

The Relations Between Ancient China's Taoism And Modern Mathematics & Physics

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www.zhongyuyan.org

December 18, 2015

I have mainly analyzed the mathematical meaning of non-classical mathematical theory for three fundamental physics equations - Maxwell's equations, Dirac's equations, Einstein's equations from the quantized core theory of ancient China's Taoism, and found they have some structures described in the core of the theory of ancient China's Taoism, especially they all obviously own the yin-yang induction structure. This reveals the relations between the ancient China's Taoism and modern mathematics and physics in a way, which may help us to understand some problems of the fundamental theory of physics.

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1 Introduction

In this article, I will briefly discuss the relations between the ancient Chinese Taoism and modern mathematics&physics. This is extracted from the book I'm writing whose name is *Tao's mathematical theory* ¹, I have tried to explore the mathematical principles underlying the Ancient China's Taoism(hereinafter referred to as ACT) which has a long history and rich contents, and tried to derive the contents in modern mathematics and physics. To make us focus on the relations between ACT and the modern physics, I here only extract the contents which are most closely related to the physics.

I am not a professional scholar, in fact, my previous job is a software engineer, I have never been professional education of mathematics and physics, but I have a long experience in self-study of mathematics (in fact, all my knowledge about computers is also almost entirely come through self-study), and kept on thinking the basic meaning of mathematics. After about three years ago, I found a unusual direct relation between ACT and modern mathematics, then I have begun a systematic thinking on the related issues, the results of these thinking so had formed the draft of *Tao's mathematical theory*. About a year ago I gave up the job of software engineer and specialized in writing *Tao's mathematical theory*, in this process, I have tried to look for where the hidden meaning in physics. ACT has always claimed that their task is to explain the universe and nature, and sure itself did this, which is just like the modern sciences. Of course, compared with the modern science, ACT declines, and is widely considered to be a non-scientific error, but it has ever own the reputation as same as the reputation of the modern science in today China.

However, ACT in China today still has many staunch believers. In addition, compared to modern Chinese scholars, the famous ancient Chinese thinkers's character are trustworthy. These prompted me to think seriously whether ACT as it claimed, contains the great truths in the universe. If that is true, because it and the modern science both are discourse the same universe and nature, then they must have in common, which is the power driving me to look for the relations between ACT and the modern physics.

Surprisingly, the study based on this idea really got some almost magical results. In few words, I have found that the three basic equations in the modern physics: Maxwell's equations, Dirac's equations, Einstein's equations, all have a direct close relationship with the theory of yin and yang, the five elements, the eight trigrams, etc. Further, we can get some wonderful inspiration from these ancient doctrines when considering some basic theories, such as quantum mechanics, the standard model, etc.

In this research, I was looking for a lot of English tutorials of PDF format from

¹The draft of this book can be downloaded from the www.zhongyuyan.org/DaoXue/Daoxue.php.

the network to try to understand exactly what is it saying in physics, although I have read consciously a lot of books about physics before, but have not really understand how does the physics work. Thanks to there are very much excellent lectures on the network, which made me more in-depth understanding of the physics, but my knowledge of mathematics and physics is still not enough. Now I'm tired, and lack of the resources for continuing to the research, although there are many problems to be solved. But I'm sure I have now had my own understanding of the physics, very aware of the contents and the opinions to be expressed, and would like to spread it, share it with the friends in the world, because I think these things are very interesting and thus it may be useful and worthy of researching. So I now translate these into English, and let it get the free dissemination on the Internet, just like those countless files which have gave me a lot of benefits. Unfortunately, I lack the experience of writing thesis, especially writing experience in English, so I try to write them out as simple and straightforward as possible, but themselves are very simple although very magical, this is also the the pursuit of simple ideas in ACT. Furthermore, since the contents of physics for the intentioned reader should be easy and I can not trespass, so they are simply quoted.

The main contents of this article are understanding the three basic physics equations from the ACT perspective: Maxwell's equations of classical electrodynamics, Dirac's equations of relativity quantum mechanics, and Einstein's equations of general relativity. For the first two I've done a lot of thinking, so they occupy a major part in this article. By contrast, my grasp for Einstein's equations are inadequate, so here it is a brief content, but the viewpoint about it is the same peculiar. In addition, I will also discuss some of my ideas and questions on the quantum electrodynamics, the gauge theory and the standard model, and even supersymmetry inspired from the viewpoints of ACT.

Section 2 discusses the thoughts of yin and yang, and Tao in ancient China. Section 3 discusses the theory of the five elements, and gives the method for understanding and deriving the Maxwell's equations from the structure of a special kind of graph. Section 4 discusses the theory of the eight trigrams, and gives the method for understanding and deriving the Maxwell's equations, Dirac's equations, Einstein's equations. Section 5 discusses the features of QED presented under viewpoints of ACT, and where I have wrote down some of my questions about the gauge theory and supersymmetry. Section 6 is the end of the article, where I made some thinking on the history of ACT.

In addition, in order to introduce some interesting and maybe physics-related contents in *Tao's mathematical theory*, I have listed them in the Appendix. Appendix 1 describes the relations between the period representation in ACT and the theory of five elements. Appendix 2 describes the try of using the eight trigrams to represent the macroscopic physical motion, which imitate most common traditional practice in ACT. Appendix 3 describes the meaning of magic square in ACT and my personal

understanding about it.

2 Tao And YinYang

Tao is a concept that has a long history in China, the earliest extant literature on it is *Tao Te Ching* which has been completed at two thousand five hundred years ago, and *Book of Changes* which has been completed more earlier. Indeed Tao is not just a concept, there has formed a huge system of thoughts around it in the history of ancient civilization in China. In this article I will identify this system of thoughts by Ancient China's Taoism (abbreviated to as ACT). ACT's core view is that the Tao is the root of all things, and uses yin and yang as a basic tool for interpretation on Tao. This is embodied in one sentence in the *Book of Changes*:

One yin and one yang, this is said to be Tao.²

We will in the later find that this is indeed not a empty talk, but has a solid basis in physics.

Another main point of ACT is that the Tao is based on the mathematical theory to construct the world. *Tao Te Ching* says:

Tao produces one, one produces two, two produces three, three produces all things.³

Book of Changes says:

Tai Chi produces two Yis, two Yis produce four Xiangs, four Xiangs produce eight Guas, eight Guas determine the good and bad, the good and bad produce great cause. ⁴

These two sentences, although very simple, but their mathematical meaning is very basic, since the former corresponds to a sequence of integers:

$$0, 1, 2, 3, \dots \quad (1)$$

and the later also corresponds to a sequence of integers:

$$1 = 2^0, 2 = 2^1, 4 = 2^2, 8 = 2^3, \dots \quad (2)$$

²The chinese is "一阴一阳之谓道。".

³The chinese is "道生一，一生二，二生三，三生万物。".

⁴The chinese is "太极生两仪，两仪生四象，四象生八卦，八卦定吉凶，吉凶生大业。". Tai Chi corresponds to "太极", means very very extreme. Yi corresponds to "仪", means apperance. Xiang corresponds to "象", means image. Gua(or trigram) corresponds to "卦", means hung.

The first sequence corresponds to the addition, linear; the second sequence corresponds to the multiplication, the exponential, nonlinear. It's well known in physics, linear and nonlinear pattern is an important feature for classifying the physical motions. From the viewpoint of ACT, both of them are the "one yin and one yang" too.

Although the mathematical principle here is primitive and simple, but they do highlight that ACT believes the mathematics enjoyed the distinguished status in its doctrine. *Tao Te Ching* has no more discussion of mathematics, but the text of *Book of Changes* is in full use of the symbols which are called by binary today as its descriptive language of the whole doctrine, and all the other literals are just the interpretation for them. This is a very mysterious fact, it is not also modern information technology using binary to represent all the information? The traditional of emphasis on mathematics is inherited by the ancient Chinese ACT scholars and carry forward, so they had developed a variety of doctrine which are as a whole called "ShuShu"⁵ and are closely related to the arithmetic. But the mathematics they had used is very limited, in fact, from a modern point of view these mathematics can be understood as the elementary theory of integers. Although the ancient Chinese had grasped developed mathematics in different areas, such as algebra about linear equations and geometry based on Pythagorean, but they almost did not appear in these doctrines. This had clearly restricted the development of ACT, so that they had lost the opportunity to evolve into modern science, we can clearly see this point later.

Perhaps their more fatal flaw is not academic per se, but rather their values, being accurate is to say that the values of those who later inherited ACT, a variety of Shushu that they had developed all have a strong purpose to seek wealth and status, and almost absolutely have nothing to do with the knowledge about nature itself, which is in fact out of ACT's original meaning, because the original highest pursuit of ACT is the understanding for human on Tao and nature.

This highest value has been clearly expressed in the *Book of Changes* and *Tao Te Jing*. *Tao Te Jing*, said: "Human follow the earth'law, the earth follow the sky'law, the sky follow the Tao's law, the Tao follow nature's law."⁶ *Book of Changes*, said:"Formerly, sage have created the Yi doctrine. They praise the great divinities and found the yarrow, represent the yin and yang by number such as three for yang and two for yin, observe changes based on yin and yang so that established the Gua, analysis the state of yin and yang such as rigid or soft, so constructed the Yao, follow the Tao's character and understand the reasoning of necessity and responsibility, exhaust the reason and humanity so as to achieve the comprehension for life. "⁷

⁵The chinese is "术数", it means mysterious technology mathematics, always relations to prediction about destiny.

⁶The original text is "人法地，地法天，天法道，道法自然。".

⁷The original text is "昔者，圣人之作易也：幽赞神明而生蓍；参天两地而倚数；观变於阴阳而

Actually, this is the spirit of modern science. In contrast, the low values of the ACT successors had made them been more and more far away from the Tao of nature in the universe.

I would like first to express my view about physics: physics is actually a theory about the fundamental yin and yang in nature. We will see later the evidences that support this view and make it reasonable. In fact, physics is very suitable to be used to explain the theory of yin and yang theory of ACT, because in physics there are many "one yin and one yang" examples. For example, the stationary is yin, the motion is yang; the mass is yin, energy is yang; the location is yin, the speed is yang; the uniform motion is yin, the accelerated motion is yang; the potential is yin, the kinetic energy is yang. As another example, the curve is yin, the straight is yang; the curved spacetime is yin, the flat space-time is yang; the angular momentum is yin, the momentum is yang; the spin angular momentum is yin, the orbital angular momentum is yang. Another example, the magnetism is yin, the electricity is yang; material is yin, radiation is the yang; the particle is yin, the wave is yang; positive charge is yin, negative charge is yang; the atom's electron cloud is the yang, the nucleus is yin; dark matter is yin, the matter is the yang; the dark energy is yin, the energy is yang. We can extend this list virtually unlimited.

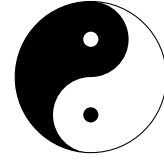


Fig. 1: The diagram of growth and decline for yin and yang

Similar with physics, ACT is very much concerned about the change of things, because of predictions about the future is a fundamental pursuit of all varieties of Shushu, but ACT is lack of powerful mathematical tools, they almost always use a cycle of finite integer structure \mathbb{Z}_n to describe the change. Of course, modern physics has also been used to describe the changes with harmonious oscillators. However, this defect of ACT can be made up by modern mathematical in accordance with physics methods, which is an interesting discovery given in my book.

There are four major points in the theory about yin and yang in ACT:

- (1) Tao produces yin and yang. Tao in fact is the relationship between two symmetrical parts which are named after yin and yang.
- (2) Yin and yang are companions. Solitary yin no life, alone yang does not grow, yin and yang are always interdependent.
- (3) Yin and Yang are constantly changing with growth and decline. Relative to the observer, the yin and yang are always constantly changing, and these changes can be quite difficult to predict even measure. A common form of these changes is the shift.

立卦；发揮於剛柔而生爻；和順於道德而理於義；穷理尽性以至於命。”，it is very obscure.

- (4) Yin and yang generates all things. Anything, that is substances or advanced system consisted of matter and their relationship, is the phenomenon constituted by both yin and yang.

ACT often uses black to represent yin, and uses white to represent yang, of course, the most famous representation is Taiji Figure, as shown in Fig1.

For using the methods of physics to quantify the theory of yin and yang in ACT, maybe the best solution is using the Schrödinger's equation [2, p. 21] in quantum mechanics

$$i\hbar\frac{\partial\phi}{\partial t} = \hat{H}\phi, \quad (3)$$

as the basic equations of motion describing the motion of yin and yang. To this end, we put the wave function in quantum mechanics interpreted as yin and yang pair: we can treat the real part of the wave function as yang, and the imaginary part of the wave function is regarded as a yin. This means that $\phi = \phi_{yang} + i\phi_{yin}$. After the above equation (3) was expanded, we get:

$$\begin{aligned} \hbar\frac{\partial\phi_{yin}}{\partial t} &= -\hat{H}\phi_{yang} \\ \hbar\frac{\partial\phi_{yang}}{\partial t} &= \hat{H}\phi_{yin} \end{aligned} \quad (4)$$

This can obviously allows us to see the quantitative relationship between yin and yang has a natural consensus with the yin and yang theory in ACT. Here we are in fact let the yin and yang represented as real numbers, this is not arbitrary, it can be seen as a basic assumption that the quantization theory of yin and yang.

From the viewpoint of ACT, Schrödinger's equation establishes the equivalence relation between matter moving in the external space and the motion of yin and yang in the inner of the matter. \hat{H} correspond to the external factors, it is yang; and ϕ corresponds to the internal factors of the matter, it is yin. Explanation here may not allow us to immediately see the magical view of ACT, and appears more like a human subjective creation, but the next facts are likely to change the attitude of the skeptics at the moment.

Here is the last I want to declare in this section, : Do not ask me what is yin and what is yang. So do not ask me about its weight, its size, its density, its composition, its color, its flavor and so on. So do not ask me about its location and distribution, it is at what speed in what kind of trajectory, and so on. This statement applies to all other ACT concept introduced in the latter, they are often not anything, but it is often anything. They are the pattern of things existing and changing, we will clearly see their presence inside the basic equations of physics.

3 The Five Elements

The Five Elements Theory⁸(abbreviated as Wuxing), is a basic thought in ACT, in fact, we can see its direct impact in almost all of the ancient Chinese theory.

3.1 The traditional theory

Its basic theory is five elements⁹:

$$\text{金, 木, 水, 火, 土} \tag{5}$$

and their two inter-relationships, one of which is called mutual restriction, another is called mutual synergism¹⁰, they are all ordered. The relationship is jus the below 10 ordered pairs:

$$\begin{aligned} \text{mutual restriction pairs:} & \text{金 R 木, 木 R 土, 土 R 水, 水 R 火, 火 R 金,} \\ \text{mutual synergism pairs:} & \text{金 S 水, 水 S 木, 木 S 火, 火 S 土, 土 S 金.} \end{aligned} \tag{6}$$

The names of the five elements in the general context all have the common sense: '金(metal)' means metals and their products; '木(wood)' means plants and wood; '水(water)' means liquid water or a flow of liquid; '火(fire)' means fire; '土(soil)' means soil. However, if attached to such a literal interpretation, then we leave the original meaning of the Wuxing in ACT too far. According to graph theory's viewpoint, Wuxing can be represented as a directed graph as shown in Fig.2. In this graph, each node corresponds to an element in Wuxing and the two kinds of edges correspond to two kinds of relationships in Wuxing: the solid line corresponds to the relationship of mutual restriction, mutual synergism relationship corresponds to the dotted line, you should pay attention to this relationship is orded, so we have to use the arrow to represent the order.

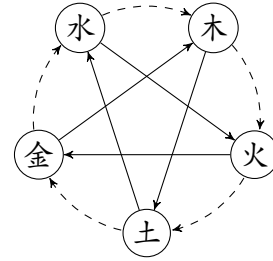


Fig. 2: The directed graph of the five elements

The ancient Chinese had developed a very rich doctrine based on such a simple mathematical structure. In ACT, Wuxing can be used to understand the relationship

⁸The chinese is 五行, means five move or walker.
⁹They are always correpond to five planet in ancient chinese astronomy: Venus, Jupiter, Mercury, Mars, Saturn. In chinese these five planets are named the same word but append a word 星.
¹⁰The chinese for mutual restriction is "相克", means mutual attack, The chinese for mutual synergism is "相生", means mutual produce.

between objects, mutual synergism corresponds to the relationship between objects that is good, and helpful for growing, while mutual restriction corresponds to the relationship between objects that has a destructive and conflicting effect. It can also be used to represent space, such as the West corresponds to 金, Orient corresponds to 木, northern corresponds to 水, south corresponds to 火, the center corresponds to 土. It is also associated with time, such as 木 corresponds to spring, 火 corresponds to summer, 金 corresponds to fall, 水 corresponds to winter, 土 corresponds to the last month of the four seasons (Another opinion is that 土 corresponds to a time interval between summer and autumn). It can also be used to represent the properties of objects, e.g., which corresponds to the five colors, five tunes, five tastes like. Wuxing in ACT in fact assumes that there is a system of Wuxing inside each thing, but also each thing lives in an outside system of Wuxing. From a physical viewpoint, it has similarities with the viewpoint of field theory .

Another main point of Wuxing is that it has established some common state models for the five elements, the simplest of which is a five-states model consisting five states: vigorous, gaining help, recuperate, captivity, death¹¹. In another famous state model, there are twelve states. These two state models both have a cyclic structure. Five elements in Wuxing automatically according to the same order complete the transition in these states with rhythm of time, but at the same time the status of each element is generally different. Divination method using Wuxing firstly set the Wuxing property of the things which are to be calculated, then it basing on the Wuxing property of time and space to judge the states of things, and therefore concluded the respective results of the parties(they are all attributed to the Wuxing and therefore have the Wuxing relations between them.) which were involved in the interaction after the interaction procedure.

Using the terminology of modern mathematics, the five elements in Wuxing are equivalent to a sine or cosine function defined on the time axis, but the phase angle of each function is different. Therefore, the five elements in Wuxing are the simple harmonic oscillators which are oscillating with different phase angles, so they are similar to the free waves described by Schrodinger's equation. Given its characteristics similar to field theory, we can understand them as quantum field theory in ACT, the two relations in which correspond to the creation and annihilation operators operator in quantum field theory. Wuxing is very popular for use in various branches in ACT, just as Fourier transform is widely used in mathematics and physics.

¹¹The chinese is: 旺, 相, 休, 囚, 死.

Question: In ACT, all calculations are finited, and there is full of infinite in quantum field theory, if we can use a finited mathematical model to construct a finited quantum field theory, then use these finited quantum field theories to approach the infinite quantum field theory, just like local-global principle in mathematics ^a?

^ahttps://en.wikipedia.org/wiki/Hasse_principle

3.2 The graph structure and Maxwell's equations

Here our focus is not those Wuxing theories about specific things in ACT, but the hidden mathematical structure in Wuxing. We can easily see that its graph has a lot of symmetries, and symmetries is the basic object in the modern physics research. I have argued why are there just five elements in Wuxing from the viewpoint of yin and yang in my book. The basic idea of this argument is the assumption that starting from me, then consider my relationship with the others. Suppose my relationships with others are divided into two kinds: one is a beneficial relationship; the other relationship is detrimental. Each relationship is divided into two kinds again: one is I take the initiative and the others passive; the second is I passive and the others active. If we look for the smallest closed system in which such a relationship exists between the various members equally, then the number of members in the system must be 5.

To generalize this theory, starting from the characteristic of this graph in which there are two circular loops, I found that a more general theory should be the additive group structure of \mathbb{Z}_p based on prime p . Therefore, in this viewpoint, each element in Wuxing corresponds to an element in \mathbb{Z}_p , for example:

$$\text{土} \rightarrow 0, \text{金} \rightarrow 1, \text{水} \rightarrow 2, \text{木} \rightarrow 3, \text{火} \rightarrow 4, \quad (7)$$

In this representation, the mutual synergism relationship is represented as an operator $+1$ in \mathbb{Z}_5 , and mutual restriction relation is represented as an operator $+2$ in \mathbb{Z}_5 , as shown in Fig 3.

In ACT, these five elements also have an integer representation, it is:

$$\text{土} \rightarrow 5, \text{金} \rightarrow 4, \text{水} \rightarrow 1, \text{木} \rightarrow 3, \text{火} \rightarrow 2, \quad (8)$$

This is different from our opinion, but we can found there is some kind of correspondence between these two. ACT uses this integer representation to explain the relationship between the magic square and Wuxing.

In order to use analytics to study the relationship between the quantities given in this graph, I introduced a central node to come in this graph, as shown in Fig 4. The

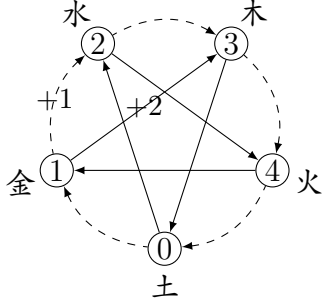


Fig. 3: The representation of five elements in \mathbb{Z}_5

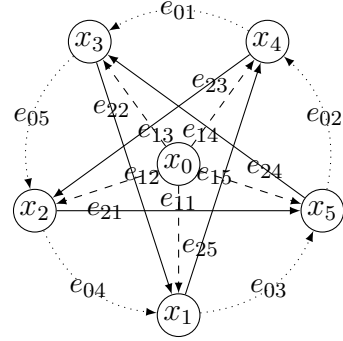


Fig. 4: The Wuxing graph with center

central node x_0 will be used to indicate the time parameter, we can denote this graph by $\overset{\circ}{\mathbb{X}}_p$. When I consider the case of $\overset{\circ}{\mathbb{X}}_3$, and found that it can be used to understand the Maxwell's equations, their graphs are in 5a and 5b.

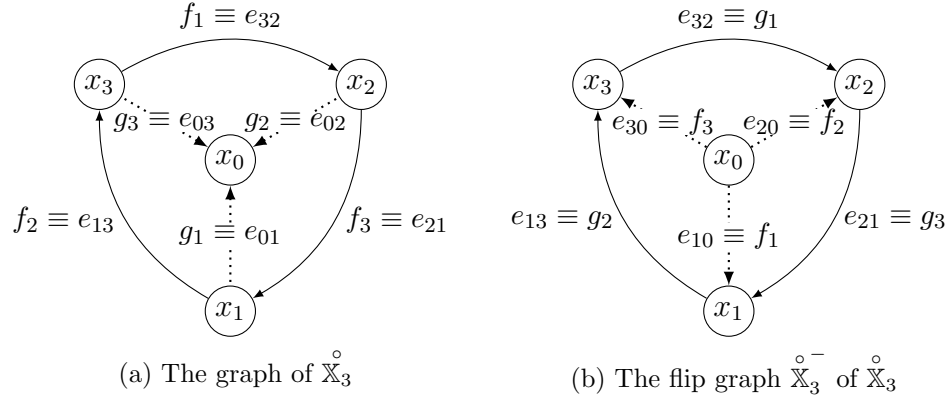


Fig. 5: Thr graph structure of $\overset{\circ}{\mathbb{X}}_3$

To do this, just make the following assumptions:

- (1) Each node corresponds to a real number, denoted by $x_i, 0 \leq i \leq 3$. Each edge $e_{ij}, 0 \leq i \neq j \leq 3$ corresponds to a function, and their arguments are $x = (x_0, x_1, x_2, x_3)$.
- (2) For each node x_i , each edge entering or out of it corresponds to a quantity, that is, for node x_i , the edge into it e_{ji} corresponds to $+\frac{\partial e_{ji}}{\partial x_j}$, the edge out of it e_{ij} corresponds to $-\frac{\partial e_{ij}}{\partial x_j}$, the sum of this quantities corresponds to incoming and

outgoing edges generates a quantity, j^i , corresponding to this node:

$$\sum_j \frac{\partial e_{ji}}{\partial x_j} - \sum_j \frac{\partial e_{ij}}{\partial x_j} = j^i. \quad (9)$$

- (3) Each $\overset{\circ}{\mathbb{X}}_3$ has a flip figure, $\overset{\circ-}{\mathbb{X}}_3$, as shown in 5b, it is produced by swapping the three wheel radiation edges(g_1, g_2, g_3) with three wheel loop edges(f_1, f_2, f_3) under TCD(Through Center Dual, ie, each edge corresponds to the edge which would be crossed when it is extended throug the center x_0 .), and then reversing the wheel radiation edge resulted from the swapping. Corresponding graph also has the balance equations described by (9).

Under these assumptions, we can write down the following equations by Fig 5a:

$$\frac{\partial g_1}{\partial x_0} + \frac{\partial g_2}{\partial x_1} + \frac{\partial g_3}{\partial x_3} = j^0 \quad (10a)$$

$$-\frac{\partial g_1}{\partial x_0} + \frac{\partial f_3}{\partial x_2} - \frac{\partial f_2}{\partial x_3} = j^1 \quad (10b)$$

$$-\frac{\partial g_2}{\partial x_0} + \frac{\partial f_1}{\partial x_3} - \frac{\partial f_3}{\partial x_1} = j^2 \quad (10c)$$

$$-\frac{\partial g_3}{\partial x_0} + \frac{\partial f_2}{\partial x_1} - \frac{\partial f_1}{\partial x_2} = j^3 \quad (10d)$$

write down the following equations by Fig 5b:

$$-\frac{\partial f_1}{\partial x_1} - \frac{\partial f_2}{\partial x_2} - \frac{\partial f_3}{\partial x_3} = h^0 \quad (11a)$$

$$\frac{\partial f_1}{\partial x_0} + \frac{\partial g_3}{\partial x_2} - \frac{\partial g_2}{\partial x_3} = h^1 \quad (11b)$$

$$\frac{\partial f_2}{\partial x_0} + \frac{\partial g_1}{\partial x_3} - \frac{\partial g_3}{\partial x_1} = h^2 \quad (11c)$$

$$\frac{\partial f_3}{\partial x_0} + \frac{\partial g_2}{\partial x_1} - \frac{\partial g_1}{\partial x_2} = h^3 \quad (11d)$$

If we let $h^i = 0, 1 \leq i \leq 4, g_1 \rightarrow \frac{1}{c^2}E_x, g_2 \rightarrow \frac{1}{c^2}E_y, g_3 \rightarrow \frac{1}{c^2}E_z, f_1 \rightarrow B_x, f_2 \rightarrow B_y, f_3 \rightarrow B_z, j^0 \rightarrow \frac{\rho}{\varepsilon_0}$, then we get the electromagnetic field equations [1, p. 19]. This process is not perfect, its shortcomings is exposed when generalized to $\overset{\circ}{\mathbb{X}}_p, p > 3$ with more nodes, because there are multiple loops, TCD map above will face how to choose wheel loops to perform such flip transformations.

If in accordance with the electromagnetic field 4-potential theory, generalizing the equations to $\overset{\circ}{\mathbb{X}}_p$ is simple. This can be easily implemented as follows: First, define

the function A_i which is defined on x for each node x_i , then define

$$\frac{\partial A_i}{\partial x_j} - \frac{\partial A_j}{\partial x_i}, \quad (12)$$

for each edge e_{ij} . Finally, the equations are established by setting the balance conditions in each node.

But I have tried to use various methods to establish equations by the structure of graph, although ultimately have not found the method which can result in a simple second-order partial differential equations by SDS method¹². But I finally found an interesting way to construct the equations of motion, which uses a definition called by the generalized cross product.

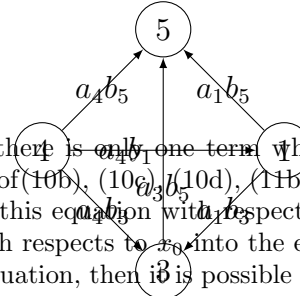
For example, suppose $a = (a_1, a_2, a_3, a_4, a_5)$, $b = (b_1, b_2, b_3, b_4, b_5) \in \mathbb{R}^5$, so their generalized cross product

$$c \equiv (c_1, c_2, c_3, c_4, c_5) = a \times b \quad (13)$$

is defined as

$$\begin{aligned} c_1 &= a_3b_5 - b_3a_5 + a_3b_2 - b_3a_2 + a_3b_4 - b_3a_4 + \\ &\quad a_5b_2 - b_5a_2 + a_5b_4 - b_5a_4 + a_2b_4 - b_2a_4, \\ c_2 &= a_4b_1 - b_4a_1 + a_4b_3 - b_4a_3 + a_4b_5 - b_4a_5 + \\ &\quad a_1b_3 - b_1a_3 + a_1b_5 - b_1a_5 + a_3b_5 - b_3a_5, \\ c_3 &= a_5b_2 - b_5a_2 + a_5b_4 - b_5a_4 + a_5b_1 - b_5a_1 + \\ &\quad a_2b_4 - b_2a_4 + a_2b_1 - b_2a_1 + a_4b_1 - b_4a_1, \\ c_4 &= a_1b_3 - b_1a_3 + a_1b_5 - b_1a_5 + a_1b_2 - b_1a_2 + \\ &\quad a_3b_5 - b_3a_5 + a_3b_2 - b_3a_2 + a_5b_2 - b_5a_2, \\ c_5 &= a_2b_4 - b_2a_4 + a_2b_1 - b_2a_1 + a_2b_3 - b_2a_3 + \\ &\quad a_4b_1 - b_4a_1 + a_4b_3 - b_4a_3 + a_1b_3 - b_1a_3. \end{aligned} \quad (14)$$

Then there are $a \cdot c = b \cdot c = 0$. This method is in fact calculated the sum of the products between nodes as the digram similar to Fig 6. Fig 6 is the illustration of calculating the first component of the c . This algorithm has a part of properties of the cross product, there are



¹² SDS(Second Differentiate then Substitute) method means there is only one term which is a function differentiating with respects to time x_0 in each equation of (10b), (10c), (10d), (11b), (11c), and (11d). For example, it is g_1 (10b), so when we differentiat this equation with respects to x_0 , we can substitute the derivative of the other functions f_3 , f_2 with respects to x_0 into the equation and make the euquation into a second order partial differential equation, then it is possible to make this equation becomes all about g_1 of second order partial differential equation, here it is indeed the case. This method can be extended to other equations with similar features. The calculation structure of a component in the generalized cross product

also many interesting arithmetic properties. According to it we can construct a generalized curl $\nabla \times$, so that we can follow the form of

$$\begin{aligned}
 \nabla \cdot \vec{\mathbf{E}} &= \frac{1}{\varepsilon_p} \rho \\
 -\frac{1}{c_p^2} \frac{\partial \vec{\mathbf{E}}}{\partial t} + \nabla \times \vec{\mathbf{B}} &= \mu_p \vec{\mathbf{j}} \\
 \nabla \cdot \vec{\mathbf{B}} &= 0 \\
 \frac{\partial \vec{\mathbf{B}}}{\partial t} + \nabla \times \vec{\mathbf{E}} &= 0
 \end{aligned} \tag{15}$$

to establish the equations of motion for the general $\overset{\circ}{\mathbb{X}}_p$, but resulted equations by this method have not simple second order form.

Although the use of $\overset{\circ}{\mathbb{X}}_3$ to represent Maxwell's equations is not perfect, it is still able to give us a lot of inspirations about electro-

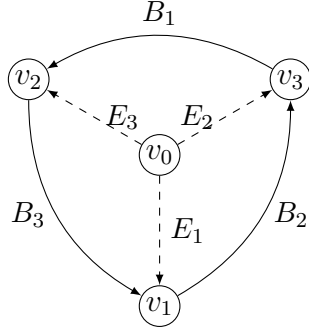


Fig. 7: Lorentz force and $\overset{\circ}{\mathbb{X}}_3$

magnetic fields and space-time. For example, we can use it to represent Lorentz force [1, p. 14] formula

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}). \tag{16}$$

This formula can be illustrated by the Fig 7 like this: The three nodes on the wheel loop in the graph correspond to the three spatial dimensions, x_1, x_2, x_3 , it is labeled by v_1, v_2, v_3 , which means that these three nodes also correspond to the speed of the particle. Wheel center node is labeled by $v_0 \equiv 1$, and it corresponds to the time dimension x_0 . Three wheel radiation edges emitted from the wheel center correspond to the electric field components $E_i, 1 \leq i \leq 3$, and the wheel loop edges correspond to three magnetic field components $B_i, 1 \leq i \leq 3$. Each edge exert force on both

nodes of its two ends, the force on one end is defined as the quantity corresponding to another end multiplied by the quantity corresponding to the edge. Then Lorentz force component in each dimension F_i is defined as the sum of all quantities which correspond to the edges associated with x_i multiplied by the charge q , the contribution from in-edge is positive, while the contribution from out-edge is negative.

Since the angular momentum operator in quantum mechanics also bases on this structure, so using it to understand the angular momentum is also helpful for us. Angular momentum operator [2, p. 127] is defined as

$$\begin{aligned}\hat{L}_1 &\equiv \hat{x}_2 \frac{\partial}{\partial x_3} - \hat{x}_3 \frac{\partial}{\partial x_2} \\ \hat{L}_2 &\equiv \hat{x}_3 \frac{\partial}{\partial x_1} - \hat{x}_1 \frac{\partial}{\partial x_3} \\ \hat{L}_3 &\equiv \hat{x}_1 \frac{\partial}{\partial x_2} - \hat{x}_2 \frac{\partial}{\partial x_1}\end{aligned}\tag{17}$$

We can illustrate the definition according to Fig 8 like this: let the wheel center node

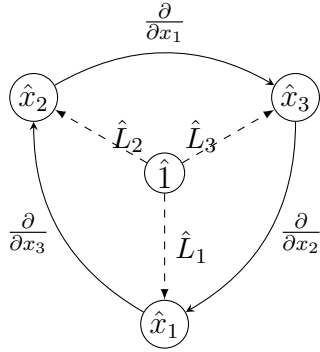


Fig. 8: The definition of angular and \mathbb{X}_3 in the complete graph

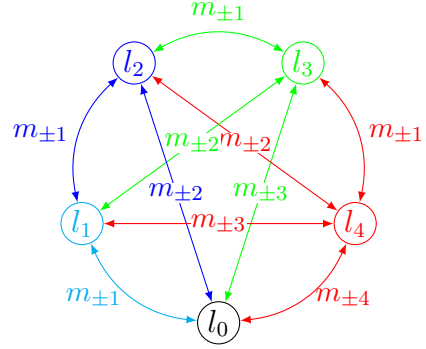


Fig. 9: The eigenstates of angular operator in the complete graph

be $\hat{x}_0 \equiv \hat{1}$, that is identity operator. For each node \hat{x}_i on the wheel loop, let's for each of its in-edges e_{ji} define $+\hat{x}_j \hat{e}_{ji}$, for each of its out-edges e_{ij} , define $-\hat{x}_j \hat{e}_{ij}$, where \hat{e}_{ji} , \hat{e}_{ij} is the corresponding operator on the edge e_{ij} , as shown in Fig 8. Then, we assume that for each node \hat{y}_i , the sum of all expressions which correspond to the edges of the node must be 0, then we can write out the following three balance equations for each node:

$$\begin{aligned}\hat{1}\hat{L}_1 + \hat{x}_3 \frac{\partial}{\partial x_2} - \hat{x}_2 \frac{\partial}{\partial x_3} &= 0 \\ \hat{1}\hat{L}_2 + \hat{x}_1 \frac{\partial}{\partial x_3} - \hat{x}_3 \frac{\partial}{\partial x_1} &= 0 \\ \hat{1}\hat{L}_3 + \hat{x}_2 \frac{\partial}{\partial x_1} - \hat{x}_1 \frac{\partial}{\partial x_2} &= 0\end{aligned}\tag{18}$$

This is just the definition of angular momentum operator above.

Another interesting fact is that we can put the simultaneously eigenkets of $L^2 = \hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_3^2$ and \hat{L}_3 into a complete graph, as shown in 9, where each node, $l_k, 0 \leq k \leq 4$, represents the state $|k, 0\rangle$, while the $2k$ bidirectional edges connected to it correspond to a state $|k, s\rangle, -k \leq s \leq k, s \neq 0$.

In relativistic electrodynamics, the 4-vector potential of electromagnetic field is naturally convenient to represent by $\overset{\circ}{\mathbb{X}}_3$, and it has a gauge symmetry, corresponding gauge group is $U(1)$, which seems to correspond to the loop in \mathbb{X}_3 . So I have a question:

\mathbb{X}_p with $\frac{p-1}{2}$ loops corresponds with a gauge group $SU(\frac{p-1}{2})$?

Mentioned above is the first interesting approach to understand the physics that we get from ACT. The most important achievement here is that we probably should not regard the four dimensions of space-time as four numbers unrelated, but regarded them as a whole as a structure. There is a structure in each space-time point, and this structure determines by a simple integer. Special relativity has taught us that there is a special dependent relationship between the time and the space. In fact, the previous figures all are related with special relativity. And subtly, Poincaré group [9, p. 22] [4, p. 5] [10] derived from special relativity can be very appropriately illustrated by Fig 10 : where, $J_i, 1 \leq i \leq 3$, are rotate transformations, and three $K_i, 1 \leq i \leq 3$,

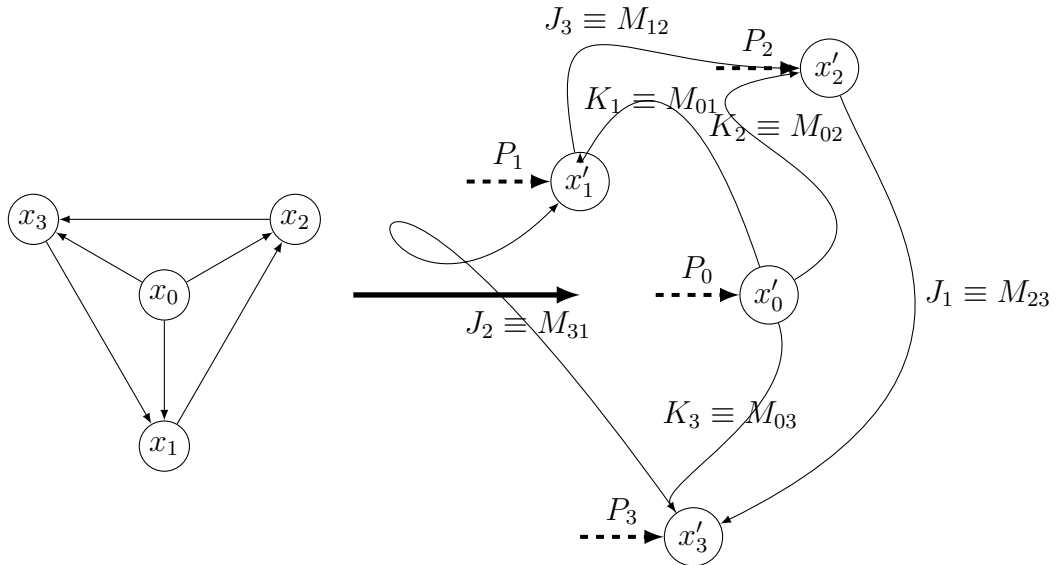


Fig. 10: The Poincaré group and $\overset{\circ}{\mathbb{X}}_3$

are boost transformations, $P_i, 0 \leq i \leq 3$, are the space-time translations.

Perhaps it is because the structure of space-time has special relevance with properties of integers, so any spatial dimension is impossible.

3.3 Interaction and a math problem

Wuxing provides us with many instructive on thinking the interaction between the objects. In ACT, in the basic sense Wuxing is used to explain the relationship between the objects, or interactions. On the whole, there are two kinds of interaction, but in the local of each element, there are four kinds of interaction, because each kind of interaction are directional. When an action is applied to another element by one element, it will eventually suffer the effects of the reaction, because along the edge that is acted on, there exists a path back to itself.

In physics, force is defined by a directional vector, but Newton's third law tells us that the force between objects is mutual. However, Newton's third law does not tell us how this is happening. Maybe we can assume that there exsist such a closed loop mechanism inside the law of interactions and the law of conservation.

According to this view, I think about the following questions: when an element exert effects on another, how many path, which is of length n and back to the elelment, does the action have. Since in the \mathbb{X}_p , each edge is represented by a $+a, 1 \leq a \leq \frac{p-1}{2}$, so it reduces to the solution number of following equation, $S(p, n)$:

$$\sum_{k=1}^n a_k \equiv 0 \pmod{p} \quad 1 \leq a_k \leq \frac{p-1}{2}. \quad (19)$$

I have found by calculations that the soltuions number of this equation for a fixed p have a linear recursion relations in n , and the characteristic polynomial of the linear recurrence relation formula $F(p)$ is:

$$F_p(z) = \prod_{k=1}^{\frac{p-1}{2}} (z^k - 1). \quad (20)$$

This relationship for general positive integers m is also true, it is

$$F_m(z) = \prod_{k=1}^{\frac{m-1}{2}} (z^{\frac{k}{(k,m)}} - 1) \quad (21)$$

Here (k, m) is the greatest common divisor of k and m .

For $p = 5$, the sequence of $S(5, n), n \geq 3$, is $\{a_k\}_{k \geq 1}$:

$$3, 6, 8, 11, 13, 16, 18, 21, 23, 26, 28, 31, 33, 36, 38, 41, 43, \dots \quad (22)$$

Its recurrence formula is

$$a_{k+3} = -a_k + a_{k+1} + a_{k+2} \quad (23)$$

The characteristic polynomial of the recurrence formula is

$$F_5(z) = z^3 - z^2 - z + 1 = (z - 1)(z^2 - 1). \quad (24)$$

I have not proved this formula, who would like to give me a simple proof?

4 The Eight Trigrams

The Eight Trigrams (hereinafter abbreviated as Bagua¹³) has a special status in ACT, it is the basic symbols of the language in *Book of Changes*, in fact, it also gives the specific theories about yin and yang. In ACT, every Gua¹⁴ consists of some ordered positions, and Yao¹⁵ lives on the positions. Every Yao is yin Yao, denoted by -- -- , or yang Yao, denoted by — — , and the entire Gua usually is denoted by a heap of Yaos arranged by the order of from down to up. For example, ☰ denotes a Gua of three Yaos, its name is 兑. From the viewpoint of modern mathematics, a Gua is a binary digit, every Yao is a variable whose value space is $\mathbb{F}_2 = \{0, 1\}$. In the following, we use 0 to denote yin Yao, use 1 to denote yang Yao, use 01 sequence with the order of from right to left to represent Gua, for example, previous Gua 兑 is denote as 011. But in ACT, Gua is far from so simple, instead it has a very rich meaning.

According to the number of postions used in a Gua, there are six Gua sapces in ACT. That is the two Yis Gua space, \mathbb{F}_2^1 , consisting of yin Yao (-- -- or 0) and yang Yao (— — or 1), it has only one Yao; the four Xiangs Gua space, \mathbb{F}_2^2 , with two Yaos, the names of the Guas in it are: Taiyin(☷ or 00), Shaoyin(☱ or 01), Shaoyang(☳ or 10), Taiyang(☰ or 11)¹⁶; the eight Guas space, \mathbb{F}_2^3 , with three Yaos, the names of the Guas in it are: Qian(☰ or 111), Kun(☷ or 000), Zhen(☳ or 001), Kan(☵ or 010), Gen(☶ or 100), Xun(☴ or 110), Li(☲ or 101), Dui(☱ or 011)¹⁷; sixty four Guas space, \mathbb{F}_2^{3+3} , with 6 Yaos, but these Yaos are divided into two groups of up Gua and down Gua equally. Each of the Guas in the space also have a special name, but we omitted here. We let these spaces are abbreviated as $\mathbb{F}_2^1 \equiv \mathbb{G}_1$, $\mathbb{F}_2^2 \equiv \mathbb{G}_2$, $\mathbb{F}_2^3 \equiv \mathbb{G}_3$,

¹³The chinese is 八卦.

¹⁴The chinese is 卦, the english meaning is binary.

¹⁵The chinese is 爻, the english meaning is bit.

¹⁶The chinese for these name is: Taiyin, 太阴, means extreme yin; Shaoyin, 少阴, means young yin; Shaoyang, 少阳, means young yang; Taiyang, 太阳, means extreme yang.

¹⁷The chinese for these names: Qian is 乾, Kun is 坤, Zhen is 震, Kan is 坎, Gen is 艮, Xun is 巽, Li is 离, Dui is 兑.

$\mathbb{F}_2^{3+3} \equiv \mathbb{G}_3^3$. There is also another Gua space in ACT, it really is to let the value of the Yao in each Gua in \mathbb{G}_3^3 is the element in \mathbb{G}_2 , but here the elements in \mathbb{G}_2 are usually denoted by $\text{---}\times$ (called by old yin), --- (called by young yin), --- (called by young yang), $\text{---}\circ$ (called by old yang) respectively, but we also use $\overset{\times}{0}, 0, 1, \overset{\circ}{1}$ to denote the four symbols respectively. Old yin and old yang both are called moving or changing Yao, in divination they represent changes. We denote this Gua space by $\mathbb{G}_3^3 \times \mathbb{G}_2$. In general, when each Yao corresponds to another Gua space \mathbb{G}_m we always use $\times \mathbb{G}_m$ to denote it.

4.1 The traditional theory

ACT is mysterious, because there is almost no reasoning process about the theory. ACT's successor almost never ponder their reasoning from the general principle, in many cases, their interpretation for the classical theory can only be understood as an association helping for memory. In the Bagua theory, this case is more serious, there are too many theories which are difficult to understand the origin of it. Here we will only focus on two Bagua theories: one is a Xiantian theory of Bagua, another theory is Houtian theory of Bagua¹⁸.

The main difference between these two theories is that they assign a different order, and specify a different location on the plane for the elements in \mathbb{G}_3 . The order of the Bagua to which Xiantian theory assigns is equivalent to modern binary theory, the order is:

$$\begin{array}{cccccccc} \text{乾,} & \text{兑,} & \text{离,} & \text{震,} & \text{巽,} & \text{坎,} & \text{艮,} & \text{坤,} \\ 111, & 011, & 101, & 001, & 110, & 010, & 100, & 000 \end{array} \quad (25)$$

where the first line is the chinese names of Bagua, and the second row is the corresponding binary representation. The number with distinguished significance is always put on the front in ACT. If we exchange yin-yang symbols and reverse the position order, then we can see this order is same with the order of binary digits. In Houtian theory, the Bagua is arranged by a sequence of pairs consisting of two mutual NOT Guas:

$$\begin{array}{cccccccc} \text{乾,} & \text{坤,} & \text{震,} & \text{巽,} & \text{坎,} & \text{离,} & \text{艮,} & \text{兑,} \\ 111, & 000, & 001, & 110, & 010, & 101, & 011, & 100, \end{array} \quad (26)$$

The point is that it has divided the eight Guas into two symmetrical subsequence, that is, the subsequence corresponding to odd-numbered positions, and the subsequence corresponding to even-numbered positions((assuming first number is 1). Because of this structure, the eight Guas gain the famous names, these names actually are the

¹⁸Xiantian in chinese is 先天, literally means before-sky, so it means before birth, even includes before universe birth, the english meaning is inborn, innate. Houtian in chinese is 后天, literally means after-sky, so it means after birth, even includes after universe birth, the english meaning is acquired, postnatal.

call of the family members, the Guas appear on the odd sub-sequence correspond to male members of the family, the names of which in turn are father, eldest son, middle son, young son; the Guas appear on the even sub-sequence correspond to female members of the family, the names of which in turn are mother, eldest daughter, middle daughter, young daughter.

Another difference between the two theories is that they specify the elements of \mathbb{G}_3 with different locations on the plane, Fig 11 is specified by Xiantian theory, and Fig 12 is specified by Houtian theory. The reason for these two definitions both are difficult to understand, apparently Fig 11 is easier to understand, where the two centrosymmetric Guas are mutual NOT. It should be noted, in ACT, the location is always relation to the orientation which is positioned by the compass direction, i.e. the direction of the geomagnetic field. In addition, they are also related to the magic square. Although I try to give an explanation for this, but the results of the research could not able to give the perfect answer.

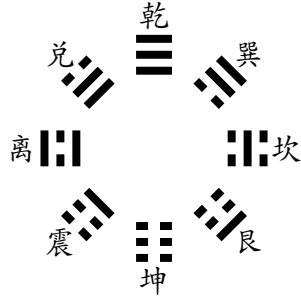


Fig. 11: The orientations of the eight trigrams in the plane for Xiantian theory

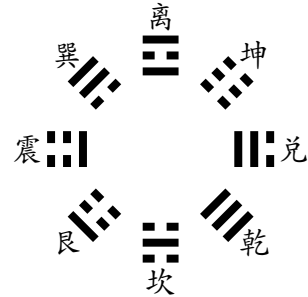


Fig. 12: The orientations of the eight trigrams in the plane for Houtian theory

But I found that the Xiantian theory and the Houtian theory is two theories of "one yin and one yang". In Xiantian theory, all Guas are equal, which correspond to the tree structure, as shown in 13, the representation is also often appeared in the literature; or corresponds to a cube, as shown in 14. And Houtian theory has a center or a root, which corresponds to the circular structure or cyclic structure, which can be shown in Fig 15 and Fig 16.

In ACT, there is a relation between Wuxing and Bagua, which is that the eight trigrams are mapped to the five elements, as shown in Tab. 1. I was trying to find a variety of ways to understand this relationship, the final conclusion is that using Fig 17 to understand the mapping is the most reasonable. In this understanding, the mapping from Bagua to Wuxing is still expressing the the Houtian structure of Bagua, but here the both parents Guas are Kangua(010) and Ligua(101).

While the theory about $\mathbb{G}_3 \mapsto \mathbb{X}_5$ is mysterious, but we can clearly establish the correspondence between \mathbb{X}_3 and \mathbb{G}_3 . This is just let the three Yaos in \mathbb{G}_3 correspond

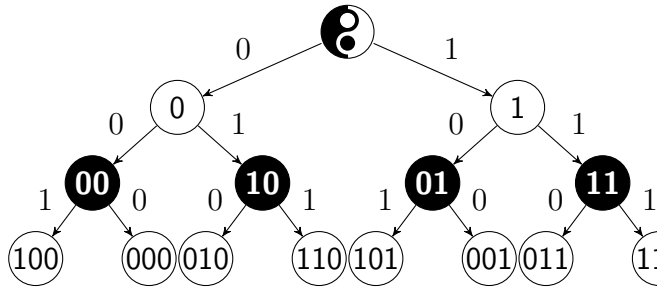


Fig. 13: The tree structure for the Xiantian theory

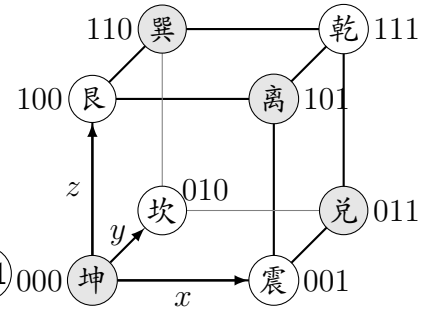


Fig. 14: The cube structure for the Houtian theory

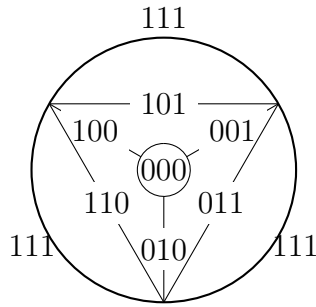


Fig. 15: The line circular diagram for Houtian theory

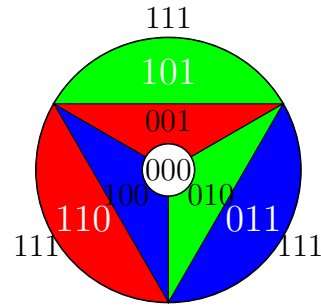


Fig. 16: The area circular diagram for Houtian theory

to the three nodes in \mathbb{X}_3 , and addition operators in \mathbb{X}_3 correspond to the circular shift operators in \mathbb{G}_3 , which is actually the isomorphism between two additive groups of \mathbb{Z}_3 . Moreover, we tend to see there is a close relationship between Houtian theory and the cyclic structure in \mathbb{G}_3 .

As we have introduced a node x_0 in \mathbb{X}_3 , we also introduce a Yao with numbered by 0 in \mathbb{G}_3 , but it is hidden. After \mathbb{G}_3 has been introduced such a hidden Yao, it is denoted by $\mathring{\mathbb{G}}_3$. Similar to the case in $\mathring{\mathbb{X}}_3$, the hidden Yao in $\mathring{\mathbb{G}}_3$ will be used to represent the time, while the other three Yaos are used to represent three spatial dimensions.

eight trigrams (\mathbb{G}_3)	000	100	001	111	010	001	110	101
five elements (\mathbb{X}_5)	土	土	金	金	水	木	木	火

Table 1: The map from the eight trigrams to the five elements

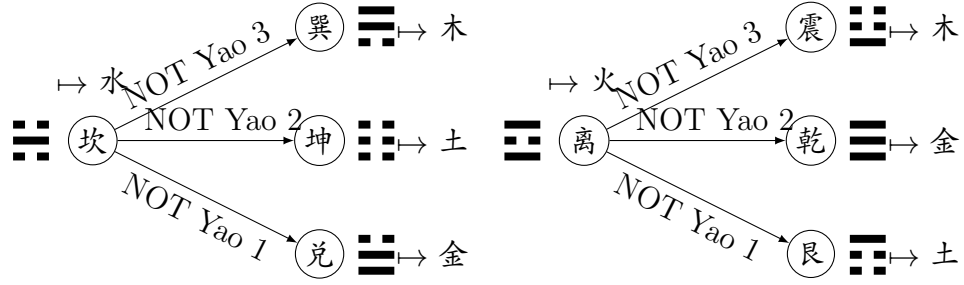


Fig. 17: The map from the eight trigrams to the five elements

4.2 The relation between Houtian theory and Maxwell's equations

In order to see the relation between Houtian theory and the Maxwell's equations[1, p. 19], let's correspond each component of the electric field \vec{E} and each component of the magnetic field \vec{B} to the six offspring Guas in Houtian theory.

$$\begin{aligned}
 E_1 &\mapsto M_{001}, E_2 \mapsto M_{010}, E_3 \mapsto M_{100} \\
 B_1 &\mapsto M_{110}, B_2 \mapsto M_{101}, B_3 \mapsto M_{011}
 \end{aligned}
 \tag{27}$$

where M_g , the g represents a Gua in the six offspring Guas, can be understood as another name for each component in \vec{E} and \vec{B} . We can also understand the mapping completely from Gua theory, that is, each Yao (including the hidden Yao) corresponds to a real space \mathbb{R}^1 , and each Gua g corresponds to a function M_g defined on Yao space \mathbb{R}^4 .

After using the new names for the components of the electric and magnetic field,

eight equations of Maxwell's equations in vacuum is:

$$\begin{aligned}
0 \cdot \frac{\partial M_{111}}{\partial x_0} + \frac{\partial M_{110}}{\partial x_1} + \frac{\partial M_{101}}{\partial x_2} + \frac{\partial M_{011}}{\partial x_3} &= 0 \\
\frac{\partial M_{110}}{\partial x_0} + 0 \cdot \frac{\partial M_{111}}{\partial x_1} + \frac{\partial M_{100}}{\partial x_2} - \frac{\partial M_{010}}{\partial x_3} &= 0 \\
\frac{\partial M_{101}}{\partial x_0} - \frac{\partial M_{100}}{\partial x_1} + 0 \cdot \frac{\partial M_{111}}{\partial x_2} + \frac{\partial M_{001}}{\partial x_3} &= 0 \\
\frac{\partial M_{011}}{\partial x_0} + \frac{\partial M_{010}}{\partial x_1} - \frac{\partial M_{001}}{\partial x_2} + 0 \cdot \frac{\partial M_{111}}{\partial x_3} &= 0 \\
0 \cdot \frac{\partial M_{000}}{\partial x_0} + \frac{\partial M_{001}}{\partial x_1} + \frac{\partial M_{010}}{\partial x_2} + \frac{\partial M_{100}}{\partial x_3} &= 0 \\
-\frac{\partial M_{001}}{\partial x_0} + 0 \cdot \frac{\partial M_{000}}{\partial x_1} + \frac{\partial M_{011}}{\partial x_2} - \frac{\partial M_{101}}{\partial x_3} &= 0 \\
-\frac{\partial M_{010}}{\partial x_0} - \frac{\partial M_{011}}{\partial x_1} + 0 \cdot \frac{\partial M_{000}}{\partial x_2} + \frac{\partial M_{110}}{\partial x_3} &= 0 \\
-\frac{\partial M_{100}}{\partial x_0} + \frac{\partial M_{101}}{\partial x_1} - \frac{\partial M_{110}}{\partial x_2} + 0 \cdot \frac{\partial M_{000}}{\partial x_3} &= 0
\end{aligned} \tag{28}$$

From here we can see that each equation follows the same pattern. To see this, we also need to let the four space-time parameters correspond to the Yaos, that is, for each Gua $g \in \mathbb{G}_3$, let

$$\begin{aligned}
x_0 &\mapsto \text{the hidden Yao of } M_g, \\
x_k &\mapsto \text{the } k\text{-th Yao of } M_g, 1 \leq k \leq 3.
\end{aligned} \tag{29}$$

This has already correspond each equation in (28) to a Gua g , it is the Gua which differentiate with respect to x_0 , that is, the Gua involved in the first term in the equation. Moreover, in this equation, the k -th term after the first item is related to the derivative of the Gua, which is obtained by g NOT its k -th Yao, with respect to x_k . We call this pattern of the equations by **Houtian pattern**. In this equations, 乾(111) and 坤(000) is a special, it's the same as with the case in Houtian theory.

Conversely, we can build eight equations by Houtian pattern, if we can determine the coefficients of each term in each equation. We will later discuss the issue on the coefficients. In the Maxwell's equations, their values are one of three $\pm 1, 0$, which is exactly the three possible values corresponding to the spin of bosons.

This is a very interesting fact. Therefore, we can understand the electromagnetic field from the Houtian theory of Bagua: its electric field is consisting of sons 001, 010, 100 belonging to yang, and mother 000 belonging to yin, its electric field magnetic field is consisting of daughters 110, 101, 011 belonging to yin, and father 111 belonging to yang. According to the electromagnetic induction relationship in physics, that is,

the laws of physics described by Faraday's law, Gauss's law and Ampere's law, which is very appropriate. Especially,

$$\begin{aligned} 001 \oplus 010 \oplus 100 &= 111, \\ 110 \oplus 101 \oplus 011 &= 000, \end{aligned} \quad (30)$$

which very properly describes the characteristics of the direction of the electric field and magnetic[20]. The direction of the magnetic field is the direction along which charged particle will not be exerted on by magnetic force, thus suitable to represent by 000. The direction of the electric field is the direction along which charged particle will be exerted on by electric force, thus suitable to represent by 111. According to this view, we get another kind of understanding about the cross product, it represents the mutual NOT relations between in the offspring Guas in Houtian.

Another interesting fact is that if we follow the correspondence between the male Guas and the female Guas, that is:

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial^2}{\partial t^2}, \quad \frac{\partial}{\partial x} \rightarrow \frac{\partial^2}{\partial y \partial z}, \quad \frac{\partial}{\partial y} \rightarrow \frac{\partial^2}{\partial z \partial x}, \quad \frac{\partial}{\partial z} \rightarrow \frac{\partial^2}{\partial x \partial y}, \quad (31)$$

Then we can establish the following second-order partial differential equations:

$$\begin{aligned} \frac{\partial^2 E_x}{\partial y \partial z} + \frac{\partial^2 E_y}{\partial z \partial x} + \frac{\partial^2 E_z}{\partial x \partial y} &= 0 \\ -\frac{\partial^2 E_x}{\partial t^2} + \frac{\partial^2 E_z}{\partial z \partial x} - \frac{\partial^2 E_y}{\partial x \partial y} &= 0 \\ -\frac{\partial^2 E_y}{\partial t^2} + \frac{\partial^2 E_x}{\partial x \partial y} - \frac{\partial^2 E_z}{\partial y \partial z} &= 0 \\ -\frac{\partial^2 E_z}{\partial t^2} + \frac{\partial^2 E_y}{\partial y \partial z} - \frac{\partial^2 E_x}{\partial z \partial x} &= 0 \\ \frac{\partial^2 B_x}{\partial y \partial z} + \frac{\partial^2 B_y}{\partial z \partial x} + \frac{\partial^2 B_z}{\partial x \partial y} &= 0 \\ \frac{\partial^2 B_x}{\partial t^2} + \frac{\partial^2 B_z}{\partial z \partial x} - \frac{\partial^2 B_y}{\partial x \partial y} &= 0 \\ \frac{\partial^2 B_y}{\partial t^2} + \frac{\partial^2 B_x}{\partial x \partial y} - \frac{\partial^2 B_z}{\partial y \partial z} &= 0 \\ \frac{\partial^2 B_z}{\partial t^2} + \frac{\partial^2 B_y}{\partial y \partial z} - \frac{\partial^2 B_x}{\partial z \partial x} &= 0 \end{aligned} \quad (32)$$

Use SDS method, we can form the above equations into the following fourth-order equations:

$$\frac{\partial^4 B_x}{\partial t^4} + \frac{\partial^4 B_x}{\partial x^2 \partial z^2} + \frac{\partial^4 B_x}{\partial x^2 \partial y^2} + \frac{\partial^4 B_x}{\partial y^2 \partial z^2} = 0, \quad (33)$$

In relativistic electrodynamics, the electromagnetic field is reduced to the 4-potential $A = (A_0, A_1, A_2, A_3)$:

$$\begin{aligned} E_1 &= \frac{\partial A_1}{\partial x_0} - \frac{\partial A_0}{\partial x_1}, & E_2 &= \frac{\partial A_2}{\partial x_0} - \frac{\partial A_0}{\partial x_2}, & E_3 &= \frac{\partial A_3}{\partial x_0} - \frac{\partial A_0}{\partial x_3}, \\ B_1 &= \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}, & B_2 &= \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}, & B_3 &= \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}, \end{aligned} \quad (34)$$

This corresponds to the processes where four Xiangs produce eight Guas in ACT. We can express it as the following Fig 18. Thus, the physical quantities of the

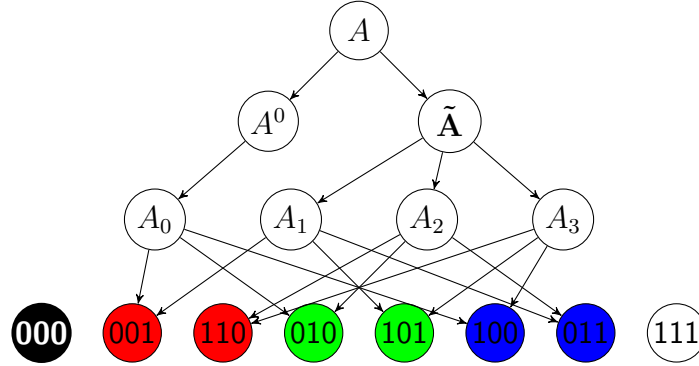


Fig. 18: The generating tree for Houtia theory

electromagnetic field is represented as a tensor:

$$(F_{\mu\nu}) = \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right)_{4 \times 4} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}. \quad (35)$$

Using the viewpoint under which we derive the motion equations of electromagnetic field, we can draw the Fig 19, where the four Xiangs are the four nodes in the graph, correspond to the space-time parameters $x = (x_0, x_1, x_2, x_3)$ and 4-potential $A = (A_0, A_1, A_2, A_3)$:

$$\begin{aligned} 00 &\mapsto x_0, A_0 \\ 01 &\mapsto x_1, A_1 \\ 11 &\mapsto x_2, A_2 \\ 10 &\mapsto x_3, A_3 \end{aligned} \quad (36)$$

the six offspring Guas correspond to the edges in the graph and correspond to the six components of the electromagnetic field by (27). If using a digital circuit views, then

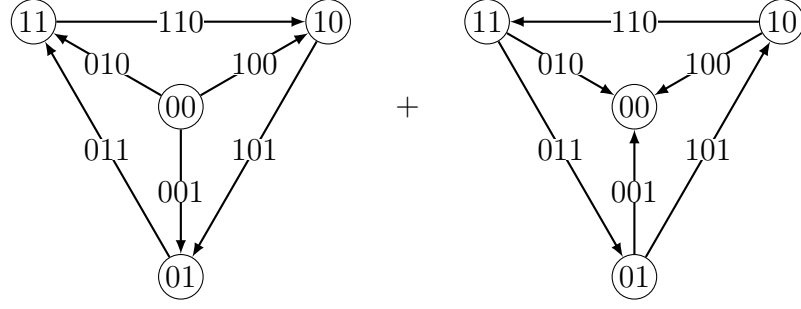


Fig. 19: The decomposition of the electromagnetic tensor

we will find the graph corresponds to exactly two shift register circuit¹⁹:

$$\begin{aligned} 00 &\rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow \dots \\ 001 &\rightarrow 010 \rightarrow 100 \rightarrow 001 \rightarrow \dots \end{aligned} \quad (37)$$

The direction of the edges in the two graphs of $\overset{\circ}{\mathbb{X}}_3$ in Fig 19 is reversed. If for each edge $\overrightarrow{N_s N_t}$ in the two graphs above, we define a quantity $\frac{\partial A_t}{\partial x_s}$, and let the left graph associated with a positive sign +1, let the right graph associated with a negative sign -1, then the tensor of the electromagnetic field is the superposition of the corresponding quantities on the two graphs, namely

$$(F_{\mu\nu}) = (Q_{\mu\nu}^+) + (Q_{\mu\nu}^-), \quad (38)$$

where

$$(Q_{\mu\nu}^+) = \begin{bmatrix} 0 & \frac{\partial A_1}{\partial x_0} & \frac{\partial A_2}{\partial x_0} & \frac{\partial A_3}{\partial x_0} \\ -\frac{\partial A_1}{\partial x_0} & 0 & \frac{\partial A_2}{\partial x_1} & \frac{\partial A_3}{\partial x_1} \\ -\frac{\partial A_2}{\partial x_0} & -\frac{\partial A_2}{\partial x_1} & 0 & \frac{\partial A_3}{\partial x_1} \\ -\frac{\partial A_3}{\partial x_0} & -\frac{\partial A_1}{\partial x_1} & -\frac{\partial A_3}{\partial x_2} & 0 \end{bmatrix}, \quad (Q_{\mu\nu}^-) = \begin{bmatrix} 0 & -\frac{\partial A_0}{\partial x_1} & -\frac{\partial A_0}{\partial x_2} & -\frac{\partial A_0}{\partial x_3} \\ \frac{\partial A_0}{\partial x_1} & 0 & -\frac{\partial A_1}{\partial x_2} & -\frac{\partial A_1}{\partial x_3} \\ \frac{\partial A_0}{\partial x_2} & \frac{\partial A_1}{\partial x_2} & 0 & -\frac{\partial A_2}{\partial x_3} \\ \frac{\partial A_0}{\partial x_3} & \frac{\partial A_3}{\partial x_1} & \frac{\partial A_2}{\partial x_3} & 0 \end{bmatrix}. \quad (39)$$

According to Maxwell's equations, the four Xiangs satisfy the second order partial differential equations:

$$\nabla^2 A_i = 0, \quad 0 \leq i \leq 3. \quad (40)$$

An important property of A is that when it is transformed by gauge transformation

$$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Gamma(x), \quad \Gamma \text{ is an arbitrary differentiable function,} \quad (41)$$

the physical quantities, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, generated by A does not change. This is obviously due to the results of

$$\frac{\partial^2 \Gamma}{\partial x_i \partial x_j} = \frac{\partial^2 \Gamma}{\partial x_j \partial x_i}. \quad (42)$$

¹⁹https://en.wikipedia.org/wiki/Ring_counter.

In general, for a $f = (f_0, f_1, f_2, f_3)$, if we require

$$\begin{aligned} \frac{\partial(A_i + f_i)}{\partial x_j} - \frac{\partial(A_j + f_j)}{\partial x_i} &= \frac{\partial A_i}{\partial x_j} - \frac{\partial A_j}{\partial x_i} \\ \implies \frac{\partial f_i}{\partial x_j} &= \frac{\partial f_j}{\partial x_i} \end{aligned} \quad (43)$$

then we eventually can be reduced to the $\nabla\Gamma$ of a $\Gamma(x)$. This shows that A can be further reduced to a quantity, it is perhaps the meaning of Tai Chi.

4.3 The relation between Xiantian theory and Dirac's equations

Follow the same method in the previous section, we can find the relation between Xiantian theory and Dirac's equations[3, p. 81][4, p. 56]. The form of the Diracs's equations is:

$$i\gamma^\mu \partial_\mu \phi = m\phi, \quad (44)$$

As we all know, Dirac's equations has three different representations. Here we first use Dirac's gamma matrices notation, which is

$$\begin{aligned} \gamma_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \gamma_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \\ \gamma_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \gamma_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (45)$$

We will discuss the other two forms in the latter.

We set $\phi_k = u_k + iv_k, 1 \leq k \leq 4$, that is, u_k is the real part of ϕ_k , v_k is the imaginary part of ϕ_k , and then let the these eight components denoted by

$$\begin{aligned} u_1 &\mapsto D_{000}, u_2 \mapsto D_{101}, u_3 \mapsto D_{100}, u_4 \mapsto D_{001}, \\ v_1 &\mapsto D_{011}, v_2 \mapsto D_{110}, v_3 \mapsto D_{111}, v_4 \mapsto D_{010}, \end{aligned} \quad (46)$$

separately, then after the equations (44) were expanded with the new names, we get:

$$\begin{aligned}
\frac{\partial D_{000}}{\partial x_0} + \frac{\partial D_{001}}{\partial x_1} + \frac{\partial D_{010}}{\partial x_2} + \frac{\partial D_{100}}{\partial x_3} &= mD_{011} \\
\frac{\partial D_{101}}{\partial x_0} + \frac{\partial D_{100}}{\partial x_1} - \frac{\partial D_{111}}{\partial x_2} - \frac{\partial D_{001}}{\partial x_3} &= mD_{110} \\
-\frac{\partial D_{100}}{\partial x_0} - \frac{\partial D_{101}}{\partial x_1} - \frac{\partial D_{110}}{\partial x_2} - \frac{\partial D_{000}}{\partial x_3} &= mD_{111} \\
-\frac{\partial D_{001}}{\partial x_0} - \frac{\partial D_{000}}{\partial x_1} + \frac{\partial D_{011}}{\partial x_2} + \frac{\partial D_{101}}{\partial x_3} &= mD_{010} \\
-\frac{\partial D_{011}}{\partial x_0} - \frac{\partial D_{010}}{\partial x_1} + \frac{\partial D_{001}}{\partial x_2} - \frac{\partial D_{111}}{\partial x_3} &= mD_{000} \\
-\frac{\partial D_{110}}{\partial x_0} - \frac{\partial D_{111}}{\partial x_1} - \frac{\partial D_{100}}{\partial x_2} + \frac{\partial D_{010}}{\partial x_3} &= mD_{101} \\
\frac{\partial D_{111}}{\partial x_0} + \frac{\partial D_{110}}{\partial x_1} - \frac{\partial D_{101}}{\partial x_2} + \frac{\partial D_{011}}{\partial x_3} &= mD_{100} \\
\frac{\partial D_{010}}{\partial x_0} + \frac{\partial D_{011}}{\partial x_1} + \frac{\partial D_{000}}{\partial x_2} - \frac{\partial D_{110}}{\partial x_3} &= mD_{001}
\end{aligned} \tag{47}$$

Observing the above eight equations, we would find the same pattern for each equation: each equation corresponds to a Gua $g \in \mathbb{G}_3$, it is the one which differentiate with respect to time parameter x_0 , it is also the Gua involved in the first term in the left side of the equation, the k -th term after the first term at the left side of the equation are all related to the derivative of the Gua, which is obtained by NOT the k -th Yao of g , on x_k , $1 \leq k \leq 3$. The mass term on the right side of the equation corresponds to the Gua is obtained by NOT the lowest two Yaos of g . We call this change pattern as **Xiantian pattern**. It differs from the Houtian pattern is that coefficients all are ± 1 , but not 0, which happens to be the values set of the spin of fermions, and there is a mass term related to a Gua on the right side of the equation.

In the equations all Guas are equal, and therefore consistent with the Xiantian theory. It is worth noting that the equations describing the motion equation of a fermion field whose spin direction towards the direction of x_3 , and the two Guas corresponding to the real and imaginary part of each ϕ_k exactly are same as the third Yao (corresponding to x_3), but the first and second Yaos are different. It is very magical, because in popular opinion, the spinor is considered to have no relation to the space-time parameters, but we see that it has a direct relation to the space-time parameters.

Conversely, we can write down the Dirac's equations based on Xiantian pattern, if the coefficients in the equation can all be determined. We will also discuss the issue on the coefficients in the equations in the later.

Dirac's theory constructs an important quantity – electric current 4-vector – from

the spinors, its formula is:

$$\begin{aligned}
\bar{\psi}\gamma^0\psi &= v_4^2 + u_4^2 + v_3^2 + u_3^2 + v_2^2 + u_2^2 + v_1^2 + u_1^2 \\
&= D_{000}^2 + D_{001}^2 + D_{010}^2 + D_{011}^2 + D_{100}^2 + D_{101}^2 + D_{110}^2 + D_{111}^2, \\
\bar{\psi}\gamma^1\psi &= 2(u_1u_4 + v_1v_4 + u_2u_3 + v_2v_3) \\
&= 2(D_{000}D_{001} + D_{010}D_{011} + D_{100}D_{101} + D_{110}D_{111}), \\
\bar{\psi}\gamma^2\psi &= 2(u_1v_4 - v_1u_4 + v_2u_3 - u_2v_3) \\
&= 2(D_{000}D_{010} - D_{001}D_{011} + D_{100}D_{110} - D_{101}D_{111}) \\
\bar{\psi}\gamma^3\psi &= 2(u_1u_3 - v_2v_4 - u_2u_4 + v_1v_3) \\
&= 2(D_{000}D_{100} - D_{001}D_{101} - D_{010}D_{110} + D_{011}D_{111}),
\end{aligned} \tag{48}$$

From the viewpoint of ACT, which is just the relations between four Xiangs and eight Guas. According to the denotions of all terms in Guas, we can see each product term in $\bar{\psi}\gamma^k\psi$ is the product of two Guas which are mutual NOT at the k -th Yao. Viewing from the cube of Bagua, two terms, which are connected by one of the four red edges, make the product, as shown in 20. Thus eight nodes with equal status in the process

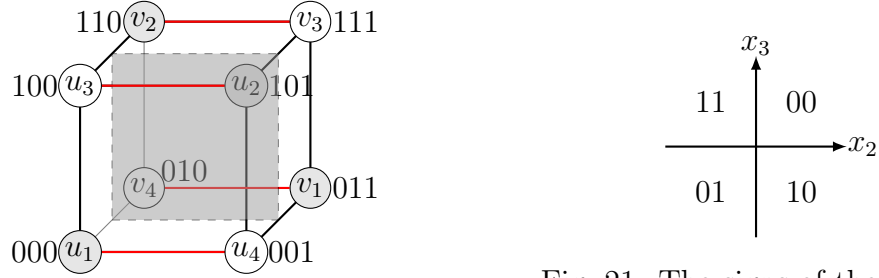


Fig. 20: The calculation structure of the components of current vector

Fig. 21: The signs of the terms in

of constitute the electric current vector. In fact, the electric current vector can be decomposed into the sum of eight four Xiangs $Q_g, g \in \mathbb{G}_3$:

$$(\bar{\psi}\gamma_0\psi, \bar{\psi}\gamma_1\psi, \bar{\psi}\gamma_2\psi, \bar{\psi}\gamma_3\psi) = \sum_{g \in \mathbb{G}_3} Q_k, \tag{49}$$

where

$$\begin{aligned}
Q_{000} &= (D_{000}^2, D_{000}D_{001}, +D_{000}D_{010}, +D_{000}D_{100}), \\
Q_{100} &= (D_{100}^2, D_{100}D_{101}, +D_{100}D_{110}, +D_{100}D_{000}), \\
Q_{001} &= (D_{001}^2, D_{001}D_{001}, -D_{001}D_{011}, -D_{001}D_{101}), \\
Q_{101} &= (D_{101}^2, D_{101}D_{100}, -D_{101}D_{111}, -D_{101}D_{001}), \\
Q_{010} &= (D_{010}^2, D_{010}D_{011}, +D_{010}D_{000}, -D_{110}D_{100}), \\
Q_{110} &= (D_{110}^2, D_{110}D_{111}, +D_{110}D_{100}, -D_{110}D_{010}), \\
Q_{011} &= (D_{011}^2, D_{011}D_{010}, -D_{011}D_{001}, +D_{011}D_{111}), \\
Q_{111} &= (D_{111}^2, D_{111}D_{110}, -D_{111}D_{101}, +D_{111}D_{011}).
\end{aligned} \tag{50}$$

Their graphical representation is 22, and the signs corresponding to the four Xiangs is determined by the lowest two Yaos of its Gua, as shown in Fig 21.

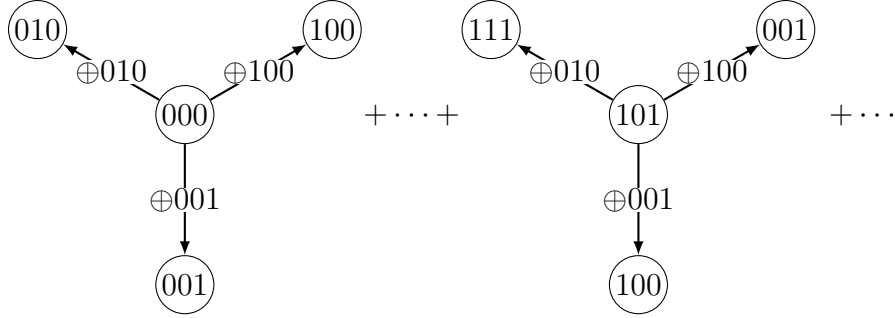


Fig. 22: The decomposition of the current vector

We now examine the gauge invariance of electrical current vector, which is actually derived from the transforming between the real and imaginary parts of each component of the spinor, that is,

$$\phi'_k = e^{i\theta} \phi_k, 1 \leq k \leq 4, \tag{51}$$

or

$$u'_k = \cos \theta u_k - \sin \theta v_k, v'_k = \sin \theta u_k + \cos \theta v_k, \tag{52}$$

Thus this is the symmetry between D_g and $D_{g \oplus 011}$. The reason that this transforme makes the electrical current vector unchanged actually lies in the fact that the following three expressions in this transformation remain the same:

$$\begin{aligned}
u'_s v'_t - v'_s u'_t &= (\cos \theta u_s - \sin \theta v_s)(\sin \theta u_t + \cos \theta v_t) - (\sin \theta u_s + \cos \theta v_s)(\cos \theta u_t - \sin \theta v_t) \\
&= \cos \theta \sin \theta u_s u_t + \cos^2 \theta u_s v_t - \sin^2 \theta v_s u_t - \sin \theta \cos \theta v_s v_t \\
&\quad - \sin \theta \cos \theta u_s u_t + \sin^2 \theta u_s v_t - \cos^2 \theta v_s u_t + \cos \theta \sin \theta v_s v_t \\
&= (\cos^2 \theta + \sin^2 \theta) u_s v_t - (\cos^2 \theta + \sin^2 \theta) v_s u_t = u_s v_t - v_s u_t.
\end{aligned} \tag{53}$$

$$\begin{aligned}
u'_s u'_t + v'_s v'_t &= (\cos \theta u_s - \sin \theta v_s)(\cos \theta u_t - \sin \theta v_t) + (\sin \theta u_s + \cos \theta v_s)(\sin \theta u_t + \cos \theta v_t) \\
&= \cos^2 \theta u_s u_t - \cos \theta \sin \theta u_s v_t - \sin \theta \cos \theta v_s u_t + \sin^2 \theta v_s v_t \\
&+ \sin^2 \theta u_s u_t + \sin \theta \cos \theta u_s v_t + \cos \theta \sin \theta v_s u_t + \cos^2 \theta v_s v_t \\
&= \cos^2 \theta u_s u_t + \sin^2 \theta u_s u_t + \sin^2 \theta v_s v_t + \cos^2 \theta v_s v_t = u_s u_t + v_s v_t.
\end{aligned} \tag{54}$$

$$\begin{aligned}
(u'_s)^2 + (v'_s)^2 &= (\cos \theta u_s - \sin \theta v_s)^2 + (\sin \theta u_s + \cos \theta v_s)^2 \\
&= \cos^2 \theta u_s^2 - 2 \cos \theta \sin \theta u_s v_s + \sin^2 \theta v_s^2 + \sin^2 \theta u_s^2 + 2 \cos \theta \sin \theta u_s v_s + \cos^2 \theta v_s^2 \\
&= \cos^2 \theta u_s^2 + \sin^2 \theta v_s^2 + \sin^2 \theta u_s^2 + \cos^2 \theta v_s^2 \\
&= u_s^2 + v_s^2.
\end{aligned} \tag{55}$$

These three expressions kept unchanged under the gauge transformation is actually require that $D_{g_1} D_{g_2}$ is always accompanied by the $D_{g_1 \oplus 011} D_{g_2 \oplus 011}$ with the proper sign.

Although (48) has given the procedure for calculating four Xiangs from eight Guas ϕ , its computational structure is shown in Fig 23, however, the reverse procedure of

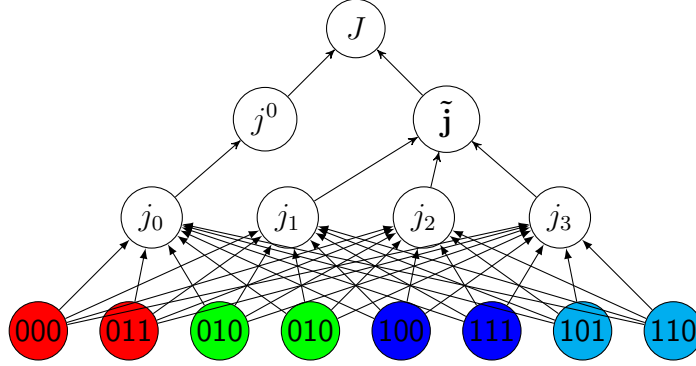


Fig. 23: The calculation structure of the current vector

calculating four images j from eight Guas ϕ is difficult, which is contrary to the case in Maxwell's equations, where the procedure of calculating eight Guas from four Xiangs is simpler.

Electric current vector satisfies the conservation equation:

$$\partial_\mu j^\mu = 0. \tag{56}$$

4.4 The Gua representation of Einstein's equations

Before discussing the coefficients problem of Xiantian pattern and Houtian pattern, let us look at the features of Einstein's equations which can be found in the perspective

of the changes of Guas. Although we do will find some magical phenomenon, but I do not have adequate research, so I can only give a simple explanation here.

In order to understand the Einstein's equations from the viewpoint of Gua, we must use the Gua space $\bar{\mathbb{G}}_3 \equiv \overset{\circ}{\mathbb{G}}_3$, but it is somehow equal to $\mathbb{G}_3 \times \mathbb{G}_2$. We first review the Gua representation for six quantities of electromagnetic field in (27). In physics, the electromagnetic field is considered to be a tensor, which is a matrix in $\mathbb{R}^{4 \times 4}$ and its components is of the form

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}, \quad (57)$$

and from the viewpoint of graph it corresponds to the edges in the graph. In fact, the Gua representation (27) we have gived is exactly the kind of representation based on the graph structure and implemented by binary. To see this point, let's represent the 16 locations in the four by four square matrix with the string of symbols like

$$b_3 b_2 b_1 . b_0, \quad (58)$$

where $b_k = 1$ iff $k = \mu$ or $k = \nu$, if $\mu = \nu$, then we use the $\overset{\circ}{1}$ to represent it. So each of the symbols in $b_k, 0 \leq k \leq 1$ can take a vaule in 3 values: 0, 1, $\overset{\circ}{1}$. We can extend this set by appending the $\overset{\times}{0}$, making the case here same as Gua space $\mathbb{G}_3 \times \mathbb{G}_2$ in ACT. A symmetric or anti-symmetric tensor $G_{\mu\nu}$ can be represented by:

$$\begin{bmatrix} 000.\overset{\circ}{1} & 001.0 & 010.0 & 100.0 \\ 001.0 & 00\overset{\circ}{1}.0 & 011.0 & 101.0 \\ 010.0 & 011.0 & 0\overset{\circ}{1}0.0 & 101.0 \\ 100.0 & 101.0 & 101.0 & \overset{\circ}{1}00.0 \end{bmatrix} \quad (59)$$

If we ignore the $.b_0$ in $b_3 b_2 b_1 . b_0$ and just take $b_3 b_2 b_1$, then we get the Gua representation for electromagnetic field (27). We call this notation of tensor by the Gua representation of tensor, and the representation which omitted the $.b_0$ is referred to the oमित Gua representation. According to this view, we get a different understanding of the tensor, because the component of the tensor represents a interlock on two Yaos or a automatic outbreak on one Yao in the Gua space.

Metric is also a tensor, therefore, it also has such a representation, only diagonal elements is not 0, but very important, which is contrary to the case in electromagnetic fields. This may indicate a profound connection between the two. We now represent

the $g_{\alpha\beta}$ with the omitted Gua representation as

$$g_{\alpha\beta} \equiv \begin{bmatrix} 000_g & 001_g & 010_g & 100_g \\ 001_g & 00\overset{\circ}{1}_g & 011_g & 101_g \\ 010_g & 011_g & 01\overset{\circ}{0}_g & 110_g \\ 100_g & 101_g & 110_g & 10\overset{\circ}{0}_g \end{bmatrix}. \quad (60)$$

Next, our focus is to understand the formula for calculating the Riemann curvature tensor in the local inertial frame[8, p. 78]:

$$R_{\alpha\beta\mu\nu} = \frac{1}{2}[g_{\alpha\nu,\beta\mu} - g_{\alpha\mu,\beta\nu} + g_{\beta\mu,\alpha\nu} - g_{\beta\nu,\alpha\mu}], \quad (61)$$

It is noteworthy that there is a similar formula in the representation theory of Lorentz group[9, p. 29]:

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i\eta_{\mu\rho}M_{\nu\sigma} - i\eta_{\nu\sigma}M_{\mu\rho} + i\eta_{\mu\sigma}M_{\nu\rho} + i\eta_{\nu\rho}M_{\mu\sigma}. \quad (62)$$

According to the definition of the Riemann curvature tensor, we have[8, p. 78]

$$R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu} = -R_{\alpha\beta\nu\mu} = R_{\mu\nu\alpha\beta}, \quad (63)$$

This means that the Riemann curvature tensor has the intrinsically equivalence relations between the four space-time positions(denoted as 1,2,3,4), which can be represented by the following subgroup $R_4 = \{e, a, b, c, ac, bc, abc\}$ of the four-symmetric group S_4 :

$$\begin{aligned} e &= \begin{pmatrix} 1234 \\ 1234 \end{pmatrix}, a = \begin{pmatrix} 1234 \\ 2134 \end{pmatrix}, b = \begin{pmatrix} 1234 \\ 1243 \end{pmatrix}, ab = \begin{pmatrix} 1234 \\ 2143 \end{pmatrix}, \\ c &= \begin{pmatrix} 1234 \\ 3412 \end{pmatrix}, ac = \begin{pmatrix} 1234 \\ 4312 \end{pmatrix}, bc = \begin{pmatrix} 1234 \\ 3421 \end{pmatrix}, abc = \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} \end{aligned} \quad (64)$$

the multiplication table of R_4 is:

$$a^2 = b^2 = c^2 = e, ab = ba, ac = ca, bc = cb. \quad (65)$$

We map the eight elements in R_4 to the eight Guas as shown in Tab 2, then the multiply between them is just the XOR operation in binary arithmetic. In 2, the signs in the parenthesis on the first row are obtained from (63), we find that the signs of the components of Riemann curvature corresponding to the male Guas and female Guas in Houtian theory are just the opposite.

In the above we have seen that the magical relation between the Riemann curvature and Bagua. Now we come to a further understanding of the computing structure of (61), so let's consider Fig 24, where each node corresponds to a space-time parameters,

e(+)	a(-)	b(-)	ab(+)	c(+)	ac(-)	bc(-)	abc(+)
000	001	010	011	101	100	111	110

Table 2: The map from the transformation group of Riemann curvature tensor to the eight trigrams

The two diagonal lines in the graph, which are $\overrightarrow{\alpha\beta}$ and $\overrightarrow{\mu\nu}$, are a key feature of this graph, they are marked by two dotted lines. According to this figure we can easily write out the formula (61): start from α , according to the clockwise along the four sides of the square, for each edge E , calculate the second derivative of the component of the metric corresponding to E with respect to the other two nodes which are not the endpoints of E in the graph, then sum all the quantities obtained in accordance with the sign of each side.

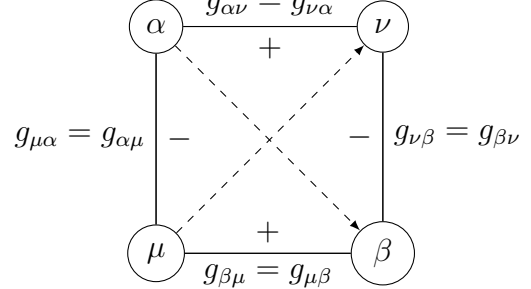


Fig. 24: The calculation for the components of the Riemann tensor

Since each component is determined by such a graph, each graph is determined by two edges $\overrightarrow{\alpha\beta}$ and $\overrightarrow{\mu\nu}$, but we can represent the two edges by two Guas g_1 and g_2 respectively, so we can represent the components of the Riemann curvature by two Guas, we denote it as $R_{g_1}^{g_2}$. According to the symmetry of the Riemann curvature, the nonzero components of the Riemann curvature are the following 36:

$$\begin{aligned}
& R_{001}^{001}, R_{010}^{010}, R_{100}^{100}, R_{110}^{110}, R_{101}^{101}, R_{011}^{011}, \\
& \{R_{010}^{001}, R_{001}^{010}\}, \{R_{100}^{001}, R_{001}^{100}\}, \{\underline{R_{110}^{001}}, \underline{R_{001}^{110}}\}, \{R_{101}^{001}, R_{001}^{101}\}, \{R_{011}^{001}, R_{001}^{011}\}, \\
& \{R_{100}^{010}, R_{010}^{100}\}, \{R_{110}^{010}, R_{010}^{110}\}, \{\underline{R_{101}^{010}}, \underline{R_{010}^{101}}\}, \{R_{011}^{010}, R_{010}^{011}\}, \{R_{110}^{100}, R_{100}^{110}\}, \\
& \{R_{101}^{100}, R_{100}^{101}\}, \{\underline{R_{011}^{100}}, \underline{R_{100}^{011}}\}, \{R_{101}^{110}, R_{110}^{101}\}, \{R_{011}^{110}, R_{110}^{011}\}, \{R_{011}^{101}, R_{101}^{011}\}.
\end{aligned} \tag{66}$$

where the Guas those are used as indexes are all the offspring Guas in Houtian theory, which is also similar to the case in the electromagnetic field. However, according to the symmetry of Riemann curvature components, $R_{g_1}^{g_2} = R_{g_2}^{g_1}$, while according to Bianchi identities[8, p. 79]

$$R_{\alpha\beta\mu\nu;\lambda} + R_{\alpha\beta\lambda\mu;\nu} + R_{\alpha\beta\nu\lambda;\mu} = 0, \tag{67}$$

there are only two of the three pairs of components are independent in the above pairs marked by red, their feature appearance on Gua is that each pair consists of

two Guas which are mutual NOT. Thus the total number of independent components are:

$$6 + \frac{1}{2}(6 \times 5) - 1 = 21 - 1 = 20. \quad (68)$$

Conversely, we can construct the graph similar to the above by any two offspring Guas in Houtian theory, and then calculate the corresponding Riemann curvature component, but here need to pay attention to there is a yang Yao lives on the hidden 0-th Yao in the three daughter Guas. Of course, we can also fully use binary arithmetic to define such a computing structure, but this is different from the previous simple binary arithmetic with which we are familiar, which mainly consists of NOT operations or XOR operations.

Further, we examine Ricc tensor $R_{\mu\nu}$, its formula is [8][p. 93]:

$$R_{\mu\nu} = g^{\kappa\alpha} R_{\kappa\mu\alpha\nu}, \quad (69)$$

From the standpoint of Gua, this is the process for reducing three Yaos Gua from Six Yaos Gua. Let the Gua representation of $R_{\mu\nu}$ is

$$\begin{bmatrix} 000_R & 001_R & 010_R & 100_R \\ 001_R & 00\overset{\circ}{1}_R & 011_R & 101_R \\ 010_R & 011_R & 0\overset{\circ}{1}0_R & 110_R \\ 100_R & 101_R & 110_R & \overset{\circ}{1}00_R \end{bmatrix}. \quad (70)$$

Then the scalar curvature is [8, p. 94]:

$$R = R_{\alpha}^{\alpha} = R_0^0 + R_1^1 + R_2^2 + R_3^3. \quad (71)$$

This seems to directly reduced to the Tai Chi from the Bagua. Then according to the definition of Einstein's tensor[8, p. 96]

$$G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \right), \quad (72)$$

finally, we can write out the Einstein's tensor as

$$\begin{bmatrix} 000_R - \frac{R}{2}000_g & 001_R - \frac{R}{2}001_g & 010_R - \frac{R}{2}010_g & 100_R - \frac{R}{2}100_g \\ 001_R - \frac{R}{2}001_g & 00\overset{\circ}{1}_R - \frac{R}{2}00\overset{\circ}{1}_g & 011_R - \frac{R}{2}011_g & 101_R - \frac{R}{2}101_g \\ 010_R - \frac{R}{2}010_g & 011_R - \frac{R}{2}011_g & 0\overset{\circ}{1}0_R - \frac{R}{2}0\overset{\circ}{1}0_g & 110_R - \frac{R}{2}110_g \\ 100_R - \frac{R}{2}100_g & 101_R - \frac{R}{2}101_g & 110_R - \frac{R}{2}110_g & \overset{\circ}{1}00_R - \frac{R}{2}\overset{\circ}{1}00_g \end{bmatrix}. \quad (73)$$

This means that we have representend Einstein's tensor as a physical quantity corresponding to Gua space, the relation between the physical quantity and the Gua space

is mainly determined by the definition of the Riemann curvature tensor (61). We can call this Gua space as Einstein's Gua space.

Finally, Einstein's equations [8, p. 96]

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (74)$$

can be understood as the balance between the Einstein's Gua space and the other Gua spaces, while the Gua spaces corresponding to $T_{\mu\nu}$ include the Maxwell's Gua space.

From the foregoing discussion, we can see that everywhere in the time and space there is a clear Gua space structure. If we use \mathbb{G}_1 to represent it, then using 1 to represent

three spatial dimensions, using 0 to represent the time dimension is suitable. If we use \mathbb{G}_2 to represent it, then taking into account the case there is a movement direction, using Fig 25 to represent it is appropriate.

In this figure, 11 corresponds to the the direction of movement of \vec{v} , 01 and 10 represent the other two spatial dimensions, and 00 represents time dimension. If we use \mathbb{G}_3 to represent it, then using the following Figure 26 to represent the four-dimensional physical quantity in space-time is appropriate, such as the 4-potential for the electromagnetic field, the 4-current for the fermion field. This seems to imply that such quantities are always in pairs, whether dark matter and dark energy[21] with this?

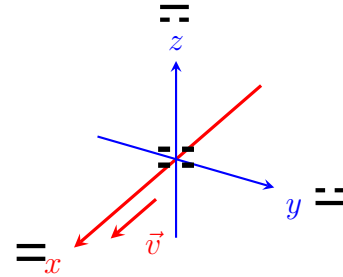


Fig. 25: The four Xiangs and the space-time coordinates

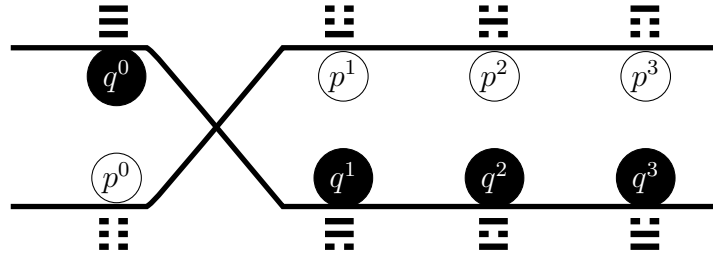


Fig. 26: Energy and dark energy, matter and dark matter?

4.5 The unified change patterns in Gua space

Now we come to solve the problem of how to determine the coefficients encountered in establishing Maxwell's equations and Dirac's equations according to Xiantian pattern

and Houtian pattern respectively. This is the problem ever encountered when I want to establish the Maxwell's equations and the Dirac's equations in general Gua space.

A reasonable solution to this problem is that we admit a **SDRB(Second Differentiate Return Back) principle**: when a Gua $g_0 \in \mathbb{G}_n$ has propagated the effect to the other Guas $g_k \in \mathbb{G}_n, 1 \leq k \leq n$ by the first order differentiate, all the Guas will return this effect back to the Gua g_0 which initiates the differentiate operation, that is, after using the SDS method for the equations, we get a simple second order partial differential equations from the linear first order partial differential equations.

For the Xiantian pattern, we first write out:

$$s_{000}^0 \frac{\partial g_{000}}{\partial x_0} + s_{000}^1 \frac{\partial g_{001}}{\partial x_1} + s_{000}^2 \frac{\partial g_{010}}{\partial x_2} + s_{000}^3 \frac{\partial g_{100}}{\partial x_3} = mg_{000}^m \quad (75a)$$

$$s_{001}^0 \frac{\partial g_{001}}{\partial x_0} + s_{001}^1 \frac{\partial g_{000}}{\partial x_1} + s_{001}^2 \frac{\partial g_{011}}{\partial x_2} + s_{001}^3 \frac{\partial g_{101}}{\partial x_3} = mg_{001}^m \quad (75b)$$

$$s_{010}^0 \frac{\partial g_{010}}{\partial x_0} + s_{010}^1 \frac{\partial g_{011}}{\partial x_1} + s_{010}^2 \frac{\partial g_{000}}{\partial x_2} + s_{010}^3 \frac{\partial g_{110}}{\partial x_3} = mg_{010}^m \quad (75c)$$

$$s_{011}^0 \frac{\partial g_{011}}{\partial x_0} + s_{011}^1 \frac{\partial g_{010}}{\partial x_1} + s_{011}^2 \frac{\partial g_{001}}{\partial x_2} + s_{011}^3 \frac{\partial g_{111}}{\partial x_3} = mg_{011}^m \quad (75d)$$

$$s_{100}^0 \frac{\partial g_{100}}{\partial x_0} + s_{100}^1 \frac{\partial g_{101}}{\partial x_1} + s_{100}^2 \frac{\partial g_{110}}{\partial x_2} + s_{100}^3 \frac{\partial g_{000}}{\partial x_3} = mg_{100}^m \quad (75e)$$

$$s_{101}^0 \frac{\partial g_{101}}{\partial x_0} + s_{101}^1 \frac{\partial g_{100}}{\partial x_1} + s_{101}^2 \frac{\partial g_{111}}{\partial x_2} + s_{101}^3 \frac{\partial g_{001}}{\partial x_3} = mg_{101}^m \quad (75f)$$

$$s_{110}^0 \frac{\partial g_{110}}{\partial x_0} + s_{110}^1 \frac{\partial g_{111}}{\partial x_1} + s_{110}^2 \frac{\partial g_{100}}{\partial x_2} + s_{110}^3 \frac{\partial g_{010}}{\partial x_3} = mg_{110}^m \quad (75g)$$

$$s_{111}^0 \frac{\partial g_{111}}{\partial x_0} + s_{111}^1 \frac{\partial g_{110}}{\partial x_1} + s_{111}^2 \frac{\partial g_{101}}{\partial x_2} + s_{111}^3 \frac{\partial g_{011}}{\partial x_3} = mg_{111}^m \quad (75h)$$

where $s_b^i \in R, b \in \mathbb{G}_3, 0 \leq i \leq 3, g_b$ is the function on x , and g_b^m corresponds to b , it is also a $g_{b'}$, in Dirac's equations $b' = b \oplus 011$.

For the first equation (75a) using SDS method, we get:

$$\begin{aligned}
& s_{000}^0 \frac{\partial g_{000}}{\partial x_0} + s_{000}^1 \frac{\partial g_{001}}{\partial x_1} + s_{000}^2 \frac{\partial g_{010}}{\partial x_2} + s_{000}^3 \frac{\partial g_{100}}{\partial x_3} = m g_{011} \\
\implies & s_{000}^0 \frac{\partial^2 g_{000}}{\partial x_0^2} + s_{000}^1 \frac{\partial^2 g_{001}}{\partial x_0 \partial x_1} + s_{000}^2 \frac{\partial^2 g_{010}}{\partial x_0 \partial x_2} + s_{000}^3 \frac{\partial^2 g_{100}}{\partial x_0 \partial x_3} = m \frac{\partial g_{011}}{\partial x_0} \\
\implies & s_{000}^0 \frac{\partial^2 g_{000}}{\partial x_0^2} + s_{000}^1 \frac{\partial}{\partial x_1} \left(-\frac{s_{001}^1}{s_{001}^0} \frac{\partial g_{000}}{\partial x_1} - \frac{s_{001}^2}{s_{001}^0} \frac{\partial g_{011}}{\partial x_2} - \frac{s_{001}^3}{s_{001}^0} \frac{\partial g_{101}}{\partial x_3} + \frac{m}{s_{001}^0} g_{010} \right) \\
& + s_{000}^2 \frac{\partial}{\partial x_2} \left(-\frac{s_{010}^1}{s_{010}^0} \frac{\partial g_{011}}{\partial x_1} - \frac{s_{010}^2}{s_{010}^0} \frac{\partial g_{000}}{\partial x_2} - \frac{s_{010}^3}{s_{010}^0} \frac{\partial g_{110}}{\partial x_3} + \frac{m}{s_{010}^0} g_{001} \right) \\
& + s_{000}^3 \frac{\partial}{\partial x_3} \left(-\frac{s_{100}^1}{s_{100}^0} \frac{\partial g_{101}}{\partial x_1} - \frac{s_{100}^2}{s_{100}^0} \frac{\partial g_{110}}{\partial x_2} - \frac{s_{100}^3}{s_{100}^0} \frac{\partial g_{000}}{\partial x_3} + \frac{m}{s_{100}^0} g_{111} \right) = m \frac{\partial g_{011}}{\partial x_0} \\
\implies & s_{000}^0 \frac{\partial^2 g_{000}}{\partial x_0^2} - \frac{s_{001}^1 s_{000}^1}{s_{001}^0} \frac{\partial^2 g_{000}}{\partial x_1^2} - \frac{s_{010}^2 s_{000}^2}{s_{010}^0} \frac{\partial^2 g_{000}}{\partial x_2^2} - \frac{s_{100}^3 s_{000}^3}{s_{100}^0} \frac{\partial^2 g_{000}}{\partial x_3^2} \\
& - \left(\frac{s_{001}^2 s_{000}^1}{s_{001}^0} + \frac{s_{010}^1 s_{000}^2}{s_{010}^0} \right) \frac{\partial^2 g_{011}}{\partial x_1 \partial x_2} - \left(\frac{s_{001}^3 s_{000}^1}{s_{001}^0} + \frac{s_{100}^1 s_{000}^3}{s_{100}^0} \right) \frac{\partial^2 g_{101}}{\partial x_1 \partial x_3} \\
& - \left(\frac{s_{010}^3 s_{000}^2}{s_{010}^0} + \frac{s_{100}^2 s_{000}^3}{s_{100}^0} \right) \frac{\partial^2 g_{110}}{\partial x_2 \partial x_3} \\
& = m \left(\frac{\partial g_{011}}{\partial x_0} - \frac{s_{000}^1}{s_{001}^0} \frac{\partial g_{010}}{\partial x_1} - \frac{s_{000}^2}{s_{010}^0} \frac{\partial g_{001}}{\partial x_2} - \frac{s_{000}^3}{s_{100}^0} \frac{\partial g_{111}}{\partial x_3} \right)
\end{aligned}$$

So get the neat conditions according to the left side on the last step above

$$\frac{s_{001}^2 s_{000}^1}{s_{001}^0} + \frac{s_{010}^1 s_{000}^2}{s_{010}^0} = 0 \implies s_{000}^1 s_{001}^2 s_{010}^0 + s_{000}^2 s_{010}^1 s_{001}^0 = 0 \quad (77a)$$

$$\frac{s_{001}^3 s_{000}^1}{s_{001}^0} + \frac{s_{100}^1 s_{000}^3}{s_{100}^0} = 0 \implies s_{000}^1 s_{001}^3 s_{100}^0 + s_{000}^3 s_{100}^1 s_{001}^0 = 0 \quad (77b)$$

$$\frac{s_{010}^3 s_{000}^2}{s_{010}^0} + \frac{s_{100}^2 s_{000}^3}{s_{100}^0} = 0 \implies s_{000}^2 s_{010}^3 s_{100}^0 + s_{000}^3 s_{100}^2 s_{010}^0 = 0 \quad (77c)$$

General constraint equation is: let germinal Gua is g , and g_i represents the Gua which is obtained by NOT the i -th Yao in g , g_j represents the Gua which is obtained by NOT the j -th Yao in g , then the constraint satisfied by the coefficients is

$$s_g^i s_{g_i}^j s_{g_j}^0 + s_g^j s_{g_j}^i s_{g_i}^0 = 0, 1 \leq i \neq j \leq 3. \quad (78)$$

We now examine the three constraints obtained from the right side on the last

step above:

$$-\frac{s_{000}^1}{s_{001}^0} = \frac{s_{011}^1}{s_{011}^0} \implies s_{000}^1 s_{011}^0 + s_{011}^1 s_{001}^0 = 0 \quad (79a)$$

$$-\frac{s_{000}^2}{s_{010}^0} = \frac{s_{011}^2}{s_{011}^0} \implies s_{000}^2 s_{011}^0 + s_{011}^2 s_{010}^0 = 0 \quad (79b)$$

$$-\frac{s_{000}^3}{s_{100}^0} = \frac{s_{011}^3}{s_{011}^0} \implies s_{000}^3 s_{011}^0 + s_{011}^3 s_{100}^0 = 0 \quad (79c)$$

General constraint equation is: let germinal Gua is g , its mass Gua is g_m , g_k is the Gua obtained by NOT the k -th, $1 \leq k \leq 3$, Yao in g , then the constraint is

$$s_g^k s_{g_m}^0 + s_{g_m}^k s_{g_k}^0 = 0. \quad (80)$$

According to the constraints (78) and (80) we can write out all the constraint equations on the coefficients.

We now examine the coefficients associated with the second order derivative of the germinal Gua on the left side of the equation with respect to each Yao, they have a simple form. Let germinal Gua is g , g_k is the Gua which is obtained by NOT the k -th Yao in g , then the coefficients associated with its second order derivative with respect to x_k , $1 \leq k \leq 3$, after multiplied by s_g^0 is

$$d_k = -\frac{s_g^k s_{g_k}^k}{s_g^0 s_{g_k}^0}. \quad (81)$$

For each gua g , we call the quad tuple, which is composed of the +1 corresponding by its 0-th term and the three numbers given above, as the mother-son sign of g , denote it as $\text{sign}(g)$.

We now examine the solutions of the equations given by (78) and (80). This is a quadratic polynomial equations, for simplicity, we search only solutions within the space consists of $\{-1, +1\}$. The calculation results show that the equations has total $2^{11} = 2048$ solutions, and in each solution all the mother-son sign of the Guas are same. Thus each solution corresponds to only one mother-son sign, we called it the mother-son sign of the solution. According to the calculation results, all solutions have a total of eight kinds of mother-son sign, they are:

$$\begin{aligned} & (+1, -1, -1, -1), (+1, -1, -1, +1), (+1, -1, +1, -1), (+1, -1, +1, +1) \\ & (+1, +1, -1, -1), (+1, +1, -1, +1), (+1, +1, +1, -1), (+1, +1, +1, +1) \end{aligned} \quad (82)$$

that is, this is the set of mother-son sign which is consists of the mother-son signs obtained by filling all possible values in the last three position in the mother-son sign. Moreover, the number of solutions corresponding to each of these mother-son sign are

all 256. That is, there are $2048 = 256 \times 8$. But according to special relativity, we should choose the 256 solutions whose mother-son sign are $\text{sign} = (+1, -1, -1, -1)$. I have no further study of these solutions, but they should be equivalent to the Dirac's representation. It is worth noting that using any kind of eight $\oplus g, g \in \mathbb{G}_3$ forms to construct the mass term in equations will not affect the structure of the solutions.

Similarly, we can determine the coefficients associated with each term in the equations established with Houtian pattern.

First, according to Houtian pattern write out the equations:

$$0 + s_{000}^1 \frac{\partial g_{001}}{\partial x_1} + s_{000}^2 \frac{\partial g_{010}}{\partial x_2} + s_{000}^3 \frac{\partial g_{100}}{\partial x_3} = mg_{000}^m \quad (83a)$$

$$s_{001}^0 \frac{\partial g_{001}}{\partial x_0} + 0 + s_{001}^2 \frac{\partial g_{011}}{\partial x_2} + s_{001}^3 \frac{\partial g_{101}}{\partial x_3} = mg_{001}^m \quad (83b)$$

$$s_{010}^0 \frac{\partial g_{010}}{\partial x_0} + s_{010}^1 \frac{\partial g_{011}}{\partial x_1} + 0 + s_{010}^3 \frac{\partial g_{110}}{\partial x_3} = mg_{010}^m \quad (83c)$$

$$s_{011}^0 \frac{\partial g_{011}}{\partial x_0} + s_{011}^1 \frac{\partial g_{010}}{\partial x_1} + s_{011}^2 \frac{\partial g_{001}}{\partial x_2} + 0 = mg_{011}^m \quad (83d)$$

$$s_{100}^0 \frac{\partial g_{100}}{\partial x_0} + s_{100}^1 \frac{\partial g_{101}}{\partial x_1} + s_{100}^2 \frac{\partial g_{110}}{\partial x_2} + 0 = mg_{100}^m \quad (83e)$$

$$s_{101}^0 \frac{\partial g_{101}}{\partial x_0} + s_{101}^1 \frac{\partial g_{100}}{\partial x_1} + 0 + s_{101}^3 \frac{\partial g_{001}}{\partial x_3} = mg_{101}^m \quad (83f)$$

$$s_{110}^0 \frac{\partial g_{110}}{\partial x_0} + 0 + s_{110}^2 \frac{\partial g_{100}}{\partial x_2} + s_{110}^3 \frac{\partial g_{010}}{\partial x_3} = mg_{110}^m \quad (83g)$$

$$0 + s_{111}^1 \frac{\partial g_{110}}{\partial x_1} + s_{111}^2 \frac{\partial g_{101}}{\partial x_2} + s_{111}^3 \frac{\partial g_{011}}{\partial x_3} = mg_{111}^m \quad (83h)$$

where the meaning of $s_b^k, 0 \leq k \leq 3, g_b^m, m, b \in \mathbb{G}_3$ are same as the case in Xiantian patter above. Using SDS method, we can get the conditions need to be satisfied by these coefficients: let germinal Gua is g , its generation is n , its root is g_r , where

$$\begin{aligned} n(001) = n(110) = 1, n(010) = n(101) = 2, n(100) = n(011) = 3, \\ g_r(001) = g_r(010) = g_r(100) = 111, g_r(110) = g_r(101) = g_r(011) = 000, \end{aligned} \quad (84)$$

then the two related conditions are

$$s_g^k s_{g_k}^n \pm s_{g_r}^k s_{g_k}^0 = 0, 1 \leq k \leq 3, k \neq n, \quad (85)$$

where g_k is Gua which is obtained by NOT the k -th Yao in g , and for each k the sign must agree to take a positive or negative.

The constraints related to the mass term is

$$s_g^k + s_{g_m}^k s_{g_k}^0 = 0, 1 \leq k \leq 3, k \neq n, \quad (86)$$

where g_m is the mass Gua corresponding to g , that is $g_m = g$ or g_m is the NOT Gua of g .

The calculation results show that the equations given by these constraints, if not consider mass constraints, there are $2^{19} = 524288$ solutions. If we require that the coefficients in the equations corresponding to QianGua(111) and KunGua(000) are all +1, then there is $2^{13} = 8192$ solutions. In the case of considering the mass constraints, generating the mass Gua through either XOR 000 or XOR 111, the resulting equations have only $2^7 = 128$ solutions.

If we want to generalized the method by which establishing the equations based on the change pattern of Gua sapce to the general Gua space, then we will face the problem about how to determine the change pattern and how to determine the coefficients. According to the previous experience, we can follow the steps to establish the equations in the general Gua space:

- (1) We can choose one from the two basic patterns. If we select the Xiantian pattern, then we should first determine the two mutual NOT Gua which are the parents Guas. Then the coefficients associated with the parents Guas should be taken to be zero. If the pattern will be generalized to the Gua space $\mathbb{G}_n, n > 3$, then we will maybe face some flaws, because there are a plurality of circular shift loop, and this may be a significant difference. If we select the Xiantian pattern, then all the coefficients are not zero in all equations. Of course, in addition to the two, other pattterns are also possible.
- (2) Regardless of which pattern is selected, we determine all coefficients in all of the equations by SDS method. Of course, in general Gua space, the constraint conditions on the coefficients may not be satisfied, including the constraints given by the two basic pattern. However, when more coefficients are allowed to be 0, these constraints can certainly be satisfied, because the method for establishing the equations based on the higher order spinors is a meaningful example.

It should be noted, change patterns in Gua space discussed here actually establish the following correspondence for each Gua $g \in \mathbb{G}_3$,

$$\frac{\partial}{\partial x_0} \mapsto \oplus 000, \frac{\partial}{\partial x_1} \mapsto \oplus 001, \frac{\partial}{\partial x_2} \mapsto \oplus 010, \frac{\partial}{\partial x_3} \mapsto \oplus 100, \quad (87)$$

But there are other forms of equivalence correspondence, such as I have considered this correspondence:

$$\frac{\partial}{\partial x_0} \mapsto \oplus 000, \frac{\partial}{\partial x_1} \mapsto \oplus 001, \frac{\partial}{\partial x_2} \mapsto \oplus 011, \frac{\partial}{\partial x_3} \mapsto \oplus 111, \quad (88)$$

But apparently (87) is the simplest and most natural.

In addition, Dirac's equation(44) also has two forms of Weyl and Majorana²⁰. In Weyl's form, four gamma matrix are taken as:

$$\begin{aligned} \gamma^0 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \\ \gamma^3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (89)$$

After expanded the equations defined by them with separating the real and imaginary parts, we can get the following eight equations:

$$\begin{aligned} \left(\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_3} \right) u_1 - \frac{\partial u_2}{\partial x_1} - \frac{\partial v_2}{\partial x_2} &= 0 \\ \left(\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_3} \right) u_2 - \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial x_2} &= 0 \\ \left(-\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_3} \right) v_1 + \frac{\partial v_2}{\partial x_1} - \frac{\partial u_2}{\partial x_2} &= 0 \\ \left(-\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_3} \right) v_2 + \frac{\partial v_1}{\partial x_1} + \frac{\partial u_1}{\partial x_2} &= 0 \end{aligned} \quad (90)$$

and

$$\begin{aligned} \left(\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_3} \right) u_3 + \frac{\partial u_4}{\partial x_1} + \frac{\partial v_4}{\partial x_2} &= 0 \\ \left(\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_3} \right) u_4 + \frac{\partial u_3}{\partial x_1} - \frac{\partial v_3}{\partial x_2} &= 0 \\ \left(-\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_3} \right) v_3 - \frac{\partial v_4}{\partial x_1} + \frac{\partial u_4}{\partial x_2} &= 0 \\ \left(-\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_3} \right) v_4 - \frac{\partial v_3}{\partial x_1} - \frac{\partial u_3}{\partial x_2} &= 0 \end{aligned} \quad (91)$$

The first four (90) are about u_1, v_1, u_2, v_2 , the later four (91) are about u_3, v_3, u_4, v_4 . Due to the special nature caused by $\frac{\partial}{\partial x_3}$ in this case, our correspondence between differentiation operation and NOT operation can not be established on \mathbb{G}_3 , but if we deal x_3 as same as x_0 , then we can for each of the two parts establish the correspondence between differentiation operation and NOT operation in \mathbb{G}_2 respectively. For

²⁰https://en.wikipedia.org/wiki/Gamma_matrices.

example, we can encode the quantities like this:

$$\begin{aligned} u_1 &\mapsto 000, u_2 \mapsto 001, v_1 \mapsto 011, v_2 \mapsto 010, \\ u_3 &\mapsto 100, u_4 \mapsto 101, v_3 \mapsto 111, v_4 \mapsto 110. \end{aligned} \quad (92)$$

where we encode the 3-th Yao of u_1, v_1, u_2, v_2 as 0, and encode the 3-th Yao of u_3, v_3, u_4, v_4 as 1.

The four gamma matrices in Dirac's equations in Majaorna's form are:

$$\begin{aligned} \gamma^0 &= \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \gamma^1 = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}, \gamma^2 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \\ \gamma^3 &= \begin{bmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{bmatrix}, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix}. \end{aligned} \quad (93)$$

after expanded, we will also find the phenomenon which is similar to the phenomenon presented in the the Dirac's equations in form of Weyl's:

$$\begin{aligned} \left(-\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_2}\right) u_1 + \frac{\partial u_4}{\partial x_1} + \frac{\partial u_3}{\partial x_3} &= mu_4 \\ \left(\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_2}\right) u_2 - \frac{\partial u_3}{\partial x_1} + \frac{\partial u_4}{\partial x_3} &= mu_3 \\ \left(-\frac{\partial}{\partial x_0} + \frac{\partial}{\partial x_2}\right) u_3 + \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_3} &= mu_2 \\ \left(\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_2}\right) u_4 - \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_3} &= mu_1 \end{aligned} \quad (94)$$

but here it is $\frac{\partial}{\partial x_2}$ similar to $\frac{\partial}{\partial x_0}$. We can follow the correspondence between the differentiation operation and NOT operation establish the following encoding for u_1, u_2, u_3, u_4 :

$$u_1 \mapsto 00, u_4 \mapsto 01, u_3 \mapsto 10, u_2 \mapsto 11. \quad (95)$$

In the viewpoints of ACT, we can see the difference between the three forms of the Dirac's equations from a different angle, apparently the the original Dirac's form is more natural, there is a particular spatial dimension in two other forms.

We now can uniformly understand the meaning of Schrodinger's equation, Pauli's equation, Dirac's equation from the viewpoints of ACT. Schrodinger's equation has established the relation between the motion in the external space and the motion in intrinsic Gua space \mathbb{G}_1 for objects. Pauli's equation has established the relation

between the motion in the external space and the motion in intrinsic Gua space \mathbb{G}_2 for objects. And Dirac's equation has established the relation between the motion in the external space and the motion in intrinsic Gua space \mathbb{G}_3 for objects. This seems to imply there is a mirror image of a whole time and space in each space-time point, it is also an ancient perspective in ACT, but it is generally considered be idealism.

According to this method we can establish the motion equations for \mathbb{G}_n with more Yaos, but it means higher spatial dimensions, but after all, our experience is that the higher dimensions of the universe does not exist. So we may need from another angle to consider the issue.

In ACT, there is two Gua spaces with more than 3 Yaos, that is \mathbb{G}_3^3 and $\mathbb{G}_3^3 \times \mathbb{G}_2$. \mathbb{G}_3^3 is formed by the Guas with 6 Yaos which is constructed by combining two Guas with 3 Yaos according to the vertical structure, where the 64 Guas is also the main contents of *Book of Changes*. $\mathbb{G}_3^3 \times \mathbb{G}_2$ is formed by let each Yao take value in \mathbb{G}_2 based on \mathbb{G}_3^3 , which is the Gua space used by augurs.

In divination, the basic theories about two Gua space is the same, the up and down Gua in the Gua with 6 Yaos is usually understood as two opposite things, for example in the analysis model for divination about marriage, one Gua is used to represent the man, and the other represents the woman. The dynamic Yao in the $\mathbb{G}_3^3 \times \mathbb{G}_2$ is often used to judge the variation tendency, and is the bridge linking with different Guas with 6 Yaos.

What is it corresponds to in physics? I have not found a clear answer. For example, we use $g_{b_1}^{b_2}$ to denote a Gua in \mathbb{G}_3^3 , if we require that when each Yao parameter(x_k) changes, it gets a linear combination of two derivative of two Guas obtained by NOT a single bit in the Yao, one of which is $g_{b_1}^{b_2^k}$ and the other is $g_{b_1^k}^{b_2}$, with respect to x_k , where b_i^k is the Gua with 3 Yaos which is obtained by NOT the k -th Yao of b_i , $i = 1, 2$. Then the equations will have the following form, for example let $b_1 = 000$, $b_2 = 000$:

$$\begin{aligned} \frac{\partial g_{000}^{000}}{\partial x_0} + \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_1} \right) A_{000}^1 \begin{pmatrix} g_{000}^{001} \\ g_{001}^{000} \end{pmatrix} + \left(\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_2} \right) A_{000}^2 \begin{pmatrix} g_{000}^{010} \\ g_{010}^{000} \end{pmatrix} \\ + \left(\frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_3} \right) A_{000}^3 \begin{pmatrix} g_{000}^{100} \\ g_{100}^{000} \end{pmatrix} = 0 \end{aligned} \quad (96)$$

where A_{000}^k , $1 \leq k \leq 3$ is the matrix in $\mathbb{R}^{2 \times 2}$. Thus we will get 64 equations, which will make the situation becomes very complex. This may be related to the theory of quantum entanglement and gauge field.

Einstein's equation tells us another change pattern in Gua space, according to (66), Einstein's equations can be regarded as the description of the motion of Guas with 6 Yaos which are formed by the six offspring Guas in Houtian theory pairs with each other or itself.

5 QED And The Standard Model

In QED[5, 6, 7], Maxwell's equation corresponds to the boson field(of photon), Dirac's equation corresponds to the fermion field(of electron), the two together correspond to a interact field, and form the theory of quantum electrodynamics. The Lagranian of the interaction field is[3, p. 139]

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j_{\mu}A^{\mu} - \bar{\psi}(\partial^{\mu}\gamma_{\mu} + m)\psi \quad (97)$$

We can obtain two equations of motion from it:

$$\begin{aligned} \partial_{\lambda}F^{\lambda\nu} &= -j_{\nu} \\ i\partial^{\mu}\gamma_{\mu}\psi - m\psi &= e\gamma_{\mu}A^{\mu}\psi \end{aligned} \quad (98)$$

This is the equations of motion of the Maxwell's field and the Dirac's field after the interaction respectively. From which we see the two fields enter into the equation of free motion for four Xiangs in each other by its own four Xiangs, so that the electric current vector is substituted by the electron wave function described by the Dirac's field, and the effects of the motion of the electric current vector in Dirac's equations corresponds to the 4-potential in the Maxwell's field. This makes the Maxwell's field and Dirac's field form yet another wonderful model of "one yin and one yang", the two are a pair of mutualistic.

According to the Bagua structures of the two fields given earlier, we can express the Bagua structure of the interaction field as Fig. 27. In this figure, the part

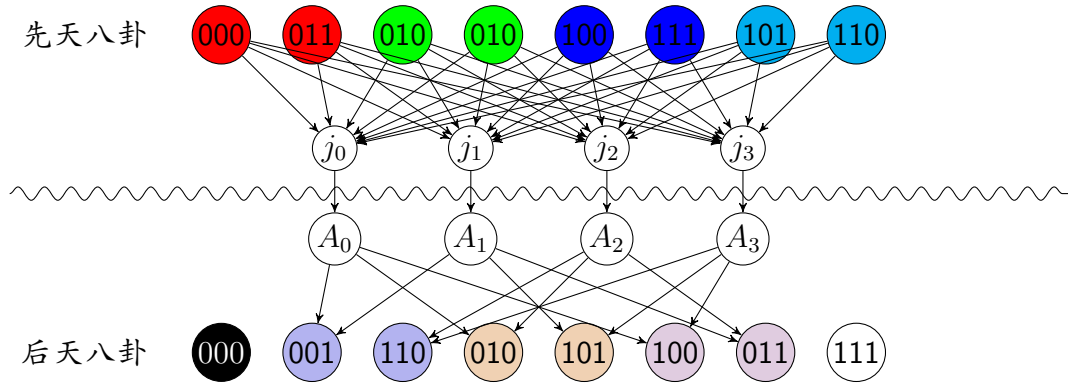


Fig. 27: QED and the unification of Xiantian and Houtian theories

above the wavy line expresses the Xiantian Bagua and its four Xiangs, the part under the wavy line expresses Houtian Bagua and its four Xiangs, and the directions of the arrows between each node correspond to the direction of computing. Thus, we have to amaze that the names of Xiantian and Houtian in ACT actually express the

meaning of this structure cleverly, which makes me somewhat believe that the most important theories in ACT come from extraterrestrial civilization or are create by the ancient saints with extraordinary wisdom.

QED has successfully unified the Maxwell's field and Dirac's filed, in other words, unified the Houtian theory and Xiantian theory for Bagua, this is another powerful example for the opinion that the physics is the ideal subject used for explaining the ACT. Maxwell's field is the intermedia for interactions between the Dirac's fields, so it is dynamic, is yang; with this in relative terms, Diarc's field is stationary, is yin. The electron described by Dirac's field in a particular state will emit a photon, which is just the "extreme yin will produce yang"; The photon described by Maxwell's field carries energy, under the right conditions can eject the electrons, which is just the "extreme yang will produce yin". The relation between the Maxwell's field and the Dirac's field in QED perfectly illustrates the meaning of "one yin and one yang, this is said to be Tao".

We have already seen a lot of duality between the Maxwell's field and Dirac's field, where once again wish to summarize:

- (1) Each Gua in Xiantian theory is very equal, but the parents Guas and the offspring Guas in Houtian theory have a very different position.
- (2) The eight Guas in Xiantian and Houtian theory in fact all follow the same pattern of change, which is the SDRB principle. In the equations established according to the Xiantian theory, the coefficients associated with all Guas are nonzero, but in the equations established according to the Houtian theory, the coefficients associated with the parents Guas are zero, the other are nonzero.
- (3) The equations of motion for Xiantian and Houtian theory all obey the special relativity structrue of $\overset{\circ}{\mathbb{X}}_3$, but their status in the graph are different. In the $\overset{\circ}{\mathbb{X}}_3$ of Xiantian theory, the offspring Guas correspond to the edges in the graph; in the $\overset{\circ}{\mathbb{X}}_3$ of Houtian theory, each Gua corresponds to a node in the graph.
- (4) The eight Guas in Xiantian and the eight Guas in Houtian are in different ways reduced to the four Xiangs respectively. The four Xiangs in Xiantian theory are actually the superposition of the eight tetrahedrons corresponding to the eight vertices in the cube for Xiantian theory, and the four Xiangs in Houtian theory are the superposition of the two $\overset{\circ}{\mathbb{X}}_3$ with opposite directions for Houtian theory. For the degree of difficulty for calculating eight Guas or four Xiangs from each other, in Xiantian theory calculating four Xiangs from eight Guas is easy, otherwise difficult; and the case in Houtian theory is the opposite. The basic computing in this calculation used in Xitian theroy is mainly multiplication, and in Houtian theory it is differentiation.

- (5) Each Gau in Xitian theory is paired with the other by NOT the lowest two Yaos(in other words, the two Yaos corresponding to the plane whose normal direction is the spin axis), that is, the real part and the imaginary part of the corresponding component of the spinor. Each Gua in the Houtian is paired with the other by NOT all the Yaos, that is, the electric and magnetic field components in the same dimension.
- (6) The Guas in the Xiantian as a whole is divided into two parts by the highest Yao(in other words, the direction of the spin axis), this two parts substantially corresponds to the positive matter and antimatter. The Guas in the Houtian as a whole is divided into electric and magnetic fields by the difference between male Guas and female Guas.
- (7) The characteristics such as quantity distribution of positive matter and antimatter distinguished in the Dirac's theory corresponds to Xiantian theory are asymmetric in nature, which in turn produces the violation of various symmetries, such as P symmetry, CP symmetry CPT symmetry[16] and the like. The asymmetry in Maxwell's electromagnetic theory corresponding to Houtian theory is also obvious, such as electric charge exist, and magnetic charge has not been detected.
- (8) The physical quantity corresponding to Xiantian theory is the quantities for uncertain phenomenon, that is, the probability of the substance appear or hide, which corresponds to the "it is hard to measure yin and yang", so it is Xiantian("before birth"). The physical quantity corresponding to Houtian theory is the quantities which are familiar for human senses, which corresponds to the sharply distinction between yin and yang, so it is Houtian("after birth").

From the viewpoint of ACT, the QED theory which unifies the Maxwell's field and the Dirac's field corresponds to the Gua space with six Yaos \mathbb{G}_3^3 , that is the famous 64 Guas with six Yaos(hexagram) in the *Book of Changes*, where we let down Gua(trigram) space, \mathbb{G}_3 , corresponds to the Dirac's field, while the up Gua space, \mathbb{G}_3^3 , corresponds to Maxwell's field, which makes calculation direction between the two fields is consistent with the order of bottom-up defined in ACT.

The physics believes that the key feature of the QED is that it has the local gauge invariance. We have seen the global invariance of QED before, that is, in the Dirac's theory, after the quantities, corresponding to the pair of two Guas formed by one acting as real part and the other acting as imaginary part, are transformed as (52), where the parameter θ is not a space-time-dependent parameters constant, the resulted equations are invariant. In the Maxwell's theory, after the 4-potential is transformed as (41), where the function satisfy $\nabla^2\Gamma(x) = 0$, the resulted equations are invariant. Local gauge-invariant is that after both are performed with gauge

transform depending on the space and time parameters the resulted equations are also invariant[4, p. 284],

$$\begin{aligned}\psi(x) &\rightarrow e^{-ie\Gamma(x)}\psi(x), \\ A^\mu(x) &\rightarrow A^\mu(x) + \partial^\mu\Gamma(x),\end{aligned}\tag{99}$$

All transformations with the parameters of time and space performed on the Guas in Xiantian form the group $U(1)$. Physics further generalize this approach to $SU(n)$ theory[18, 19], which means that put the more fermion fields, or spinors as a whole array to accept this gauge transformation. One result of this approach is that we can derive naturally some bosons described by the Maxwell's fields, these bosons play a similar role with the photon in the explanation of the physical phenomena. This in fact is yet another example of the "one yin and one yang". Using this method, physics based on the combination of gauge groups $U(1)$, $SU(2)$ and $SU(3)$ establishes the standard model[11, 12] which gives a unified description of the elementary particles, and this theory is considered as the most successful physical theory.

This idea immediately reminds us the Gua space $\mathbb{G}_3^3 \times \mathbb{G}_2$ in ACT. I'm tempted to ask:

- (1) Since there is a clear correspondence between \mathbb{G}_3^3 and QED, then whether there is a correspondence between $\mathbb{G}_3^3 \times \mathbb{G}_2$ and the electroweak unification theory[13, 14, 15] of $U(1) \times SU(2)$, and whether there is a correspondence between $\mathbb{G}_3^3 \times \mathbb{G}_3$ and the QCD[5, 6, 7, 17] of $SU(3)$?
- (2) $SU(n)$ considers the combination of n fermions's wave function, that is, it uses $8n$ real quantities to represent the states of the combined system, which is actually a combination of additions, whether the truth is that it should be a combination of multiplication, that is, it should use 2^{3n} real quantities to represent the states?
- (3) Since \mathbb{G}_3 and \mathbb{G}^3 all have obvious equations of motion, then what is the equation of motion for $\mathbb{G}_3 \times \mathbb{G}_n$ and $\mathbb{G}^3 \times \mathbb{G}_n$? How about \mathbb{G}_n and \mathbb{G}^n ?
- (4) Whether there is relation between $\mathbb{G}_3 \times \mathbb{G}_n$, $\mathbb{G}^3 \times \mathbb{G}_n$ and quantum entanglement?
- (5) Whether there is a relation between the elementary particles in the Standard Model of divided into three generations and the structure of three generations in Houtian theory?
- (6) Whether the true meaning of supersymmetry theory should reflect the two different change pattern, just like the Xiantian pattern and Houtian pattern in \mathbb{G}_3 ?

- (7) If there is a physically Gua space \mathbb{G}_n in the space-time point , then what is the meaning contained in it on the relation between substance and information? what is the relation between the conditon for triggering the change of Yaos in the Gua and the Planck constant?
- (8) From Fig 18 and Fig 23, we see neither of them is a a standard binary tree, the structres in their second floor all are 1+3 type branch, this is jus the difference between time and space, but the time node and space nodes shows some equality on the physical quantities corresponding to the eight Guas in the last floor. whether this is the ultimate root of various asymmetric of the physical quantities in the the universe?
- (9) The double helix structure of DNA in biology is another famous example of "one yin and one yang", cells divide and replicate by it. Whether there is a similar mechanism in the nuclear fission and fusion in physics? That is to say whether this procedures correspond to, for example, fission:

$$\mathbb{G}_3 \times \mathbb{G}_2 \rightarrow \mathbb{G}_3 + \mathbb{G}_3, \quad (100)$$

fusion:

$$\mathbb{G}_3 + \mathbb{G}_3 \rightarrow \mathbb{G}_3 \times \mathbb{G}_2, \quad (101)$$

replication:

$$\mathbb{G}_3 \rightarrow \mathbb{G}_3 \times \mathbb{G}_2. \quad (102)$$

6 Conclusion

All th above is the contents which are direct relative to the modern physics and I came to realize in the process of my writing *Tao's mathematical theory*. In fact, ACT holds two basic thouthts about all things in the universe: one is the thoutht of cycle, this thoutht always look upon the change as a cycle process, by dividing the process into some stages to understand the change. This thoutht is more concerned about the change such as cycle of the seasons, without concern for the objects moving along the trajectory in space and time which is the major study objects in physics. Another is the thoutht of yin and yang, this thoutht is always decompose the objects and their movement process into two opposing parts, and explain the changes by the relations between the two parts. Wuxing and Bauga is the most perfect concept which have integrated these two thouthts, which constitute the core of the theory of ACT. They are highly abstract, but the strange thing is that we can find there is a profound relation between them and the modern physics.

If we do some meditation on modern physics, so this is not surprising. Because physics often decomposes the motion into the sum of simple periodic motions by Fourier transform, and the symmetry in motion has also become the important subject of physics, all this coincidence with the ACT theory whose representative is the Wuxing and Yinyang. Of course, the relations given in this article is just a coincidence, or the inevitable result of the laws of nature, I do not know, but I hope these ideas can get more specialized research by the physicists.

If these relations are true, it means that the physical quantities always present in every space-time point with the structure of Gua space, and then I want to know that there are how many kinds of Gua space in each space-time point, how many kinds of change pattern do they have, how do they interact(in ACT, the interaction between yin and yang is always expressed by the words "coitus" used between the man and woman.), what is the relation between them and the various forces in physics.

Different from the western thinkers are good at thinking about the movement and change process of the things based on the geometry, ancient Chinese thinkers essentially ignored the meaning from the geometry. Clearly, from the Newtonian mechanics to Einstein's theory of relativity, the basic theory of physics is almost completely established on the basis of the geometry analysis of the trajectory of the object in the time and space. This kind of thought is very important and great, it is also very successful. Einstein even attempt to establish a theory to unite all the physics based on geometry, for example, he try to attribute electromagnetic fields also to the geometry just as he did with gravity. As we all know, this attempt failed.

The physics theory established on quantum mechanics is largely out of the scope of geometry, for example, we can not use the concept of classic trajectory to understand the movement of electrons inside atoms, but only be able to use some discrete values to represent them. Therefore, we can say that the theory of quantum mechanical in a sense is the proof of the ancient Chinese thinker's opinion have their own merit, although the fact they had not leave a deep theory on geometry is regrettable. Ancient Chinese thinkers prefer to ponder the invisible presence, they put these call metaphysical²¹, and the touchable, measurable phenomena, called by immetaphysical²². The classic expression on this in *Book of Changes* is "Therefore the things above the shape is called by Tao, the things below shape is called by implement."²³ That is to say, the ancient Chinese thinker think that the invisible presence would be regarded as the root causes of physical existence. The expression of this in *Tao Te Ching* is "All things are born from 'have', 'have' born from none."²⁴ Another book, *LieZi* had described the principle of 'have' producing from none, the principle divided the procedure of 'have' producing from none into four stages: extreme

²¹The chinese is 形而上, means above the shape.

²²The chinese is 形而下, means below the shape.

²³The Chinese is 是故形而上者谓之道, 形而下者谓之器.

²⁴The Chinese is 万物生于有, 有生于无.

change²⁵, which is a state before the advent of Qi²⁶, then there is no the limit from geometry, therefore, we can not hear, see, and track it, but its beginning to change leads to the generation process of shape; extreme initial, this state is the beginning of shape; extreme native²⁷, this state is the beginning of quality²⁸. And, there is another state called by mess²⁹ after the Qi, shape, and quality is completed, in this state where these three are combined with together, not separate.³⁰ The viewpoints of *LieZi* on the constitutes of shape is also interesting, he believes that "形之所形者实矣, 而形形者未尝有", that is, the reason for the things own its shape is because the materials filled in it, but the filling things themselves have no shape. These thoughts make ancient Chinese thinkers believe that the invisible presence is more valuable because more timeless, more real, and the more closer to the Tao, so they will naturally despise tangible existence. But these thoughts of preferring on intangible and the understanding methodsinsights emphasizing on awareness method also makes ACT often be labeled with idealistic, pseudo-science, and ignorance and so on by many stupid ignorants and their followers in China today.

This distinction between tangible and intangible in physics should be also seen as another example of "one yin and one yang".

No matter the relations between ACT and modern physics is true or not, but they can make us feel that ACT has a profound extraordinary understanding on the nature, their distance from the science does not seem far away. However, ACT's core theory, although early in the two thousand years basically mature, but from then until now it has no longer been further breakthrough development. As science was introduced to China in modern times, it began to recede from the China's mainstream scholarship, gradually being forgotten. Obviously, such a historical process of ACT is an issue which makes people interested in.

As mentioned earlier ACT's successor had not be able to use and develop effective mathematical tools and their values are so low, these are the important factors for ACT failed to become a more deeper and more complete theory. In addition, we must be traced back to Chinese political for the reasons. After Qin Shi Huang unified China in 221 BC, he had established a political system which is inherited over two thousand years. This political system, although in some periods maintain the order

²⁵The chinese is 易.

²⁶The chinese is 气.

²⁷The chinese is 朴.

²⁸The chinese is 质.

²⁹The chinese is 浑沦.

³⁰The Chinese is 昔者圣人因阴阳以统天地。夫有形者生于无形，则天地安从生？故曰：有太易，有太初，有太始，有太素。太易者，未见气也；太初者，气之始也；太始者，形之始也；太素者，质之始也。气形质具而未相离，故曰浑沦。浑沦者，言万物相浑沦而未相离也。视之不见，听之不闻，循之不得，故曰易也。易无形埒，易变而为一，一变而为七，七变而为九。九变者，穷也，乃复变而为一。一者，形变之始也。清轻者上为天，浊重者下为地，冲和气者为人；故天地含精，万物化生。

making the society peace, but because it is often very unfair, so this positive effect is always short. More far-reaching is that its harm for the development and progress of Chinese civilization is enormous. Because it strongly opposed to the freedom of thought, the rulers are always trying to implement policy of keeping the people in ignorance and weakness to weaken the power of the people, so that they have gained a fragile comparative advantage and attempt to consolidate their rule. So since the beginning of the era of Qin Shi Huang, all the China's dynasties were implementing the harsh ideological control policy, for this reason, many people with independent thinking and talent had always been marginalized and even persecuted, killed. Also, because the political system often makes the social resources be concentrated to the ruling center, gradually people are efficiently tamed into the minions of the power, more and more accustomed to tolerate the oppression from the rich and powerful, and their independent creativity are more and more decadent. Naturally, we see a clear downward trend in the thinking ability of the Chinese people parallel to this historical process. In the last few hundred years the negative effect of this political development to the extreme, and the Chinese people's creativity also fallen to the lowest point, and this was also the era of the rise of Western science and ACT completely downhill.

This political legacy remains in modern China to keep producing the tragedy and suffering. This is the experience beared by every governed, I have a deep understanding of this. Modern China's rulers are still trying to control strictly the ideological, they are repeatedly instilling a variety of hypocritical political preaching into the students in the classroom from primary to university, ironically, they are in the name of "universal truth" to preached, and accustomed to using many words such as "先进性"³¹ to brag about it. In fact, this is a big joke, their effects only are wasting the time of the young students and and deceiving them thus weakening their capacity, of course, this is just the conspiracy what they want to achieve. The Great Firewall is another typical example, it built a wall for the Chinese network to make China's network separated from many great Internet sites such as Google, Wikipedia etc, which caused me a lot of trouble because all my research and even the software development before are very dependent on these sites. Clearly, its harms for Chinese Internet, and even for China is far from so simple as causing trouble to the free web browsing, but same as with various Chinese ancient cage policy causing poison endless. The irony is that, they take the name of their wall as similar as the wall built by Qin Shi Huang, that is the Great Wall, although they have a completely different in the material, but I think the most fundamental difference between them is that their defensive orientation is the opposite. The Great Wall of the Qin Shi Huang is used for defending the invasion from the northern barbarians, that is, its direction is from inside to outside; and their Great FireWall is used to intercept the Chinese people accessing information from abroad, that is, its direction is from outside to inside. That is the Great Wall of

³¹Their "theorists" express the words as "advanced sex" in the paper with an abstract in English.

defense direction is rotated by 180 degrees, which is maybe the largest "evolution" of the Great Wall in the past over 2000 years. According to the argument of the ruler, the reason for they have built this wall ringing the people is to protect people from the outside world against poisonous information and to protect this country, but they are willing to sent their families abroad to study and live, spend the public funds to travel abroad and splurge, even transfer out vast wealth. This is a very ugly human nature, but it is ever growing in China's ruling class, has poisoned generations of Chinese people over two thousand years, this should be the chronic illness in Chinese civilization.

"One yin and yang, this is said to be Tao" after all is eternal, Chinese people always say "one thing face down one thing(this means everything has its vanquisher)", modern technological civilization from the West is just a panacea for this disease, it was introduced to China should be the most fortunate of Chinese people over the latest centuries, it is the only savior saving Chinese people from suffering.

ACKNOWLEDGEMENTS

The research described in this article is based entirely on the Internet, especially for the required knowledge of physics, I have downloaded about ten thousands of files of PDF format on physics and mathematics. So I want to first acknowledge the Internet, which contains some well-known sites such as Google search engine, Wikipedia etc.

However, since I was inside the circle established by the Great firewall, those sites which had provided the mirrors for the google search engine and always soon had been disappeared are vital for me to get those files, I am grateful for them. HandianNet³² and GuShiWenNet³³ are two respectable sites inside the wall, they are very helpful for me to read the Chinese ancient literature.

Of course, those who have placed their books openly on the sites, so that I was able to get them freely are what I would like to thank here in particular. Although I only read carefully the small part of the documents obtained, but these have made me understand basically what is the physics saying. There are so many files which have given me great help, I just listed a handful of them later in the bibliography which I've read and are impressive on. At the beginning of this study I focused on learning the tutorials such as [1, 2, 3], and [22, 24, 23] later have added me more knowledge, I have almost completely translated them into Chinese. It is through this process I let myself into the state of studying physics, these tutorials are very helpful, they let me know a lot of details of physics, and unlock a lot of confusion.

³²<http://www.zdic.net/>.

³³<http://www.gushiwen.org/>.

Numerous developers of free software on Internet is another class of people here to thank. My previous software development work benefit the infinite from they. In the research, I have downloaded a lot of free software, where the most important of which are TexStudio and Maxima. In the procedure, I have learned to use TexStudio to edit Latex document, and using it to draw figures is a great thing, Luque Manuel, one of the developers of pst-solides3d, had ever very enthusiastically helped me draw a figure. Maxima's computing functions are very well, which makes me often do not have to write the C ++ programs to do a variety of calculations, the help given by Mr Barton Willis, one of its developers, is unforgettable.

Internet is really a great thing, although I can use only part of it, but even so, it helped me to finish the research described in this article with the power of individuals by a poor man's way. So my mind have gained the liberation and joy which never happen before, this is really a happy thing.

Finally, the birth of this article must be attributed to my parents, just for them and the the ancient Chinese civilization passed on them, I have a different life experience, and I also won a different understanding of the world, these factors ultimately constitute my motivation to write this article.

Appendix A. Peroid Theory

On the basis of the five elements theory, ACT had developed much more richer period theory. The most famous is Tiangan³⁴, Dizhi³⁵, and the combination of both which is named after Jiazi.

Tiangan period corresponds to 10 elements of the periodic sequence $A = \{a_i\}_{i=1}^{10}$,

$$\begin{aligned} & \text{甲, 乙, 丙, 丁, 戊, 己, 庚, 辛, 壬, 癸.} \\ & \text{Jia, Yi, Bing, Ding, Wu, Ji, Gen, Xin, Ren, Gui.} \end{aligned} \tag{103}$$

Each element of Tiangan period a_i in A has an image in five elements, which can be expressed as the following Fig 28. The images of period elements all are represented by



Fig. 28: Tiangan period mapping to Wuxing

the color corresponding to five elements (cyan corresponds to wood, red corresponds to fire, yellow corresponds to earth, white corresponds to gold, black corresponds to water, the following two figures with the same meaning.).

³⁴The Chinese is 天干, means sky-stem.

³⁵The Chinese is 地支, means earthly-branch.

Dizhi period corresponds to 12 elements of the periodic sequence $B = \{b_i\}_{i=1}^{12}$:

$$\begin{aligned} & \text{子, 丑, 寅, 卯, 辰, 巳, 午, 未, 申, 酉, 戌, 亥.} \\ & \text{Zi, Chou, Yin, Mao, Chen, Si, Wu, Wei, Shen, You, Xu, Hai.} \end{aligned} \quad (104)$$

Each element of Dizhi period $b_i \in B$ also has an image in five elements, which can be expressed as the following Fig29.

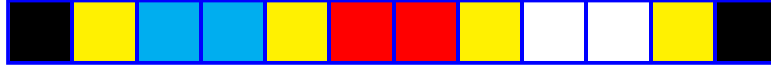


Fig. 29: Dizhi period mapping to Wuxing

Parallel extension of sequence A and sequence B corresponds to another periodic sequence called by Jiazi, which is the sequence $C = \{a_i b_j\}$ consisting of sixty elements, the name of its first element is just Jiazi (composed by the first element in Tiangan period and the first element in Dizhi period). These elements also has images in five elements, the image of each is called its Nayin³⁶. This mapping is actually repeated four times by the subsequence which consists of the first 15 elements, and the map for the subsequence to the five elements can be expressed as the following Fig 30. The

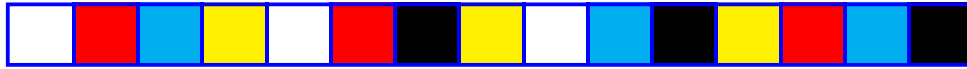


Fig. 30: Jiazi period mapping to Wuxing

best explanation for this mapping maybe: it consists of the sequence

$$C_0 = \text{火, 金, 土, 水, 木,} \quad (105)$$

and the right circular shift of the sequence by one position

$$C_{+1} = \text{木, 火, 金, 土, 水,} \quad (106)$$

and the left circular shift of the sequence by one position

$$C_{-1} = \text{金, 土, 水, 木, 火,} \quad (107)$$

in which the positions $3k + 1, 1 \leq x \leq 5$ correspond to the sequence C_{-1} , the positions $3k + 2, 1 \leq k \leq 5$ correspond to the sequence C_0 , and the positions $3k + 3, 1 \leq k \leq 5$ correspond to the sequence C_{+1} .

A twelve states model of Wuxing is closely related to Dizhi period theory, and it is widely used in divination. In ACT these period theories are mainly used to represent

³⁶The Chinese is 纳音, means absorbing tune.

time, but in feng shui they are also used to represent angles in the plane. In the theories of ACT, time is always represented by these period element, so each time corresponds to a tuple consisting of Wuxing, and because each object has Wuxing attribution, so ACT believes these can be used to judge the states of every object. This in fact is the basic principles used by most of the Shushu(Divination) to make prediction.

Appendix B. Macroscopic Motion Represented By Bagua

Book of Changes in its title, it is the book on things change. It is primarily used as a tool of divination in ACT, but they are also be understood as a philosophy book. We have seen that the use of *Book of Changes* to describe the basic elements of physics equations can indeed give us spectacular views, but it is mainly used to describe the macroscopic motion and macro scene.

Its eight basic trigrams usually corresponds to eight macroscopic material systems in nature. Qiangua(111) corresponds to the sky, Kungua(000) corresponding to the earth, Gengua(100) corresponds to the mountain, Duigua(011) corresponds to the marsh, Ligua(101) corresponds to fire, Kangua(010) corresponds to the water, Zhen-gua(001) corresponds to the thunder, Xungua(110) corresponds to the wind. Here the two material systems represented by two mutual NOT Guas have the opposite relations.

From the physics viewpoint, we can also let the eight basic trigrams correspond to eight categories of macroscopic motion, each category has a a certain degree of correlation with the original meaning given by *Book of Changes*. These eight categories of motion can be simply explained as follows:

- 1 Qian-category motion is defined as diffusion or outbreak from a space-time point uniformly towards the outside of the spherical surrounding it. A typical example is the fission of the nucleus. Its Gua imagination³⁷ is shown in Fig 31a.
- 2 Kun-category motion is defined as in a spherical spatial and temporal scope, matter and energy aggregates to the center. A typical example is the fusion of nucleus. Its Gua imagination is shown in Fig 31b.
- 3 Zhen-category motion is defined as two objects which move in the same direction and collide. A typical example is the collision between objects. Its Gua imagination is shown in Fig 31c.
- 4 Xun-category motion is defined as two objects which move in reverse direction and separate. A typical example is the expansion of the gas. Its Gua imagination is shown in Fig 31d.

³⁷The chinese is 象, means image, shape, scenery.

- 5 Kan-category motion is defined as an object move in the vicinity of the minimum point of a potential field. A typical example is the oscillatory movement of objects in the potential well. Its Gua imagination is shown in Fig 32a.
- 6 Li-category motion is defined as matter and energy radiate towards a particular direction from a certain region of spacetime. A typical example is the combustion radiating heat and light. Its Gua imagination is shown in Fig 32b.
- 7 Gen-category motion is defined as an object with a certain velocity deceleration or even stop after encountered obstacles, an then trapped in a space-time point. A typical example is the braking deceleration, friction, and such as the motion of reducing kinetic, increasing potential in the potential field. Its Gua imagination is shown in Fig 32c.
- 8 Dui-category motion is defined as objects get freedom from restraint or accelerate by the extern force of the push. A typical example is the motion of increasing kinetic, reducing potential in the potential field. Its Gua imagination is shown in Fig 32d.

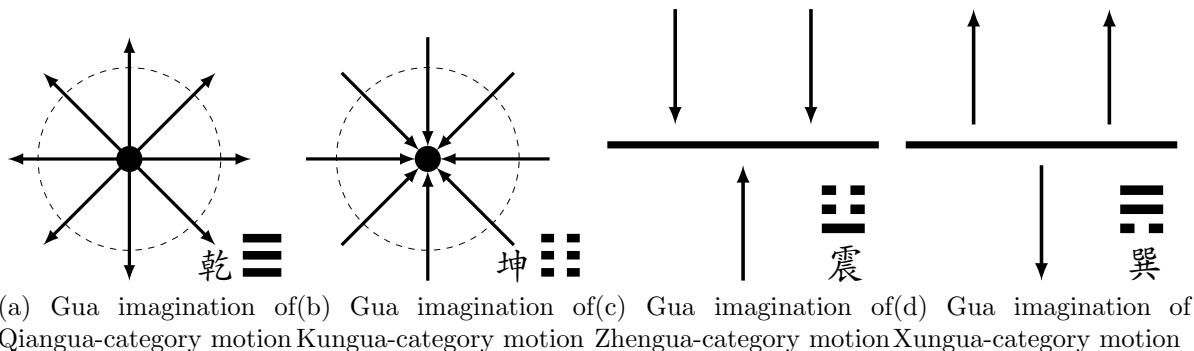


Fig. 31: Gua imagination of macroscopic motion 1

Appendix C. Magic Square Theory

In ACT, the famous magic square is related to Wuxing and Bagua, especially the magic square and Bagua often acquire orientation corresponding in the plane, as shown in Fig 33, this is puzzling.

My thought about the relation between ACT and physics just start from being

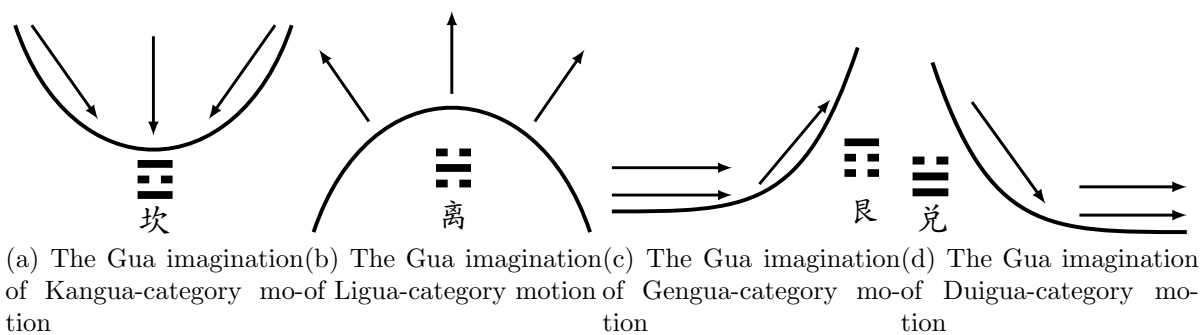


Fig. 32: The Gua imagination of macroscopic motion 2

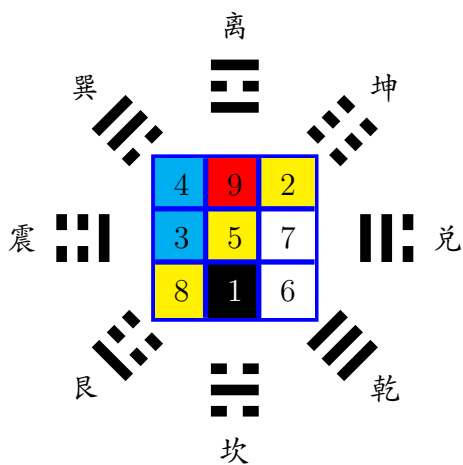


Fig. 33: The relations between magic square and Wuxing & Bagua

aware of the eight equations of magic square:

$$\begin{aligned}
 2 + 5 + 8 &= 15 \mapsto 000 \\
 2 + 7 + 6 &= 15 \mapsto 001 \\
 9 + 5 + 1 &= 15 \mapsto 010 \\
 4 + 3 + 8 &= 15 \mapsto 100 \\
 4 + 5 + 6 &= 15 \mapsto 111 \\
 4 + 9 + 2 &= 15 \mapsto 110 \\
 3 + 5 + 7 &= 15 \mapsto 101 \\
 8 + 1 + 6 &= 15 \mapsto 011
 \end{aligned}
 \tag{108}$$

maybe be related to the Maxwell's equations, but later I had not found a more obvious relation between them. As indicated above, there is a clear correspondence

between it and the Houtian structure of Bagua: three horizontal rows from top to bottom correspond to 110, 101, 011 respectively, three vertical columns from right to left correspond to 001, 010, 100 respectively, while the diagonal line upper left to lower right corresponds to the 111, the diagonal line from the upper right to the lower left corresponds to 000.

But indeed there is some relation between magic square and \mathbb{X}_3 , which could be seen from the following graphs in Fig 34a and Fig 34b, matrix forms of these two

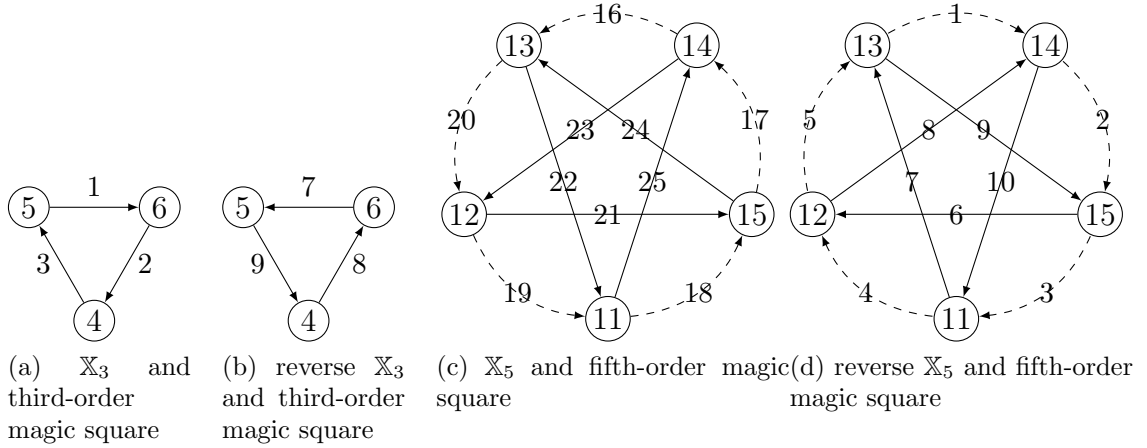
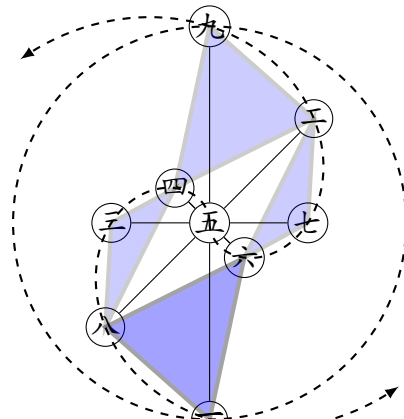


Fig. 34: \mathbb{X}_p and the magic square

graphs is just a magic square, each loop in the graph and its reverse both correspond to three consecutive integers. This relation can be directly extended to \mathbb{X}_p , for example, for $p = 5$, we can write each integer on the edge according to the loops structure in the graph and the TCD map3, as shown in the Fig 34c and Fig 34d. The fifth-order magic square matrix corresponding to these graphs is:

$$L_5 = \begin{bmatrix} 11 & 4 & 7 & 25 & 18 \\ 19 & 12 & 5 & 8 & 21 \\ 22 & 20 & 13 & 1 & 9 \\ 10 & 23 & 16 & 14 & 2 \\ 3 & 6 & 24 & 17 & 15 \end{bmatrix} \quad (109)$$

An interesting inspiration about magic square is, if you let nine numbers in the magic square all correspond to the nine points on the plane like this: let 5 into the origin, while the other 8 numbers in accordance with the polar coordinates into eight points, making their radial coordinate are the absolute value of the difference with 5, and the angle



coordinates are their position within the squares, then we can get the spiral pattern, as shown in Fig 35 on the right side. So maybe magic square corresponds to a certain physical quantity related with the spiral. For example, we can consider the general magic square satisfying following conditions:

$$\begin{aligned}
x_{11} + x_{12} + x_{13} &= n \\
x_{21} + x_{22} + x_{23} &= n \\
x_{31} + x_{32} + x_{33} &= n \\
x_{11} + x_{21} + x_{31} &= n \\
x_{12} + x_{22} + x_{32} &= n \\
x_{13} + x_{23} + x_{33} &= n \\
x_{11} + x_{22} + x_{33} &= n \\
x_{13} + x_{22} + x_{31} &= n
\end{aligned} \tag{110}$$

The solution space of the equations has two degrees of freedom, denoted by u, v , then the general magic square can be expressed with u, v as

$$M = \begin{bmatrix} \frac{2n-3u}{3} & \frac{3u+3v-n}{3} & \frac{2n-3v}{3} \\ \frac{3u-3v+n}{3} & \frac{n}{3} & \frac{-3u+3v+n}{3} \\ v & n-v-u & u \end{bmatrix}, \tag{111}$$

magic square is in fact the result of making $u = 6, v = 8, n = 15$ here. Then M has three eigenvalues: $\lambda_1 = n, \lambda_2 = -L, \lambda_3 = L$, with

$$L = \sqrt{-3v^2 + 2nv + 3u^2 - 2nu}. \tag{112}$$

Eigenvector of λ_1 is $(1, 1, 1)$, it may be understood as the direction of propagation of electromagnetic fields. The other two eigenvalues is not necessarily a real number, their eigenvector is not generally spatial directions. But you can see they are related to the two degrees of freedom on the plane which normals with the $(1, 1, 1)$, so these two degrees of freedom can be understood as related to the photon polarization.

If we required that this matrix is symmetric, then M generally have the following form:

$$M = \begin{bmatrix} -u + \frac{2}{3}n & u & \frac{n}{3} \\ u & \frac{n}{3} & -u + \frac{2}{3}n \\ \frac{n}{3} & -u + \frac{2}{3}n & u \end{bmatrix}, \tag{113}$$

Then it has three real eigenvalues

$$\begin{aligned}\lambda_1 &= n, \\ \lambda_2 &= -\frac{3^{3/2}u - \sqrt{3}n}{3}, \\ \lambda_3 &= \frac{3^{3/2}u - \sqrt{3}n}{3},\end{aligned}\tag{114}$$

Corresponding eigenvectors are:

$$\begin{aligned}\vec{e}_1 &= (1, 1, 1) \\ \vec{e}_2 &= \left(1, -\frac{6u + (2\sqrt{3} - 2)n}{(3^{3/2} + 3)u + 2n}, -\frac{(3^{3/2} - 3)u + (4 - 2\sqrt{3})n}{(3^{3/2} + 3)u + 2n}\right) \\ \vec{e}_3 &= \left(1, \frac{6u + (-2\sqrt{3} - 2)n}{(3^{3/2} - 3)u - 2n}, -\frac{(3^{3/2} + 3)u + (-2\sqrt{3} - 4)n}{(3^{3/2} - 3)u - 2n}\right)\end{aligned}\tag{115}$$

Then we can understand the \vec{e}_1 as the propagation direction of the electromagnetic field, and understand the \vec{e}_2 as the electric field direction, understand \vec{e}_3 as the magnetic field direction.

Similar to the above spiral, we can get more interesting shapes: 36a-36f.

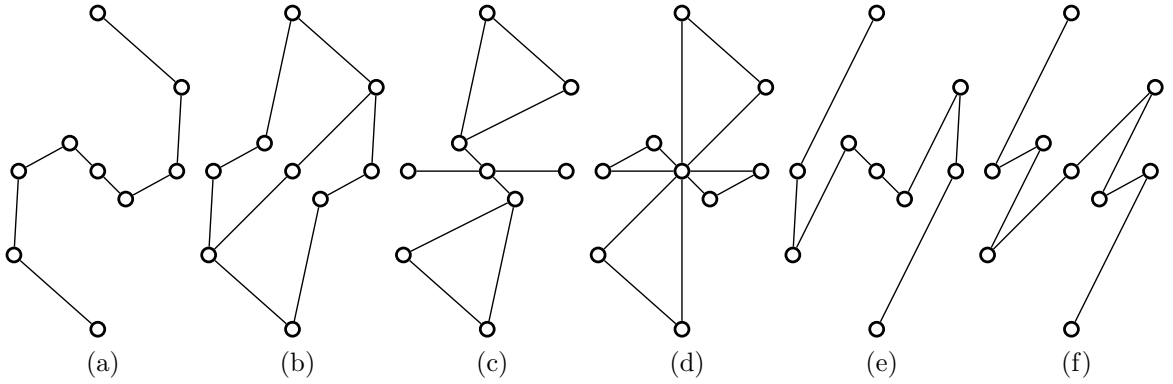


Fig. 36: The interesting shape for the magic square spiral.

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