IMPROVED IMAGE INTERPOLATION USING BILATERAL FILTER FOR WEIGHTED LEAST SQUARE ESTIMATION

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ABSTRACT

New edge-directed interpolation (NEDI) consists two steps. The two steps are parameter and data estimation. The second step can be replaced by a recently proposed technique called soft-decision to consider the consistency of image structure during this data estimation. The original idea of both steps is to assume equal variances for all estimation errors, such that an ordinary least squares (OLS) estimator can be used. Due to the existence of noise, different object layers, changing in image structures, different spatial distance to the missing data, etc, we observe that the estimation errors of data samples have unequal variances. Hence, a weighted least square (WLS) estimator should be used for both steps. The bilateral filter, which can accurately remove noise and preserve image structure, has been used to model successfully the weights of squared residuals, such that we can apply it to both steps of the estimation. Experimental results show that the average PSNR of this improved interpolation method is 0.47 dB and 0.23 dB higher than two similar approaches, NEDI and Soft-decision Adaptive Interpolation (SAI) using 24 natural images from Kodak. The subjective results show improvement as well.

Index Terms: Interpolation, edge-directed, image processing

1. INTRODUCTION

Image interpolation aims at producing a high-resolution (HR) image from its low-resolution (LR) counterpart. Conventional polynomial-based interpolation methods like bilinear and bicubic interpolations [1], which fail to interpolate discontinuities like edges, are often used due to their simplicity rather than quality.

There are two major classes of edge-directed interpolation methods which address the edge reconstruction. The first class is called explicit methods [2-3]. The second class is called implicit methods [5-8], which do not explicitly estimate the edge orientation and generally provide better results than the explicit class. Li and Orchard [7] used the Wiener filter to estimate parameters using the mean least square error optimization. Zhang and Wu [8] also extended Li's method by proposing a soft-decision adaptive interpolation to interpolate a block of pixels and constrain the consistency of the image structure within the block. However, the globally estimated parameters used in [8] may not be suitable for all pixels within the block. Hence, we begin with localized parameter estimation and only make use of soft-decision method for pixel-wise interpolation at a later stage.

Weighted least square (WLS) estimator has been proven to be the best linear unbiased estimator (BLUE) if the variances of estimation errors are unequal but there is no correlations existing among the errors, and the squared residuals are weighted by the inverse variances of errors [9]. In this paper, we improve Li and Orchard's NEDI approach [7], which has two steps of estimation. The second step is the data estimation (interpolation), which can be replaced by the soft-decision method in [8] to consider the consistency of image structure. Both steps of estimation assume that the variances of estimation errors are equal. However this assumption is generally not true, due to the existence of noise, different object layers, changing in image structures, different spatial distances to the missing data, etc. We can improve the estimations by using the WLS estimator, where the weights of squared residuals are modeled by the bilateral filter [10] which can accurately remove noise and preserve the image structure in consideration. Experimental results show that this improved interpolation gives better objective and subjective qualities. The organization of the paper is as follow. Section 2 describes the formulation of linear interpolation and the proposed improvements. Section 3 presents the experimental results and section 4 concludes the paper.

2. LINEAR INTERPOLATION

Let us briefly describe the formulation of a 2D linear interpolation method called NEDI as an introduction. The formulation of NEDI using the original covariance method is shown in [7]. The 4-order linear estimation model is given by

$$x_i = \sum_{t=1}^{4} a_t \cdot x_{i\Delta t} + \varepsilon_i, \quad for \quad i = 1,2,...,P \quad (1)$$

where ε_i is the estimation error, P is the number of samples, a_t is the true model parameters and each available data sample x_i has four neighboring data $x_{i\Delta t}$. Figure 1a shows the spatial position of $x_{i\Delta t}$ and x_i. If the estimation errors ε_i are zero-mean, uncorrelated and have equal variances, the estimated model parameters $\hat{a}_t$ can be found by minimizing the sum of squared residuals

$$\min_{\{\hat{a}_t\}} \sum_{i=1}^{P} \left[x_i - \sum_{t=1}^{4} \hat{a}_t \cdot x_{i\Delta t} \right]^2 \quad (2)$$

where the matrix form of (2) is given by

$$\min_{\boldsymbol{a}} \left\| \boldsymbol{x} - \boldsymbol{x_a a} \right\|_2^2 \quad (3)$$

and the definitions of the matrices are defined by

$$\boldsymbol{x} = \{x_i\}^T, \boldsymbol{x_a} = \{x_{i\Delta t}\}^T, \boldsymbol{a} = \{\hat{a}_t\}^T \quad (4)$$

The sizes of matrices $\boldsymbol{x}$, $\boldsymbol{x_a}$ and $\boldsymbol{a}$ are Px1, Px4 and 4x1 respectively. Therefore, the close form solution of (3) is given by

$$\boldsymbol{a} = (\boldsymbol{x_a}^T \boldsymbol{x_a})^{-1} \boldsymbol{x_a}^T \boldsymbol{x} \quad (5)$$

where $\boldsymbol{a}$ is called the ordinary least squares (OLS) estimator which is the best linear unbiased estimator (BLUE) according to Gauss–Markov theorem. Due to "geometry duality" [7], the missing data y can be estimated (interpolated) by its four neighboring data $y_{\Delta t}$ as follows

$$y = \sum_{t=1}^{4} \hat{a}_t \cdot y_{\Delta t} \quad (6)$$

Equations (5) and (6) show the original parameter estimation and data estimation steps of NEDI in [7]. For the missing data between two available LR data (e.g. the missing data between $y_{\Delta 1}$ and $y_{\Delta 2}$ in Figure 1b), the spatial positions of the neighbors and missing data are rotated by 45 degree and scaled by $1/\sqrt{2}$. More details can be found in [7]. Moreover, the linear interpolation method described in this paper can be applied to high activity areas only, which can be detected using a local variance larger than 8, as used in NEDI. The number of sample is P=64, i.e. 8x8 window, as used in NEDI.

2.1. Weighted least square optimization

Due to the existence of noise, different object layers, changing of image structures, different spatial distances to the missing data, etc, the variances of estimation errors are often unequal. If the errors are moreover uncorrelated, the estimated model parameter $\boldsymbol{a}$ is the best linear unbiased estimator by minimizing a weighted sum of squared residuals, where the weight is proportional to the inverse variance σ_i^{-2} of the estimation error as explained in [9].

$$\min_{\{\hat{a}_t\}} \sum_{i=1}^{P} \sigma_i^{-2} \left[\left(x_i - \sum_{t=1}^{4} \hat{a}_t \cdot x_{i\Delta t} \right) \right]^2 \quad (7)$$

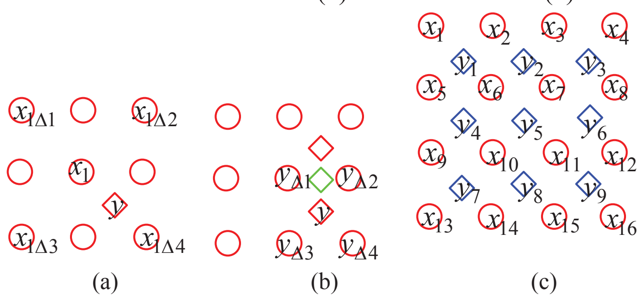
where the matrix form of (7) is given by

$$\min_{\boldsymbol{a}} (\boldsymbol{x} - \boldsymbol{x_a a})^T \boldsymbol{W} (\boldsymbol{x} - \boldsymbol{x_a a}) \quad (8)$$

and the diagonal matrix $\boldsymbol{W}$ has a size of PxP and is defined as $\boldsymbol{W} = diag\{\sigma_i^{-2}\}$. The closed form solution of (8) is given by

$$\boldsymbol{a} = (\boldsymbol{x_a}^T \boldsymbol{W} \boldsymbol{x_a})^{-1} \boldsymbol{x_a}^T \boldsymbol{W} \boldsymbol{x} \quad (9)$$

where $\boldsymbol{a}$ is the weighted least square (WLS) estimator. Since the true model parameter is undetermined due to ill-posed problem in (6), we rely on the PSNR of the estimated data y using $\boldsymbol{a}$ and the original data (LR pixels), i.e. the reconstructed HR image, to evaluate the performance of WLS and OLS estimators. Experimental results in Section 3 show that a significant improvement of 0.35 dB PSNR is achieved if WLS estimator in (9) is used instead of (5).



○ Available low-resolution data ◇ Missing high-resolution data

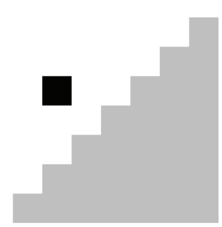
Fig.1: The spatial positions of LR and HR pixels.

2.2. Bilateral filter to model the weight

The bilateral filter [10], which can accurately remove noise and preserve image structure, has been successfully used to model the weights of squared residuals for WLS optimization [11]. In this paper, we apply it to the two steps of estimation. The bilateral filter comprises of two exponential terms to separately penalize the spatial distance and radiometric distance between the sample and the pixel of interest, such that

$$\sigma_i^{-2} \equiv e^{-\left\| z_i - z \right\|_2^2 / 2\sigma_a^2} \cdot e^{-(x_i - \hat{y})^2 / 2\sigma_b^2} \quad (10)$$

where the 2x1 vectors $\boldsymbol{z}_i$ and $\boldsymbol{z}$ represent the coordinates of sample i and the pixel of interest. Since y is the pixel of interest which is not available, it can be approximated by $\hat{y}$ using bilinear interpolation, etc. Experimental results show that setting $2\sigma_a^2$ and $2\sigma_b^2$ in (10) to 20 and 15000 respectively give the best average PSNR. Figure 2 shows a common scenario. The noisy sample in black has a significantly lower weight than surrounding samples.



1.1	1.2	1.4	1.4	1.4	1.4	1.2	1
1.2	1.4	1.7	1.8	1.8	1.7	1.3	1.1
1.4	1.7	2	2.2	2.2	2	1.5	1.2
1.4	1.8	***0.9***	2.7	2.7	2	1.6	1.3
1.4	1.8	2.2	2.7	2.4	2	1.6	1.3
1.4	1.7	2	2	2	1.7	1.5	1.2
1.2	1.3	1.5	1.6	1.6	1.5	1.3	1.1
1	1.1	1.2	1.3	1.3	1.2	1.1	1

Fig.2: 64 (=8x8) samples' (left) weights in percentage (right)

2.3. Consistency of the estimated data

The estimation of y (interpolation step) using (6) can be further improved if the consistency of the estimated data is considered, i.e. the estimated data is constrained to estimate back the available data. We modify the recently proposed soft-decision technique [8] from the original OLS to the WLS estimator using the same bilateral filter to weight the squared residual due to the same reasons (noise, changing image structure, etc). We can apply this improved technique during the data estimation. Let us use y_k to represent a block of missing data as shown in Figure 1c, where y_5 is equal to y in (6). The first linear estimation model is given by

$$y_k = \sum_{t=1}^{4} \hat{a}_t \cdot y_{k\Delta t} + \varepsilon_k \ , \quad for \quad k = 1,2,...,9 \tag{11}$$

where the estimated parameters $\hat{a}_t$ can be found using (9) and the neighboring data $y_{k\Delta t}$ of the missing data y_k correspond to some available LR data x_i, as shown in Figure 1c. For example, $y_{1\Delta 1} = x_1$, $y_{1\Delta 2} = x_2$, $y_{1\Delta 3} = x_5$ and $y_{1\Delta 4} = x_6$. The second linear estimation model for consistency is given by

$$x_i = \sum_{t=1}^{4} \hat{a}_t \cdot x_{i\Delta t} + \varepsilon_i \ , \quad for \quad i = 6,7,10,11 \tag{12}$$

where the neighboring data $x_{i\Delta t}$ of the available data x_i correspond to some missing data y_k, as shown in Figure 1c. For example, $x_{6\Delta 1} = y_1$, $x_{6\Delta 2} = y_2$, $x_{6\Delta 3} = y_4$ and $x_{6\Delta 4} = y_5$. Therefore, the available data x_i is estimated by the missing data y_k. If the variances of estimation errors ε_i and ε_k are unequal and the errors are uncorrelated (the same assumption in Section 2.1), the estimated missing data $\hat{y}_k$ is the best linear unbiased estimator by minimizing the weighted sum of squared residuals,

$$\min_{\{\hat{y}_k\}} \sum_{k=1}^{9} \sigma_k^{-2} \left[\hat{y}_k - \sum_{t=1}^{4} \hat{a}_t \cdot \hat{y}_{k\Delta t} \right]^2 + \sum_{i=6,7,10,11} \sigma_i^{-2} \left[x_i - \sum_{t=1}^{4} \hat{a}_t \cdot x_{i\Delta t} \right]^2 \tag{13}$$

where the weights are estimated by (10). The approximation of $\hat{y}_k$ using bilinear interpolation is required for the calculation of σ_k^{-2} in (10). Note that the neighboring data $x_{i\Delta t}$ contains some missing data $\hat{y}_k$ that have to be estimated.

Table 1 PSNR (dB) of various methods

Images	Bicubic[1]	NEDI [7] using eqn (5, 6)	Proposed using eqn (9, 6)	Proposed using eqn (9, 18)	SAI [8]	[6]
24 Kodak images	29.406	29.443	29.796	29.913	29.683	29.412

In matrix form, (13) can be written as

$$\min_{\hat{y}} (\boldsymbol{C}\hat{\boldsymbol{y}} - \boldsymbol{D}\boldsymbol{x})^T \boldsymbol{W} (\boldsymbol{C}\hat{\boldsymbol{y}} - \boldsymbol{D}\boldsymbol{x}) \tag{14}$$

where the definitions of the matrix are given by

$$\hat{\boldsymbol{y}} = \{\hat{y}_k\}^T, \boldsymbol{x} = \{x_i\}^T, \boldsymbol{W} = diag\{\sigma_k^{-2}, \sigma_i^{-2}\} \tag{15}$$

and the sizes of the matrix $\hat{\boldsymbol{y}}$, $\boldsymbol{x}$ and $\boldsymbol{W}$ are 9x1, 16x1 and 13x13 respectively. $\boldsymbol{D}$ is a 13x16 matrix as defined below:

$$\boldsymbol{D} = \begin{bmatrix} a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & 0 & a_3 & a_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

where $a_t = \hat{a}_t$ for simplicity. $\boldsymbol{C}$ is a 13x9 matrix and $\boldsymbol{C} = \begin{bmatrix} \boldsymbol{I}_{9x9} \\ \boldsymbol{C}_1 \end{bmatrix}$. $\boldsymbol{I}_{9x9}$ is a 9x9 identity matrix and the definition of $\boldsymbol{C}_1$ is given by

$$\boldsymbol{C}_1 = \begin{bmatrix} a_1 & a_2 & 0 & a_3 & a_4 & 0 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 & a_3 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & 0 & a_3 & a_4 & 0 \\ 0 & 0 & 0 & 0 & a_1 & a_2 & 0 & a_3 & a_4 \end{bmatrix} \tag{17}$$

The closed form solution of (14) is thus given by

$$\hat{\boldsymbol{y}} = (\boldsymbol{C}^T \boldsymbol{W} \boldsymbol{C})^{-1} \boldsymbol{C}^T \boldsymbol{W} \boldsymbol{D} \boldsymbol{x} \tag{18}$$

where only the center point y_5 (=y in (6)) is replaced by the estimated value, $\hat{y}_5$. If the ordinary least squares (OLS) estimator (i.e. $\sigma_k^{-2} = \sigma_i^{-2} = 1$) instead of weighted least square (WLS) estimator is used, and the improvement of PSNR using (18) rather than (6) is 0.072 dB. Instead, if the WLS estimator is used, the improvement is 0.117 dB.

3. EXPERIMENTAL RESULTS

The test was run on an Intel-i7, 3GHz computer system. 24 natural images from Kodak of size 768x512 were down-sampled and up-sampled two times. We implemented the bicubic interpolation [1], NEDI [7], Soft-decision Adaptive Interpolation (SAI) [8] (using λ =0.5 and safe guard threshold=15000) and the proposed method using non-optimized C++ codes. Note that the proposed method adopts a similar safe guard technique [8] (which contributes to an improvement of 0.02dB PSNR), to reject inappropriate cases for linear interpolation. The method of [6] is provided by the authors via website. The execution time (for 1 image) of bicubic interpolation, NEDI, SAI, the proposed method using (9) and (6) and proposed method using (9) and (18) are 0.05,12,11,30,65 seconds respectively. Although the computation of proposed methods is high, there is much room for optimization. For example, iteration methods like gradient descent can be used to speed up the calculation of (9) and (18).

Let us justify the use of weighted least square estimator (WLS) for parameter estimation in (9). As shown in Table 1, the improvement of PSNR of the reconstructed HR image using (9) rather than (5) is 0.35 dB in average. The PSNR is further improved by 0.11 dB if the image consistency is considered using (18) rather than (6). The overall method using (9) and (18) produces the highest PSNR (dB) among some sophisticated algorithms [6-8] in the literature.

Three methods with highest PSNR are compared. Figure 3 shows that the objective measurements in Table 1, the results of which agree with the subjective evaluation. The proposed overall method has the best subjective quality, as well as PSNR (dB) measurements. SAI [8] is weak in the reconstruction of details compared with the proposed method, especially in the high frequency region pointed by the arrows, while NEDI [7] produces some obvious noises around the edges pointed by the arrows.

4. CONCLUSION

In this paper, we have presented an improved edge-directed interpolation algorithm using weighted least square estimator for both parameter estimation and data estimation, where the weights are modeled by the bilateral filter. Experimental results show that the proposed interpolation algorithm has better performance compared to sophisticated algorithms in the literature in terms of objective and subjective measurements. An enhancement work is to develop an automatic scheme to find the locally optimal variances of two exponential terms in equation 10, which is a direction for future work.

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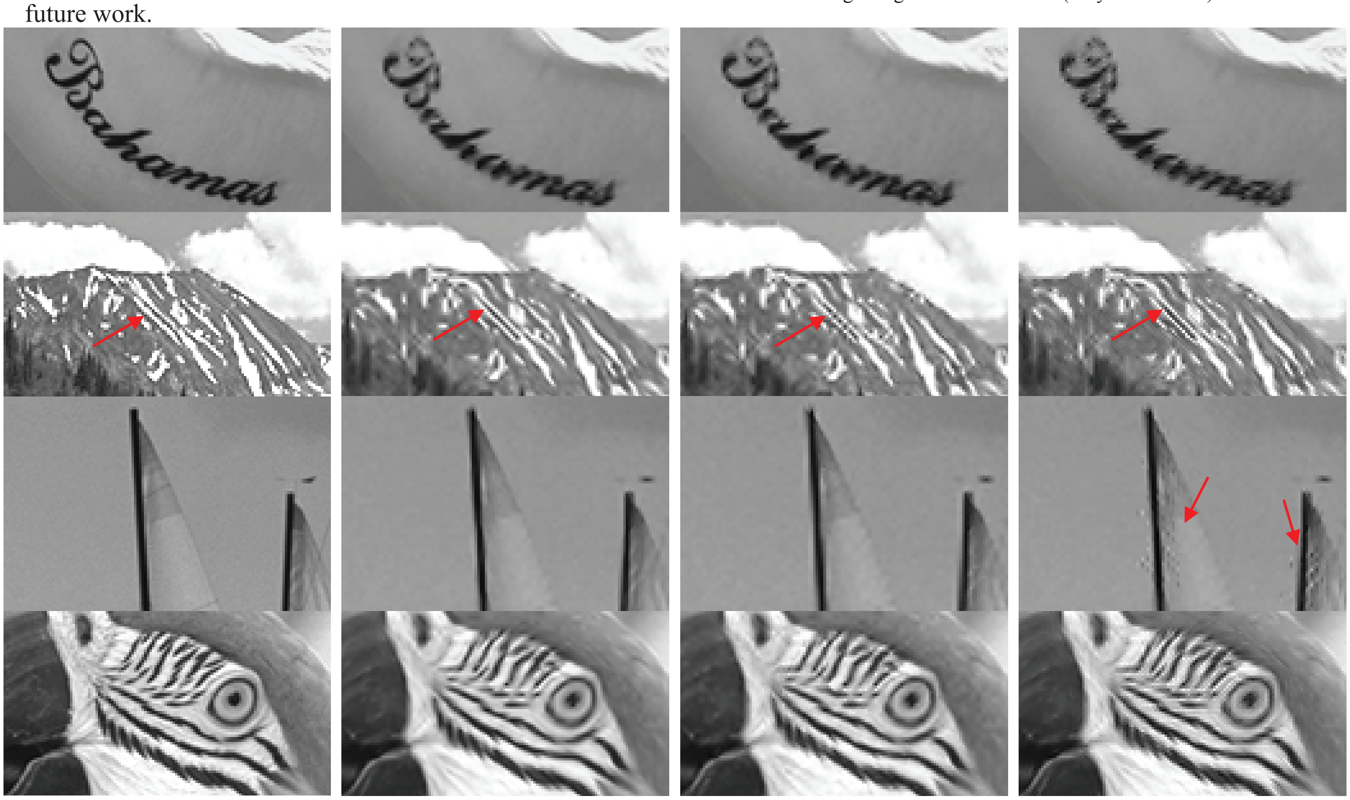

(a) Original **(b) Proposed overall method** **(c) SAI [8]** **(d) NEDI [7]**

Fig.3: Portions of interpolated results.