - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{16}$$

In the usual way one obtains the solution to (16) as,

$$ds^2 = \left(\frac{R_c - \alpha}{R_c}\right)dt^2 - - \left(\frac{R_c}{R_c - \alpha}\right)dR_c^2 - R_c^2(d\theta^2 + \sin^2\theta d\varphi^2)\,,$$

$$\alpha = 2m\,,$$

which by using (15) becomes,

$$ds^2 = \left(\frac{\sqrt{C} - \alpha}{\sqrt{C}}\right)dt^2 - - \left(\frac{\sqrt{C}}{\sqrt{C} - \alpha}\right)\frac{C'^2}{4C}\left[\frac{r - r_0}{|r - r_0|}\right]^2 dr^2 - C(d\theta^2 + \sin^2\theta d\varphi^2)\,,$$

that is,

$$ds^2 = \left(\frac{\sqrt{C} - \alpha}{\sqrt{C}}\right)dt^2 - - \left(\frac{\sqrt{C}}{\sqrt{C} - \alpha}\right)\frac{C'^2}{4C}dr^2 - C(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{17}$$

which is the line-element derived by Abrams [7] by a different method. Alternatively one could set $r = R_c$ in (6), as Hilbert in his work [8] effectively did, to obtain the familiar Droste/Weyl/(Hilbert) line-element,

$$ds^2 = \left(\frac{r - \alpha}{r}\right)dt^2 - \left(\frac{r}{r - \alpha}\right)dr^2 - - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{18}$$

and then noting, as did J. Droste [9] and A. Eddington [10], that r^2 can be replaced by a general analytic function of r without destroying the spherical symmetry of (18). Let that function be $C(D(r))$, $D = |r - r_0|$, and so equation (17) is again obtained. Equation (18) taken literally is an *incomplete* particular solution since the boundary on the r-parameter has not yet been rigorously established, but equation (17) provides a way by which the form of $C(D(r))$ might be determined to obtain a means by which all particular solutions, in terms of an infinite sequence, may be constructed, according to the general prescription of Eddington. Clearly, the correct form of $C(D(r))$ must naturally yield the Droste/Weyl/(Hilbert) solution, as well as the true Schwarzschild solution [11], and the Brillouin solution [12], amongst the infinitude of particular solutions that the field equations admit. (Fiziev [13] has also shown that there exists an infinite number of solutions for the point-mass and that the Hilbert black hole is not consistent with general relativity.)

In the gravitational field only the circumference χ of a great circle is a measurable quantity, from which R_c and R_p

are calculated. To obtain the metric for the field in terms of χ, use (15) in (17) to yield,

$$ds^2 = \left(1 - \frac{2\pi\alpha}{\chi}\right) dt^2 - \left(1 - \frac{2\pi\alpha}{\chi}\right)^{-1} \frac{d\chi^2}{4\pi^2} - \\ - \frac{\chi^2}{4\pi^2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (19) \\ \alpha = 2m\,.$$

Equation (19) is independent of the r-parameter entirely. Since only χ is a measurable quantity in the gravitational field, (19) constitutes the correct solution for the gravitational field of the simple point-mass. In this way (19) is truely the *only* solution to Einstein's field equations for the simple point-mass.

The only assumptions about r that I make are that the point-mass is to be located somewhere, and that somewhere is r_0 in parameter space (M_s, g_s), the value of which must be obtained rigorously from the geometry of equation (17), and that a test particle is located at some $r \neq r_0$ in parameter space, where $r,\ r_0 \in \Re$.

The geometrical relationships between the components of the metric tensor of (1) must be precisely the same in (6), (17), (18), and (19). Therefore, the circumference χ of a great circle on (17) is given by,

$$\chi = 2\pi\sqrt{C(D(r))}\,,$$

and the proper distance (proper radius) $R_p(r)$ on (6) is,

$$R_p(r) = \int \sqrt{B(D(r))}dr\,.$$

Taking $B(D(r))$ from (17) gives,

$$R_p(D) = \int \sqrt{\frac{\sqrt{C}}{\sqrt{C}-\alpha}\,\frac{C'}{2\sqrt{C}}}\,dr = \\ = \sqrt{\sqrt{C(D)}\left(\sqrt{C(D)}-\alpha\right)} + \qquad (20) \\ + \alpha \ln\left|\frac{\sqrt{\sqrt{C(D)}} + \sqrt{\sqrt{C(D)}-\alpha}}{K}\right|, \\ D = |r - r_0|\,, \\ K = \text{const.}$$

The relationship between r and R_p is,

$$as\ r \to r_0^{\pm},\ R_p(r) \to 0^+\,,$$

or equivalently,

$$as\ D \to 0^+,\ R_p(r) \to 0^+\,,$$

where r_0 is the parameter space location of the point-mass. Clearly $0 \leqslant R_p < \infty$ always and the point-mass is invariantly located at $R_p(r_0) \equiv 0$ in (M_g, g_g), a manifold with boundary.

From (20),

$$R_p(r_0) \equiv 0 = \sqrt{\sqrt{C(r_0)}\left(\sqrt{C(r_0)}-\alpha\right)} + \\ + \alpha \ln\left|\frac{\sqrt{\sqrt{C(r_0)}} + \sqrt{\sqrt{C(r_0)}-\alpha}}{K}\right|,$$

and so,

$$\sqrt{C(r_0)} \equiv \alpha,\ K = \sqrt{\alpha}.$$

Therefore (20) becomes

$$R_p(r) = \sqrt{\sqrt{C(|r-r_0|)}\left(\sqrt{C(|r-r_0|)}-\alpha\right)} + \\ + \alpha \ln\left|\frac{\sqrt{\sqrt{C(|r-r_0|)}} + \sqrt{\sqrt{C(|r-r_0|)}-\alpha}}{\sqrt{\alpha}}\right|, \qquad (21) \\ r, r_0 \in \Re\,,$$

and consequently for (19),

$$2\pi\alpha < \chi < \infty\,.$$

Equation (21) is the required mapping. One can see that r_0 cannot be determined: in other words, r_0 is entirely arbitrary. One also notes that (17) is consequently singular only when $r = r_0$ in which case $g_{00} = 0$, $\sqrt{C_n(r_0)} \equiv \alpha$, and $R_p(r_0) \equiv 0$. There is no value of r that makes $g_{11} = 0$. One therefore sees that the condition for singularity in the gravitational field is $g_{00} = 0$; indeed $g_{00}(r_0) \equiv 0$.

Clearly, contrary to the orthodox claims, r does not determine the geometry of the gravitational field directly. It is not a radius in the gravitational field. The quantity $R_p(r)$ is the non-Euclidean radial coordinate in the pseudo-Riemannian manifold of the gravitational field around the *point* $R_p = 0$, which corresponds to the parameter *point* r_0.

Now in addition to the established fact that, in the case of the simple (i.e. non-rotating) point-mass, the lower bound on the radius of curvature $\sqrt{C(D(r_0))} \equiv \alpha$, $C(D(r))$ must also satisfy the no matter condition so that when $\alpha = 0$, $C(D(r))$ must reduce to,

$$C(D(r)) \equiv \left|r - r_0\right|^2 = (r - r_0)^2\,; \qquad (22)$$

and it must also satisfy the far-field condition (spatially asympotically flat),

$$\lim_{r\to\pm\infty} \frac{C(D(r))}{(r - r_0)^2} \to 1. \qquad (23)$$

When $r_0 = 0$ equation (22) reduces to,

$$C(|r|) \equiv r^2 ,$$

and equation (23) reduces to,

$$\lim_{r \to \pm\infty} \frac{C(|r|)}{r^2} \to 1.$$

Furthermore, $C(r)$ must be a strictly monotonically increasing function of r to satisfy (15) and (21), and $C'(r) \neq 0$ $\forall\ r \neq r_0$ to satisfy (17) from (2a). The only general form for $C(D(r))$ satisfying all the required conditions (the Metric Conditions of Abrams [7]), from which an infinite sequence of particular solutions can be obtained [1] is,

$$C_n(D(r)) = \left(\left|r - r_0\right|^n + \alpha^n\right)^{\frac{2}{n}} , \qquad (24)$$

$$n \in \Re^+, \quad r \in \Re, \quad r_0 \in \Re ,$$

where n and r_0 are arbitrary. Then clearly, when $\alpha = 0$, equations (7) are recovered from equation (17) with (24), and when $r_0 = 0$ and $\alpha = 0$, equation (1) is recovered.

According to (24), when $r_0 = 0$ and $r \geqslant r_0$, and n is taken in integers, the following infinite sequence of particular solutions obtains,

$C_1(r) = (r + \alpha)^2$ (Brillouin's solution [12])
$C_2(r) = r^2 + \alpha^2$
$C_3(r) = (r^3 + \alpha^3)^{\frac{2}{3}}$ (Schwarzschild's solution [11])
$C_4(r) = (r^4 + \alpha^4)^{\frac{1}{2}}$, etc.

When $r_0 = \alpha$ and $r \in \Re^+$, and n is taken in integers, the following infinite sequence of particular solutions is obtained,

$C_1(r) = r^2$ (Droste/Weyl/(Hilbert) [9, 14, 8])
$C_2(r) = (r - \alpha)^2 + \alpha^2$
$C_3(r) = [(r - \alpha)^3 + \alpha^3]^{\frac{2}{3}}$
$C_4(r) = [(r - \alpha)^4 + \alpha^4]^{\frac{1}{2}}$, etc.

The Schwarzschild forms obtained from (24) satisfy Eddington's prescription for a general solution.

By (17) and (24) the circumference χ of a great circle in the gravitational field is,

$$\chi = 2\pi\sqrt{C_n(r)} = 2\pi \left(|r - r_0|^n + \alpha^n\right)^{\frac{1}{n}} , \qquad (25)$$

and the proper radius $R_p(r)$ is, from (21),

$$R_p(r) = \sqrt{\left(|r-r_0|^n+\alpha^n\right)^{\frac{1}{n}}\left[\left(|r-r_0|^n+\alpha^n\right)^{\frac{1}{n}} - \alpha\right]} + \alpha \ln\left|\frac{\left(|r-r_0|^n+\alpha^n\right)^{\frac{1}{2n}} + \sqrt{\left(|r-r_0|^n+\alpha^n\right)^{\frac{1}{n}} - \alpha}}{\sqrt{\alpha}}\right| . \qquad (26)$$

According to (24), $\sqrt{C_n(D(r_0))} \equiv \alpha$ is a scalar invariant, being independent of the value of r_0. Nevertheless the field is singular at the point-mass. By (21),

$$\lim_{r \to \pm\infty} \frac{R_p^2}{|r - r_0|^2} = 1 ,$$

and so,

$$\lim_{r \to \pm\infty} \frac{R_p^2}{C_n(D(r))} = \lim_{r \to \pm\infty} \frac{\frac{R_p^2}{|r-r_0|^2}}{\frac{C_n(D(r))}{|r-r_0|^2}} = 1 .$$

Now the ratio $\frac{\chi}{R_p} > 2\pi$ for *all* finite R_p, and

$$\lim_{r \to \pm\infty} \frac{\chi}{R_p} = 2\pi ,$$

$$\lim_{r \to r_0^\pm} \frac{\chi}{R_p} = \infty ,$$

so $R_p(r_0) \equiv 0$ is a quasiregular singularity and cannot be extended. The singularity occurs when parameter $r = r_0$, irrespective of the values of n and r_0. Thus, there is no sense in the orthodox notion that the region $0 < r < \alpha$ is an *interior* region on the Hilbert metric, since $r_0 \neq 0$ on that metric. Indeed, by (21) and (24) $r_0 = \alpha$ on the Hilbert metric. Equation (26) amplifies the fact that it is the *distance* $D = |r - r_0|$ that is mapped from parameter space into the proper radius (distance) in the gravitational field, and a distance *must* be $\geqslant 0$.

Consequently, strictly speaking, r_0 is not a singular point in the gravitational field because r is merely a parameter for the radial quantities in (M_g, g_g); r is neither a radius nor a coordinate in the gravitational field. No value of r can really be a singular point in the gravitational field. However, r_0 is mapped invariantly to $R_p = 0$, so $r = r_0$ *always* gives rise to a quasiregular singularity in the gravitational field, at $R_p(r_0) \equiv 0$, reflecting the fact that r_0 is the boundary on the r-parameter. Only in this sense should r_0 be considered a singular point.

The Kretschmann scalar $f = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ for equation (17) with equation (24) is,

$$f = \frac{12\alpha^2}{[C_n(D(r))]^3} = \frac{12\alpha^2}{\left(|r - r_0|^n + \alpha^n\right)^{\frac{6}{n}}} . \qquad (27)$$

Taking the near-field limit on (27),

$$\lim_{r \to r_0^\pm} f = \frac{12}{\alpha^4} ,$$

so $f(r_0) \equiv \frac{12}{\alpha^4}$ is a scalar invariant, irrespective of the values of n and r_0, invalidating the orthodox assumption that the singularity must occur where the curvature is unbounded. Indeed, no curvature singularity can arise in the gravitational field. The orthodox analysis claims an unbounded curvature singularity at $r_0 = 0$ in (18) purely and simply by its invalid initial assumptions, *not* by mathematical imperative. It incorrectly assumes $\sqrt{C_n(r)} \equiv R_p(r) \equiv r$, then with its additional invalid assumption that $0 < r < \alpha$ is valid on the Hilbert metric, finds from (27),

$$\lim_{r \to 0^+} f(r) = \infty \,,$$

thereby satisfying its third invalid assumption, by *ad hoc* construction, that a singularity occurs only where the curvature invariant is unbounded.

The Kruskal-Szekeres form has no meaning since the r-parameter is not the radial coordinate in the gravitational field at all. Furthermore, the value of r_0 being entirely arbitrary, $r_0 = 0$ has no particular significance, in contrast to the mainstream claims on (18).

The value of the r-parameter of a certain spacetime event depends upon the coordinate system chosen. However, the proper radius $R_p(D(r))$ and the curvature radius $\sqrt{C_n(D(r))}$ of that event are independent of the coordinate system. This is easily seen as follows. Consider a great circle centred at the point-mass and passing through a spacetime event. Its circumference is measured at χ. Dividing χ by 2π gives,

$$\frac{\chi}{2\pi} = \sqrt{C_n(D(r))} \,.$$

Putting $\frac{\chi}{2\pi} = \sqrt{C_n(D(r))}$ into (21) gives the proper radius of the spacetime event,

$$R_p(r) = \sqrt{\frac{\chi}{2\pi}\left(\frac{\chi}{2\pi} - \alpha\right)} + \alpha \ln \left| \frac{\sqrt{\frac{\chi}{2\pi}} + \sqrt{\frac{\chi}{2\pi} - \alpha}}{\sqrt{\alpha}} \right| \,,$$

$$2\pi\alpha \leqslant \chi < \infty,$$

which is independent of the coordinate system chosen. To find the r-parameter in terms of a particular coordinate system set,

$$\frac{\chi}{2\pi} = \sqrt{C_n(D(r))} = \left(\left|r - r_0\right|^n + \alpha^n\right)^{\frac{1}{n}} \,,$$

so

$$|r - r_0| = \left[\left(\frac{\chi}{2\pi}\right)^n - \alpha^n\right]^{\frac{1}{n}} \,.$$

Thus r for any particular spacetime event depends upon the arbitrary values n and r_0, which establish a coordinate system. Then when $r = r_0$, $R_p = 0$, and the great circumference $\chi = 2\pi\alpha$, irrespective of the values of n and r_0. A truly coordinate independent description of spacetime events has been attained.

The mainstream insistence, on the Hilbert solution (18), without proof, that the r-parameter is a radius of sorts in the gravitational field, the insistence that its r can, without proof, go down to zero, and the insistence, without proof, that a singularity in the field must occur only where the curvature is unbounded, have produced the irrational notion of the black hole. The fact is, the radius *always* does go down to zero in the gravitational field, but that radius is the *proper radius* R_p ($R_p = 0$ corresponding to a coordinate radius $D = 0$), not the curvature radius R_c, and certainly not the r-parameter.

There is no escaping the fact that $r_0 = \alpha \neq 0$ in (18). Indeed, if $\alpha = 0$, (18) *must* give,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

the metric of Special Relativity when $r_0 = 0$. One cannot set the lower bound $r_0 = \alpha = 0$ in (18) and simultaneously keep $\alpha \neq 0$ in the components of the metric tensor, which is effectively what the orthodox analysis has done to obtain the black hole. The result is unmitigated nonsense. The correct form of the metric (18) is obtained from the associated Schwarzschild form (24): $C(r) = r^2$, $r_0 = \alpha$. Furthermore, the proper radius of (18) is,

$$R_p(r) = \int_{\alpha}^{r} \sqrt{\frac{r}{r - \alpha}}\, dr \,,$$

and so

$$R_p(r) = \sqrt{r\,(r - \alpha)} + \alpha \ln \left| \frac{\sqrt{r} + \sqrt{r - \alpha}}{\sqrt{\alpha}} \right| .$$

Then,

$$r \to \alpha^+ \Rightarrow D = |r - \alpha| = (r - \alpha) \to 0 \,,$$

and in (M_g, g_g),

$$r^2 \equiv C(r) \to C(\alpha) = \alpha^2 \Rightarrow R_p(r) \to R_p(\alpha) = 0 \,.$$

Thus, the r-parameter is mapped to the radius of curvature $\sqrt{C(r)} = \frac{\chi}{2\pi}$ by ψ_1, and the radius of curvature is mapped to the proper radius R_p by ψ_2. With the mappings established the r-parameter can be mapped directly to R_p by $\psi(r) = \psi_2 \circ \psi_1(r)$. In the case of the simple point-mass the mapping ψ_1 is just equation (24), and the mapping ψ_2 is given by (21).

The local acceleration of a test particle approaching the point-mass along a radial geodesic has been determined by N. Doughty [15] at,

$$a = \frac{\sqrt{-g_{rr}}\,(-g^{rr})\,|g_{tt,r}|}{2g_{tt}} \,. \tag{28}$$

For (17) the acceleration is,

$$a = \frac{\alpha}{2C_n^{\frac{3}{4}}\left(C_n^{\frac{1}{2}} - \alpha\right)^{\frac{1}{2}}} .$$

Then,

$$\lim_{r \to r_0^{\pm}} a = \infty,$$

since $C_n(r_0) \equiv \alpha^2$; thereby confirming that matter is indeed present at the point $R_p(r_0) \equiv 0$.

In the case of (18), where $r \in \Re^+$,

$$a = \frac{\alpha}{2r^{\frac{3}{2}} \left(r - \alpha\right)^{\frac{1}{2}}},$$

and $r_0 = \alpha$ by (24), so,

$$\lim_{r \to \alpha^+} a = \infty\,.$$

Y. Hagihara [16] has shown that all those geodesics which do not run into the boundary at $r = \alpha$ on (18) are complete. Now (18) with $\alpha < r < \infty$ is a particular solution by (24), and $r_0 = \alpha$ is an arbitrary point at which the point-mass is located in parameter space, therefore all those geodesics in (M_g, g_g) not running into the point $R_p(r_0) \equiv 0$ are complete, irrespective of the value of r_0.

Modern relativists do not interpret the Hilbert solution over $0 < r < \infty$ as Hilbert did, instead making an arbitrary distinction between $0 < r < \alpha$ and $\alpha < r < \infty$. The modern relativist maintains that one is entitled to just "choose" a region. However, as I have shown, this claim is inadmissible. J. L. Synge [17] made the same unjustified assumptions on the Hilbert line-element. He remarks,

> "*This line-element is usually regarded as having a singularity at $r = \alpha$, and appears to be valid only for $r > \alpha$. This limitation is not commonly regarded as serious, and certainly is not so if the general theory of relativity is thought of solely as a macroscopic theory to be applied to astronomical problems, for then the singularity $r = \alpha$ is buried inside the body, i. e. outside the domain of the field equations $R_{mn} = 0$. But if we accord to these equations an importance comparable to that which we attach to Laplace's equation, we can hardly remain satisfied by an appeal to the known sizes of astronomical bodies. We have a right to ask whether the general theory of relativity actually denies the existence of a gravitating particle, or whether the form (1.1) may not in fact lead to the field of a particle in spite of the apparent singularity at $r = \alpha$.*"

M. Kruskal [18] remarks on his proposed extension of the Hilbert solution into $0 < r < 2m$,

> "*That this extension is possible was already indicated by the fact that the curvature invariants of the Schwarzschild metric are perfectly finite and well behaved at $r = 2m$*.*"

which betrays the very same unproven assumptions.

G. Szekeres [6] says of the Hilbert line-element,

> "*. . . it consists of two disjoint regions, $0 < r < 2m$, and $r > 2m$, separated by the singular hypercylinder $r = 2m$.*"

which again betrays the same unproven assumptions.

I now draw attention to the following additional problems with the Kruskal-Szekeres form.

(a) Applying Doughty's acceleration formula (28) to the Kruskal-Szekeres form, it is easily found that,

$$\lim_{r \to 2m^-} a = \infty.$$

But according to Kruskal-Szekeres there is *no matter* at $r = 2m$. Contra-hype.

(b) As $r \to 0$, $u^2 - v^2 \to -1$. These loci are spacelike, and therefore *cannot* describe *any* configuration of matter or energy.

Both of these anomalies have also been noted by Abrams in his work [7]. Either of these features alone proves the Kruskal-Szekeres form inadmissible.

The correct geometrical analysis excludes the interior Hilbert region on the grounds that it is not a region at all, and invalidates the assumption that the r-parameter is some kind of radius and/or coordinate in the gravitational field. Consequently, the Kruskal-Szekeres formulation is meaningless, both physically and mathematically. In addition, the so-called "Schwarzschild radius" (*not* due to Schwarzschild) is also a meaningless concept - it is not a radius in the gravitational field. Hilbert's $r = 2m$ is indeed a point, i. e. the "Schwarzschild radius" is a point, in both parameter space and the gravitational field: by (21), $R_p(2m) = 0$.

The *form* of the Hilbert line-element is given by Karl Schwarzschild in his 1916 paper, where it occurs there in the equation he numbers (14), in terms of his "auxiliary parameter" R. However Schwarzschild also includes there the equation $R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}$, having previously established the range $0 < r < \infty$. Consequently, Schwarzschild's auxiliary parameter R (which is actually a curvature radius) has the lower bound $R_0 = \alpha = 2m$. Schwarzschild's R^2 and Hilbert's r^2 can be replaced with any appropriate analytic function $C_n(r)$ as given by (24), so the range and the boundary on r will depend upon the function chosen. In the case of Schwarzschild's particular solution the range is $0 < r < \infty$ (since $r_0 = 0$, $C_3(r) = \left(r^3 + \alpha^3\right)^{\frac{2}{3}}$) and in Hilbert's particular solution the range is $2m < r < \infty$ (since $r_0 = 2m$, $C_1(r) = r^2$).

The geometry and the invariants are the important properties, but the conventional analysis has shockingly erred in its geometrical analysis and identification of the invariants, as a direct consequence of its initial invalidated assumptions about the r-parameter, and clings irrationally to these assumptions to preserve the now sacrosanct, but nonetheless ridiculous, black hole.

The only reason that the Hilbert solution conventionally breaks down at $r=\alpha$ is because of the initial arbitrary and incorrect assumptions made about the parameter r. There is no *pathology of coordinates* at $r=\alpha$. If there is anything pathological about the Hilbert metric it has nothing to do with coordinates: the etiology of a pathology must therefore be found elewhere.

There is no doubt that the Kruskal-Szekeres form is a solution of the Einstein vacuum field equations, however that does not guarantee that it is a solution to the problem. There exists an infinite number of solutions to the vacuum field equations which do not yield a solution for the gravitational field of the point-mass. Satisfaction of the field equations is a necessary but insufficient condition for a potential solution to the problem. It is evident that the conventional conditions (see [19]) that must be met are inadequate, viz.,

1. *be analytic*;
2. *be Lorentz signature*;
3. *be a solution to Einstein's free-space field equations*;
4. *be invariant under time translations*;
5. *be invariant under spatial rotations*;
6. *be (spatially) asymptotically flat*;
7. *be inextendible to a wordline L*;
8. *be invariant under spatial reflections*;
9. *be invariant under time reflection*;
10. *have a global time coordinate.*

This list must be augmented by a boundary condition at the location of the point-mass, which is, in my formulation of the solution, $r \to r_0^{\pm} \Rightarrow R_p(r) \to 0$. Schwarzschild actually applied a form of this boundary condition in his analysis. Marcel Brillouin [12] also pointed out the necessity of such a boundary condition in 1923, as did Abrams [7] in more recent years, who stated it equivalently as, $r \to r_0 \Rightarrow C(r) \to \alpha^2$. The condition has been disregarded or gone unrecognised by the mainstream authorities. Oddly, the orthodox analysis violates its own stipulated condition for a global time coordinate, but quietly disregards this inconsistency as well.

Any constants appearing in a valid solution must appear in an invariant derived from the solution. The solution I obtain meets this condition in the invariance, at $r=r_0$, of the circumference of a great circle, of Kepler's 3rd Law [1, 2], of the Kretschmann scalar, of the radius of curvature $C(r_0)=\alpha^2$, of $R_p(r_0)\equiv 0$, and not only in the case of the point-mass, but also in all the relevant configurations, with or without charge.

The fact that the circumference of a great circle approaches the finite value $2\pi\alpha$ is no more odd than the conventional oddity of the change in the arrow of time in the "interior" Hilbert region. Indeed, the latter is an even more violent oddity: inconsistent with Einstein's theory. The finite limit of the said circumference is consistent with the geometry resulting from Einstein's gravitational tensor. The variations of θ and φ displace the proper radius vector, $R_p(r_0)\equiv 0$, over the spherical surface of finite area $4\pi\alpha^2$, as noted by Brillouin. Einstein's theory admits nothing more pointlike.

Objections to Einstein's formulation of the gravitational tensor were raised as long ago as 1917, by T. Levi-Civita [20], on the grounds that, from the mathematical standpoint, it lacks the invariant character actually required of General Relativity, and further, produces an unacceptable consequence concerning gravitational waves (i.e they carry neither energy nor momentum), a solution for which Einstein vaguely appealed *ad hoc* to quantum theory, a last resort obviated by Levi-Civita's reformulation of the gravitational tensor (which extinguishes the gravitational wave), of which the conventional analysis is evidently completely ignorant: but it is not pertinent to the issue of whether or not the black hole is consistent with the theory as it currently stands on Einstein's gravitational tensor.

3 The geometry of the simple point-charge

The fundamental geometry developed in section **2** is the same for all the configurations of the point-mass and the point-charge. The general solution for the simple point-charge [2] is,

$$ds^2 = \left(1-\frac{\alpha}{\sqrt{C_n}}+\frac{q^2}{C_n}\right)dt^2 - \left(1-\frac{\alpha}{\sqrt{C_n}}+\frac{q^2}{C_n}\right)^{-1}\times \\ \times \frac{{C'_n}^2}{4C_n}dr^2 - C_n(d\theta^2+\sin^2\theta d\varphi^2)\,, \tag{29}$$

$$C_n(r) = \left(\left|r-r_0\right|^n+\beta^n\right)^{\frac{2}{n}},$$

$$\beta = m+\sqrt{m^2-q^2}\,,\quad q^2<m^2\,,$$

$$n\in\Re^+,\quad r,r_0\in\Re\,.$$

where n and r_0 are arbitrary.

From (29), the radius of curvature is given by,

$$R_c = \sqrt{C_n(r)} = \left(\left|r-r_0\right|^n+\beta^n\right)^{\frac{1}{n}},$$

which gives for the near-field limit,

$$\lim_{r\to r_0^{\pm}}\sqrt{C_n(r)} = \sqrt{C_n(r_0)} = \beta = m+\sqrt{m^2-q^2}\,.$$

The expression for the proper radius is,

$$R_p(r) = \sqrt{C(r)-\alpha\sqrt{C(r)}+q^2} + \\ + m\ln\left|\frac{\sqrt{C(r)}-m+\sqrt{C(r)-\alpha\sqrt{C(r)}+q^2}}{\sqrt{m^2-q^2}}\right|.$$

Then

$$\lim_{r\to r_0^\pm} R_p(r) = R_p(r_0) \equiv 0\,.$$

The ratio $\frac{\chi}{R_p} > 2\pi$ for all finite R_p, and

$$\lim_{r\to\pm\infty} \frac{\chi}{R_p(r)} = 2\pi\,,$$

$$\lim_{r\to r_0^\pm} \frac{\chi}{R_p(r)} = \infty\,,$$

so $R_p(r_0) \equiv 0$ is a quasiregular singularity and cannot be extended.

Now, since the circumference χ of a great circle is the only measurable quantity in the gravitational field, the unique solution for the field of the simple point-charge is,

$$\begin{aligned} ds^2 &= \left(1 - \frac{2\pi\alpha}{\chi} + \frac{4\pi^2 q^2}{\chi^2}\right) dt^2 - \\ &- \left(1 - \frac{2\pi\alpha}{\chi} + \frac{4\pi^2 q^2}{\chi^2}\right)^{-1} \frac{d\chi^2}{4\pi^2} - \\ &- \frac{\chi^2}{4\pi^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \end{aligned} \qquad (30)$$

$$2\pi\left(m + \sqrt{m^2 - q^2}\right) < \chi < \infty\,.$$

Equation (30) is entirely independent of the r-parameter.

In terms of equation (29), the Kretschmann scalar takes the form [21],

$$f(r) = \frac{8\left[6\left(m\sqrt{C_n(r)} - q^2\right)^2 + q^4\right]}{C_n^4(r)}\,, \qquad (31)$$

so

$$\lim_{r\to r_0^\pm} f(r) = f(r_0) = \frac{8\left[6\left(m\beta - q^2\right)^2 + q^4\right]}{\beta^8} = \frac{8\left[6\left(m^2 + m\sqrt{m^2-q^2} - q^2\right)^2 + q^4\right]}{(m + \sqrt{m^2-q^2})^8}\,,$$

which is a scalar invariant. Thus, no curvature singularity can arise in the gravitational field of the simple point-charge.

The standard analysis incorrectly takes $\sqrt{C_n(r)} \equiv R_p(r) \equiv r$, then with this assumption, and the additional invalid assumption that $0 < r < \infty$ is true on the Reissner-Nordstrom solution, obtains from equation (31) a curvature singularity at $r = 0$, satisfying, by an *ad hoc* construction, its third invalid assumption that a singularity can only arise at a point where the curvature invariant is unbounded.

Equation (29) is singular *only* when $g_{00} = 0$; indeed $g_{00}(r_0) \equiv 0$. Hence, $0 \leqslant g_{00} \leqslant 1$.

Applying Doughty's acceleration formula (28) to equation (29) gives,

$$a = \frac{\left|m\sqrt{C_n(r)} - q^2\right|}{C_n(r)\sqrt{C_n(r) - \alpha\sqrt{C_n(r)} + q^2}}\,.$$

Then,

$$\lim_{r\to r_0^\pm} a = \frac{\left|m\beta - q^2\right|}{\beta^2\sqrt{\beta^2 - \alpha\beta + q^2}} = \infty\,,$$

confirming that matter is indeed present at $R_p(r_0) \equiv 0$.

4 The geometry of the rotating point-charge

The usual expression for the Kerr-Newman solution is, in Boyer-Lindquist coordinates,

$$\begin{aligned} ds^2 &= \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\varphi\right)^2 - \\ &- \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\varphi - a\,dt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \end{aligned} \qquad (32)$$

$$a = \frac{L}{m}, \quad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

$$\Delta = r^2 - r\alpha + a^2 + q^2, \quad 0 < r < \infty\,.$$

This metric is alleged to have an event horizon r_h and a static limit r_b, obtained by setting $\Delta = 0$ and $g_{00} = 0$ respectively, to yield,

$$r_h = m \pm \sqrt{m^2 - a^2 - q^2}$$

$$r_b = m \pm \sqrt{m^2 - q^2 - a^2\cos^2\theta}\,.$$

These expressions are conventionally quite arbitrarily taken to be,

$$r_h = m + \sqrt{m^2 - a^2 - q^2}$$

$$r_b = m + \sqrt{m^2 - q^2 - a^2\cos^2\theta}\,,$$

apparently because no-one has been able to explain away the meaning of the the "inner" horizon and the "inner" static limit. This in itself is rather disquieting, but nonetheless accepted with furtive whispers by the orthodox theorists. It is conventionally alleged that the "region" between r_h and r_b is an ergosphere, in which spacetime is dragged in the direction of the of rotation of the point-charge.

The conventional taking of the r-parameter for a radius in the gravitational field is manifest. However, as I have shown, the r-parameter is neither a coordinate nor a radius in the gravitational field. Consequently, the standard analysis is erroneous.

I have already derived elsewhere [2] the general solution for the rotating point-charge, which I write in most general form as,

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 -$$
$$-\frac{\sin^2\theta}{\rho^2}\left[\left(C_n + a^2\right)d\varphi - adt\right]^2 - \frac{\rho^2}{\Delta}\frac{C_n'^{\,2}}{4C_n}dr^2 - \rho^2 d\theta^2\,,$$
$$C_n(r) = \left(\left|r - r_0\right|^n + \beta^n\right)^{\frac{2}{n}}\,, \quad n \in \Re^+\,, \qquad (33)$$
$$r,\ r_0 \in \Re, \quad \beta = m + \sqrt{m^2 - q^2 - a^2\cos^2\theta}\,,$$
$$a^2 + q^2 < m^2, \quad a = \frac{L}{m}, \quad \rho^2 = C_n + a^2\cos^2\theta\,,$$
$$\Delta = C_n - \alpha\sqrt{C_n} + q^2 + a^2\,,$$

where n and r_0 are arbitrary.

Once again, since only the circumference of a great circle is a measurable quantity in the gravitational field, the unique general solution for all configurations of the point-mass is,

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 -$$
$$-\frac{\sin^2\theta}{\rho^2}\left[\left(\frac{\chi^2}{4\pi^2} + a^2\right)d\varphi - adt\right]^2 - \frac{\rho^2}{\Delta}\frac{d\chi^2}{4\pi^2} - \rho^2 d\theta^2\,,$$
$$a^2 + q^2 < m^2, \quad a = \frac{L}{m}, \quad \rho^2 = \frac{\chi^2}{4\pi^2} + a^2\cos^2\theta\,, \qquad (34)$$
$$\Delta = \frac{\chi^2}{4\pi^2} - \frac{\alpha\chi}{2\pi} + q^2 + a^2\,,$$
$$2\pi\left(m + \sqrt{m^2 - q^2 - a^2\cos^2\theta}\right) < \chi < \infty\,.$$

Equation (34) is entirely independent of the r-parameter.

Equation (34) emphasizes the fact that the concept of a point in pseudo-Euclidean Minkowski space is not attainable in the pseudo-Riemannian gravitational field. A point-mass (or point-charge) is characterised by a proper radius of zero and a finite, non-zero radius of curvature. Einstein's universe admits of nothing more pointlike. The relativists have assumed that, insofar as the point-mass is concerned, the Minkowski point can be achieved in Einstein space, which is not correct.

The radius of curvature of (33) is,

$$\sqrt{C_n(r)} = \left(|r - r_0|^n + \beta^n\right)^{\frac{1}{n}}\,, \qquad (35)$$

which goes down to the limit,

$$\lim_{r\to r_0^{\pm}}\sqrt{C_n(r)} = \sqrt{C_n(r_0)} = \beta = m + \sqrt{m^2 - q^2 - a^2\cos^2\theta}\,, \qquad (36)$$

where the proper radius $R_p(r_0) \equiv 0$. The standard analysis incorrectly takes (36) for the "radius" of its static limit.

It is evident from (35) and (36) that the radius of curvature depends upon the direction of radial approach. Therefore, the spacetime is not isotropic. Only when $a = 0$ is spacetime isotropic. The point-charge is always located at $R_p(r_0) \equiv 0$ in $(M_g,\ g_g)$, irrespective of the value of n, and irrespective of the value of r_0. The conventional analysis has failed to realise that its r_b is actually a varying radius of curvature, and so incorrectly takes it as a measurable radius in the gravitational field. It has also failed to realise that the location of the point-mass in the gravitational field is not uniquely specified by the r-coordinate at all. The point-mass is *always* located just where $R_p = 0$ in (M_g, g_g) and its "position" in (M_g, g_g) is otherwise meaningless. The test particle has already encountered the source of the gravitational field when the radius of curvature has the value $C_n(r_0) = \beta$. The so-called ergosphere also arises from the aforesaid misconceptions.

When $\theta = 0$ the limiting radius of curvature is,

$$\sqrt{C_n(r_0)} = \beta = m + \sqrt{m^2 - q^2 - a^2}\,, \qquad (37)$$

and when $\theta = \frac{\pi}{2}$, the limiting radius of curvature is,

$$\sqrt{C_n(r_0)} = \beta = m + \sqrt{m^2 - q^2}\,,$$

which is the limiting radius of curvature for the simple point-charge (i. e. no rotation) [2].

The standard analysis incorrectly takes (37) as the "radius" of its event horizon.

If $q = 0$, then the limiting radius of curvature when $\theta = 0$ is,

$$\sqrt{C_n(r_0)} = \beta = m + \sqrt{m^2 - a^2}\,, \qquad (38)$$

and the limiting radius of curvature when $\theta = \frac{\pi}{2}$ is,

$$\sqrt{C_n(r_0)} = \beta = 2m = \alpha\,,$$

which is the radius of curvature for the simple point-mass.

The radii of curvature at intermediate azimuth are given generally by (36). In all cases the near-field limits of the radii of curvature give $R_p(r_0) \equiv 0$.

Clearly, the limiting radius of curvature is minimum at the poles and maximum at the equator. At the equator the effects of rotation are not present. A test particle approaching the rotating point-charge or the rotating point-mass equatorially experiences the effects only of the non-rotating situation of each configuration respectively. The effects of the rotation manifest only in the values of azimuth other than $\frac{\pi}{2}$. There is no rotational drag on spacetime, no ergosphere and no event horizon, i. e. no black hole.

The effects of rotation on the radius of curvature will necessarily manifest in the associated form of Kepler's 3rd Law, and the Kretschmann scalar [22].

I finally remark that the fact that a singularity arises in the gravitational field of the point-mass is an indication that a material body cannot collapse to a point, and therefore such a model is inadequate. A more realistic model must be sought in terms of a non-singular metric, of which I treat elsewhere [23].

Dedication

I dedicate this paper to the memory of Dr. Leonard S. Abrams: (27 Nov. 1924 — 28 Dec. 2001).

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