The scales of atomic and particle physics in relation to Planck scale

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The scales of atomic and particle physics are shown to be related to Planck scale through multiplication by small integer powers of two numbers. One of the numbers is the fine structure constant; the other is of geometric origin. The scales include the Bohr radius and related scales (the reduced Compton wavelength, the classical electron radius, the electron mass and the Hartree energy), the pion charge radius and strong interaction time scale, the up and top quark masses, and the GUT scale.

1 Introduction

All massive particles, including atomic nuclei, have been shown to occupy mass levels that descend in geometric sequence from the Planck Mass [1, 2, 3]. There are three Planck sequences: Sequence 1, of common ratio 1/π; Sequence 2, of common ratio 2/π; and Sequence 3, of common ratio 1/e. The GUT scale, Higgs field vacuum expectation value, quark doublet mass scales and the energy equivalent of the Rydberg constant have been shown to lie upon coincident levels within the Planck sequences [3]. Since the energy equivalent of the Rydberg constant ($R_\infty hc$, which is equal to the ionization energy of the hydrogen atom) lies upon levels that descend from the Planck energy, the relationship to Planck scale of another atomic scale, the Bohr radius (a measure of the size of the hydrogen atom), was investigated. A simple relationship, based on the common ratio of one of the sequences, was found. The reduced Compton wavelength, the classical electron radius, the mass of the electron and the Hartree energy ($E_h = 2R_\infty hc$, an atomic unit of energy, equal to the electric potential energy of the hydrogen atom) could then be written in terms of the relationship. As the result of some simple reasoning and inspection, several particle physics scales have been found to be related to Planck scale within the same scheme. Some relationships uncovered in previous work were then observed to fall within the scheme. The results are discussed.

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2 The Bohr radius and related scales

The Bohr radius, a_0 = 52.917721092(17) x 10^{-12} *m* [4], is related to the Planck length, l_{Planck} = 1.616199(97) x 10^{-35} *m* [4], by the equation

$$a_0 = \left(\frac{\pi}{2}\right)^{125} l_{Planck} \tag{1}$$

More precisely, the power of π/2 in (1) is 125.00062(13).

The reduced Compton wavelength is given by

$$\frac{\lambda}{2\pi} = \propto \left(\frac{\pi}{2}\right)^{125} l_{Planck} \tag{2}$$

where ∝ is the fine structure constant. The classical electron radius is given by

$$r_e = \propto^2 \left(\frac{\pi}{2}\right)^{125} l_{Planck} \tag{3}$$

The mass of the electron is given by

$$m_{electron} = \propto^{-1} \left(\frac{\pi}{2}\right)^{-125} m_{Planck} \tag{4}$$

The Hartree energy (27.21138505(60) eV [4]) is given by

$$E_h = \propto \left(\frac{\pi}{2}\right)^{-125} E_{Planck} \tag{5}$$

All experimental values are provided in Table 1, in comparison with the values derived from the Planck model.

3 The up and top quark masses

The up quark mass has been found to be related to the Hartree energy by the equation

$$m_{up}c^2 = \left(\frac{\pi}{2}\right)^{25} E_h \tag{6}$$

suggesting that the mass of the up quark is a manifestation of potential energy, removed to a different scale. As a consequence of (6), the up quark mass is related to the Planck Mass by the equation

$$m_{up} =\propto \left(\frac{\pi}{2}\right)^{-100} m_{Planck} \tag{7}$$

From both (6) and (7), $m_{up} = 2.18\, MeV/c^2$. The Particle Data Group's evaluation of the up quark mass is $2.3^{+0.7}_{-0.5}\, MeV/c^2$ [5]. In the review 'Quark Masses' by A V Manohar and C T Sachrajda, in [5], the up quark mass is given as 2.15(15) *MeV/c*^{2}.

The top quark mass is related to the Hartree energy by the equation

$$m_{top}c^2 = \left(\frac{\pi}{2}\right)^{50} E_h \tag{8}$$

and to the Planck Mass by the equation

$$m_{top} =\propto \left(\frac{\pi}{2}\right)^{-75} m_{Planck} \tag{9}$$

From both (8) and (9), $m_{top} = 174.1\, GeV/c^2$. The Particle Data Group's evaluation of the top quark mass is $173.5 \pm 1.4\, GeV/c^2$ [5].

4 The pion charge radius

The masses of the lightest charged lepton and the lightest quark have both been shown to be related to Planck scale through multiplication by integer powers of $(\pi/2)^{25}$ and $\propto$. The mass of the lightest charged hadrons ($\pi^{\pm}$) is not related to Planck scale in this way. However, the pion charge radius, 0.672(8) x 10^{-15} m [5], is given by

$$r_\pi = \left(\frac{\pi}{2}\right)^{100} l_{Planck} \tag{10}$$

and consequently the strong interaction time scale is given by

$$r_\pi/c = \left(\frac{\pi}{2}\right)^{100} t_{Planck} \tag{11}$$

From (10), $r_\pi = 0.661 \times 10^{-15} m$. From (11), $r_\pi/c = 2.21 \times 10^{-24} s$.

5 The GUT scale

The GUT scale, $\sim 2 \times 10^{16}\ GeV$, is related to the mass of the electron by the equation

$$E_{GUT} \approx \left(\frac{\pi}{2}\right)^{100} m_{electron} c^2 \tag{12}$$

Perfect equality in (12) would imply a GUT scale of $2.09 \times 10^{16}\ GeV$; a GUT scale of this value would be related to Planck scale by the equation

$$E_{GUT} = \propto^{-1} \left(\frac{\pi}{2}\right)^{-25} E_{Planck} \tag{13}$$

6 The Higgs field vacuum expectation value

Unlike the other scales considered here, the Higgs field VEV (~246 GeV) is not related to Planck scale through multiplication by integer powers of $(\pi/2)^{25}$ and $\propto$, although couplings with the Higgs field are incorporated within the Planck model [2, 6]. In a recent paper [7], the following relationship between the Higgs field VEV and the up-down quark doublet mass scale was shown graphically:

$$m_{u-d}c^2 \approx \left(\frac{\pi}{2}\right)^{-25} E_{VEV} \tag{14}$$

In the light of the new findings, (14) provides more evidence for the incorporation of the mass generation mechanism into a model that encompasses gravity.

7 Vector meson-baryon mass differences

Partnerships have been identified between many pairs of particles or multiplets [1, 8, 9]. Partnerships are arranged symmetrically about mass levels, and the mass difference of the partners has been shown, in many instances, to be related to the value of a mass level. Partnerships exist between the constituent particles of isospin doublets and also between other particles, of the same or different spin. The charged leptons have been shown to have pseudoscalar meson partners; the partnerships are centred upon 'principal' (integer level number) mass levels whose level numbers are multiples of 3 [3, 10, 11]. The uds baryons Λ and Σ^0 also form a partnership centred upon a principal mass level, in Sequence 2; the mass difference of the two baryons is equal to the mass of a principal level in Sequence 2 [1]. Partnerships are often manifest when strange hadrons are involved. The following vector meson-baryon partnerships, identified in [1], are arranged symmetrically about (non-principal) mass levels in Sequence 2; the mass differences characterizing the partnerships are equal to the mass of a principal level, of level number 100:

$$m_{\Sigma} - m_{K^*} = \left(\frac{\pi}{2}\right)^{-100} m_{Planck} \tag{15}$$

$$m_{\Xi} - m_{\varphi} = \left(\frac{\pi}{2}\right)^{-100} m_{Planck} \tag{16}$$

In (15), the K* mesons are represented by the mass value of the interstitial level in Sequence 2, as are the Ξ baryons in (16). The Σ baryons in (15) are represented by the mass of Σ^0. The hadron mass differences in (15) and (16), calculated from the Particle Data Group's evaluations

[5], are 298.9(5) *MeV/c²* and 298.1(5), respectively. The value of $(\pi/2)^{-100}m_{Planck}$ is 298.3 *MeV/c²*.

Since the quarks contribute in similar extent to the mass of the hadrons in each partnership:

$$K^{*}: u\bar{s}, \bar{u}s;\ d\bar{s}, \bar{d}s$$
$$\Sigma: uus, uds, dds$$

$$\varphi: c_{1(u\bar{u}+d\bar{d})} + c_2(s\bar{s})$$
$$\Xi: uss;\ dss$$

(15) and (16) suggest that the strong interaction energy making up much of the hadron mass is greater for the baryon than the vector meson by an amount equal to 298 MeV, a unit of energy in the Planck model.

8 Discussion

Two distance scales have been shown to be related to Planck scale through multiplication by integer powers of $(\pi/2)^{25}$ alone: the Bohr radius (the size of the hydrogen atom) and the pion charge radius (the size of the lightest charged hadron). The other scales are closely allied to these two scales. The significance of the factor $(\pi/2)^{25}$ may be understood with reference to previous observations.

The scales of the Standard Model have been shown to arise at coincident levels within sequences of common ratio $(\pi/2)^{38}$ and π^{15}, incorporated into Sequences 1 and 2, that ascend and descend from coincident levels at 5.12 MeV [12]. Since $(\pi/2)^{38} \approx \pi^{15}$, level and sublevel[2] near-coincidence occurs at regular intervals in a geometric sequence. A 'Higgs Sequence', populated by strange hadrons, has also been identified [2, 6]. This sequence descends from the Higgs field VEV with common ratio $(\pi/2)^{-5} \approx \pi^{-2}$ and is incorporated into Sequences 1 and 2. Recently, stable charged particles have been shown to occupy the levels and sublevels of two sequences, of common ratio $(\pi/2)^{-5}$ and $e^{-3.75}$, incorporated into Sequences 2 and 3 [7]. The

[2] Half-level, quarter-level, etc.

‘mass levels of stability’ are now understood to lie within sequences of common ratio $(\pi/2)^{-25} \approx e^{-11.25}$.

Hadron ‘mass construction’ equations have been proposed [1], in which units of mass from Sequence 2, of common ratio 2/π, feature. Particle mass differences have been found to be equal to the values of levels within Sequence 2 [8]. We have seen here that interaction energy may be constructed from units of energy from Sequence 2. That the Bohr radius and the pion charge radius are derived from Planck scale through powers of 2/π should not be surprising. Prominent scales are found at coincident levels within the fundamental sequences [12]. The Bohr radius and the pion charge radius are related to Planck scale by the factors $(\pi/2)^{125} \approx \pi^{50}$ and $(\pi/2)^{100} \approx \pi^{40}$, respectively.

We have shown that Sequences 1, 2 and 3 may derive from the geometry of spacetime [2, 7, 12, 13]. In the model developed, 4-branes wrap a Planck-scale space and physics is located upon 3-branes at their intersections.

The relationships of atomic and particle physics scales to Planck scale have been compiled in Table 2.

9 References

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Scale	Value calculated in the Planck scheme	Measured value
Bohr radius	52.90×10^{-12} m	52.92×10^{-12} m [4]
Reduced Compton wavelength	386.1×10^{-15} m	386.2×10^{-15} m [4]
Classical electron radius	2.817×10^{-15} m	2.818×10^{-15} m [4]
pion charge radius	0.6614×10^{-15} m	$0.672(8) \times 10^{-15}$ m [5]
electron mass	0.5111 MeV/c^2	0.5110 MeV/c^2 [5]
up quark mass	2.177 MeV/c^2	$2.3^{+0.7}_{-0.5}$ MeV/c^2 [5] 2.15(15) MeV/c^2 [5]
top quark mass	174.1 GeV/c^2	173.5 ± 1.4 GeV/c^2 [5]
Hartree energy	27.22 eV	27.21 eV [4]
GUT scale	2.092×10^{16} GeV	2×10^{16} GeV
Strong interaction time scale	2.206×10^{-24} s	$2.24(3) \times 10^{-24}$ s*

Table 1: Values of scales calculated in the Planck scheme compared with measured values. The errors in the calculated values arise largely from uncertainty in the Planck scale, $\sim 6\times10^{-5}$, and are insignificant here. Measured values are given to four significant figures where possible.

* Derived from the pion charge radius.

Length scales	
Bohr radius	$a_0 = \left(\frac{\pi}{2}\right)^{125} l_{Planck}$
Reduced Compton wavelength	$\frac{\lambda}{2\pi} = \propto \left(\frac{\pi}{2}\right)^{125} l_{Planck}$
Classical electron radius	$r_e = \propto^2 \left(\frac{\pi}{2}\right)^{125} l_{Planck}$
pion charge radius	$r_\pi = \left(\frac{\pi}{2}\right)^{100} l_{Planck}$
Mass scales	
electron mass	$m_{electron} = \propto^{-1} \left(\frac{\pi}{2}\right)^{-125} m_{Planck}$
up quark mass	$m_{up} = \propto \left(\frac{\pi}{2}\right)^{-100} m_{Planck}$
top quark mass	$m_{top} = \propto \left(\frac{\pi}{2}\right)^{-75} m_{Planck}$
Energy scales	
Hartree energy (27.211 eV)	$E_h = \propto \left(\frac{\pi}{2}\right)^{-125} E_{Planck}$
GUT scale (2 x 10^{16} GeV)	$E_{GUT} = \propto^{-1} \left(\frac{\pi}{2}\right)^{-25} E_{Planck}$
Time scale	
Strong interaction time scale	$r_\pi / c = \left(\frac{\pi}{2}\right)^{100} t_{Planck}$

Table 2: The scales of atomic and particle physics in relation to Planck scale