Important Taylor Series

The following are some important Taylor series representations you should memorize. Intervals of convergence (i.e. intervals on which the representations are valid) are also given.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^3 + x^4 + \cdots , \qquad -1 < x < 1;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots , \qquad -\infty < x < \infty;$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots , \qquad -\infty < x < \infty;$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots , \qquad -\infty < x < \infty;$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots , \qquad -1 \le x \le 1;$$

$$\ln(1-x) = \sum_{n=1}^{\infty} -\frac{x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots , \qquad -1 < x < 1.$$