

## Volatility, the Macroeconomy, and Asset Prices

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### ABSTRACT

How important are volatility fluctuations for asset prices and the macroeconomy? We find that an increase in macroeconomic volatility is associated with an increase in discount rates and a decline in consumption. We develop a framework in which cash flow, discount rate, and volatility risks determine risk premia and show that volatility plays a significant role in explaining the joint dynamics of returns to human capital and equity. Volatility risk carries a sizable positive risk premium and helps account for the cross section of expected returns. Our evidence demonstrates that volatility is important for understanding expected returns and macroeconomic fluctuations.

RECENT ECONOMIC ANALYSIS emphasizes the important role of macroeconomic volatility movements in determining asset prices and macroquantities. In the asset pricing model of Bansal and Yaron (2004), an increase in aggregate volatility lowers asset prices and, importantly, shocks to volatility carry a separate risk premium. A growing literature in macroeconomics also highlights the effect of volatility on macroquantities.<sup>1</sup> In this paper, we show that variation in macroeconomic volatility is indeed an important and separate risk that significantly affects the macroeconomy (aggregate consumption) and asset prices. To guide our analysis, we develop a dynamic asset pricing framework in which the stochastic discount factor and therefore the risk premium are determined

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<sup>1</sup> Bloom (2009) analyzes the effect of time-varying volatility for investment; Ramey and Ramey (1995), Gilchrist and Williams (2005), Gilchrist, Sim, and Zakrajsek (2010), Basu and Bundick (2012), Fernandez-Villaverde and Rubio-Ramirez (2013), and Justiniano and Primiceri (2008) discuss the relationship between volatility and output; Caldara et al. (2012) consider volatility risk in the context of a production-based asset pricing model.

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by three sources of risk: cash flow risk, discount rate risk, and volatility risk. Our empirical work yields three central findings: (i) an increase in volatility is associated with a rise in discount rates and a decline in future consumption; (ii) volatility risk plays a significant role in accounting for the joint dynamics of returns to human capital and equity; and (iii) volatility risk carries a sizeable positive risk premium and helps explain the level of and cross-sectional dispersion in expected returns. Taken together, our evidence suggests that volatility risk is important for understanding the dynamics of asset prices and macroeconomic fluctuations.

We document that, in the data, both macroeconomic- and return-based volatility measures feature persistent predictable variation, which makes volatility news a potentially important source of economic risk. The earlier works of Bollerslev and Mikkelsen (1996) in the context of market return volatility and Kandel and Stambaugh (1991), McConnell and Quiros (2000), Stock and Watson (2002), and Bansal, Khatchatrian, and Yaron (2005) in the context of macroeconomic volatility provide supporting evidence of predictable low-frequency variation in volatility. We incorporate this evidence in our theoretical framework and evaluate the implications of volatility risk for consumption, returns to human capital and equity, and the cross-sectional dispersion in risk premia.

In our dynamic asset pricing model with time-varying macroeconomic volatility (which we refer to as Macro-DCAPM-SV model), the stochastic discount factor and therefore the risk premium are determined by three sources of risk: cash flow risk, discount rate risk, and volatility risk. To identify the underlying economic risks using the standard VAR-based methodology, we model the return on aggregate wealth as a weighted average of returns to human capital and financial wealth and assume that the expected return to the human component of wealth is linear in economic states.<sup>2</sup> We estimate the model using observed macro and financial market data and find that, empirically, high macro-volatility states are high risk states associated with significant consumption declines, high risk premia, and high discount rates. The documented positive relationship between ex-ante volatility and discount rates results in a positive correlation between returns to human capital and financial wealth. This implication is consistent with standard economic theory, in which the two assets are positively correlated as they both represent claims to aggregate cash flows. In contrast, in a constant-volatility setting, Lustig and Van Nieuwerburgh (2008) find that returns to human capital and equity are strongly negatively correlated. Our evidence suggests that their puzzling finding can be resolved when time variation in economic volatility is taken into account.

Specifically, in the model with constant volatility under benchmark preferences of risk aversion equal to five and intertemporal elasticity of substitution

<sup>2</sup>The importance of the human capital component of wealth for explaining equity prices has been illustrated in earlier work by Jagannathan and Wang (1996), Campbell (1996), and Santos and Veronesi (2006).

(IES) equal to two, the correlation between realized returns to human capital and equity is  $-0.6$ , and the correlation in their five-year expected returns is  $-0.5$ . In our Macro-DCAPM-SV model that incorporates volatility risk, the two assets tend to move together: the correlation in realized returns to human capital and financial wealth is about  $0.2$ , and the correlation in their five-year expected returns is  $0.4$ . We show that the inclusion of volatility risk has important implications for the time-series dynamics of the underlying economic shocks. In particular, in our volatility risk-based model, discount rates are high and positive in the recent recessions of 2001 and 2008, which is consistent with a sharp increase in economic volatility and risk premia during those times. In contrast, the constant-volatility specification generates negative discount rate news in the two recessions. We also show that model specifications that ignore volatility risk imply a counterfactual positive correlation between expected consumption and discount rates.

We document that volatility risk carries positive and economically significant risk premia, and helps explain the level and cross-sectional variation in expected returns. The model-implied market price of volatility risk is  $-1$ . We show that in the data all equity portfolios as well as the return to human capital have negative exposure to aggregate volatility risk. Thus, the compensation for volatility risk in equities is positive. Quantitatively, the model-implied risk premia of the wealth portfolio, human capital, and equity are  $2.6\%$ ,  $1.4\%$ , and  $7.4\%$ , respectively. Volatility risk accounts for about one-third of the total risk premium of human capital, and about one-half of risk premia of the aggregate and financial wealth portfolios. We show that the Macro-DCAPM-SV model is able to account for the observed dispersion in risk premia across book-to-market- and size-sorted portfolios. The value spread in the model is  $5.5\%$  compared with  $5.9\%$  in the data; the size spread is  $7.1\%$  and  $7.4\%$  in the model and in the data, respectively.

The key time-series and cross-sectional implications of our volatility-based model continue to hold if the aggregate wealth portfolio is measured simply by the return to the stock market.<sup>3</sup> Consistent with the findings in our benchmark macro model, we find that, in the market-based (Market-DCAPM-SV) specification, all equity portfolios have negative volatility betas, that is, equity prices fall on positive news about volatility. Given that investors attach a negative price to volatility shocks, volatility risk carries a positive risk premium. We also document strong comovement between the risk premium and ex-ante market volatility at both the short and especially the long horizons, which reflects a positive correlation between discount rate and volatility risks. Further, we find that in periods of recessions and especially those with significant economic stress, such as the Great Recession, both discount rate news and volatility news are large and positive. On average, volatility and discount rate risks

<sup>3</sup> Substituting the return to aggregate wealth with the return to the stock market has a long tradition in the static CAPM literature. It has also been practiced in empirical work based on recursive preferences (e.g., Epstein and Zin (1991), among others). Unobservability of the return to aggregate wealth is discussed in Roll (1977).

account for about 35% of the overall risk premia in the cross section, and almost 50% of the total premium of the market portfolio. Our evidence based on the Macro-DCAPM-SV model and the Market-DCAPM-SV specification is consistent in that, in both cases, volatility and discount rates are strongly positively correlated and volatility risk contributes positively to equity premia.

We show that ignoring volatility risk may result in quantitatively large biases in state prices and misleading inferences about the underlying sources of risk. The dynamics of consumption, discount rate, and volatility news are intimately linked in equilibrium. Ignoring time variation in volatility leads to distortions in this equilibrium relationship and consequently to distortions in the joint dynamics of the other two risks. Using our benchmark Macro-DCAPM-SV model, we find that the dynamics of the stochastic discount factor extracted by relying solely on financial market data and ignoring volatility risk, as in Campbell (1996), are significantly biased. In general, volatility of the implied stochastic discount factor and hence the implied risk premia are substantially biased downwards.

The rest of the paper is organized as follows. In Section I we present a theoretical framework for the analysis of volatility risk and its implications for consumption dynamics and the stochastic discount factor. In Section II we empirically implement the Macro-DCAPM-SV model, quantify the role of volatility risk in the data, and discuss the model implications for the market, human capital, and wealth portfolios, as well as a cross section of equity returns. Section III discusses implications of the Market-DCAPM-SV specification and the role of volatility risk for explaining the cross section of assets. Section IV provides concluding remarks.

## I. Theoretical Framework

In this section, we consider a general economic framework with recursive utility and time-varying economic uncertainty and derive the implications for innovations to current and future consumption growth, returns, and the stochastic discount factor. We show that accounting for fluctuations in economic uncertainty is important for correct inferences about economic news, and that ignoring volatility risk can alter the implications for the financial markets.

### A. Consumption and Volatility

We adopt a discrete-time specification of the endowment economy where the agent's preferences are described by a Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989) and Weil (1989). The lifetime utility of the agent,  $U_t$ , satisfies

$$U_t = \left[ (1 - \delta)C_t^{1-\frac{1}{\psi}} + \delta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \quad (1)$$

where  $C_t$  is the aggregate consumption level,  $\delta$  is the subjective discount factor,  $\gamma$  is the coefficient of relative risk aversion, and  $\psi$  is the Intertemporal Elasticity of Substitution (IES). For notational ease, we denote  $\theta = (1 - \gamma)/(1 - \frac{1}{\psi})$ . When  $\gamma = 1/\psi$ , the preferences collapse to a standard expected power utility.

As shown in Epstein and Zin (1989), the stochastic discount factor  $M_{t+1}$  can be written in terms of the log consumption growth rate,  $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$ , and the log return to the consumption asset (wealth portfolio),  $r_{c,t+1}$ . In logs,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}. \quad (2)$$

The standard Euler condition

$$E_t [M_{t+1}R_{t+1}] = 1 \quad (3)$$

allows us to price any asset in the economy. Assuming that the stochastic discount factor and the consumption asset return are jointly log-normal, the Euler equation for the consumption asset leads to

$$E_t \Delta c_{t+1} = \psi \log \delta + \psi E_t r_{c,t+1} - \frac{\psi - 1}{\gamma - 1} V_t, \quad (4)$$

where we define  $V_t$  to be the conditional variance of the stochastic discount factor plus the consumption asset return:

$$\begin{aligned} V_t &= \frac{1}{2} \text{Var}_t(m_{t+1} + r_{c,t+1}) \\ &= \frac{1}{2} \text{Var}_t m_{t+1} + \text{Cov}_t(m_{t+1}, r_{c,t+1}) + \frac{1}{2} \text{Var}_t r_{c,t+1}. \end{aligned} \quad (5)$$

The volatility component  $V_t$  is equal to the sum of the conditional variance of the discount factor and the consumption return and the conditional covariance between the two, which are directly related to movements in aggregate volatility and risk premia in the economy. Hence,  $V_t$  is a measure of aggregate economic volatility. In our subsequent discussion, we show that, under further model restrictions,  $V_t$  is proportional to the conditional variance of future aggregate consumption, and the proportionality coefficient is always positive and depends only on the coefficient of risk aversion. As can be seen from equation (4), economic volatility shocks are not separately reflected in expected consumption when there is no stochastic volatility in the economy (i.e.,  $V_t$  is a constant) or when the IES parameter is one,  $\psi = 1$ . The case without variation in volatility is considered by Campbell (1996), Campbell and Vuolteenaho (2004), and Lustig and Van Nieuwerburgh (2008). In this paper, we argue that variation in volatility is important for interpreting movements in consumption and the asset markets.

We use the equilibrium restriction in equation (4) to derive immediate consumption news. The return to the consumption asset  $r_{c,t+1}$ , which enters the equilibrium condition in equation (4), satisfies the usual budget constraint

$$W_{t+1} = (W_t - C_t)R_{c,t+1}. \tag{6}$$

A standard log-linearization of the budget constraint yields

$$r_{c,t+1} = \kappa_0 + wc_{t+1} - \frac{1}{\kappa_1}wc_t + \Delta c_{t+1}, \tag{7}$$

where  $wc_t \equiv \log(W_t/C_t)$  is the log wealth-to-consumption ratio, and  $\kappa_0$  and  $\kappa_1$  are the linearization parameters. Solving the recursive equation forward, the immediate consumption innovation can be written as the revision in expected future returns to the consumption asset minus the revision in expected future cash flows:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j r_{c,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1}. \tag{8}$$

Using the expected consumption relation in equation (4), we can further express the consumption shock in terms of immediate consumption return news,  $N_{R,t+1}$ , revisions to expected future returns (discount rate news),  $N_{DR,t+1}$ , as well as future volatility news,  $N_{V,t+1}$  :

$$N_{C,t+1} = N_{R,t+1} + (1 - \psi)N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1}N_{V,t+1}, \tag{9}$$

where for convenience we denote

$$\begin{aligned} N_{C,t+1} &\equiv c_{t+1} - E_t c_{t+1}, & N_{R,t+1} &\equiv r_{c,t+1} - E_t r_{c,t+1}, \\ N_{DR,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j r_{c,t+j+1} \right), & N_{V,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j V_{t+j} \right), \\ N_{CF,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) = N_{DR,t+1} + N_{R,t+1}. \end{aligned} \tag{10}$$

To highlight the intuition for the relationship between consumption, asset prices, and volatility, let us define future expected consumption news,  $N_{ECF,t+1}$ , as follows:

$$N_{ECF,t+1} = (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right). \tag{11}$$

Note that the consumption innovation in equation (9) implies that future expected consumption news can be decomposed into discount rate news to the wealth portfolio and economic volatility news:

$$N_{ECF,t+1} = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}. \tag{12}$$

In a similar way, we can decompose the shock to the wealth-to-consumption ratio into expected consumption news and volatility news:

$$\begin{aligned} (E_{t+1} - E_t)wc_{t+1} &= N_{\text{ECF},t+1} - N_{\text{DR},t+1} \\ &= \left(1 - \frac{1}{\psi}\right) \left(N_{\text{ECF},t+1} - \frac{1}{\gamma - 1}N_{\text{V},t+1}\right). \end{aligned} \quad (13)$$

When the IES is equal to 1, the substitution effect is equal to the income effect, so future expected consumption news moves one-to-one with discount rate news. As the two types of news exactly offset each other, the wealth-to-consumption ratio is constant so that the agent consumes a constant fraction of total wealth. When the IES is not equal to 1 and aggregate volatility is time-varying, movements in expected consumption no longer correspond to movements in discount rates.<sup>4</sup> In Sections II and III, we show that, in the data, “bad” economic times are associated with low future expected growth, high risk premia, and high uncertainty, that is, volatility news comoves significantly positively with discount rate news and negatively with cash flow news. This evidence is consistent with the economic restriction in equation (12) in the presence of volatility risk and an IES above 1. Note that, when volatility news is ignored, equation (12) implies that future consumption news and discount rate news are perfectly positively correlated so that bad times of high volatility and high discount rates correspond to good times of positive future consumption news. This stands in stark contrast to empirical observations and economic intuition, and highlights the importance of volatility risk to correctly interpret movements in consumption and asset prices.

### B. Asset Prices and Volatility

The innovation to the stochastic discount factor implied by the representation in equation (2) is given by

$$\begin{aligned} N_{\text{M},t+1} \equiv m_{t+1} - E_t m_{t+1} &= -\frac{\theta}{\psi}(\Delta c_{t+1} - E_t \Delta c_{t+1}) \\ &\quad + (\theta - 1)(r_{c,t+1} - E_t r_{c,t+1}). \end{aligned} \quad (14)$$

Substituting out the consumption shock using equation (9), we obtain that the stochastic discount factor is driven by future cash flow news,  $N_{\text{CF},t+1}$ , future discount rate news,  $N_{\text{DR},t+1}$ , and volatility news,  $N_{\text{V},t+1}$ :

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{\text{CF},t+1} + N_{\text{DR},t+1} + N_{\text{V},t+1}. \quad (15)$$

<sup>4</sup> Time-varying risk aversion as in Campbell and Cochrane (1999), as well as incomplete markets and/or market segmentation as in Basak and Cuoco (1998), Guvenen (2009), and Povala (2012), would also induce time-varying risk premia. The ability of these alternative models to answer questions explored in this paper is left for future research.

As shown in the above equation, the market price of cash flow risk is  $\gamma$ , and the market prices of volatility and discount rate news are equal to  $-1$ . Notably, volatility risk is present at any value of the IES. Thus, even though with the IES equal to 1 volatility news does not directly affect the consumption innovation as shown in equation (9), the stochastic discount factor still carries volatility risk. Ignoring volatility risk will therefore lead to incorrect inferences and can significantly affect interpretation of the asset markets.<sup>5</sup>

Given this decomposition for the stochastic discount factor, we can rewrite the expression for the ex-ante economic volatility  $V_t$  in (5) as follows:

$$\begin{aligned} V_t &= \frac{1}{2} \text{Var}_t(m_{t+1} + r_{c,t+1}) \\ &= \frac{1}{2} \text{Var}_t(-\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} + N_{R,t+1}) \\ &= \frac{1}{2} \text{Var}_t((1 - \gamma)N_{CF,t+1} + N_{V,t+1}), \end{aligned} \tag{16}$$

where, in the last equation, we use the identity that the sum of the immediate and discount rate news on the wealth portfolio is equal to cash flow news. Consider the case in which the variance of volatility news  $N_{V,t+1}$  and its covariance with cash flow news are constant (i.e., volatility shocks are homoskedastic). In this case,  $V_t$  is driven by the variance of current and future consumption news, where the proportionality coefficient is determined only by the coefficient of risk aversion:

$$V_t = \text{const} + \frac{1}{2}(1 - \gamma)^2 \text{Var}_t(N_{CF,t+1}). \tag{17}$$

Hence, volatility news corresponds to news in the future variance of long-run consumption shocks; in this sense,  $V_t$  is a measure of ex-ante economic volatility. Further, note that, when there is a single consumption volatility factor, we can identify  $V_t$  from the rescaled volatility of immediate consumption news,  $V_t = \text{const} + \frac{1}{2}(1 - \gamma)^2 \chi \text{Var}_t(\Delta c_{t+1})$ , where  $\chi$  is the scaling factor equal to the ratio of the variance of long-run consumption growth to the variance of current consumption growth,

$$\chi = \text{Var}(N_{CF})/\text{Var}(N_C). \tag{18}$$

We impose this structural restriction to identify economic volatility shocks in our empirical work.

Using the Euler equation, we obtain that the risk premium on any asset is equal to the negative covariance between the asset's return  $r_{i,t+1}$  and the stochastic discount factor:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \text{Var}_t r_{i,t+1} = \text{Cov}_t(-m_{t+1}, r_{i,t+1}). \tag{19}$$

<sup>5</sup> It is easily seen that, when  $\theta = 1$ , equation (14) reduces to the familiar power utility case, and it follows that the decomposition of the stochastic discount factor in (15) in that case reduces to  $-\gamma N_{C,t+1}$ .



Hence, knowing the exposures (betas) of returns to the fundamental sources of risk, we can calculate the risk premium on the asset and decompose it into risk compensation for cash flow news, discount rate news, and volatility news:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \text{Var}_t r_{i,t+1} = \gamma \text{Cov}_t(r_{i,t+1}, N_{CF,t+1}) - \text{Cov}_t(r_{i,t+1}, N_{DR,t+1}) - \text{Cov}_t(r_{i,t+1}, N_{V,t+1}). \quad (20)$$

This risk premia restriction is an asset pricing model with three distinct sources of risk. Cash flow news corresponds to news about future consumption, discount rate news corresponds to news about the expected return to aggregate wealth, and volatility news corresponds to news about macroeconomic volatility. As stated above, we denote this specification as the Macro-DCAPM-SV model. This specification emphasizes the importance of measuring news using macroeconomic data. In Section II we provide a way to estimate such a model and evaluate its empirical implications. It is important to note that the measurement of the return to aggregate wealth entails measuring the return to human capital along with the return to financial assets. Consequently, our Macro-DCAPM-SV approach has implications for the joint dynamics of the return to human capital, the market return, and the entire cross section of asset returns. Through the Macro-DCAPM-SV model, we can evaluate the puzzling negative correlation between returns to human capital and the market as highlighted in Lustig and Van Nieuwerburgh (2008), as well the implications for the cross section of asset returns.

A significant body of empirical work in finance replaces the return to aggregate wealth with the financial market return, essentially assuming that the two returns are the same (see, for example, the empirical work based on the static CAPM).<sup>6</sup> Epstein and Zin (1991) use this approach in the context of recursive preferences. While returns to aggregate and financial wealth may not entirely coincide, the availability of extensive stock market data makes it a convenient and easy-to-implement approach. To provide a comprehensive account of volatility risk and its importance for asset prices, we also entertain this approach in Section III. In this specification of the model, cash flow news, discount rate news, and volatility news are measured using available data on the value-weighted stock market return and its realized volatility. We refer to this reduced-form specification as the Market DCAPM with stochastic volatility and denote it as Market-DCAPM-SV. By its very nature, this approach cannot be used to address issues related to the comovement of human capital and financial market returns, yet we can still exploit it to highlight the implications of time-varying volatility for the cross section of equity returns.

<sup>6</sup> Bansal, Kiku, and Yaron (2007) show that the gap between the aggregate wealth return and the market return can be quite substantial. Hence, this approximation can significantly alter risk premia implications.

## II. Macro-DCAPM-SV Model

In this section, we develop and implement a Macro-DCAPM-SV model to quantify the role of the volatility channel for asset markets. As the aggregate consumption return (i.e., aggregate wealth return) is not directly observed in the data, we assume that it is a weighted combination of the returns to the stock market and human capital. This allows us to adopt a standard VAR-based methodology to extract the innovations to the consumption return, volatility, and the stochastic discount factor to assess the importance of the volatility channel for returns to human capital and equity.

### A. Econometric Specification

Denote by  $X_t$  a vector of state variables that include annual real consumption growth  $\Delta c_t$ , real labor income growth  $\Delta y_t$ , the real market return  $r_{d,t}$ , the market price-dividend ratio  $z_t$ , and the realized variance measure  $RV_t$ :

$$X_t = (\Delta c_t \ \Delta y_t \ r_{d,t} \ z_t \ RV_t)'. \quad (21)$$

For parsimony, we focus on a minimal set of economic variables in our benchmark empirical analysis, and, in Section II. E, we confirm that our main results are robust to the choice of predictive variables, volatility measurements, and estimation strategy.

The vector of state variables  $X_t$  follows a VAR(1) specification, which we refer to as Macro VAR:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (22)$$

where  $\Phi$  is a persistence matrix and  $\mu_X$  is an intercept. Shocks  $u_{t+1}$  are assumed to be conditionally normal with a time-varying variance-covariance matrix  $\Omega_t$ .

To identify the fluctuations in aggregate economic volatility, we include as one of the state variables a realized variance measure based on the sum of squares of monthly industrial production growth over the year:

$$RV_{t+1} = \sum_{j=1}^{12} \Delta i p_{t+j/12}^2. \quad (23)$$

Constructing the realized variance from monthly data helps us capture more accurately fluctuations in aggregate macroeconomic volatility in the data, and we use industrial production because high-frequency real consumption data are not available for a long sample. For robustness, we check that our results do not materially change if we instead construct the measure based on the realized variance of annual consumption growth. To ensure consistency, we rescale industrial production-based realized variance to match the average level of consumption variance.

The expectations of  $RV_{t+1}$  implied by the dynamics of the state vector capture the ex-ante macroeconomic volatility in the economy. Following the derivations

in Section I, economic volatility  $V_t$  is proportional to the ex-ante expectation of the realized variance  $RV_{t+1}$  based on the VAR(1):

$$\begin{aligned} V_t &= V_0 + \frac{1}{2}\chi(1 - \gamma)^2 E_t RV_{t+1} \\ &= V_0 + \frac{1}{2}\chi(1 - \gamma)^2 i'_v \Phi X_t, \end{aligned} \tag{24}$$

where  $V_0$  is an unimportant constant that disappears in the expressions for shocks,  $i_v$  is an indicator vector that extracts the realized variance measure from  $X_t$ , and  $\chi$  is a scale parameter that captures the link between aggregate consumption volatility and  $V_t$ . In the model with volatility risk, we fix the value of  $\chi$  to the ratio of the variances of cash flow news to immediate consumption news, consistent with the theoretical restriction in Section I. In the specification where volatility risk is absent, the parameter  $\chi$  is set to zero. Then, following standard VAR-based derivations, the revisions in future expectations of economic volatility can be calculated as

$$N_{V,t+1} = \frac{1}{2}\chi(1 - \gamma)^2 i'_v Q u_{t+1}, \tag{25}$$

where  $Q$  is the matrix of the long-run responses,  $Q = \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1}$ .

The VAR specification implies that shocks to the immediate market return,  $N_{R,t+1}^d$ , and future market discount rate news,  $N_{DR,t+1}^d$ , are given by<sup>7</sup>

$$N_{R,t+1}^d = i'_r u_{t+1}, \quad N_{DR,t+1}^d = i'_r Q u_{t+1}, \tag{26}$$

where  $i_r$  is an indicator vector that extracts the market return component from the set of state variables  $X_t$ . While the market return is directly observed and market return news can be extracted directly from the VAR(1), in the data, we can only observe labor income and not the total return to human capital. We make the following identifying assumption, identical to Lustig and Van Nieuwerburgh (2008), that the expected return to labor income is linear in the state variables:

$$E_t r_{y,t+1} = \alpha + b' X_t, \tag{27}$$

where  $b$  captures the loadings of the expected return to human capital on the economic state variables. Given this restriction, news to future discounted human capital returns,  $N_{DR,t+1}^y$ , is given by

$$N_{DR,t+1}^y = b' \Phi^{-1} Q u_{t+1}, \tag{28}$$

<sup>7</sup> In what follows, we use superscript “d” to denote shocks to the market return and superscript “y” to identify shocks to the human capital return. Shocks without the superscript refer to the consumption asset, consistent with the notation in Section I.

and the immediate shock to the return to labor income,  $N_{R,t+1}^y$ , can be computed as follows:

$$\begin{aligned} N_{R,t+1}^y &= (E_{t+1} - E_t) \left( \sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1} \right) - N_{DR,t+1}^y \\ &= i'_y (I + Q) u_{t+1} - b' \Phi^{-1} Q u_{t+1}, \end{aligned} \quad (29)$$

where an indicator vector  $i_y$  extracts labor income growth from the state vector  $X_t$ .

To construct the aggregate consumption return (i.e., aggregate wealth return), we follow Jagannathan and Wang (1996), Campbell (1996), Lettau and Ludvigson (2001), and Lustig and Van Nieuwerburgh (2008) and assume that it is a portfolio of returns to the stock market and returns to human capital:

$$r_{c,t} = (1 - \omega)r_{d,t} + \omega r_{y,t}. \quad (30)$$

The share of human wealth in total wealth  $\omega$  is assumed to be constant. It immediately follows that the immediate and future discount rate news on the consumption asset are equal to the weighted average of the corresponding news to the human capital and market returns, with weight parameter  $\omega$ :

$$\begin{aligned} N_{R,t+1} &= (1 - \omega)N_{R,t+1}^d + \omega N_{R,t+1}^y, \\ N_{DR,t+1} &= (1 - \omega)N_{DR,t+1}^d + \omega N_{DR,t+1}^y. \end{aligned} \quad (31)$$

These consumption return innovations can be expressed in terms of the parameters and shocks of Macro VAR, and the vector of the expected labor return loadings  $b$  following equations (26) to (29).

Finally, we can combine the expressions for the volatility news and return news on the consumption asset to back out the implied consumption shock following equation (9):

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= N_{R,t+1} + (1 - \psi)N_{DR,t+1} + \frac{\psi-1}{\gamma-1} N_{V,t+1} \\ &= \underbrace{[(1 - \omega)i'_r Q + \omega(i'_y(I + Q) - b' \Phi^{-1} Q)]}_{N_{R,t+1}} u_{t+1} \\ &\quad + (1 - \psi) \underbrace{[(1 - \omega)i'_r Q + \omega b' \Phi^{-1} Q]}_{N_{DR,t+1}} u_{t+1} + \underbrace{\left( \frac{\psi-1}{\gamma-1} \right) \frac{1}{2} \chi (1 - \gamma)^2 i'_v Q}_{N_{V,t+1}} u_{t+1} \\ &\equiv q(b)' u_{t+1}. \end{aligned} \quad (32)$$

The vector  $q(b)$  defined above depends on the model parameters; in particular, it depends linearly on the expected labor return loadings  $b$ . On the other hand, as consumption growth itself is one of the state variables in  $X_t$ , it follows that the consumption innovation from Macro VAR satisfies

$$c_{t+1} - E_t c_{t+1} = i'_c u_{t+1}, \quad (33)$$

**Table I**  
**Data Summary Statistics**

This table reports summary statistics for real consumption growth, real labor income growth, the real market return, the stock market price-dividend ratio, and realized variance. Realized variance is based on the sum of squared monthly industrial production growth rates over the year, rescaled to match the unconditional variance of consumption growth. The sample comprises annual observations from 1930 to 2010. Consumption growth, labor income growth, and market return statistics are in percent; realized variance is multiplied by 10,000.

	Mean	<i>Std. Dev.</i>	AR(1)
Consumption growth	1.86	2.18	0.48
Labor income growth	2.01	3.91	0.39
Market return	5.70	19.64	−0.01
Price-dividend ratio	3.38	0.45	0.88
Realized variance	4.76	11.13	0.44

where  $i_c$  is an indicator vector that extracts consumption growth out of the state vector  $X_t$ . We impose the important consistency requirement that the model-implied consumption shock in equation (32) matches the VAR consumption shock in (33), so that

$$q(b) \equiv i_c, \quad (34)$$

and solve the above equation, which is linear in  $b$ , to back out the unique expected human capital return loadings  $b$ . That is, in our approach, the specification for the expected labor return ensures that the consumption innovation implied by the model is identical to the consumption innovation in the data.

### *B. Data and Estimation*

In our empirical analysis, we use an annual sample from 1930 to 2010. Real consumption corresponds to real per capita expenditures on nondurable goods and services, and real income is real per capita disposable personal income; both series are taken from the Bureau of Economic Analysis. Market return data comprise the value-weighted stock market portfolio from CRSP. Summary statistics for these variables are presented in Table I. The average labor income and consumption growth rates are about 2%. Labor income is more volatile than consumption growth, but the two series comove quite closely in the data, with a correlation coefficient of 0.80. The average log market return is 5.7%, and its volatility is 20%. The realized variance of industrial production is quite persistent and volatile in the data, and is strongly countercyclical: in recessions, the realized variance is, on average, 40% above its unconditional mean. Further, the realized variance comoves negatively with the market price-dividend ratio: the correlation coefficient between the two series is  $-0.25$ , so that asset prices fall at times of high macroeconomic volatility, consistent with the findings in Bansal, Khatchatrian, and Yaron (2005).

**Table II**  
**Macro VAR Estimates**

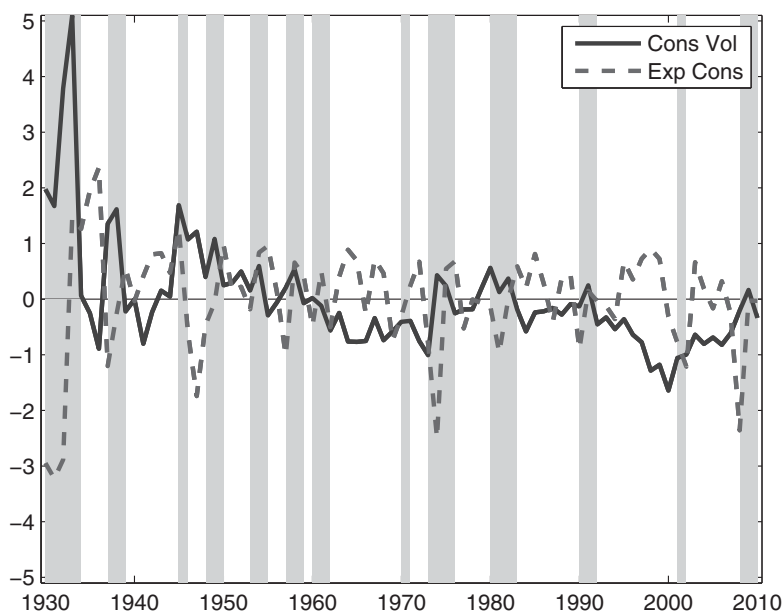
This table reports OLS estimates of the persistence matrix, and the predictive  $R^2$  for each of the variables in the Macro VAR. The economic variables in the Macro VAR include real consumption growth  $\Delta c_t$ , real labor income growth  $\Delta y_t$ , the real market return  $r_{d,t}$ , the market price-dividend ratio  $z_t$ , and realized variance  $RV_t$ . Realized variance is based on the sum of squared monthly industrial production growth rates over the year, rescaled to match the unconditional variance of consumption growth. The sample comprises annual observations from 1930 to 2010. Robust standard errors are in brackets.

	$\Delta c_t$	$\Delta y_t$	$r_{dt}$	$z_t$	$RV_t$	$R^2$
$\Delta c_{t+1}$	0.221 [0.090]	0.148 [0.036]	0.057 [0.012]	0.002 [0.003]	2.660 [1.092]	0.61
$\Delta y_{t+1}$	-0.272 [0.271]	0.506 [0.146]	0.078 [0.026]	0.005 [0.007]	3.014 [3.430]	0.37
$r_{d,t+1}$	-3.095 [0.899]	1.126 [0.377]	0.068 [0.086]	-0.076 [0.038]	-10.906 [10.557]	0.08
$z_{t+1}$	-3.588 [0.937]	0.978 [0.644]	-0.230 [0.144]	0.921 [0.042]	-9.029 [10.955]	0.80
$RV_{t+1}$	-0.007 [0.005]	-0.006 [0.004]	0.001 [0.0004]	-0.001 [0.0003]	0.310 [0.091]	0.35

We estimate the Macro VAR specification in (22) using equation-by-equation OLS. For robustness, we also consider a generalized least square (GLS) approach in which we incorporate the information in the conditional variance of the residuals; the results are very similar, and are discussed in the robustness section. The Macro VAR estimation results are reported in Table II. The magnitudes of the  $R^2$ s in the regressions vary from 10% for the market return to 80% for the price-dividend ratio. Notably, the consumption and labor income growth rates are quite predictable in this rich setting, with  $R^2$ s of 60% and nearly 40%, respectively. Because of the correlation between the variables, it is hard to interpret individual slope coefficients in the regression. Note, however, that ex-ante consumption volatility is quite persistent with an autocorrelation coefficient of 0.63 at an annual frequency, and it loads significantly negatively on the market price-dividend ratio. Notably, the ex-ante volatility process is less volatile and more persistent than realized volatility.

We plot ex-ante consumption volatility and the expected consumption growth rate in Figure 1. Our evidence underscores persistent fluctuations in ex-ante macroeconomic volatility and a gradual decline in volatility over time, which is similar to the findings in McConnell and Quiros (2000) and Stock and Watson (2002).<sup>8</sup> Notably, the volatility process is strongly countercyclical: its correlation with the NBER recession indicator is -40%, and it is -30% with the expected real consumption growth from the Macro VAR. Consistent with this evidence, the future expected consumption news implied by the Macro VAR,  $N_{\text{ECF}}$ , is sharply negative at times of high volatility. As shown in Table III,

<sup>8</sup> Time-series dynamics of the conditional volatility of consumption growth are also discussed in Duffee (2005) and Bansal, Kiku, and Yaron (2007).

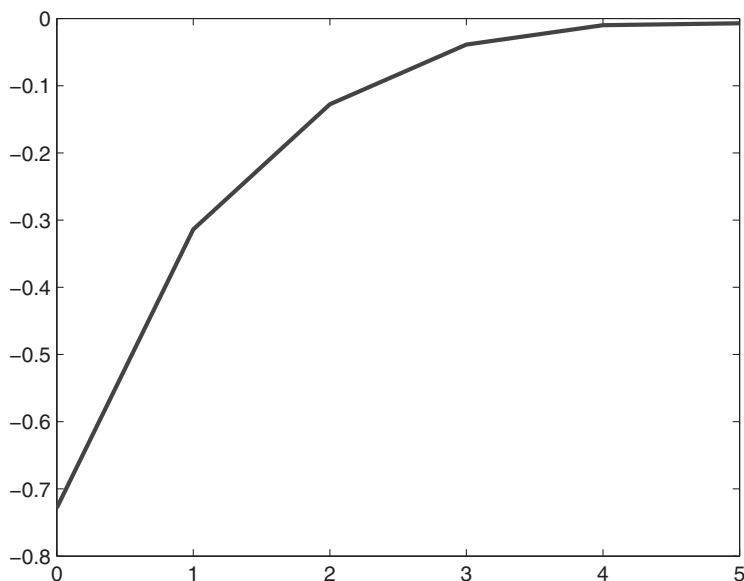


**Figure 1. Volatility and expected consumption growth from Macro VAR.** The figure plots time series of consumption volatility (solid line) and expected consumption growth (dashed line) implied by the Macro VAR estimates. The two lines are normalized to have zero mean and unit variance; shaded areas represent NBER recession dates.

**Table III**  
**Role of Volatility for Economic News**

This table reports standard deviations and correlations of economic news with volatility news, and the average magnitude of the economic news in the lowest 25% and highest 25% volatility news periods. Economic news includes future expected consumption news  $N_{ECF}$ , volatility news  $N_V$ , stochastic discount factor news  $N_M$ , and discount rate news on the market return  $N_{DR}^d$ , labor return  $N_{DR}^y$ , and wealth return  $N_{DR}$ . The news is constructed based on the Macro-DCAPM-SV model. The “With Vol Risk” columns show the economic news in the model specification with volatility risk, while the “No Vol Risk” columns document the implications when volatility risk is absent. The coefficient of risk aversion is set to  $\gamma = 5$ , and the elasticity of intertemporal substitution parameter is  $\psi = 2$ .

	$N_{ECF}$	$N_V$	$N_{DR}^d$	With Vol Risk			No Vol Risk		
				$N_{DR}^y$	$N_{DR}$	$N_M$	$N_{DR}^y$	$N_{DR}$	$N_M$
Std. Dev.	5.27	30.06	14.56	2.58	3.97	47.79	4.70	2.63	26.71
Corr. with $N_V$	-0.27	1.00	0.86	0.27	0.77	0.84	-0.85	-0.27	0.24
Volatility News Periods:									
Lowest 25%	2.16	-33.01	-12.72	-0.63	-3.05	-44.99	4.53	1.08	-7.86
Highest 25%	-1.69	37.37	16.41	0.68	3.82	50.21	-5.16	-0.85	8.16

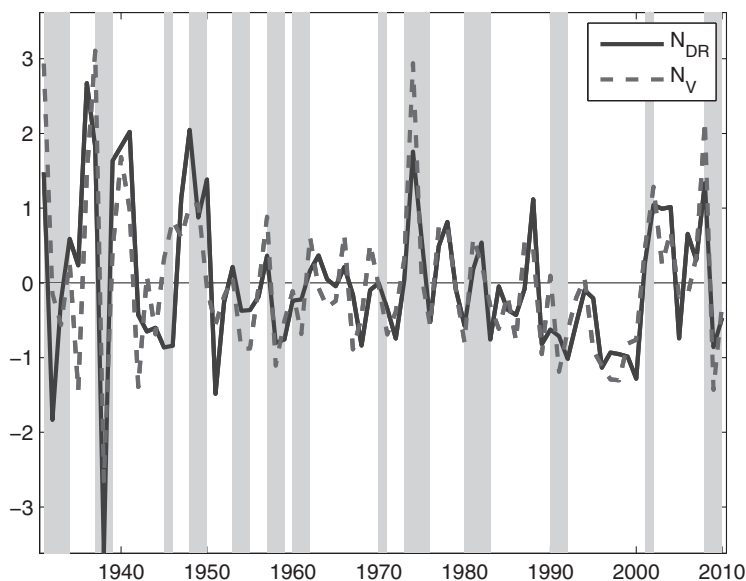


**Figure 2. Consumption response to volatility shock from Macro VAR.** The figure shows an impulse response of consumption growth to a one-standard-deviation shock to the volatility of consumption implied by the Macro VAR estimates. Consumption growth is annual, in percent.

future expected consumption news is, on average,  $-1.70\%$  in high (top 25%) versus  $2.2\%$  in low (bottom 25%) volatility times. To show the dynamic impact of volatility news on consumption, we compute the impulse response of consumption growth to a one-standard-deviation shock in ex-ante consumption volatility,  $\text{Var}_t \Delta c_{t+1}$  (see Appendix A. for details on the computations). Based on our estimation results, a one-standard-deviation volatility shock corresponds to a  $(1.95\%)^2$  increase in ex-ante consumption variance. As shown in Figure 2, consumption growth declines by almost 1% on the impact of positive volatility news and remains negative up to five years in the future. The response of labor income growth is similar: labor income growth drops by 2% on the impact of positive volatility news, and the response remains negative up to five years in the future.

Given the estimates of our Macro VAR specification, we can compute volatility news and stock market return news following the derivations in Section II.A. To derive the implications for the return to human capital and the wealth portfolio, we set the coefficient of risk aversion  $\gamma$  to five and the IES parameter  $\psi$  to two. The share of human wealth in overall wealth  $\omega$  is set to 0.8, the average value used in Lustig and Van Nieuwerburgh (2008). In the full model specification featuring volatility risk, we fix the volatility parameter  $\chi$  to the ratio of the volatilities of long run to immediate consumption news, according to the restriction in equation (18). To discuss the model implications in the absence of volatility risk, we set  $\chi$  equal to zero.





**Figure 3. Discount rate news and volatility news.** The figure plots time series of discount rate news on the wealth portfolio  $N_{DR}$  and volatility news  $N_V$  implied by the Macro-DCAPM-SV model. The two lines are normalized to have zero mean and unit variance; shaded areas represent NBER recession dates.

In the full model with volatility risk, volatility news is strongly correlated with discount rate news in the data. As documented in Table III, the correlation between volatility news and discount rate news to the market reaches nearly 90%, and the correlations of volatility news with discount rate news to the labor return and the wealth portfolio are 30% and 80%, respectively. A high correlation between volatility news and discount rate news to the wealth return is evident in Figure 3. These findings are consistent with the intuition of the economic long-run risks (LRR) model of Bansal and Yaron (2004), where a significant component of discount rate news is driven by shocks to consumption volatility. On the other hand, when volatility risk is absent, discount rate news no longer reflects fluctuations in volatility, but rather tracks revisions in future expectations of consumption. As a result, the correlation between the implied discount rate news and our earlier measure of volatility news becomes  $-0.85$  for the labor return and  $-0.30$  for the wealth portfolio. The implied discount rate news ignoring volatility risk is very different from the discount rate news when volatility risk is taken into account. For example, when volatility risk is accounted for, the measured discount rate news is 5.3% in the recession of 2008 and 1% in 2001. Without the volatility channel, however, it would appear that discount rate news is negative at those times: the measured discount rate shock is  $-2.8\%$  in 2008 and  $-0.7\%$  in 2001. Thus, ignoring the volatility channel, the discount rate on the consumption asset can be significantly misspecified due

**Table IV**  
**Labor, Market, and Aggregate Wealth Return Correlations**

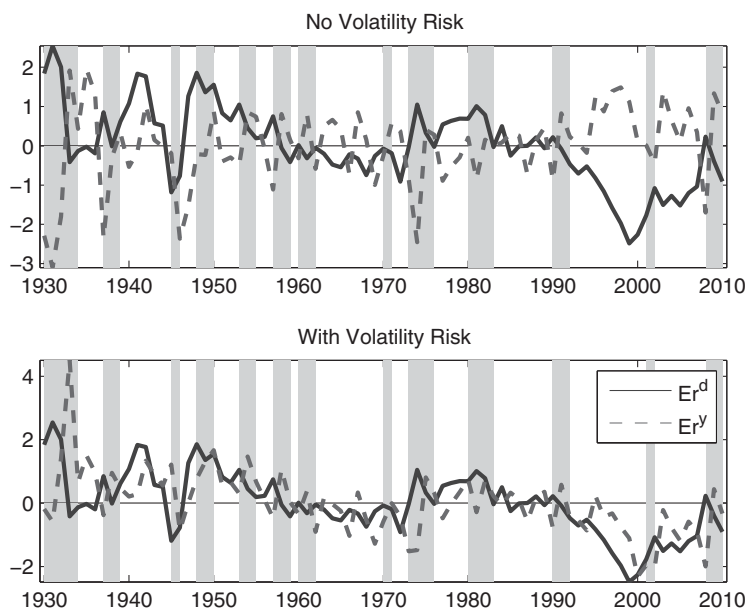
This table reports correlations between immediate return shocks, discount rate shocks, and five-year expected returns to the market, labor, and aggregate wealth.  $N_R$ ,  $N_{DR}$ , and  $E_t r_{t \rightarrow t+5}$  denote immediate return news, discount rate news, and the five-year expected return to the wealth portfolio, respectively. Subscripts “y” and “d” denote the corresponding news for the labor return and stock market return. The news is constructed based on the Macro-DCAPM-SV model. The “With Vol Risk” column shows the correlations in the model specification with volatility risk, while the “No Vol Risk” column documents the correlations when volatility risk is absent. The coefficient of risk aversion is set to  $\gamma = 5$ , and the elasticity of intertemporal substitution parameter is  $\psi = 2$ .

		No Vol Risk	With Vol Risk
<b>Market and Labor Return:</b>			
Immediate Shocks	$\text{Corr}(N_R^d, N_R^y)$	-0.61	0.19
Discount Shocks	$\text{Corr}(N_{DR}^d, N_{DR}^y)$	-0.72	0.25
Five-Year Expectations	$\text{Corr}(E_t r_{t \rightarrow t+5}^d, E_t r_{t \rightarrow t+5}^y)$	-0.50	0.39
<b>Market and Wealth Return:</b>			
Immediate Shocks	$\text{Corr}(N_R^d, N_R)$	0.46	0.80
Discount Shocks	$\text{Corr}(N_{DR}^d, N_{DR})$	0.08	0.86
Five-Year Expectations	$\text{Corr}(E_t r_{t \rightarrow t+5}^d, E_t r_{t \rightarrow t+5})$	0.27	0.79
<b>Wealth and Labor Return:</b>			
Immediate Shocks	$\text{Corr}(N_R, N_R^y)$	0.42	0.74
Discount Shocks	$\text{Corr}(N_{DR}, N_{DR}^y)$	0.64	0.70
Five-Year Expectations	$\text{Corr}(E_t r_{t \rightarrow t+5}, E_t r_{t \rightarrow t+5}^y)$	0.70	0.87

to the omission of the volatility risk component, altering the dynamics of the wealth returns as we discuss in Section II.C.

### C. Macro-DCAPM-SV: Labor and Market Returns

Table IV reports evidence on the correlations between immediate and future returns to the market, human capital, and wealth portfolio. Without the volatility risk channel, shocks to the market and human capital returns are significantly negatively correlated, consistent with the evidence in Lustig and Van Nieuwerburgh (2008). Indeed, as shown in the top panel of the table, the correlations between immediate stock market and labor income return news,  $N_{R,t+1}^d$  and  $N_{R,t+1}^y$ , discount rate news,  $N_{DR,t+1}^d$  and  $N_{DR,t+1}^y$ , and expected five-year returns,  $E_t r_{t \rightarrow t+5}^d$  and  $E_t r_{t \rightarrow t+5}^y$ , range between  $-0.50$  and  $-0.72$ . All these correlations turn positive when the volatility channel is present: the correlation for immediate return news increases to 0.19, and, for discount rates and expected five-year returns, it goes up to 0.25 and 0.39, respectively. Figure 4 plots the implied time series of long-term expected returns to the market and human capital. A negative correlation between the two series is evident in the model specification that ignores volatility risk. The evidence for the comovement of returns is similar for the wealth and human capital, and for the market and wealth, as shown in the middle and lower panels of Table IV. Because the wealth return is a weighted average of the market and human capital returns,



**Figure 4. Five-year expected market and labor returns.** The figure plots time series of five-year expected returns to the market (solid line) and human capital (dashed line) implied by the Macro-DCAPM-SV model. The “With Vol Risk” panel shows implied returns in the model specification with volatility risk, while the “No Vol Risk” panel plots the returns in the model specification when volatility risk is absent. The coefficient of risk aversion is set to  $\gamma = 5$ . The two lines are normalized to have zero mean and unit variance; shaded areas represent NBER recession dates.

these correlations are, in fact, positive without the volatility channel. These correlations increase considerably and become closer to one when volatility risk is introduced. For example, the correlation between market return news and wealth return news is 80% with the volatility risk channel, while without volatility risk this correlation is 46%. Similarly, the correlations between five-year expected returns of the market and aggregate wealth portfolios are 79% and 27% with and without volatility risk, respectively.

To understand the role of volatility risk for the properties of the wealth portfolio, consider again the consumption equation in (12), which explicitly accounts for the different discount rates for human capital and the market:

$$\begin{aligned}
 (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) &= \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \\
 &= \psi (\omega N_{DR,t+1}^y + (1 - \omega) N_{DR,t+1}^d) - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}.
 \end{aligned}
 \tag{35}$$

When volatility risk is not taken into account,  $N_V = 0$  and all the variation in future cash flows is driven by news to discount rates on the market and human capital. However, as shown in Table III, in the data consumption growth is

**Table V**  
**Labor, Market, and Aggregate Wealth Return Risk Premium**

This table reports the total risk premium on the labor, market, and wealth return, and its decomposition into cash flow, discount rate, and volatility risk premium components.  $N_M$  denotes the stochastic discount factor shock, and  $N_{CF}$ ,  $N_{DR}$ , and  $N_V$  denote cash flow news, discount rate news on the wealth portfolio, and volatility news, respectively.  $N_R$ ,  $N_R^y$  and  $N_R^d$  denote immediate return news to wealth, labor, and the stock market, respectively. The news is constructed based on the Macro-DCAPM-SV model. The “With Vol Risk” column shows the risk premia in the model specification with volatility risk, while the “No Vol Risk” column documents the risk premia when volatility risk is absent. The coefficient of risk aversion is set to  $\gamma = 5$ , and the elasticity of intertemporal substitution parameter is  $\psi = 2$ .

		No Vol Risk	With Vol Risk
<b>Market Return:</b>			
Risk Premium	$\text{Cov}(-N_M, N_R^d)$	2.40	7.40
Cash Flow Risk Premium	$\text{Cov}(\gamma N_{CF}, N_R^d)$	2.64	2.64
Vol Risk Premium	$\text{Cov}(-N_V, N_R^d)$	0	4.44
Discount Rate Risk Premium	$\text{Cov}(-N_{DR}, N_R^d)$	-0.23	0.32
<b>Wealth Return:</b>			
Risk Premium	$\text{Cov}(-N_M, N_R)$	0.88	2.62
Cash Flow Risk Premium	$\text{Cov}(\gamma N_{CF}, N_R)$	0.96	1.22
Vol Risk Premium	$\text{Cov}(-N_V, N_R)$	0	1.34
Discount Rate Risk Premium	$\text{Cov}(-N_{DR}, N_R)$	-0.08	0.06
<b>Labor Return:</b>			
Risk Premium	$\text{Cov}(-N_M, N_R^y)$	0.50	1.43
Cash Flow Risk Premium	$\text{Cov}(\gamma N_{CF}, N_R^y)$	0.54	0.87
Vol Risk Premium	$\text{Cov}(-N_V, N_R^y)$	0	0.56
Discount Rate Risk Premium	$\text{Cov}(-N_{DR}, N_R^y)$	-0.04	0.00

much smoother than asset returns: the volatility of cash flow news is about 5% relative to 15% for discount rate news on the market. Hence, to explain relatively smooth variation in cash flows in the absence of volatility news, discount rate news to human capital must offset a large portion of discount rate news to the market, which manifests in the large negative correlation between the two returns as documented in Table IV. On the other hand, when volatility news is taken into account, it removes the risk premia fluctuations from the discount rates and isolates the news in expected cash flows. Indeed, a strong positive correlation between volatility news and discount rate news in the data is evident in Table III. This allows the model to explain the link between consumption and asset markets without forcing a negative correlation between labor and market returns.

We use the extracted news components to identify the innovation to the stochastic discount factor, according to equation (15), and document the implications for the risk premia in Table V. At our calibrated preference parameters, in the model with volatility risk, the risk premium on the market is 7.4%; it is 2.6% for the wealth portfolio and 1.4% for the labor return. Interestingly, the risk premium for the wealth return is very similar to the estimates in Lustig, Nieuwerburgh, and Verdellhan (2011), who use a different empirical approach

**Table VI**  
**Role of Preferences for Aggregate Returns**

This table shows the implied correlations between labor and market returns (for immediate news  $N_R$ , discount rate news  $N_{DR}$ , and five-year expected returns  $Er$ ), and the risk premia on market, labor, and aggregate wealth returns, for various parameter values for the intertemporal elasticity of substitution  $\psi$  and the human capital share  $\omega$ . The news is constructed based on the Macro-DCAPM-SV model. The “With Vol Risk” columns show the correlations and risk premia in the model specification with volatility risk, while the “No Vol Risk” columns document the correlations and risk premia when volatility risk is absent. The coefficient of risk aversion coefficient is set to  $\gamma = 5$ .

$\psi$	With Vol Risk						No Vol Risk					
	Lbr-Mkt Corr			Risk Premia			Lbr-Mkt Corr			Risk Premia		
	$N_R$	$N_{DR}$	$Er$	Mkt	Lbr	Wlth	$N_R$	$N_{DR}$	$Er$	Mkt	Lbr	Wlth
	$\omega = 0.80$											
0.5	-0.89	-0.54	-0.34	5.02	-5.16	-3.13	-0.81	-0.19	0.07	1.68	-1.42	-0.80
1.0	-0.94	-0.43	-0.15	6.60	-1.43	0.18	-0.94	-0.43	-0.15	2.16	-0.31	0.18
1.5	-0.30	-0.17	0.14	7.13	0.40	1.75	-0.77	-0.60	-0.34	2.32	0.21	0.63
2.0	0.19	0.25	0.39	7.40	1.43	2.62	-0.61	-0.72	-0.50	2.40	0.50	0.88
2.5	0.36	0.52	0.51	7.56	2.08	3.17	-0.52	-0.79	-0.62	2.45	0.68	1.03
3.0	0.43	0.61	0.57	7.66	2.53	3.55	-0.45	-0.85	-0.71	2.48	0.81	1.14
	$\omega = 0.85$											
0.5	-0.86	-0.49	-0.33	4.55	-3.96	-2.69	-0.75	-0.13	0.07	1.50	-1.11	-0.71
1.0	-0.90	-0.33	-0.10	5.98	-0.85	0.17	-0.90	-0.33	-0.10	1.92	-0.13	0.18
1.5	-0.03	0.03	0.23	6.46	0.66	1.53	-0.64	-0.49	-0.27	2.06	0.32	0.58
2.0	0.34	0.47	0.50	6.69	1.51	2.29	-0.46	-0.62	-0.41	2.13	0.58	0.81
2.5	0.45	0.68	0.62	6.84	2.04	2.76	-0.36	-0.71	-0.53	2.17	0.74	0.95
3.0	0.51	0.74	0.66	6.93	2.41	3.09	-0.30	-0.77	-0.63	2.20	0.85	1.05

to measure the wealth-to-consumption ratio. Most of the risk premium comes from the cash flow and volatility risks. The contribution of the volatility risk ranges from about 40% of the total risk premium on the labor return to 50% for the wealth portfolio and 60% for the market. Cash flow news contributes about 35% to the total risk premium on the market, and 60% to the risk premium on the labor return. Both cash flow and volatility risks contribute about equally to the risk premium on the consumption asset. The discount rate shocks contribute virtually nothing to the risk premia across all the assets. Without the volatility channel, the risk premia are 2.4% for the market, 0.9% for the wealth return, and 0.5% for the labor return, and essentially all of the risk premium reflects compensation for the cash flow risk.

The main results in the paper obtain using the benchmark preference parameters  $\gamma = 5$  and  $\psi = 2$  and share of human capital  $\omega = 0.80$ . In Table VI, we document the model implications for IES parameters ranging from 0.5 to 3.0, and for  $\omega = 0.85$ . Without the volatility channel, the correlation between labor and market returns is negative and large at all considered values for the preference parameters, consistent with the evidence in Lustig and Van Nieuwerburgh (2008). In the model with volatility risk, the IES must be sufficiently above 1

to generate a positive link between labor and market returns—with an IES below 1, the volatility component no longer offsets risk premia variation in the construction of consumption news, which makes the labor-market return correlations even lower than in the case without volatility risk. At our benchmark value of human capital share  $\omega = 0.80$ , the IES parameter of two enables us to achieve positive correlations. For higher values of human capital share,  $\omega = 0.85$ , the correlations turn positive at lower IES. The evidence is similar for other values of the risk aversion parameter. Higher coefficients of risk aversion lead to a higher risk premium, which is why we choose moderate values of  $\gamma$  in our analysis.

#### D. Macro-DCAPM-SV: The Cross Section of Assets

In addition to the market, human capital, and wealth returns, we consider asset pricing implications for a broader cross section of assets, which includes the five size- and five book-to-market-sorted portfolios.

To evaluate the model implications, we use the risk premia restriction as in equation (20). Specifically, we use the extracted news to construct the innovation to the stochastic discount factor and price a cross section of equity returns by exploiting the Euler equation, that is,

$$\begin{aligned} E_t[R_{i,t+1} - R_{f,t}] &= -\text{Cov}_t(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1}) \\ &= \gamma \text{Cov}_t(N_{CF,t+1}, \epsilon_{i,t+1}) - \text{Cov}_t(N_{DR,t+1}, \epsilon_{i,t+1}) \\ &\quad - \text{Cov}_t(N_{V,t+1}, \epsilon_{i,t+1}), \end{aligned} \quad (36)$$

where  $E_t[R_{i,t+1} - R_{f,t}]$  is the arithmetic risk premium of asset  $i$  and  $\epsilon_{i,t+1} \equiv r_{i,t+1} - E_t r_{i,t+1}$  is the innovation to asset  $i$ 's return. It is important to emphasize that we evaluate the model implications under the estimated Macro VAR parameters and for the benchmark risk aversion and IES of five and two, respectively, and impose the model's restrictions on the market prices of discount rate and volatility risks.

From the log-linear approximation of returns,

$$r_{i,t+1} = \kappa_{i,0} + \Delta d_{i,t+1} + \kappa_{i,1} z_{i,t+1} - z_{i,t}, \quad (37)$$

it follows that the return innovation depends on the dividend innovation and that to the price-dividend ratio. To measure cash flow risk and dividend shocks, we use an econometric approach that is similar to the one in Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Dittmar, and Kiku (2009). This economically structured approach allows for sharper identification of low-frequency aggregate risks in cash flows (dividends). For each equity portfolio, we estimate its cash flow exposure ( $\phi_i$ ) by regressing the portfolio's dividend growth rate,  $\Delta d_{i,t}$ , on the two-year moving average of consumption growth,  $\overline{\Delta c}_{t-1 \rightarrow t}$ :

$$\Delta d_{i,t} = \mu_i + \phi_i \overline{\Delta c}_{t-1 \rightarrow t} + \epsilon_{i,t}^d. \quad (38)$$

**Table VII**  
**Asset Pricing Implications of the Macro-DCAPM-SV Model**

This table shows risk premia implied by the Macro-DCAPM-SV model and risk exposures (betas) of the aggregate market and a cross section of five book-to-market- and five size-sorted portfolios. The bottom panel presents the market prices of the risks. According to the model, the market price of cash flow risk is equal to the coefficient of risk aversion  $\gamma = 5$ , and market prices of discount rate and volatility risks are fixed at  $-1$ . The “Data” column reports average returns in excess of the three-month Treasury bill rate in the 1930 to 2010 sample.

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.4	0.53	-0.32	-4.45
BM1	7.2	7.7	0.39	-0.49	-5.22
BM2	7.6	8.5	0.55	-0.42	-5.32
BM3	9.4	9.1	0.82	-0.23	-4.71
BM4	10.8	10.2	1.22	0.02	-4.11
BM5	13.1	13.2	2.31	0.70	-2.38
Size1	14.8	14.1	1.89	0.21	-4.86
Size2	13.0	11.7	1.14	-0.22	-5.75
Size3	11.6	10.6	0.92	-0.31	-5.72
Size4	10.4	8.8	0.67	-0.33	-5.06
Size5	7.4	7.0	0.51	-0.29	-4.14
			Price(CF)	Price(DR)	Price(Vol)
			5	-1	-1

The innovation to the price-dividend ratio,  $\epsilon_{i,t+1}^z$ , is obtained by regressing  $z_{i,t}$  on the VAR predictive variables. It then follows that the innovation to the return of portfolio  $i$  is given by

$$r_{i,t+1} - E_t r_{i,t+1} = \phi_i(\Delta c_{t+1} - E_t \Delta c_{t+1}) + \epsilon_{i,t+1}^d + \kappa_{i,1} \epsilon_{i,t+1}^z, \quad (39)$$

where  $\Delta c_{t+1} - E_t \Delta c_{t+1}$  is the VAR-based innovation to consumption growth. We find that, as should be the case, this return innovation is not predicted by any of the VAR predictive variables we use.<sup>9</sup>

According to equation (20), the model risk premium on any portfolio can be decomposed into the contribution of cash flow, discount rate, and volatility news. The risk compensation for each of the three risks is given by the product of the covariance between the return innovation and the corresponding news (return beta) and market price of risk; in our Macro-DCAPM-SV specification, the market price of cash flow risk is equal to the coefficient of risk aversion  $\gamma = 5$ , and it is  $-1$  for both the discount and the volatility risks. Table VII documents the risk premia in the data and the model, as well as the beta of each portfolio to cash flow, discount rate, and volatility risks. As shown in the table, our Macro-DCAPM-SV model accounts very well for the cross-sectional

<sup>9</sup> The cross-sectional evidence that we present and discuss below, including evidence on equity exposure to cash flow, discount rate, and volatility risks, is robust if instead we directly use returns to measure return innovations.

spread in the equity premium across book-to-market and size portfolios: the value spread is 5.5% in the model relative to 5.9% in the data, and the size spread is 7.1% and 7.4% in the model and the data, respectively. Most of the risk premium comes from cash flow and volatility news, while discount rate shocks contribute quite little to the overall risk premium. Specifically, the cash flow betas increase monotonically from low to high book-to-market portfolios, and from large- to small-size portfolios, so both the level and the relative contribution of cash flow risk increase with the overall risk premium in the cross section. Indeed, for the book-to-market portfolios, the cash flow risk premium increases monotonically from 2% for growth firms to 11.5% for value firms, and, in relative terms, the contribution of cash flow risk increases from 35% to almost 90% of the total premium, respectively. Similarly, for the size portfolios, the relative contribution of cash flow risk decreases from 67% of the total premium for small firms to 36% for large firms. Volatility risk accounts for the remaining part of the risk premium. The contribution of volatility risk is highest for the growth portfolio and the portfolio of large firms, where it accounts for 60% and 70% of the total premium, respectively, and it decreases to about 30% for value firms and small firms. On average, both cash flow and volatility risks account for about 50% of the total risk premium each, while discount rates contribute virtually nothing to the risk premium. The magnitude of the contributions of each source of risk in the cross section of book-to-market and size portfolios is consistent with our findings for labor, market, and consumption returns.

Without volatility risk, the magnitudes of the risk premia are much smaller: the coefficient of risk aversion has to be increased to about 15 to match the risk premia levels in the data. Without volatility risk, nearly all the risk premium is attributed to cash flow risk.

### *E. Robustness*

We conduct a number of robustness checks to ensure that our main results are not sensitive to volatility measures, choice of predictive variables, or sample period.

In our benchmark Macro-DCAPM-SV, we measure realized variance using squared monthly industrial production growth rates, scaled to match the overall consumption volatility. First, we check that our results remain broadly similar if we instead compute realized variance based on the square of annual consumption growth. Adjusting risk aversion to target the market risk premium, the implied correlation between immediate news to the stock market and the labor return is 0.20; it is 0.01 for discount rate news and 0.26 for five-year expected returns. The model fit for the cross section of returns is materially unchanged.

To evaluate the robustness to alternative VAR predictive variables, we augment the VAR to include (i) the risk-free rate, (ii) the risk-free rate and the term spread, and (iii) the risk-free rate, the term spread, and the default spread. In all cases, our result do not materially change. The implications for the



correlation between the labor return and the market return as well as for the cross section of returns are similar to the benchmark results. For example, if we include data on the interest rate, the slope of the term structure, and the default spread in our benchmark Macro VAR specification, the implied correlation between the immediate returns to human capital and the stock market is 0.13, and it is 0.07 for discount rates and 0.33 for five-year expected returns.

Our empirical evidence also remains quantitatively similar if we relax the assumption that the volatility of volatility shocks is constant and, in estimation, explicitly account for variation in conditional second moments. In a more general setup that we consider, we allow for time variation in the variance of volatility shocks and estimate the model using GLS. To keep estimation tractable, here, we assume that the dynamics of the variance-covariance matrix of the VAR innovations in equation (22) are governed by the conditional variance of the consumption innovation, that is,  $\Omega_t = \sigma_t^2 \Omega$ , where  $\sigma_t^2 = \text{Var}_t(\Delta c_{t+1})$ . Similar to our benchmark specification, in this case, variation in  $V_t$  is also proportional to  $\sigma_t^2$ ,

$$V_t = \xi \sigma_t^2, \quad (40)$$

where  $\xi$  is a nonlinear function of the underlying preference and time-series parameters and is provided in Appendix B. The estimation is carried out by imposing restrictions that guarantee positivity of the estimate of the conditional variance, and under constraints that limit variation in GLS weights to ensure sensible time-series estimates. Consistent with the evidence presented and discussed above, we find that volatility and discount rate news in this generalized specification are strongly positively correlated. Likewise, the correlation between market and human capital returns is positive and equal to 0.31 and 0.45 for realized and discount rate news, respectively. Allowing for time-varying volatility of volatility shocks increases the relative contribution of volatility risk to risk premia but the increase is fairly marginal. On average across portfolios, volatility risk accounts for about 53% of the overall risk premia compared with 50% in the benchmark case.

Finally, we check that our results are robust across subperiods. Using the postwar subsample based on our benchmark VAR gives a correlation between the immediate market return and human capital return of 0.27; the correlation is 0.26 for discount rate news and 0.52 for five-year expected returns. The cross-sectional evidence, again, remains similar.

#### *F. The Importance of Volatility and Model Misspecification*

To gain further understanding of how volatility affects inferences about consumption and the stochastic discount factor, we use our VAR estimates to compare implied consumption, discount rates, and the stochastic discount factor when one ignores the volatility term. To that end, we reconstruct the consumption and stochastic discount factor innovation series based on the

**Table VIII**  
**Misspecification of Consumption and Stochastic Discount Factor**

This table compares implications for the consumption shock, the stochastic discount factor shock, and the market risk premium under the benchmark Macro-DCAPM-SV model (columns “Vol”) and under the specification that ignores the volatility news component in the construction of these shocks (columns “Ignore Vol”). The table reports the standard deviations of the consumption shock and stochastic discount factor (SDF) shock, correlations between consumption shocks under these two specifications, correlations between stochastic discount factor shocks under these two specifications, and the levels of the market risk premium under intertemporal elasticity of substitution (IES) parameters of 2, 1, and 0.75. The coefficient of risk aversion is set to  $\gamma = 5$ . Volatility is expressed in percentage terms.

	IES = 2		IES = 1		IES = 0.75	
	Ignore Vol	Vol	Ignore Vol	Vol	Ignore Vol	Vol
<b>Implied Consumption Shock</b>						
Vol of cons. shock	7.62	1.29	1.29	1.29	2.29	1.29
Corr. with cons. shock	0.16	1.00	1.00	1.00	0.57	1.00
<b>Implied SDF Shock</b>						
Vol of SDF shock	27.84	47.79	24.15	42.71	22.02	39.33
Corr. with SDF shock	0.81	1.00	0.73	1.00	0.65	1.00
Market Risk Premium	2.96	7.40	2.16	6.60	1.63	6.07

estimated components of  $N_R$ ,  $N_{DR}$ , and  $N_V$  via the right-hand side of equations (9) and (15), respectively. In Table VIII, we show what the consumption and stochastic discount factor dynamics would be if  $N_V$  is not accounted for—that is, the implications of ignoring the volatility component. As the table shows, relative to the true dynamics of consumption, the implied consumption innovations are larger and have a correlation of about 0.16 with the true consumption dynamics. It can be shown that this larger volatility and low correlation is true whenever IES is different from one. Table VIII also shows the stochastic discount factor that would arise based on these misspecified consumption dynamics that ignore the volatility component. For all IES values, the implied stochastic discount factor is misspecified and, in general, is not as volatile as the true one and the risk premia are substantially lower. Moreover, the implied stochastic discount factor has a low correlation with the true stochastic discount factor. These findings have important implications for researchers that use financial data to extract cash flow and discount rate news to account for the market movements and the cross section of equity returns.

In Bansal et al. (2012), we show that the misspecification bias for the consumption dynamics and stochastic discount factor analyzed within our estimated VAR also arise within a plausibly calibrated economy. The calibrated economy highlights the theory outlined in Section I and allows us to analytically account for the “true” counterparts for each component in equation (9). For brevity, we defer details regarding the calibration to the Internet Appendix, but, here, we point out that we calibrate an LRR model similar to Bansal, Kiku,

and Yaron (2011) that includes time variation in uncertainty.<sup>10</sup> This model captures many salient features of macroeconomic and asset market data and, importantly, ascribes a prominent role for volatility risk.<sup>11</sup> We document that the model matches key moments of consumption and asset market data, and thus provides a realistic laboratory for our analysis. Notably, the model produces a significant positive correlation between discount rate news and volatility news: it is 83% for the consumption asset and similar for the market. Further, for both consumption and market returns, most of the risk compensation comes from cash flow and volatility news, while the contribution of discount rate news is quite small.

Using the calibrated model, we evaluate the consumption innovations implied by asset market data via the right-hand side of equation (9) when the volatility term is and is not taken into account. We show that, when the volatility component is ignored, for all values of IES different from one, true and implied consumption growth have a correlation of about 50% and the implied volatility of consumption is much higher than the true value. Further, we show that the  $N_V$  and  $N_{DR}$  news are negatively correlated with the innovation to consumption, while the analytical “true” correlations are zero. Finally, the stochastic discount factor is misspecified relative to the true one for *all* values of the IES. Specifically, the market risk premium is about 60% of the true one, and the correlations of the stochastic discount factor with return, discount rate, and cash flow news are distorted. Taken together, consistent with the VAR findings, the calibration evidence clearly demonstrates the potential pitfalls that might arise in interpreting asset pricing models and the risk sources driving asset markets if the volatility channel is ignored.

The analysis in Table VIII assumes that the researcher has access to the return to wealth,  $r_{c,t+1}$ . In many instances, however, that is not the case (e.g., Campbell and Vuolteenaho (2004), Campbell (1996)) and the return to the market  $r_{d,t+1}$  is used instead. The fact that the market return is a levered asset relative to the consumption return exacerbates the inference problems discussed earlier. This is indeed the case—in Bansal et al. (2012) we show that, when the IES is equal to 2, the volatility of the implied consumption shock is about 15% relative to the true volatility of only 2.5%.<sup>12</sup>

<sup>10</sup> The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

<sup>11</sup> See Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) for a discussion of the LRR channels for the asset markets and specifically the role of volatility risk, Bansal, Khatchatrian, and Yaron (2005) for early extensive empirical evidence on the role of volatility risk, Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010), and Drechsler and Yaron (2011) for the importance of volatility risk for derivative markets, and Bansal and Shaliastovich (2013) for the importance of volatility risk for the bond and currency markets.

<sup>12</sup> Campbell (1996, Table IX) reports the implied consumption innovations based on equation (9) when volatility is ignored and the return and discount rate shocks are read off a VAR using observed financial data. The volatility of the consumption innovations when the IES is assumed to be two is about 22%, not far from the quantity displayed in our simulated model. As in our case, lower IES values lead to somewhat smoother implied consumption innovations. While Campbell

Finally, we show that one has to be cautious in empirically assuming that volatility is constant while appealing to discount rate variation. To illustrate, consider the case in which the volatility is constant and all the economic shocks are homoskedastic. First, it immediately follows that the revision in expected future volatility news is zero,  $N_{V,t+1} = 0$ . Further, when all the economic shocks are homoskedastic, all the variances and covariances are constant, which implies that the risk premium on the consumption asset is constant as well. Thus, the discount rate shocks just capture the innovations to the future expected risk-free rates. Hence, under homoskedasticity, the economic sources of risk include the revisions in future expected cash flow and the revisions in future expected risk-free rates,

$$m_{t+1}^{\text{No Vol}} - E_t m_{t+1}^{\text{No Vol}} = -\gamma N_{\text{CF},t+1} + N_{\text{RF},t+1} \quad (41)$$

for  $N_{\text{RF},t+1} = (E_{t+1} - E_t) (\sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j})$ . When volatility is constant, the risk premia are constant and determined by the unconditional covariances of asset returns with future risk-free rate news and future cash flow news. Further, the beta of returns with respect to discount rate shocks,  $N_{\text{DR},t+1}$ , should be just equal to the return beta with respect to the future risk-free rate shocks,  $N_{\text{RF},t+1}$ . In several empirical studies in the literature (see, for example, Campbell and Vuolteenaho (2004)), the risk-free rate is assumed to be constant. It follows that the news about future discount rates is exactly zero, as is the discount rate beta, and all the risk premium in the economy is captured just by risk in future cash flows. Thus, ignoring volatility risk can significantly alter the interpretation of risk and return in financial markets.

### III. Market-DCAPM-SV Model

To further highlight the importance of volatility risk for understanding the dynamics of asset prices, we use a market-based VAR approach to news decomposition. As is frequently done in the literature, here, we assume that the wealth portfolio corresponds to the aggregate stock market and therefore is observable. This assumption allows us to measure cash flow, discount rate, and volatility news directly from available stock market data. As shown in Section II, in a more general setting that explicitly makes a distinction between aggregate and financial wealth and accounts for time variation in volatility, realized and expected returns to wealth and the stock market are highly correlated. This evidence suggests that we should be able to learn about time-series dynamics of fundamental risks and their prices from observed equity data. Furthermore, to sharpen identification of underlying risks, we extract them by exploiting both time-series and cross-sectional moment restrictions.

(1996) concludes that this evidence is more consistent with a low IES, the analysis here suggests that, in fact, this evidence is consistent with an environment in which the IES is greater than 1 and the innovation structure contains a volatility component.

The theoretical framework here is same as that in Section I with the return to the consumption asset equal to the return to the market portfolio. Hence, the equilibrium risk premium on any asset is determined by its exposure to the innovation to the market return and news about future discount rates and future volatility. The multibeta implication of our model is similar to the multibeta pricing of the intertemporal CAPM of Merton (1973), where the risk premium depends on the market beta and asset exposure to state variables that capture changes in future investment opportunities. What distinguishes our market volatility-based dynamic capital asset pricing model (Market-DCAPM-SV) from Merton's framework is that, in our model, both the relevant risk factors and their prices are identified and pinned down by the underlying model primitives and preferences. This is important from an empirical perspective as it provides us with testable implications that can be taken to the data. Also, note that, in our Market-DCAPM-SV model, which is derived from recursive preferences, the relevant economic risks comprise not only short-run fluctuations (as in the equilibrium C-CAPM of Breeden (1979)) but also risks that matter in the long run.

#### A. Market-Based Setup

As derived above, the stochastic discount rate of the economy is given by

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}. \quad (42)$$

To measure news components from equity data, we assume that the state of the economy is described by the vector

$$X_t \equiv (RV_{r,t}, z_t, \Delta d_t, ts_t, ds_t, i_t)', \quad (43)$$

where  $RV_{r,t}$  is the realized variance of the aggregate market portfolio,  $z_t$  is the log of the market price-dividend ratio,  $\Delta d_t$  is the continuously compounded dividend growth of the aggregate market,  $ts_t$  is the term spread defined as a difference in yields on the 10-year Treasury bond and three-month T-bill,  $ds_t$  is the yield differential between Moody's BAA- and AAA-rated corporate bonds, and  $i_t$  is the log of the real long-term interest rate. The data are real, sampled at an annual frequency, and span the 1930 to 2010 period. Realized variance is constructed by summing the squared monthly rates of the market return within a year. The real long-term rate is measured as the yield on the 10-year Treasury bond adjusted by inflation expectations. The latter are estimated using a first-order autoregression specification for inflation. Excess returns to the market and a cross section are constructed by subtracting the annualized rate on the three-month Treasury bill from annual nominal equity returns. Our state vector comprises variables that are often used in the return and volatility forecasting literature (see Fama and French (1988, 1989), Kandel and Stambaugh (1990), and Hodrick (1992)). We discuss the robustness of our evidence to the state specification below.

We model the dynamics of  $X_t$  via a first-order vector-autoregression,

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (44)$$

where  $\Phi$  is a  $6 \times 6$  matrix of VAR coefficients,  $\mu_X$  is a  $6 \times 1$  vector of intercepts, and  $u_{t+1}$  is a  $6 \times 1$  vector of zero-mean conditionally normal VAR innovations. Note that the dynamics of the log return to the aggregate market portfolio ( $r$ ) are implied by the dynamics of its price-dividend ratio and dividend growth:

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \kappa_1 z_{t+1} - z_t, \quad (45)$$

where  $\kappa_0$  and  $\kappa_1$  are constants of log-linearization. To construct cash flow, discount rate, and volatility news, we iterate on the VAR, using the same algebra as in Section II.A with the simplification that all news components are now directly read from the VAR since the return to the market is assumed to represent the return to the overall wealth. For example, cash flow news and discount rate news are computed as

$$N_{CF,t+1} = i'_{\Delta d}(I + Q)u_{t+1}, \quad (46)$$

$$N_{DR,t+1} = i'_{\Delta d}(I + Q)u_{t+1} - (i_{\Delta d} + \kappa_1 i_z)'u_{t+1}, \quad (47)$$

where  $i_z$  and  $i_{\Delta d}$  are the indicator vectors with an entry of one in the second and third positions, respectively, and  $Q = \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1}$ . Volatility news is computed in a similar way.

We use the extracted news to construct the innovation in the stochastic discount factor and price a cross section of equity returns by exploiting the Euler equation in equation (36). The procedure for extracting the return news follows the discussion in Section II.D using aggregate dividend instead of consumption. It is important to emphasize that we carry out estimation under the null of the model. In particular, we restrict the premium of a zero-beta asset and impose the model's restrictions on the market prices of discount rate and volatility risks (both are equal to  $-1$ ). The price of cash flow risk is estimated along with time-series parameters of the model.

To extract news and construct the innovation in the stochastic discount factor, we estimate time-series parameters and the market price of cash flow risk using GMM by exploiting two sets of moment restrictions. The first set of moments comprises the VAR orthogonality moments; the second set contains the Euler equation restrictions for the market portfolio and a cross section of five book-to-market- and five size-sorted portfolios. To ensure that the moment conditions are scaled appropriately, we weight each moment by the inverse of its variance and allow the weights to be continuously updated throughout estimation. Further details on the GMM estimation are provided in Appendix C.

**Table IX**  
**Market-Based VAR Estimates**

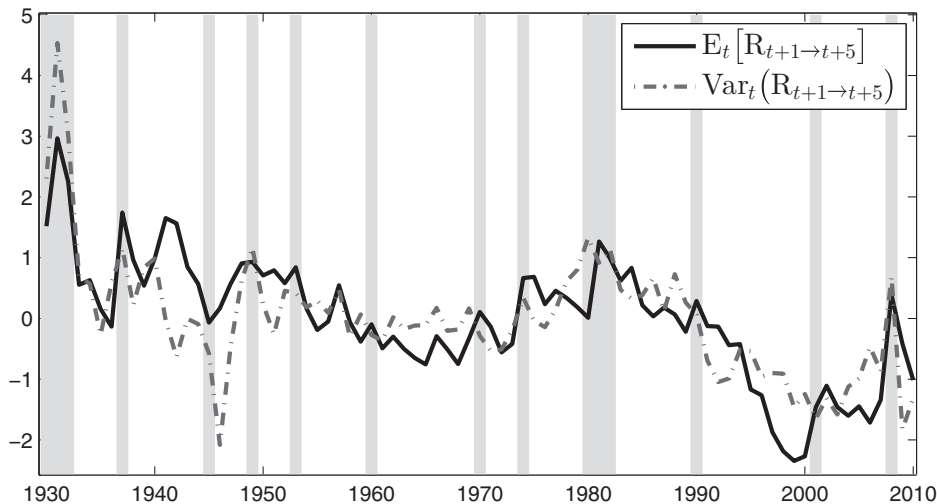
This table presents GMM estimates of the persistence matrix and the predictive  $R^2$  for each variable in the market-based VAR.  $RV_{r,t}$  denotes the realized variance of the aggregate market portfolio,  $z_t$  the log of the price-dividend ratio,  $\Delta d_t$  log dividend growth,  $ts_t$  and  $ds_t$  term and default spreads, respectively, and  $i_t$  the log of the interest rate. Robust standard errors are presented in brackets. The data used in estimation are real and cover the 1930 to 2010 period.

	$RV_{r,t}$	$z_t$	$\Delta d_t$	$ts_t$	$ds_t$	$i_t$	$R^2$
$RV_{r,t+1}$	0.274 [0.167]	-0.017 [0.012]	0.023 [0.035]	-0.696 [0.336]	2.579 [1.031]	0.353 [0.202]	0.63
$z_{t+1}$	-0.961 [0.863]	0.904 [0.074]	-0.584 [0.261]	0.856 [1.276]	4.981 [5.213]	0.593 [0.556]	0.80
$\Delta d_{t+1}$	0.715 [0.408]	0.050 [0.031]	0.161 [0.131]	2.347 [0.952]	-7.887 [3.121]	-0.407 [0.370]	0.27
$ts_{t+1}$	-0.071 [0.023]	0.004 [0.003]	-0.016 [0.011]	0.410 [0.103]	1.036 [0.147]	-0.029 [0.019]	0.50
$ds_{t+1}$	0.020 [0.017]	-0.001 [0.002]	0.009 [0.006]	-0.127 [0.034]	0.682 [0.146]	0.021 [0.024]	0.56
$i_{t+1}$	-0.352 [0.105]	0.001 [0.007]	-0.050 [0.026]	-0.197 [0.182]	2.075 [0.792]	0.613 [0.109]	0.48

### B. Ex-Ante Volatility and Discount Rate Dynamics

The GMM estimates of the market-based VAR dynamics are presented in Table IX. As shown in the first row of the table, the realized variance of the market return is highly predictable with an  $R^2$  of more than 60%. Time variation in the one-year-ahead expected variance comes mostly from variation in realized variance, term and default spreads, and the risk-free rate, all of which are quite persistent in the data. The conditional variance therefore features persistent time-series dynamics with a first-order autocorrelation coefficient of about 0.69. These persistent dynamics are consistent with empirical evidence of low-frequency fluctuations in market volatility documented in the literature (see, for example, Bollerslev and Mikkelsen (1996), among others).

We find that the extracted discount rate and volatility risks are strongly countercyclical and positively correlated. Both news tend to increase during recessions and decline during economic expansions. At the one-year horizon, the correlation between discount rate and volatility news is 0.47. This evidence aligns well with economic intuition. As the contribution of risk-free rate news is generally small, discount rate risk is mostly driven by news about future risk premia, and the latter is tied to expectations about future economic uncertainty. Consequently, discount rates and conditional volatility of the market portfolio share common time-series dynamics, especially at low frequencies. We illustrate their comovement in Figure 5 by plotting the five-year expected market return and the five-year conditional variance implied by the estimated VAR. As the figure shows, both discount rates and the conditional variance feature countercyclical dynamics and closely follow one another. The correlation between the two time series is 0.75.



**Figure 5. Discount rate and conditional variance of the market portfolio.** The figure plots time series of the five-year expected market return and five-year conditional variance of the market portfolio implied by the market-based VAR estimates. The two lines are normalized to have zero mean and unit variance; shaded areas represent NBER recession dates.

### C. Pricing Implications of the Market-DCAPM-SV Model

The cross-sectional implications of the Market-DCAPM-SV model are given in Table X. The table presents sample average excess returns of the market portfolio and the cross section, risk premia implied by the market-based model, and asset exposure to cash flow, discount rate, and volatility risks. The bottom panel of the table shows the estimate of the market price of cash flow risk. The evidence reported in the table yields several important insights. First, we find that cash flow risk plays a dominant role in explaining both the level and the cross-sectional variation in risk premia. At the aggregate market level, cash flow risk accounts for 4.8%, or in relative terms about 60%, of the total risk premium. Cash flow betas are monotonically increasing in book-to-market characteristics and monotonically declining with size. Value and small stocks in the data are more sensitive to persistent cash flow risk than are growth and large firms, which is consistent with the evidence in Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Dittmar, and Kiku (2009).

Second, we find that all assets have negative exposure to discount rate and volatility risks. That is, in the data, prices of all equity portfolios tend to fall when discount rates or volatility are expected to be high. Because the prices of discount rate and volatility risks, according to the model, are equal to  $-1$ , both risks carry positive premia. The negative market price of volatility risk is consistent with evidence reported in Drechsler and Yaron (2011) and Bollerslev, Tauchen, and Zhou (2009). These papers show that the estimate of the variance



**Table X**  
**Asset Pricing Implications of the Market-DCAPM-SV Model**

This table shows risk premia implied by the Market-DCAPM-SV model, risk exposures (betas) of the aggregate market, and a cross section of five book-to-market- and five size-sorted portfolios. The bottom panel presents the estimate of the market price of cash flow risk and the robust standard errors (in brackets). According to the model, prices of discount rate and volatility risks are fixed at  $-1$ . The “Data” column reports average returns in excess of the three-month Treasury bill rate from 1930 to 2010.

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.7	1.80	-1.67	-1.23
BM1	7.2	7.2	1.72	-1.53	-1.09
BM2	7.6	8.3	2.11	-1.59	-1.13
BM3	9.4	10.0	2.66	-1.58	-1.37
BM4	10.8	11.3	3.29	-1.58	-1.04
BM5	13.1	13.2	4.64	-0.91	-0.09
Size1	14.8	14.2	4.57	-1.55	-0.62
Size2	13.0	12.4	3.47	-1.98	-1.29
Size3	11.6	11.2	2.97	-1.94	-1.38
Size4	10.4	9.7	2.43	-1.88	-1.44
Size5	7.4	7.4	1.91	-1.44	-0.98
			Price(CF)	Price(DR)	Price(Vol)
			2.64	-1	-1
			[0.38]	[na]	[na]

risk premium defined as the difference between expected variances under the risk-neutral and physical measure is positive not only on average, but almost always in the time series.<sup>13</sup> Our findings are also consistent with the option-based evidence in Coval and Shumway (2001), who show that, in the data, average returns to zero-market-beta straddles are significantly negative.

In a recent paper, Campbell et al. (2013; CGPT, henceforth) also consider the contribution of volatility risk to the cross section of expected returns. Both papers agree that the market price of volatility risk is negative. Our paper shows that equity exposure to volatility risk is also negative (i.e., equities have negative volatility betas), and hence equities carry positive volatility risk premia. In contrast, for the post-1963 sample, CGPT argue that equities have positive volatility betas and therefore volatility risk premia in all equities are negative. Their evidence implies that agents may hold equities because doing so provides insurance against high-volatility states, that is, equities are a volatility hedge.

<sup>13</sup>Theoretically, under the models highlighted here, a negative market price of volatility risk implies a negative volatility beta of the aggregate market portfolio and hence a positive volatility premium of market equity. Boguth and Kuehn (2013) and Tedongap (2010) show the evidence of a negative market price of risk and negative equity exposure to consumption volatility risk in the cross section of equity returns. Bali and Zhou (2012), Han and Zhou (2012), and Todorov and Tauchen (2010) provide additional evidence of negative exposure of equity returns to alternative measures of volatility risks and Sohn (2009) shows empirically that the market price of volatility risk is negative.

This implication is quite hard to justify from the perspective of economic models and from the evidence of high market volatility and concurrent equity price declines (e.g., during the recession of 2008). Indeed, it is hard to understand why high volatility increases aggregate wealth (i.e., volatility betas are positive) and at the same time the return to aggregate wealth carries a negative risk premium in equilibrium. To provide an independent confirmation of our evidence regarding volatility betas, we compute equity exposure to the return to the zero-market-beta S&P100 straddle considered in Coval and Shumway (2001).<sup>14</sup> We find that straddle betas of the aggregate market portfolio and the cross section of size and book-to-market portfolios (either single or double sorted) are all significantly negative.<sup>15</sup> The evidence of negative exposure holds even when we consider shorter subsamples; for example, if we restrict the sample to the 1990 to 2000 period, straddle betas of all equity portfolios including the aggregate market remain significantly negative. This evidence suggests that equity exposure to volatility risk is reliably negative.

The evidence in Table X also shows that discount rate and volatility risks each account for about 20% of the overall market risk premium, and seem to affect the cross section of book-to-market-sorted portfolios in a similar way. Both discount rate and volatility risks matter more for the valuation of growth firms than that of value firms. Overall, our Market-DCAPM-SV model accounts for about 96% of the cross-sectional variation in risk premia, and implies a value premium of 6.1% and a size premium of about 6.8%. The cross-sectional fit of the model is illustrated in Figure 6. The estimate of the market price of cash flow risks is statistically significant,  $\hat{\gamma} = 2.64$  (SE = 0.38), and the model is not rejected by the overidentifying restrictions; the  $\chi^2$  test statistic is equal to 7.74 with a  $p$ -value of 0.65.<sup>16</sup>

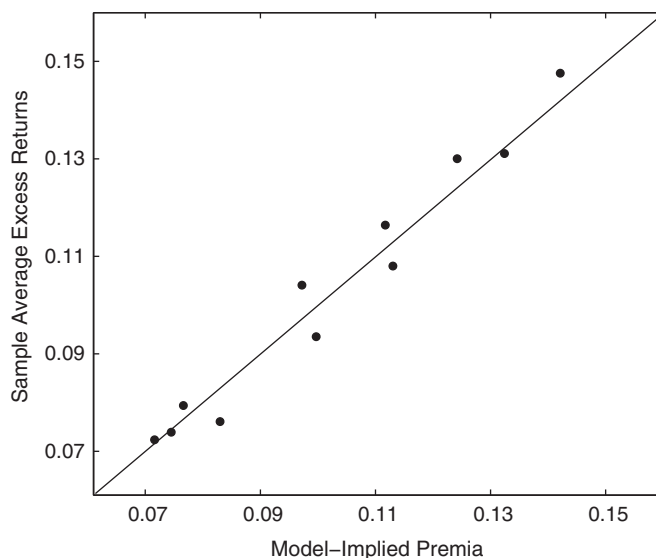
#### *D. Robustness of the Market-Based Evidence*

Our empirical evidence is fairly robust to economically reasonable changes in the VAR specification, sample period, or frequency of the data. For example, omitting the long-term bond and term spread from the VAR yields a  $p$ -value of 0.19. The estimation of the model using post-1963 quarterly sampled data results in a cross-sectional  $R^2$  of 89%, and  $\chi^2$  statistic of 15.6 with a corresponding  $p$ -value of 0.11. Across these alternative specifications, the estimates of the market price of cash flow risk continue to be significant, and the extracted discount rate and volatility risks remain strongly positively correlated. Equity

<sup>14</sup> We thank Joshua Coval and Tyler Shumway for sharing the up-to-date straddle return series with us.

<sup>15</sup> For example, the straddle beta of the market portfolio is  $-0.067$  ( $t$ -stat =  $-7.5$ ); in the cross section of size/book-to-market portfolios, straddle betas vary between  $-0.05$  and  $-0.11$  and are all significantly negative ( $t$ -statistics are all below  $-6$ ).

<sup>16</sup> It is worth noting that interpreting  $\gamma$  as risk aversion is valid only if the market return is equal to the return to the wealth portfolio. In a general environment in which financial markets are a levered claim on the aggregate economy as described in Section I and the LRR literature,  $\gamma$  will be a mixture of leverage and true risk aversion.



**Figure 6. Data and model-implied risk premia.** The figure presents a scatterplot of sample average excess returns versus model-implied risk premia of the aggregate market and a cross section of five book-to-market- and five size-sorted portfolios in the Market-DCAPM-SV model. Sample averages correspond to average returns in excess of the three-month Treasury bill rate from 1930 to 2010.

exposure to cash flow risk remains positive, and discount rate and volatility betas are consistently negative.

In Sections III.A to III.C, we assume that volatility of volatility shocks is constant. To confirm robustness of our Market-DCAPM-SV evidence, we estimate a more general setup that allows for time variation in the variance of volatility shocks.<sup>17</sup> The dynamics of the aggregate volatility component  $V_t$  under this specification are provided in Appendix B. Table XI presents the asset pricing implications of this generalized Market-DCAPM-SV setup. Consistent with the evidence from our benchmark specification, cash flow risk remains the key determinant of the level of the risk premium and its dispersion in the cross section. The contribution of volatility risk remains significant and, in fact, is slightly larger relative to the case in which volatility shocks are homoskedastic. On average, volatility and discount rate risks account for about 35% of the overall risk premia in the cross section, and for almost 50% of the total premium of the market portfolio. Similar to the implications of our benchmark model, growth stocks seem to be more sensitive to volatility (and discount rate) variation compared with value stocks.

<sup>17</sup> A similar specification with heteroskedastic market-volatility shocks is also entertained in Campbell et al. (2013).

**Table XI**  
**Asset Pricing Implications of the Generalized Market-DCAPM-SV Model**

This table shows risk premia implied by the generalized market-DCAPM-SV model, risk exposures (betas) of the aggregate market, and a cross section of five book-to-market- and five size-sorted portfolios. The bottom panel presents the estimate of the market price of cash flow risk and the robust standard errors (in brackets). According to the model, prices of discount rate and volatility risks are fixed at  $-1$ . The “Data” column reports average returns in excess of the three-month Treasury bill rate from 1930 to 2010.

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.5	1.61	-1.83	-1.80
BM1	7.2	7.2	1.61	-1.68	-1.68
BM2	7.6	8.3	2.02	-1.74	-1.75
BM3	9.4	10.1	2.64	-1.70	-2.18
BM4	10.8	11.3	3.37	-1.62	-1.73
BM5	13.1	13.1	5.07	-0.54	-0.56
Size1	14.8	14.1	4.82	-1.37	-1.25
Size2	13.0	12.4	3.48	-2.06	-2.12
Size3	11.6	11.3	2.95	-2.05	-2.23
Size4	10.4	9.8	2.33	-2.05	-2.22
Size5	7.4	7.3	1.81	-1.58	-1.46
			Price(CF)	Price(DR)	Price(Vol)
			2.37	-1	-1
			[0.96]	[na]	[na]

#### IV. Conclusion

In this paper, we show that volatility news is an important source of risk that affects the measurement and interpretation of underlying risks in the economy and financial markets. Our theoretical analysis yields a dynamic asset pricing model with three sources of risk: cash flow risk, discount rate risk, and volatility risks, each carrying a separate risk premium. We show that ignoring volatility risk may lead to sizable misspecifications of the dynamics of the stochastic discount factor and equilibrium consumption, and distorted inferences about risk and return. We find that potential distortions caused by neglecting time variation in economic volatility are indeed significant and result in upward biases in the volatility of consumption news and large downward biases in the volatility of the implied stochastic discount factor and consequently risk premia.

Consistent with existing empirical evidence, we document that both macroeconomic and return-based measures of volatility are highly persistent. Importantly, we also find that, in the data, an increase in volatility is typically accompanied by a significant decline in realized and expected consumption and an increase in risk premia. That is, high-volatility states are states of high risk reinforced by low economic growth and high discount rates. This evidence is consistent with the equilibrium relationship among volatility,

consumption, and asset prices implied by our model. A specification that ignores time variation in volatility, in contrast to the data, would imply an upward revision in expected lifetime consumption following an increase in discount rates and would clearly fail to account for a strong positive correlation between volatility and discount rate risks.

The empirical evidence we present highlights the importance of volatility risk for the joint dynamics of human capital and equity returns, and the cross-sectional risk and return trade-off. Our dynamic volatility-based asset pricing model is able to reverse the puzzling negative correlation between equity and human capital returns documented previously in the literature in the context of a homoskedastic economy. By incorporating an empirically robust positive relationship between ex-ante volatility and discount rates (a missing link in the homoskedastic case), our model implies a positive correlation between returns to human capital and equity while simultaneously matching time-series dynamics of aggregate consumption. We also show that, quantitatively, volatility risk helps explain both the level and variation in risk premia across portfolios sorted on size and book-to-market characteristics. We find that, during times of high volatility in financial markets (hence high marginal utility), equity portfolios tend to realize low returns. Therefore, equity markets carry a positive premium for volatility risk exposure.

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## Appendix

### A. Impulse Response Computations

The VAR(1) dynamics for the state variables follow

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (\text{A1})$$

where the unconditional variance-covariance matrix of shocks is  $\Omega = \Sigma \Sigma'$ .

The ex-ante consumption variance is  $\text{Var}_t \Delta c_{t+1} = v_0 + v_1' X_t$ , for  $v_1 = i_v' \Phi$ . Hence, ex-ante volatility shocks are  $v_1' u_{t+1}$ . To generate a one-standard-deviation ex-ante volatility shock, we choose a combination of primitive shocks  $\tilde{u}_{t+1}$  proportional to their impact on volatility:

$$\tilde{u}_{t+1} = \frac{(v_1' \Sigma)'}{\sqrt{v_1' \Sigma \Sigma' v_1}}. \quad (\text{A2})$$

Based on the VAR, we can compute impulse responses for the underlying variables, such as consumption growth, to consumption volatility shocks in the data.

### B. Generalized Specification

In a more generalized specification of the model, we allow for variation in the volatility of volatility shocks. For tractability, we assume that time variation

in conditional second moments of all innovations (including the innovation to the variance component) is driven by a single state variable. In particular, we assume that the state vector follows the first-order dynamics

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (\text{B1})$$

where  $u_{t+1} \sim N(0, \sigma_t^2 \Omega)$ . In the Macro VAR model discussed in Section II.E,  $\sigma_t^2 \equiv \text{Var}_t(\Delta c_{t+1})$ , and in the market-based model presented in Section III.D,  $\sigma_t^2 \equiv \text{Var}_t(N_{R,t+1})$ . The dynamics of economic volatility in this case are proportional to  $\sigma_t^2$ :

$$V_t = \xi \sigma_t^2, \quad (\text{B2})$$

where  $\xi$  can be found by exploiting the definition of  $V_t$ :

$$V_t = \xi \sigma_t^2 = \frac{1}{2} \text{Var}_t \left( (1 - \gamma) N_{\text{CF},t+1} + N_{V,t+1} \right). \quad (\text{B3})$$

It follows that

$$\xi = 0.5 (1 - \gamma)^2 [\iota_{\text{CF}} \Omega \iota'_{\text{CF}}] + (1 - \gamma) \xi [\iota_{\text{CF}} \Omega \iota'_{\sigma^2}] + 0.5 \xi^2 [\iota_{\sigma^2} \Omega \iota'_{\sigma^2}], \quad (\text{B4})$$

where  $\iota_{\text{CF}}$  and  $\iota_{\sigma^2}$  correspond to cash flow and volatility news functions, respectively. In the Macro model,

$$\iota_{\text{CF}} = [(1 - \omega) i'_d + \omega i'_y] (I - \kappa_1 \Phi)^{-1}, \quad (\text{B5})$$

$$\iota_{\sigma^2} = i'_v \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1},$$

and in the Market-based model,

$$\iota_{\text{CF}} = i'_{\Delta d} (I - \kappa_1 \Phi)^{-1}, \quad (\text{B6})$$

$$\iota_{\sigma^2} = i'_v \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1}.$$

Equation (B6) is quadratic in  $\xi$  but the admissible solution is unique. However, because of log-linearization of the model, the solution is not guaranteed to be real. To resolve this issue, in the empirical implementation, we rely on the linearized solution derived by using a first-order Taylor series approximation around  $\xi = 0$ , which is given by

$$\xi \approx \frac{0.5 (1 - \gamma)^2 \iota_{\text{CF}} \Omega \iota'_{\text{CF}}}{1 - (1 - \gamma) \iota_{\text{CF}} \Omega \iota'_{\sigma^2}}. \quad (\text{B7})$$

### C. GMM Estimation

The dynamics of the state vector are described by a first-order VAR:

$$X_t = \mu_X + \Phi X_{t-1} + u_t, \quad (\text{C1})$$

where  $X_t$  is a  $6 \times 1$  vector of the state variables,  $\mu_X$  is a  $6 \times 1$  vector of intercepts,  $\Phi$  is a  $6 \times 6$  matrix, and  $u_t$  is a  $6 \times 1$  vector of normal shocks. The VAR orthogonality moments compose the first set of moment restrictions in our GMM estimation:

$$E[h_t^{\text{VAR}}] = E \begin{bmatrix} u_t \\ u_t \otimes X_{t-1} \end{bmatrix} = 0. \quad (\text{C2})$$

The second set of moments comprises the Euler conditions for 11 portfolios (the aggregate market and the cross section of five size- and five book-to-market-sorted portfolios):

$$E[h_t^{\text{CS}}] = E[R_{i,t}^e - \text{RiskPrem}_i]_{i=1}^{11} = 0, \quad (\text{C3})$$

where  $R_{i,t}^e$  is the excess return of asset  $i$ , and  $\text{RiskPrem}_i \equiv -\text{Cov}(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1})$  is the model-implied risk premium of asset  $i$ .

Let  $\hat{h}$  denote the sample counterpart of the combined set of moment restrictions,

$$\hat{h} = \frac{1}{T} \sum_{t=1}^T \begin{bmatrix} h_t^{\text{VAR}} \\ h_t^{\text{CS}} \end{bmatrix}. \quad (\text{C4})$$

The parameters of the VAR dynamics and the market price of cash flow risk are estimated by minimizing a quadratic form of the sample moments:

$$\{\widehat{\mu}_X, \widehat{\Phi}, \widehat{\gamma}\} = \underset{\mu_X, \Phi, \gamma}{\text{argmin}} \widehat{h}' W \widehat{h}, \quad (\text{C5})$$

where  $W$  is a weight matrix. The moments are weighted by the inverse of their corresponding variances; the off-diagonal elements of matrix  $W$  are set to zero. We allow the weights to be updated throughout estimation (as in a continuously updated GMM). The reported standard errors are based on the Newey and West (1987) variance-covariance estimator.

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### Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

**Appendix S1:** Internet Appendix.